# CMENALANATHS 



# MENTAL 

# MATHS 

## CLASS

IX

# 2022-23 

DIRECTORATE OF EDUCATION
GOVT. OF NCT OF DELHI

HIMANSHU GUPTA, IAS
Director, Education \& Sports

No.PS/ $D \in / 2022 / 194$
DH. 19/5/2022


## MESSAGE

They say, 'Numbers are not just symbols on paper; numbers have life!'

It is not an exaggeration to place on record that numbers have played a pivotal role in the development and growth of human civilisation.

Numerical skills are very useful for students in their future life, especially when they appear in competitive exams.

Our Mental Maths Project aims at gradually developing and nurturing foundational numerical skills among our budding mathematicians. It started nearly two decades ago, and is striding, each passing year, on the path of progress.

Incidentally, I had an opportunity to witness the State Level Mental Maths Quiz Competition recently and I was spellbound by the speed, confidence and enthusiasm exhibited by the students. Indeed, it was to be seen to be believed!

I appreciate the dedication and hard work put in by the State Core Committee members and the Subject Experts under the able guidance of the Project Director (Mental Maths) in preparing the Question Banks and carrying this project forward with great zeal \& fervour.

(HIMANSHU GUPTA)

# विकास कालिया <br> परियोजना निदेशक ( मेंटल मैथ्स) क्षेत्रीय शिक्षा निदेशक (उत्तर \& मध्य) 

No. PO/MMP/609


सत्यमेब जयते

VIKAS KALI
PROJECT DIRECTOR (MENTAL MATHS) REGIONAL DIRECTOR OF EDUCATION (NORTH \& CENTRAL)

Dated
$02 / 01 / 2023$

## 'A Few Interesting Facts About Maths'

The word 'Mathematics' has its origin in the Greek word "Mathema' which means 'something that is learnt' or 'something that one gets to know'. In the same country (Greece), an ancient scholar Archimedes is considered to be the 'Father of Mathema' as he discovered methods to measures the areas of different shapes.
However, in our own country, we consider Aryabhatta as Father of Mathematics because of his original contributions made in Spherical Trigonometry. Some people believe that Aryabhatta invented Zero also, while some others credit another Vedic scholar Brahmgupta for this landmark discovery. The Western Scholars believe that Zero was first invented by the 'Mayans' (Mesopotamia) and a little later, by the Indians from which places, Zero travelled gradually to Cambodia, China and to the Arab world.
By the way, 'Arab' reminds me of an important branch of Maths named 'Algebra' which has its roots in the Arabian word 'Al-jabr' which means 'reunion of broken parts' (also used for reuniting broken bones)!
Algebra seeks to find out 'the missing values' and restoring them, just like restoring broken bones by providing missing links. In Algebra, we first 'imagine' values in the form of symbols like ' $x$ ' or ' $y$ ' and then, manipulate them to find out the 'actual' values. This is how even today, we find the 'missing' values or links through Algebra.

In short, we can conclude that unlike the 'inventions' of bulb, printing press or pen which were made by certain individuals, Mathematics is not an invention made by one person or by one civilisation. Its various branches were cultivated and nurtured by various individuals across various continents $\&$ civilizations and through different millennia.

As for Mental Maths, one can master Mental Maths through rigorous practice. Apart from learning Tables and Formulae by heart, one needs to learn various tricks for breaking longer calculations into smaller parts and making numbers 'round'. I am sure, our Maths Teachers will be able to identify students who have aptitude for numbers and groom them for Mental Maths Quiz Competitions.

I take this opportunity to thank all our Maths Teachers who devote so much of their extra time to prepare our students to sit for these competitions. I am also indebted to our Maths Teachers who have 'written' and 'reviewed' these question banks.

I thank my MoSs, Coordinators and the Core Team who, I think, are devoted much more than their Project Director to promoting Mental Maths among students!

Finally, I thank DBTB for the efforts they made for successful publication of these Question Banks.

Above all, I am indebted to my superior, the Director of Education, for his consistent support \& guidance.

(VIKAS KALIA)
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(MENTAL MATHS)

## ACKNOWLEDGEMENT

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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## SCHEDULE OF MENTAL MATHS QUIZ COMPETITIONS

## FOR THE YEAR 2022-2023 DIRECTORATE OF EDUCATION GOVT OF NCT OF DELHI

- Practice to students from Question Bank
- School level Quiz Competition
- Cluster level Quiz Competition
- Zonal level Quiz Competition
- District level Quiz Competition
- Regional level Quiz Competition
- State level Quiz Competition
01.04.2022 to 15.10.2022
17.10.2022 to 07.11.2022
08.11.2022 to 14.11.2022
21.11.2022 to 30.11.2022
07.12.2022 to $\mathbf{1 4 . 1 2 . 2 0 2 2}$
26.12.2022 to 31.12.2022
18.01.2023 to 31.01.2023


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## CHAPTER 1

## NUMBER SYSTEM

## POINTS TO REMEMBER:

- Natural Numbers: $\mathbf{N}=\{1,2,3,4 \ldots \ldots . . .$.$\} Also called as counting numbers.$
- Whole Numbers: $W=\{0,1,2,3 \ldots \ldots \ldots$.
- Integers: $\mathrm{Z} / \mathrm{I}=\{\ldots \ldots . . .-3,-2,-1,0,1,2,3 \ldots \ldots \ldots \ldots$.
- Rational Numbers:
$Q=\left\{x=\frac{p}{q}\right.$ where $p, q \in Z, q \neq 0$ and $\left.\operatorname{HCF}(p, q)=1\right\}$ e.g. $\frac{3}{4}, \frac{4}{5}, \frac{15}{17}, \frac{-101}{20}$
Also defined as "terminating decimals or non-terminating repeating decimal"
e.g. 2.5, 3.128, $0.3,40.157,0.333 \ldots(=0 . \overline{3})$ etc.
- Irrational Numbers:
$Q^{\prime}=$ Non-terminating, non-repeating decimals are called irrational numbers, e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi, 1.314738$. $\qquad$ 2.410112 $\qquad$ etc.
- Real Numbers (R): Rational numbers and irrational numbers taken together form real numbers.
- For any real number 'a', we have

$$
|a|=\left\{\begin{array}{c}
a, \text { if } a \geq 0 \\
-a, \text { if } a<0
\end{array} \text { and }|a| \text { is called absolute value of }{ }^{\prime} a^{\prime}\right\}
$$

- If ' $x$ ' and ' $y$ ' are any two rational numbers, then $(x+y),(x-y), x \times y, \frac{x}{y}(y \neq 0)$ are also rational numbers.
- If ' $r$ ' is a non-zero rational number and' $\boldsymbol{s}$ ' irrational number then $(r+s),(r-s)$, $r \times s, \frac{s}{r}(r \neq 0), \frac{r}{s}(s \neq 0)$ are also irrational numbers.
- For positive real numbers ' $a$ ' and ' $b$ ', the following hold:
(i) $\sqrt{a b}=\sqrt{a} \cdot \sqrt{b}$
(ii) $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$
(iii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=a-b$
(iv) $(a+\sqrt{b})(a-\sqrt{b})=\mathbf{a}^{2}-b$
(v) $(\sqrt{a}+\sqrt{b})^{2}=\mathbf{a}+2 \sqrt{a b}+b$
- Let $a>0$ be a real number and $p$ and $q$ be rational numbers then
(i) $\mathbf{a}^{p} \cdot \mathbf{a}^{q}=\mathbf{a}^{\mathbf{p + q}}$
(ii) $\left(\mathbf{a}^{p}\right)^{q}=\mathbf{a}^{p q}$
(iii) $\frac{a^{p}}{a^{q}}=\mathbf{a}^{\mathrm{p}-\mathrm{q}}$
(iv) $\mathbf{a}^{\mathbf{p}} \mathbf{b}^{\mathbf{p}}=(\mathbf{a b})^{\mathbf{p}}$
(v) $\mathbf{a}^{0}=1(a \neq 0)$


## Questions:

1. Write the unit digit of $(13)^{21}$.
2. Find the value of $\left\{8^{-4 / 3} \div 2^{-2}\right\}^{1 / 2}$
3. Find the value of $\left\{\left(23+2^{2}\right)^{2 / 3}+(140-19)^{1 / 2}\right\}^{2}$
4. If $\sqrt[3]{5^{n}}=25$, then find the value of $5 \sqrt[n]{64}$
5. Simplify:
(i) $\left[(16)^{-1 / 5}\right]^{5 / 2}$
(ii) $(0.027)^{1 / 3}$
(iii) $\left[(625)^{-1 / 2}\right]^{-1 / 4}$
(iv) $[\sqrt[3]{64}]^{-1 / 2}$
(v) $\sqrt{x^{6} y^{-6}}$
(vi) $\quad(256)^{0.16} \cdot(256)^{0.09}$
(vii) $-\sqrt{32}+32 \sqrt{2}$
(viii) $11 \sqrt{5}+2 \sqrt{125}$
(ix) $[64 / 125]^{-1 / 3}$
(x) $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^{3}\right]^{1 / 4}$

$$
\text { (xi) }\left(\frac{81}{16}\right)^{\frac{-3}{4}}\left[\left(\frac{25}{9}\right)^{\frac{-3}{2}} \div\left(\frac{5}{2}\right)^{-3}\right]
$$

6. Find the value of $x$ :
(i) $\quad 27^{x}=\frac{9}{3^{x}}$
(ii) $4^{x}-4^{x-1}=24$
(iii) $2^{5} \div 2^{x}=(\sqrt[5]{2})^{20}$
(iv) $5^{x-2} \cdot 3^{2 x-3}=135$
(v) $\quad 2^{\mathrm{x}-7} \cdot 5^{\mathrm{x}-4}=1250$
(vi) $\left(\frac{3}{5}\right)^{x} \cdot\left(\frac{5}{3}\right)^{2 x}=\frac{125}{27}$
(vii) $\left(\frac{\sqrt[4]{10}}{\sqrt{10}}\right)=\sqrt[4]{\frac{1}{5}} .2^{\mathrm{x}}$
7. Solve the following:
(i) $\left[1-\frac{1}{3}\right]\left[1-\frac{1}{4}\right]\left[1-\frac{1}{5}\right] \ldots \ldots \ldots\left[1-\frac{1}{n}\right]$
(ii) $\left(\frac{x^{a+b} \cdot x^{b+c} \cdot x^{c+a}}{x^{a} \cdot x^{b} \cdot x^{c}}\right)$
(iii) $\sqrt{x^{-1} \cdot y} \cdot \sqrt{y^{-1} \cdot z} \cdot \sqrt{z^{-1} \cdot x}$
(iv) $\sqrt{5+\sqrt{13+\sqrt{5+\sqrt{16}}}}$
(v) $\sqrt{3 \sqrt{3 \sqrt{3 \sqrt{3}}}}$

## 8. Rationalize the denominator:

(i) $\frac{2}{\sqrt{37}-6}$
(ii) $\frac{5+\sqrt{6}}{5-\sqrt{6}}$
(iii) $\frac{\sqrt{2}+1}{\sqrt{5}}$
(iv) $\frac{7 \sqrt{3}-5 \sqrt{2}}{\sqrt{48}+\sqrt{18}}$
9. If $\sqrt{2}=1.414$ find the value of $\frac{3}{\sqrt{2}}$.
10. Find: $\sqrt[4]{\frac{1}{16}}+(0.001)^{-1 / 3}-(27)^{2 / 3}$.
11. If $x^{-3}=64$, then find the value of $x^{3 / 2}+x^{0}$.
12. If $\sqrt{2}=1.4142$, find the value of $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$.
13. If $x=\frac{2}{3+\sqrt{7}}$, then find the value of $(x-3)^{4}$
14. If $\sqrt{15-a \sqrt{14}}=\sqrt{8}+\sqrt{7}$, find the value of $a$.
15. Which of the following is greater:
(i) $\sqrt{0.04}$ or $\sqrt{0.4}$
(ii) $\sqrt[3]{2}$ or $\sqrt[4]{2}$
(iii) $\sqrt[4]{\frac{1}{2}}$ or $\sqrt[4]{\frac{1}{3}}$
(iv) $\sqrt[5]{3}$ or $\sqrt[6]{30}$
16. Write the following in ascending order
(i) $\sqrt[2]{25}, \sqrt[2]{16}, \sqrt[2]{64}$
(ii) $\sqrt{3}, \sqrt[3]{2}, \sqrt[4]{4}$
(iii) $\sqrt[3]{5}, \sqrt[4]{6}, \sqrt[2]{7}$
(iv) $2^{1 / 5}, 5^{1 / 3}, 4^{1 / 5}$
17. Write the following in $p / q$ form:
(i) $23 . \overline{43}$
(ii) 0.32
(iii) 1.275
(iv) $0 . \overline{23}$
18. Write the following in $p / q$ form:
(i) $0 . \overline{23}+0 . \overline{22}$
(ii) $0 . \overline{3}+0 . \overline{4}$
(iii) $1.3 \overline{2}+0 . \overline{35}$
19. Express the following as pure surd:
(i) $3 x^{2} y \sqrt[3]{x y}$
(ii) $a^{\frac{1}{3}} \sqrt{a b^{3}}$
(iii) $\sqrt{2} \cdot \sqrt[3]{3}$
(iv) $2 x y^{2} \sqrt[5]{x}$
20. Find the rationalizing factor of:
(i) $\sqrt[3]{36}$
(ii) $\sqrt[5]{81}$
(iii) $2+\sqrt{7}$
(iv) $\sqrt{5}-\sqrt{2}$
(v) $\sqrt{120} \times \sqrt{45}$
(vi) $\sqrt{5} \times \sqrt{10}$
(vii) $\sqrt{8} \times \sqrt{6}$
(viii) $\sqrt{27} \times \sqrt{5}$
(ix) $\sqrt{16} \times \sqrt{4} \times \sqrt{3}$
(x) $5 \sqrt{8} \times 2 \sqrt{6}$
21. Write two rational numbers between:
(i) $\frac{1}{5}$ and $\frac{1}{2}$
(ii) -2 and -1
(iii) $\sqrt{2}$ and $\sqrt{3}$
(iv) $0.12122122212221 \ldots .$. and $0.14114111411114 \ldots \ldots$.
22. If $\frac{2}{7}=0 . \overline{285714}$, then find the value of
(i) $\frac{3}{7}$
(ii) $\frac{4}{7}$
(iii) $\frac{\mathbf{1}}{7}$
(iv) $\frac{5}{7}$
23. Find the value of ' $a$ ' and ' $b$ '
(i) $\frac{\sqrt{3}-1}{\sqrt{3}+1}=\mathbf{a}+b \sqrt{3}$
(ii) $\frac{\sqrt{5}-2}{\sqrt{5}+2}=\mathbf{a}+b \sqrt{5}$
24. If $\boldsymbol{x}=2+\sqrt{3}$, then find the value of
(i) $x+\frac{1}{x}$
(ii) $x-\frac{1}{x}$
25. If $x=11-2 \sqrt{30}$, find the value of
(i) $\sqrt{x}+\frac{1}{\sqrt{x}}$
(ii) $\sqrt{x}-\frac{1}{\sqrt{x}}$
26. If $x+\frac{1}{x}=2$, find the value of $x^{3}+\frac{1}{x^{3}}$.
27. If $x-\frac{1}{x}=2$, find the value of $x^{3}-\frac{1}{x^{3}}$.
28. If $x=(\sqrt{2}-1)$, then find the value of $\left(x-\frac{1}{x}\right)$. $^{3}$
29. Find one fourth of $2^{\mathbf{1 0 0}}$.
30. Find the value of $\frac{3^{40}+3^{39}+3^{38}}{3^{41}+3^{40}-3^{39}}$.
31. Select rational number and irrational number from the following:
(i) $\sqrt{\mathbf{8 0 0}}$
(ii) $(1+\sqrt{2})^{2}$
(iii) $\sqrt{3125}$
(iv) $(2 \sqrt{2}-\sqrt{5})(2 \sqrt{2}+\sqrt{5})$
(v) $\quad 9^{1 / 2}+(343)^{2 / 3}$
(vi) $(2 \sqrt{2}-\sqrt{5})(\sqrt{2}-\sqrt{5})$
(vii) $\sqrt{\frac{12}{3}}$
(viii) 0.3796
(ix) 7.478478...
(x) 3.799779977799...
(xi) $(\sqrt{2}+2)^{2}$
(xii) $(\sqrt{2}+\sqrt{3})^{2}$
(xiii) $\frac{6}{3 \sqrt{2}}$
32. Evaluate: $\frac{13 \sqrt{24}}{26 \sqrt{54}}$.
33. Evaluate: $\frac{1}{\sqrt{4}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{2}+1}$.
34. Find the value of:
(i) $\sqrt[3]{2} \times \sqrt[4]{2} \times \sqrt[12]{32}$.
(ii) $\sqrt[6]{\mathbf{1 2}} \div(\sqrt{\mathbf{3}} \times \sqrt[3]{\mathbf{2}})$.
(iii) $\sqrt{2} \times \sqrt[3]{3} \times \sqrt[4]{4}$.
35. Evaluate:
(i) $(\sqrt{5}-\sqrt{3})+(2 \sqrt{3}-\sqrt{5})+(2-\sqrt{3})$
(ii) $\sqrt{5+2 \sqrt{6}}+\sqrt{8-2 \sqrt{15}}$
36. What can be the maximum number of digits in the repeating block of digits in the decimal expansion of $\frac{1}{17}$ ?
37. In each of the following, find the value of $(x+y)$
(i) $(2 \sqrt{2}+5 \sqrt{3})+(2 \sqrt{3}-3 \sqrt{2})=x \sqrt{2}+y \sqrt{3}$
(ii) $4 \sqrt{3}-3 \sqrt{12}+2 \sqrt{75}=x \sqrt{2}+y \sqrt{3}$
(iii) $\sqrt{8}+\sqrt{32}-\sqrt{2}=x \sqrt{2}+y \sqrt{3}$
38. Give an example of pair of two irrational numbers whose:
(i) Difference is a rational number
(ii) Difference is an irrational number
(iii) Sum is a rational number
(iv) Sum is an irrational number
(v) Product is a rational number
(vi) Product is an irrational number
(vii) Quotient is a rational number
(viii) Quotient is an irrational number
39. (a). Find the value of $\sqrt{(2020)(2022)+1}$
(b). Find the value of $\sqrt{(2010)(2030)+100}$.
40. If $\frac{\sqrt{7}-1}{\sqrt{7}+1}-\frac{\sqrt{7}+1}{\sqrt{7}-1}=a-b \sqrt{7}$, find the value of $a$ and $b$.

## ANSWERS

| Q.NO. | ANSWER | Q.NO. | ANSWER |
| :---: | :---: | :---: | :---: |
| 1 | (HINT: observe the pattern of unit digit of $13^{1}, 13^{2}, 13^{3} \ldots$ ) | 8 | (i) $2(\sqrt{37}+6)$ <br> (ii) $\frac{31+10 \sqrt{6}}{19}$ <br> (iii) $\frac{\sqrt{10}+\sqrt{5}}{5}$ <br> (iv) $\frac{114-41 \sqrt{6}}{30}$ |
| 2 | 1/2 | 9 | 2.121 |
| 3 | 400 | 10 | $\frac{3}{2}$ |
| 4 | 10 | 11 | $\frac{9}{8}$ |
| 5 | (i) $\frac{1}{4}$ <br> (ii) 0.3 <br> (iii) $\sqrt{5}$ <br> (iv) $\frac{1}{2}$ <br> (v) $\frac{x^{3}}{y^{3}}$ <br> (vi) 4 <br> (vii) $28 \sqrt{2}$ <br> (viii) $21 \sqrt{5}$ <br> (ix) $\frac{5}{4}$ <br> (x) 5 <br> (xi) 1 | 12 | 0.4142 |
| 6 | (i) $\mathrm{x}=\frac{1}{2}$ <br> (ii) $\mathrm{x}=\frac{5}{2}$ <br> (iii) $x=1$ <br> (iv) $x=3$ <br> (v) $x=8$ <br> (vi) $x=3$ <br> (vii) $x=\frac{-1}{2}$ | 13 | 49 |
| 7 | (i) $\frac{2}{n}$ <br> (ii) $x^{a+b+c}$ <br> (iii) 1 <br> (iv) 3 <br> (v) $3^{\frac{15}{16}}$ | 14 | -4 |


| Q.NO. | ANSWER | Q.NO. | ANSWER |
| :---: | :---: | :---: | :---: |
| 15 | (i) $\sqrt{0.4}>\sqrt{0.04}$ <br> (ii) $\sqrt[3]{2}>$ $\sqrt[4]{2}$ <br> (iii) $\sqrt[4]{\frac{1}{2}}>\sqrt[4]{\frac{1}{3}}$ <br> (iv) $\sqrt[6]{30}>\sqrt[5]{3}$ | 22 | (a) 0.428571 <br> (b) 0.571428 <br> (c) 0.142857 <br> (d) 0.714285 |
| 16 | (i) $\sqrt{16}<\sqrt{25}<\sqrt{64}$ <br> (ii) $\sqrt[3]{2}<\sqrt[4]{4}<\sqrt{3}$ <br> (iii) $\sqrt[4]{6}<\sqrt[3]{5}<\sqrt{7}$ <br> (iv) $2^{\frac{1}{5}}<4^{\frac{1}{5}}<5^{\frac{1}{3}}$ | 23 | (i) $a=2, b=-1$ <br> (ii) $a=9, b=-4$ |
| 17 | (i) $\frac{2320}{99}$ <br> (ii) $\frac{8}{25}$ <br> (iii) $\frac{51}{40}$ <br> (iv) $\frac{23}{99}$ | 24 | (a) $x+\frac{1}{x}=4$ <br> (b) $x-\frac{1}{x}=2 \sqrt{3}$ |
| 18 | (i) $\frac{5}{11}$ <br> (ii) $\frac{7}{9}$ <br> (iii) $\frac{1659}{990}$ | 25 | $\begin{aligned} & \text { (a) } 2 \sqrt{6} \\ & \text { (b) } 2 \sqrt{5} \end{aligned}$ |
| 19 | (i) $\sqrt[3]{27 x^{7} y^{4}}$ <br> (ii) $\sqrt[6]{a^{7} b^{9}}$ <br> (iii) $\sqrt[6]{72}$ <br> (iv) $\sqrt[5]{32 x^{6} y^{10}}$ | 26 | $x=2$ |
| 20 | (i) $\sqrt[3]{6}$ <br> (ii) $\sqrt[5]{3}$ <br> (iii) $2-\sqrt{7}$ <br> (iv) $\sqrt{5}+\sqrt{2}$ <br> (v) $\sqrt{6}$ <br> (vi) $\sqrt{2}$ <br> (vii) $\sqrt{3}$ <br> (viii) $\sqrt{15}$ <br> (ix) $\sqrt{3}$ <br> (x) $\sqrt{3}$ | 27 | 14 |
| 21 | (i) $\frac{3}{10}, \frac{4}{10}$ <br> (ii) $\frac{-5}{3}, \frac{-4}{3}$ <br> (iii) 1.425, 1.567 <br> (or any suitable answer) <br> (iv) $0.125,0.13$ <br> (or any suitable answer) | 28 | - 8 |


| Q. NO. | ANSWER |
| :---: | :---: |
| 29. | $2^{98}$ |
| 30. | $\frac{13}{33}$ |
| 31. | iii,iv,v,vii, viii are Rational $\mathbf{i}, \mathbf{i i}, \mathbf{v i}, \mathbf{i x}, \mathbf{x}, \mathbf{x i}$, xii are Irrational |
| 32. | $\frac{1}{3}$ |
| 33. | 1 |
| 34. | (i) 2 <br> (ii) $\sqrt[3]{\frac{1}{3}}$ <br> (iii) $2 \sqrt[3]{3}$ |
| 35. | (i) 2 (ii) $\sqrt{2}+\sqrt{5}$ |
| 36. | 16 |
| 37. | $\begin{array}{lll}\text { (i) } 6 & \text { (ii) } 8 & \text { (iii) } 5\end{array}$ |
| 38. | (i) $\sqrt{2}-\sqrt{2}$ <br> (ii) $3 \sqrt{5}-2 \sqrt{5}$ <br> (iii) $\sqrt{2}+(-\sqrt{2})$ <br> (iv) $\sqrt{2}+\sqrt{3}$ <br> (v) $\sqrt{2} \times \sqrt{8}$ <br> (vi) $\sqrt{2} \times \sqrt{5}$ <br> (vii) $\quad \sqrt{8} \div \sqrt{2}$ <br> (viii) $\sqrt{5} \div \sqrt{3}$ <br> or Any Suitable Answer for $i$ to viii |
| 39. | a. 2021 b. 2020 |
| 40. | $a=0, b=\frac{2}{3}$ |

## CHAPTER - 2

## POLYNOMIAL

## POINTS TO REMEMBER:

- Polynomial: An algebraic expression $p(x)=a_{0}+a_{1} x+a_{2} x^{2}+\ldots \ldots \ldots \ldots \ldots+a_{n} x^{n}$ is called a polynomial in x variable.
$a_{0}, a_{1}, a_{2}, \ldots \ldots \ldots, a_{n}$ are respectively the coefficients of $x^{0}, x^{1}, x^{2} \ldots . . x^{n}$ and $a_{0}, a_{1 x}$, $\mathbf{a}_{2} \mathbf{x}^{2}, \ldots \ldots$,
$a_{n} x^{n}$ are called its terms. $a_{0}, a_{1}, a_{2}, \ldots \ldots \ldots .$. , $a_{n}$ are real numbers, while exponent of $x$ are whole number.
- Degree of a polynomial: The degree of a polynomial $\boldsymbol{p}(\mathbf{x})$ is the index of the highest power of the variable in a non - zero polynomial and is denoted by deg $p(x)$.
- Zero polynomial: The polynomial $\boldsymbol{p}(\mathbf{x})=0$ is called Zero polynomial and degree of the Zero polynomial is not defined.
- Constant polynomial: A non - zero constant is called a Constant polynomial e.g. $0.5,11,105, \mu, \quad(\mu \neq 0)$ etc. It has degree Zero.
- Linear polynomial: A polynomial of degree 1 is called Linear polynomial. Its standard form is $a x+b$, where $a$ and $b$ are real constants and $a \neq 0$. e.g. $3 x+1,5 x$.
- Quadratic polynomial: A polynomial of degree 2 is called a Quadratic polynomial. Its standard form is $\mathbf{a x}^{2}+b x+c$, where $a, b$ and $c$ are real constants and $a \neq 0$. e.g. $4 x^{2}+5 x-1$.
- Cubic polynomial: A polynomial of degree 3 is called a Cubic polynomial. Its standard form is $a x^{3}+b x^{2}+c x+d$, where $a, b, c$ and $d$ are real constants and $a \neq 0$. E.g. $\mathrm{x}^{3}+1$,
$x^{3}+3 x^{3}-5 x+1$.
- Bi-quadratic polynomial: A polynomial of degree 4 is called a Bi-quadratic polynomial. Its standard form is $a x^{4}+b x^{3}+c x^{2}+d x+e$, where $a, b, c, d$ and $e$ are real constants and $\mathbf{a} \neq 0$.
e.g. $3 x^{4}+2 x^{3}-4 x^{2}+8$.
- Monomial: A polynomial having only one non-zero term is called a Monomial. e.g. $7 \mathrm{x}, 11 \mathrm{x}^{3}, \frac{-2}{3} \mathrm{xy}, 4 \mathrm{xyz}$ etc.
- Binomial: A polynomial having two non-zero terms is called a Binomial e.g. $\mathbf{x}^{2}+1$, $3 \mathrm{p}^{3}-5 \mathrm{p}, \mathrm{x}+\mathrm{y}, \mathrm{r}^{2}+2 \mathrm{~s}^{2} \mathrm{r}$ etc.
- Trinomial: A polynomial having three non-zero terms is called a Trinomial e.g. $x^{\mathbf{3}}+x$ $-5, p+q+r, x^{2}+2 y^{2} x-3$ etc.
- Zero of a polynomial: A real number ' $a$ ' is called zero of the polynomial $p(x)$ if $\mathbf{p}(\mathbf{a})=0$.
- Remainder Theorem: If a polynomial $p(x)$ of degree $n \geq 1$ is divided by
(i) $(\mathbf{x}-\mathrm{a})$, then remainder $=\mathbf{p}(\mathbf{a})$
(ii) $(\mathbf{x}+\mathbf{a})$, then remainder $=\mathbf{p}(-\mathbf{a})$
- Factor Theorem: If $\mathbf{p}(\mathbf{x})$ is a polynomial of degree $\mathbf{n} \geq 1$ and ' $a$ ' is any real number then
(i) $(x-a)$ is a factor of polynomial $p(x)$, if $p(a)=0$
(ii) $(\mathbf{x}+\mathbf{a})$ is a factor of polynomial $p(x)$, if $p(-a)=0$
- Algebraic Expressions and Identities:
(i) $\quad(x+y)^{2}=x^{2}+2 x y+y^{2}$
(ii) $(x-y)^{2}=x^{2}-2 x y+y^{2}$
(iii) $x^{2}-y^{2}=(x+y) \cdot(x-y)$
(iv) $(x+y+z)^{2}=x^{2}+y^{2}+z^{2}+2 x y+2 y z+2 z x$
(v) $x^{3}+y^{3}=(x+y)\left(x^{2}-x y+y^{2}\right)$
(vi) $x^{3}-y^{3}=(x-y)\left(x^{2}+x y+y^{2}\right)$
(vii) $x^{3}+y^{3}+z^{3}-3 x y z=(x+y+z)\left(x^{2}+y^{2}+z^{2}-x y-y z-z x\right)$
(viii) $x^{3}+y^{3}+z^{3}-3 x y z=\frac{1}{2}(x+y+z)\left[(x-y)^{2}+(y-z)^{2}+(z-x)^{2}\right]$
(ix) If $x+y+z=0$ then $x^{3}+y^{3}+z^{3}=3 x y z$
(x) $\quad(x-y)^{3}=x^{3}-3 x^{2} y+3 x y^{2}-y^{3}$
$(x-y)^{3}=x^{3}-3 x y(x-y)-y^{3}$
(xi) $\quad(x+y)^{3}=x^{3}+3 x^{2} y+3 x y^{2}+y^{3}$
$(x+y)^{3}=x^{3}+3 x y(x+y)+y^{3}$
(xii) $\quad(x+a)(x+b)=x^{2}+(a+b) x+a b$


## QUESTIONS:

1. Which of the following are polynomials:
(i) $x^{4}-2 x^{3}+5 x+7$
(ii) $\mathrm{x}^{2}-\frac{8}{x}+3$
(iii) $(\sqrt{5} x-2)^{2}$
(iv) $(\sqrt{2} x+3)(\sqrt{2} x-3)$
(v) $\mathrm{x}^{-2}+2 \mathrm{x}^{-1}+3$
(vi) $\sqrt{3}(\mathrm{x}+\sqrt{3})$
(vii) $\frac{1}{2}+\frac{1}{3}+\frac{1}{7}$
(viii) $\frac{x^{3}+3 x^{2} y+3 x y^{2}+y^{3}}{(x+y)^{2}}$
2. Write the following polynomials in standard form and also write their degree:
(i) $p(x)=x^{5}-2 x+3 x^{3}-7$
(ii) $f(x)=(2 x-3)^{2}+(x-8)$
(iii) $\mathbf{q}(\mathbf{x})=(3 x+7)^{2}-(3 x-7)^{2}$
(iv) $p(x)=(x+2)^{3}-(x+1)$
(v) $r(x)=-9$
(vi) $f(x)=(x+\sqrt{3})(x-\sqrt{3})$
$($ vii) $g(x)=(3+\sqrt{x})(3-\sqrt{x})$
(viii) $p(x)=2 x^{2}+2^{22} x^{3}+2^{30} x+2^{11} x^{5}$
3. Classify the following polynomials as Constant, Linear, Quadratic, Cubic \& Bi-quadratic :
(i) $r(x)=3 x^{3}+4 x^{2}+2 x+1$
(ii) $g(y)=5 y^{4}-3 y^{2}-2 y+6$
(iii) $g(t)=-8 t+\frac{1}{2}$
(iv) $h(x)=2$
(v) $p(x)=x^{2}+7$

## 4. Give an example of:

(i) a polynomial of degree 0
(ii) a binomial of degree 10
(iii) a monomial of degree 15
(iv) a trinomial of degree 5
(v) a polynomial of degree 100
(vi) a polynomial of degree 4 with one zero 5
(vii) a polynomial of degree 2 having factor ( $\mathbf{x}-\mathbf{1}$ )
5. Find the value of the following polynomials as indicated:
(i) If $p(x)=3 x^{2}-5 x+4, p(2)$
(ii) If $g(y)=y^{4}-5 y^{3}+y^{2}+5, g(-1)$
(iii) If $f(t)=3 t^{2}-5 t+9, f(0)$
(iv) If $\mathbf{r}(\mathbf{y})=\mathbf{y}^{\mathbf{5}}-2 \mathrm{y}+1, \mathbf{r}(-2)$
(v) If $g(t)=t^{2}-5 t, g(2)$
6. If $f(x)=x^{2}-5 x+7$, evaluate $f(2)-f(-1)$.
7. Write the coefficient of $x$ in the following:
(i) $p(x)=x^{3}+3 x^{2}-5 x+2$
(ii) $f(x)=y x^{2}+2 y x-3 y$
(iii) $g(x)=5 x^{2}+a^{2} h x-7$
(iv) $h(x)=\frac{3}{7} x-t^{2}+y$
(v) $\mathbf{r}(\mathrm{x})=\sqrt{3} t x+7 \mathrm{x}-9$
8. Find the zeroes of the following polynomials:
(i) $p(x)=x+5$
(ii) $q(t)=2 t-5$
(iii) $\mathbf{r}(\mathbf{y})=2 \mathbf{y}$
(iv) $g(x)=x^{2}+5 x-24$
(v) $h(y)=6 y^{2}+7 y-3$
(vi) $p(t)=9 t^{2}-22 t+8$
9. Find the remainder when $p(x)$ is divided by $g(x) \&$ hence check whether $g(x)$ is a factor of $p(x)$ or not:
(i) $p(x)=x^{3}-6 x^{2}+9 x+3, g(x)=x-1$
(ii) $p(x)=2 x^{4}+9 x^{3}+6 x^{2}-11 x-6, g(x)=x+1$
(iii) $p(x)=x^{4}-x^{2}-12, g(x)=x+2$
(iv) $p(x)=x^{101}+101, g(x)=x+1$
(v) $p(x)=x^{3}+3 x^{2}+10 x, g(x)=x-2$
10. Find the value of ' $k$ ' if $g(x)$ is a factor of $f(x)$ in the following:
(i) $f(x)=x^{5}-k x^{4}+k+x-2, g(x)=(x-k)$
(ii) $f(x)=x^{3}+k x^{2}-17 x+15, g(x)=(x-3)$
(iii) $f(x)=4 x^{2}+12 x+5 k-1, g(x)=(x-1)$
(iv) $f(x)=x^{3}+3 x^{2}+k x-50, g(x)=(x-5)$
(v) $f(x)=k x^{2}-\sqrt{2} x+1, g(x)=(x-1)$
11. Find the product of the following:
(i) $(x-2)(x+6)$
(ii) $(\mathrm{x}+3)\left(5-\frac{1}{x}\right)$
(iii) $\left(y^{3}-2\right)\left(y^{2}+5\right)$
(iv) $\left(x^{2}+4-2 x\right)\left(x^{2}+4+2 x\right)$
(v) $(2 a+5 b)\left(4 a^{2}-10 a b+25 b^{2}\right)$
12. Expand the following:
(i) $(5 x-2)^{2}$
(ii) $(2 x+3)^{3}$
(iii) $(2 x-3 y+z)^{2}$
(iv) $\left(2 x-\frac{1}{2 x}\right)^{2}$
(v) $\left(x-\frac{1}{x}\right)^{3}$
(vi) $\left(x^{2}-2 x+1\right)^{2}$
13. Factorize the following:
(i) $36 a^{3} b-60 a^{2} b c$
(ii) $49 x^{2}-256 y^{2}$
(iii) $\quad 6(2 a+3 b)^{2}-8(2 a+3 b)$
(iv) $a^{3}-2 \sqrt{2} b^{3}$
(v) $4 a^{2}+b^{2}+4 a b-6 b-12 a+9$
(vi) $1+64 x^{3}$
(vii) $a^{3}-8 b^{3}+64 c^{3}+24 a b c$
(viii) $\frac{8}{27} \mathrm{x}^{3}-\frac{4}{3} \mathrm{x}^{2}+2 \mathrm{x}-1$
(ix) $2 x^{2}+y^{2}+8 z^{2}-2 \sqrt{2} x y+4 \sqrt{2} y^{3}-8 x z$
(x) $\quad 2 x y(x-y)^{2}+x^{2}(x-y)^{2}+y^{2}(x-y)^{2}$
(xi) $a^{3}-0.064$
(xii) $\quad 64 a^{3}+b^{3}+48 a^{2} b+12 a b^{2}$
(xiii) $3 x^{2}+\sqrt{3} x-2$
(xiv) $\sqrt{2} x^{2}+3 x+\sqrt{2}$
(xv) $6 a b-b^{2}+12 a c-2 b c$
(xvi) $a^{4}-81 b^{4}$
(xvii) $4 a^{2}+12 a b+9 b^{2}-8 a-12 b$
(xviii) $x^{4}+4 x^{2}+3$
(xix) $8 x^{3}-(2 x-3 y)^{3}$
(xx) $\quad a^{8}-a^{2} b^{6}$
14. Find the remainder when: $f(a)=a^{4}+4 a^{2}+16$ is divided by $p(a)=a^{2}+2 a+4$
15. If $x^{2}+5 k x-3$ and $2 x^{2}+3 p x+2 p$ are divisible by $(x-1)$, find $k+p$.
16. Evaluate the following using identities:
(i) $(102)^{2}$
(ii) $\mathbf{9 6} \times 97$
(iii) $(997)^{2}$
(iv) $104 \times 96$
(v) $(-10)^{3}+(6)^{3}+(4)^{3}$
(vi) $(99)^{3}$
17. If $-8 x^{3}+27 y^{3}+36 x^{2} y-54 x y^{2}=(k x+3 p y)^{3}$, then find $(k+p)$.
18. Complete the following expansion:

$$
(\mathbf{3} x-4 y)^{3}=\square \mathbf{x}^{3}-\square \mathbf{y}^{3}+\square \mathbf{x}^{2} \mathbf{y}-\square \mathbf{x y}^{2}
$$

19. If $3 x-2 y=3$ then find the value of $27 x^{3}-8 y^{3}-54 x^{2} y+36 x y^{2}$.
20. If $a=x-y, b=y-z, c=z-x$, then find $a^{3}+b^{3}+c^{3 .}$
21. If $2 x^{2}-13 x+15$ represents the area of a rectangle with one-dimension $x-5$, then find its other dimension.
22. If $\mathbf{a b}+\mathrm{bc}+\mathrm{ca}=0$, find the value of $\frac{1}{a^{2}-b c}+\frac{1}{b^{2}-c a}+\frac{1}{c^{2}-a c}$
23. Calculate the value of $\frac{83^{3}+17^{3}}{83^{2}-83 \times 17+17^{2}}$
24. If $\frac{x}{y}+\frac{y}{x}=-1(x, y \neq 0)$, then find the value of $x^{3}-y^{3}$.
25.Find the value of $\frac{20202020^{2}}{20202019^{2}+20202021^{2}-2} \cdot$
25. If $(x+y)=5$ and $x y=4$, find $x-y$.
26. If $a^{2}+b^{2}+c^{2}=250$ and $a b+b c+c a=3$,
then find the value of $a+b+c$.
27. Find the perimeter of rectangle whose area is $25 x^{2}-35 x+12$.
28. Find the product of $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^{2}+\frac{1}{x^{2}}\right)\left(x^{4}+\frac{1}{x^{4}}\right)$.
29. If $(x+1)$ and $(x-1)$ are factors of $a x^{3}+x^{2}-2 x+b$, find $a$ and $b$.

## ANSWERS

| Q.No. | ANSWER | $\begin{gathered} \text { Q. } \\ \text { NO. } \end{gathered}$ | ANSWER |
| :---: | :---: | :---: | :---: |
| 1. | $\begin{aligned} & \text { (I), (III), (IV), (VI), (VII), } \\ & \text { (VIII) } \end{aligned}$ | 6. | -12 |
| 2. | (i) $x^{5}+3 x^{3}-2 x-7$, degree 5 <br> (ii) $4 x^{2}-11 x+1$, degree 2 <br> (iii) $84 x$, degree 1 <br> (iv) $x^{3}+6 x^{2}+11 x+7$, degree 3 <br> (v) $-9 \mathrm{x}^{0}$, degree 0 <br> (vi) $x^{2}-3$, degree 2 <br> (vii) $-\mathrm{x}+9$, degree 1 <br> (viii) $2^{11} x^{5}+2^{22} x^{3}+2 x^{2}+2^{30} x$, degree 5 | 7. | (i) -5 <br> (ii) 2 y <br> (iii) $\mathbf{a}^{2} h$ <br> (iv) $\frac{3}{7}$ <br> (v) $\sqrt{3} t+7$ |
| 3. | (i) Cubic polynomial <br> (ii) Bi-quadratic polynomial <br> (iii) Linear polynomial <br> (iv) Constant polynomial <br> (v) Quadratic polynomial | 8. | (i) -5 <br> (ii) $\frac{5}{2}$ <br> (iii) 0 <br> (iv) $-8,3$ <br> (v) $-\frac{3}{2}, \frac{1}{3}$ <br> (vi) $\frac{4}{9}, 2$ |
| 4. | (i) $p(x)=\frac{-5}{9}$ <br> (ii) $f(x)=x^{10}-3 x^{2}$ <br> (iii) $\mathbf{g}(\mathrm{y})=3 \mathbf{y}^{15}$ <br> (iv) $h(t)=-7 t^{5}-2 t+1$ <br> (v) $q(y)=5 y^{100}-y^{82}+7 y^{17}-1$ <br> (vi) $f(x)=x^{4}-625$ <br> (vii) $p(x)=x^{2}-1$ | 9. | (i) $7, g(x)$ is not a factor of $p(x)$ <br> (ii) $4, g(x)$ is not a factor of $p(x)$ <br> (iii) $0, \quad g(x)$ is a factor of $p(x)$ <br> (iv) $100, g(x)$ is not a factor of $p(x)$ <br> (v) $40, \quad g(x)$ is not a factor of $p(x)$ |
| 5. | (i) $p(2)=6$ <br> (ii) $\mathbf{g ( - 1 )}=\mathbf{1 2}$ <br> (iii) $f(0)=9$ <br> (iv) $r(-2)=-$ <br> 27 <br> (v) $g(2)=-6$ | 10. | (i) $k=1$ <br> (ii) $k=1$ <br> (iii) $k=-3$ <br> (iv) $k=-30$ <br> (v) $k=\sqrt{2}-1$ |


| Q. NO. | ANSWER | Q. NO. | ANSWER |
| :---: | :---: | :---: | :---: |
| 11. | (i) $x^{2}+4 x-12$ <br> (ii) $5 \mathrm{x}-\frac{3}{x}+14$ <br> (iii) $\mathbf{y}^{5}+5 y^{3}-2 y^{2}-10$ <br> (iv) $\mathrm{x}^{4}+4 \mathrm{x}^{2}+16$ <br> (v) $8 a^{3}+125 b^{3}$ | 19. | 27 |
| 12. | (i) $25 x^{2}-20 x+4$ <br> (ii) $8 x^{3}+36 x^{2}+54 x+27$ <br> (iii) $4 x^{2}+9 y^{2}+z^{2}-12 x y-6 y z+4 z x$ <br> (iv) $4 x^{2}-\frac{1}{4 x^{2}}-2$ <br> (v) $x^{3}-\frac{1}{x^{3}}-3 x+\frac{3}{x}$ <br> (vi) $\mathrm{x}^{\mathbf{3}}-4 \mathrm{x}^{3}+6 \mathrm{x}^{2}-4 \mathrm{x}+1$ | 20. | $\begin{aligned} & \quad 3(x-y)(y-z) \\ & (z-x) \end{aligned}$ |
| 13. | (i) $12 a^{2} b(3 a-5 c)$ <br> (ii) $(7 x+16 y)(7 x-16 y)$ <br> (iii) $2(2 a+3 b)(6 a+9 b-4)$ <br> (iv) $(\mathbf{a}-\sqrt{2} \mathbf{b})\left(\mathbf{a}^{2}+\sqrt{2} \mathbf{a b}+2 \mathbf{b}^{2}\right)$ <br> (v) $(2 a+b-3)(2 a+b-3)$ <br> (vi) $\quad(4 x+1)\left(16 x^{2}-4 x+1\right)$ <br> (vii) $\quad(\mathrm{a}-2 \mathrm{~b}+4 \mathrm{c})\left(a^{2}+4 b^{2}+16 c^{2}+\right.$ $2 a b+8 b c-4 a c)$ <br> (viii) $\left(\frac{2}{3} x-1\right)\left(\frac{2}{3} x-1\right)\left(\frac{2}{3} x-1\right)$ <br> (ix) $\quad(-\sqrt{2} x+y+2 \sqrt{2} z)(-\sqrt{2} x+y+$ $2 \sqrt{2} z)$ <br> (x) $\quad(x+y)(x+y)(x-y)(x-y)$ <br> (xi) $\quad(a-0.4)\left(a^{2}+0.4 a+0.16\right)$ <br> (xii) $(4 a+b)(4 a+b)(4 a+b)$ <br> (xiii) $(\sqrt{3} x-1)(\sqrt{3} x+2)$ <br> (xiv) $(x+\sqrt{2})(\sqrt{2} x+1)$ <br> (xv) $\quad(6 a-b)(b+2 c)$ | 21. | (2x-3) |


|  | (xvi) $\quad(a-3 b)(a+3 b)\left(a^{2}+9 b^{2}\right)$ <br> (xvii) $(2 a+3 b)(2 a+3 b-4)$ <br> (xviii) $\left(x^{2}+1\right)\left(x^{2}+3\right)$ <br> (xix) $3 y\left(4 x^{2}+9 y^{2}-6 x y\right)$ <br> (xx) $\quad a^{2}(a-b)(a+b)\left(a^{2}+a b+\right.$ $\left.b^{2}\right)\left(a^{2}-a b+b^{2}\right)$ |  |  |
| :---: | :---: | :---: | :---: |
| 14. | $a^{2}-2 a+4$ | 22. | 0 |
| 15. | 0 | 23. | 100 |
| 16. | (i) 10404 <br> (ii) 9312 <br> (iii) 994009 <br> (iv) 9984 <br> (v) 720 <br> (vi)970299 | 24. | 0 |
| 17. | $\mathrm{k}+\mathrm{p}=\mathbf{- 1}$ | 25. | 1/2 |
| 18. | 27, 64, -108, -144 | 26. | 3 |
| 27. | 16 | 28. | (20x-14) units |
| 29. | $x^{8}-\frac{1}{x^{8}}$ | 30. | $a=2, b=-1$ |

## CHAPTER 3

## COORDINATE GEOMETRY

## POINTS TO REMEMBER

- The coordinate axes $X^{\prime} O X$ and $Y^{\prime} O Y$ are called $x$-axis and $y$-axis respectively, $X^{\prime} O X$ is the horizontal line and $Y^{\prime} O Y$ is the vertical line perpendicular to $X^{\prime} O X$.
- The coordinate axes divide the plane into four parts called quadrants.
- Point of intersection of $x$-axis and $y$-axis is called the origin and is denoted by $\mathbf{O}(0$, $0)$.
- A point is represented in a plane as $P(x, y)$.
- $O X$ and $O Y$ are called positive directions of $x$-axis and $y$-axis respectively whereas $O X^{\prime}$ and $O Y^{\prime}$ are called negative directions of $\mathbf{x}$-axis and $\mathbf{y}$-axis respectively.
- The x-coordinate of a point is also called the abscissa.
- The y-coordinate of a point is also called the ordinate.
- If a point is in I quadrant, then its coordinates are of the form (+ve, +ve), in II quadrant it is of the form (-ve, +ve), in III quadrant it is of the form (-ve, -ve) and in IV quadrant it is of the form (+ve, -ve).

- $P(x, y)$ represents a point $P$ whose coordinates are $x$ and $y$.
(a) Abscissa of point $P=P e r p e n d i c u l a r ~ d i s t a n c e ~ o f ~ p o i n t ~ P ~ f r o m ~ y-a x i s ~$
(b) Ordinate of point $P=$ Perpendicular distance of point $P$ from $x$-axis
(c) The abscissa of point $P$ is Positive if $P$ lies on the right of the $y$-axis, Zero if it is on the $\mathbf{y}$-axis and Negative if it lies on the left of $\mathbf{y}$-axis.
(d) The ordinate of point $P$ is Positive if $P$ lies above $x$-axis, Zero if it is on the $x$ - axis and Negative if it lies below the $x$-axis.
- Coordinates of a point lying on
(i) $x$-axis is of the form $(x, 0)$ and
(ii) $y$-axis are of the form $(0, y)$


## QUESTIONS

1. In which quadrant or on which axis each of the following points lie?
(a) $(-3,5)$
(b) $(5,-3)$
(c) $(2,0)$
(d) $(4,5)$
(e) $(-4,-5)$
(f) $(0,-2)$
2. Write the abscissa and ordinate of the following points.
(a) $(2,-7)$
(b) $\left(0, \frac{3}{2}\right)$
(c) $(6,4)$
(d) $(-4,-3)$
(e) $(-6,0)$
(f) $(-3,2)$
3. Write the coordinates of the following points under reflection in $x$-axis.
(a) $(3,2)$
(b) $(-5,4)$
(c) $(\mathbf{0}, \mathbf{0})$
4. Write the coordinates of the following points under reflection in $y$-axis.
(a) $(6,-3)$
(b) $(-1,0)$
(c) $(-8,-2)$
5. Write the perpendicular distance of:
(a) $(0,-4)$ from $y$-axis
(b) $(2,8)$ from $x$-axis
(c) $(-1,3)$ from $y$-axis
(d) $(3,5)$ from $x$-axis
(e) (-4, -6) from $y$-axis
6. Write the axis to which following lines are parallel.
(a) $\mathrm{y}=0$
(b) $3 x-6=0$
(c) $x=-2$
(d) $\mathrm{x}=0$
(e) $2 \mathrm{y}+8=0$
7. Find the coordinates of points whose
(a) abscissa is $\frac{-3}{2}$ and ordinate is 5
(b) ordinate is $\frac{5}{4}$ and lies on $\mathbf{y}$-axis
(c) ordinate is 5 and abscissa - 2
8. Write the type of triangle formed by joining the following points in the Cartesian plane.
(a) $\mathrm{A}(-4,0), \mathrm{B}(0,4)$ and $\mathrm{C}(4,0)$
(b) $P(0,0), Q(3,3)$ and $R(0,3)$
(c) $L(0,6), M(0,-6)$ and $N(6 \sqrt{3}, 0)$
9. Write the type of quadrilateral formed by joining the following points in the Cartesian plane.
(a) $P(3,0), Q(3,3), R(-3,3)$ and $S(-3,0)$
(b) A $(1,3), \mathrm{B}(1,-1) \mathrm{C}(7,-1)$ and $\mathrm{D}(7,3)$
(c) $\mathbf{P}(-1,0), \mathbf{Q}(0, \sqrt{3}), R(1,0)$ and $S(0,-\sqrt{3} \cdot)$
10. Find the area of the geometrical figure formed by following points
(a) $\mathrm{A}(1,2), \mathrm{B}(-4,2) \mathrm{C}(-4,-1)$ and $\mathrm{D}(1,-1)$
(b) $P(2,0), Q(3,0), R(0,-2)$ and $S(0,-3)$
(c) $\mathrm{A}(2,0), \mathrm{B}(0,8)$ and $\mathrm{O}(0,0)$
(d) $O(0,0), P(1,0)$ and $Q(0,2)$
11. Find the difference of:
(a) abscissa of $P(2,3)$ and abscissa of $Q(-2,0)$
(b) ordinate of $A(-1,5)$ and ordinate of $B(2,3)$
(c) abscissa of $L(3,5)$ and ordinate of $M(-3,-4)$
12. What will be the abscissa of a point lying on $x$-axis?
13. What will be the ordinate of a point lying on $x$-axis?
14. Find the missing vertex of the following
(a) If the points $P(1,0), Q(5,0), R(5,2)$ and $S(x, y)$ form a rectangle.
(b) If the points $A(0,0), B(2,0) C(2,2)$ and $D(x, y)$ form a square
(c) If $O(0,0)$ is the centre of the circle and points $A(0,4) B(-4,0) C(0,-4)$ and $D(x$, $y)$ lie on the circle.
15. If the points $A(2,0), B(-6,0)$ and $C(3, a-3)$ lie on the $x$-axis, then find the value of a .
16. The coordinates of two points are $A(5,3)$ and $B(-2,7)$ then find the mean of abscissa
of $A$ and ordinate of $B$.
17. Fill in the blanks:
(a) The points of intersection of the coordinate axes is called $\qquad$ -.
(b) The measure of angle between the coordinate axes is $\qquad$ .
(c) The abscissa and ordinate of the origin are $\qquad$ .
(d) The ordinate of any point on $x$-axis is $\qquad$ .
(e) The abscissa of any point on $y$-axis is $\qquad$ .
18. Into how many parts the coordinate axes divides the plane?
19. In which quadrant does $(x, y)$ lie, if $y$ is negative and $x$ is positive.
20. If the perpendicular distance of a point $A$ from $x$-axis be 4 units along the negative direction of the $y$-axis, then write the ordinate of $A$.
21. How many points lie on both the axes?
22. PQR is an equilateral triangle with coordinates of $P$ and $R$ as $(0,-5)$ and $(0,5)$ respectively. Find the length of the side of $\triangle \mathrm{PQR}$.
23. The points whose abscissa and ordinate have different signs, will lie in which quadrants?
24. Write the name of the figure obtained when the points $P(-1,-1), Q(2,3)$ and $R$ $(8,11)$ plotted on the Cartesian plane.
25. Look at the graph and answer the questions
(a) Write down the coordinates of A, B, C, D, E, F, and G
(b) Join the coordinates of $A, B, C, D$, and name the quadrilateral so formed and find its area.
(c) Write down the coordinates of $\mathbf{P}, \mathbf{Q}, \mathbf{R}, \mathbf{S}$. Join them and name the quadrilateral and find the length of diagonals.
(d) PQRS is a parallelogram. Write down the Co-ordinates of $P, Q, R, S$ and also find out the point of intersection of its diagonals.
(e) Write down the coordinates of $A, B$ and $C$. Also find the area of $\triangle \mathrm{ABC}$.
(f) A circle is drawn whose centre is $O$ and radius $=4$ units. Find the coordinates of A, B, C, D, E and F.




GRAPH FOR Q- 25 PART (c)


GRAPH FOR Q- 25 PART (e)


GRAPH FOR Q- 25 PART (d)


GRAPH FOR Q. 25 (PART (f))

## ANSWERS

| 1. (a) II Quadrant | (b) IV Quadrant |
| :--- | :--- |
| (c) X-Axis | (d) I Quadrant |
| (e) III Quadrant | (f) Y-Axis |

2. (a) Abscissa $x=2$, Ordinate $y=-7$
(b) $\mathrm{x}=0, \mathrm{y}=\frac{3}{2}$
(c) $x=6, y=4$
(d) $x=-4, y=-3$
(e) $x=-6, y=0$
(f) $\mathrm{x}=-3, \mathrm{y}=2$
3. (a) $(3,-2)$
(b) $(-5,-4)$
(c) $(0,0)$
4. (a) (-6, -3)
(b) $(1,0)$
(c) $(8,-2)$
5. (a) 0 Units
(b) 8 Units
(c) 1 Units
(d) 5 Units
(e) 4 Units
6. (a) It represents x-axis
(b) $y$-axis
(c) $y$-axis
(d) It represents $\mathbf{y}$-axis
(e) $x$-axis
7. (a) $\left(\frac{-3}{2}, 5\right)$
(b) $\left(0, \frac{5}{4}\right)$
(c) $(-2,5)$
8. (a) Isosceles triangle
(b) Isosceles right triangle
(c) Equilateral triangle
9. (a) Rectangle
(b) Rectangle
(c) Rhombus
10. (a) 15 sq. units
(b) 2.5 sq. units
(c) 8 sq. units
(d) 1 sq. units
11. (a) 4 Units
(b) 2 Units
(c) 7 Units
12. Any Number
13. 0 (Zero)
14. (a) $S(1,2)$
(b) $\mathrm{D}(\mathbf{0}, \mathbf{2})$
(c) $\mathrm{D}(4,0)$
15. $\mathbf{a}=3$
16.6
16. (a) Origin
(b) $90^{0}$
(c) 0 and 0
(d) 0
(e) 0
17. Four
18. IV Quadrant
19. -4
20. Only one i.e origin $(0,0)$
21. 10 units
22. II or IV Quadrant

## 24. LINE

25. (a) $A(3,2), \quad B(5,4), \quad C(-1,3), \quad D(-4,1), \quad E(5,0), \quad F(0,-3) \quad G(-3,-3)$,
(b) $A(-5,1), \quad B(5,1), \quad C(5,4), \quad D(-5,4)$

Quadrilateral is Rectangle and its Area $\mathbf{= 3 0}$ sq. units
(c) $P(0,3), \quad Q(-3,0), \quad R(0,-3), \quad S(3,0)$,

Quadrilateral is Square and Length of diagonal = 6 units
(d) $P(3,2), \quad Q(-2,2), \quad R(-4,-2), \quad S(1,-2)$

Point of intersection of diagonals of parallelogram is $\left(\frac{-1}{2}, 0\right)$
(e) $A(0,4), \quad B(-3,-2), \quad C(3,-2) \quad$ Area of $\triangle A B C=18$ sq. units
(f) $\mathrm{A}(2,-3.5), \mathrm{B}(-2,-3.5), \quad \mathrm{C}(-3.5,-2), \mathrm{D}(-2.5,3), \mathrm{E}(3.5,2), \quad \mathrm{F}(3.5,-2)$

## CHAPTER 4

## LINEAR EQUATIONS IN TWO VARIABLES

## POINTS TO REMEMBER

- An equation of the form $a x+b y+c=0$, where $a, b$ and $c$ are real numbers, such that $a$ and $b$ are not both zero, is called a linear equation in two variables.
- A linear equation in two variables has infinitely many solutions.
- The graph of every linear equation in two variables is a straight line.
- The graph of $x=a$ is a straight line parallel to the $y$-axis, where $a$ is any real number.

If $\mathbf{a}=\mathbf{0}$ then it represents $\mathbf{y}$-axis.

- The graph of $\mathbf{y}=\mathbf{a}$ is a straight line parallel to the $\mathbf{x}$-axis, where $\mathbf{a}$ is any real number.

If $\mathbf{a}=\mathbf{0}$ then it represents $\mathbf{x}$-axis.

- Every solution to the linear equation is a point on the graph of the linear equation.


## QUESTIONS

1. What is the standard form of linear equation in one variable?
2. What is the standard form of linear equation in two variables?
3. How many solutions does a linear equation in two variables have?
4. Compare the linear equation $3 x-8=\sqrt{2} y$ with $a x+b y+c=0 \&$ indicate the values of $a, b$ and $c$.
5. Compare $2 \mathrm{x}=-6 \mathrm{y}$ with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$ and find the values of $\mathrm{a}, \mathrm{b}$ and c .
6. Compare $x-\frac{y}{5}-10=0$ with $a x+b y+c=0$ and find the values of $a, b$ and $c$.
7. Compare $2 \mathrm{x}=0$ with $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$ and find the value of $(\mathrm{a}+\mathrm{b}+\mathrm{c})$.
8. If $x=1$ and $y=-1$ is a solution of equation $3 x+p y=6$. Find the value of $p$.
9. If $x=2$ and $y=3$ is a solution of equation $\frac{3 x+5}{3}-y=k$. Find value of $k$.
10. Find the coordinates of the point where the equation $3 x+y=6$ intersect at $x$-axis.
11. Find the coordinates of the point where the equation $2 x+y=7$ intersect at $y$-axis.
12. Find the value of $p$ so that linear equation $3 p x+y=5$ has a solution $x=\frac{2}{3}$ and $y=1$.
13. Identify that the graph of the equation $3 x-2=0$ is parallel to which axis?
14. Identify that the graph of the equation $2 y+3=0$ is parallel to which axis?
15. On the line $y=2$, what will be the value of ordinate when $x=-7$ ?
16. Express $y$ in terms of $x$ in the equation $x-2 y=4$.
17. Express $x$ in terms of $y$ in the equation $2 x+y=9$.
18. Express $y$ in the form of $x$ in the equation $3 x+5 y-11=0$. Find the points where the equation interests $y$-axis and $x$-axis.
19. Find any one solution of equation $2 x+3 y=5$.
20. Find any one solution of equation $y=\frac{x}{2}$.
21. If the point $\left(1, \frac{9}{4}\right)$ lies on the graph of the equation $m x+4 y=9$. Find the value of $m$.
22. Solve for $\mathrm{x}: \frac{3 x+2}{7}+\frac{4(x+1)}{5}=\frac{2(2 x+1)}{3}$
23. Solve for $\mathrm{x}: \frac{x-7}{4}-6=\frac{3(x+1)}{4}+2$
24. If the point $(2 k-3, k+2)$ lies on the graph of the equation $2 x+3 y+15=0$, then find the value of $k$.
25. Write the statement as a linear equation in two variables for the following:
(a) Age of $x$ exceeds the age of $\mathbf{y}$ by 7 years.
(b) After 5 years, the age of father will be two times the age of his son. If present age of father is $y$ years and that of son is $x$ years.
26. Find the common solution of both linear equation $x-2 y=1$ and $2 x+y=7$.
27. For what value of $c$, the linear equation $2 x+c y=8$ has equal values of $x \& y$ and show its solution on the graph.
28. Let $y$ varies directly as $x$. If $y=12$ when $x=4$, then write a linear equation. What is the value of $y$ when $x=5$ ?
29. Observe the graph and give the corresponding equation for it.


GRAPH FOR Q. 29 (PART (a)


GRAPH FOR Q. 29 (PART (b)

30. Observe the graph and give the corresponding equation for it.


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GRAPH FOR Q. }30\mathrm{ (PART (a)
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GRAPH FOR Q. }30\mathrm{ (PART (b)
```


## ANSWERS

| Q.NO. | ANSWER | Q.NO. | ANSWER |
| :---: | :---: | :---: | :---: |
| 1. | $\mathbf{a x}+\mathbf{b}=0, \mathbf{a} \neq 0$ | 16. | $y=\frac{x-4}{2}$ |
| 2. | $a x+b y+c=0, a, b \neq 0$ | 17. | $x=\frac{9-y}{2}$ |
| 3. | Infinite solution | 18. | $y=\frac{11-3 x}{5}$ <br> Line intersect x - axis at $\left(\frac{11}{3}, 0\right)$ <br> Line intersect $\mathbf{y}$ - axis at ( 0 , $\frac{11}{5}$ ) |
| 4. | $\mathrm{a}=3, \mathrm{~b}=-\sqrt{2}, \mathrm{c}=-8$ | 19. | $(1,1)$ or any other solution |
| 5. | $a=2, b=6, c=0$ | 20. | $(2,1)$ or any other solution |
| 6. | $a=1 b=-\frac{1}{5} c=-10$ | 21. | $\mathrm{m}=0$ |
| 7. | $a+b+c=2$ | 22. | $x=4$ |
| 8. | $p=-3$ | 23. | $\mathrm{x}=-21$ |
| 9. | $\mathrm{k}=\frac{2}{3}$ | 24. | $k=\frac{-15}{7}$ |
| 10. | (2, 0) | 25. | (a) $x-y-7=0$ <br> (b) $2 x-y+5=0$ |
| 11. | $(0,7)$ | 26. | $\mathrm{x}=3, \mathrm{y}=1,(3,1)$ |
| 12. | $\mathrm{p}=2$ | 27. | $C=\frac{8-2 x}{x}, \mathrm{x} \neq 0$ |
| 13. | y-axis | 28. | $y=3 x, y=15$ |
| 14. | $\mathbf{x}$ - axis | 29. | (a) $x=2$ <br> (b) $x=-2.5$ <br> (c) $\mathbf{y}=2$ |
| 15. | 2 | 30. | (a) $x+y=3$ <br> (b) $y-x=2$ |

## CHAPTER - 5 INTRODUCTION TO EUCLID GEOMETRY

## POINTS TO REMEMBER

- Axioms or postulates are the assumptions which are obvious universal truths. They are not proved.
- Theorems are statements which are proved using definitions, axioms, previously proved statements and deductive reasoning.
- Some of Euclid's axioms are:
(a) Things which are equal to the same thing are equal to one another.
(b) If equals are added to equals, the wholes are equal.
(c) If equals are subtracted from equals, the remainders are equal.
(d) Things which coincide with one another are equal to one another.
(e) The whole is greater than the part.
(f) Things which are double of the same things are equal to one another.
(g) Things which are halves of the same things are equal to one another.
- Euclid's postulates are following:

Postulate 1: A straight line may be drawn from any one point to any other point.
Postulate 2: A terminated line can be produced indefinitely.
Postulate 3: A circle can be drawn with any centre and any radius.
Postulate 4: All right angles are equal to one another
Postulate 5: If a straight line falling on two other straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on the side on which the sum of angles is less than two right angles.

## QUESTIONS

Answer each of the following questions using appropriate Euclid's axiom/postulate:

1. How many least number of distinct points determine a unique line?
2. In how many maximum numbers of points can two distinct lines intersect?
3. In how many maximum numbers of points can two distinct planes intersect?
4. How many lines can be determined by two distinct points in a plane?
5. How many lines can pass through a given point?
6. How many maximum number of points, can lie on a line?
7. Given two points $P$ and $Q$, how many line segments do they determine?
8. Name the line segments determined by three collinear points $X, Y$ and $Z$.
9. Does a line segment have any length?
10. Give an example of a geometrical line.
11. How many end points does a ray have?
12. Give the relation between whole and the part.
13. Two salesmen make equal sales of chocolates in the month of March. In June, each salesman doubles his sale to that of the month of March. Compare their sales in June.
14. If $a>b$ and $b>c$, then, what is the relation between $a$ and $c$ ?
15. If $a+b=10$, then find the value of $a+b+x$.
16. If $B$ lies between $A$ and $C$ and $A C=8 \mathrm{~cm}, B C=3 \mathrm{~m}$, what is $B A$ and $(A B)^{2}$ ?
17. If the point $P$ lies in between $M \& N$ and $C$ is mid-point of $M P$, then find $\mathbf{M N}$ - $\mathbf{N P}$ - MC.
18. In the figure name the following
(a) 3 line segments
(b) 4 collinear points
(c) A pair of non-intersection line segments

19. With reference to the figure given below, state which statement is true and which is false.
(i) $\mathbf{P Q}+\mathbf{Q R}=\mathbf{P R}$
(ii) $\mathbf{P R}+\mathbf{P S}=\mathbf{P S}$
(iii) Line segments PQ and PS are coincident
(iv) Points R, S, T lie on the line PQ

20. Which of the following statements are True and which are false?
(a) If two circles are equal, then their radii are equal.
(b) In figure, If $\mathbf{A B}=\mathbf{P Q}$ and $P Q=X Y$ then $A B=X Y$

(c) The edges of a surface are curves.
(d) A solid has three Dimensions.
(e) If a quantity $B$ is a part of another quantity $A$, then $A$ can be written as sum of B
and some third quantity $C$.
21. It is known that $x+y=10$ and $x=z$, then find the value of $y+z$.
22. Fill in the blanks:
(a) Line separates a plane into $\qquad$ parts.
(b) Rectilinear figure is formed by $\qquad$ .
(c) A surface is that, which has length, $\qquad$ but no $\qquad$ .
(d) A point has $\qquad$ number of dimension.
(e) A surface has $\qquad$ number of dimensions.
23. Fill in the blanks:
(a) If two squares are congruent than their $\qquad$ are equal.
(b) A pyramid is a solid figure with base as a $\qquad$ and side faces as
$\qquad$ .
(c) Two distinct $\qquad$ in a plane cannot have more than one point in common.
(d) Given line and a point, not on the line, there is one and only one perpendicular line which passes through the given point and is $\qquad$ to the line.
(e) If line $A B, A C, A D, A R$ are parallel to a line $l$, then points $A, B, C, D$ and $R$ are
$\qquad$ .
24. Multiple Choice Questions: Choose the correct Option.
(a) "Lines are parallel if they do not intersect" is stated in the form of
(i) an Axiom
(ii) a Definition
(iii) a Postulate
(iv) a Proof
(b) Which of the following needs a Proof?
(i) Theorem
(ii) Axiom
(iii) Definition
(iv) Postulate
(c) Boundaries of solids are
(i) Lines
(ii) Surfaces
(iii) Points
(iv) Curves
(d) Boundaries of surfaces are
(i) Surfaces
(ii) Lines
(iii) Points
(iv) Curves or Straight Line
25. Multiple Choice Questions: Choose the correct Option.
(a) The total number of propositions in the elements are
(i) 465
(ii) 460
(iii) 13
(iv) 55
(b) In Indus valley civilization (About 300 BC) the bricks used for construction work Were having dimensions in the ratio
(i) $1: 3: 4$
(ii) 4:2:1
(iii) 4:4:1
(iv) 4:3:2
(c) A pyramid is a solid figure, the base of which is
(i) Only a Triangle
(ii) Only a square
(iii) Only a rectangle (iv) any Polygon
(d) The side faces of a pyramid are
(i) Triangles
(ii) Squares
(iii) Polygons
(iv) Trapezium

## ANSWERS

| Q.NO. | ANSWER | Q.NO. | ANSWER |
| :---: | :---: | :---: | :---: |
| 1. | 2 | 14. | a > c |
| 2. | 1 | 15. | $10+\mathrm{x}$ |
| 3. | Infinite number of points | 16. | 5m, 25m ${ }^{2}$ |
| 4. | Unique (one) | 17. | CP |
| 5. | Infinitely many | 18. | (a) AC, BQ, RS or any other three <br> (b) C, D, Q, S <br> (c) AC, BQ or any other |
| 6. | Infinite | 19. | (a) True <br> (b) False <br> (c) False <br> (d) True |
| 7. | One | 20. | (a) True <br> (b) True <br> (c) False <br> (d) True <br> (e) True |
| 8. | XY, YZ, ZX | 21. | 10 |
| 9. | Yes (breadthless) | 22. | (a) 2 <br> (b) Straight Lines <br> (c) Breadth, Thickness <br> (d) 0 <br> (e) 2 |
| 10. | Meeting place of two walls or any other Suitable example | 23. | (a) Sides <br> (b) Polygon, Triangles <br> (c) Lines <br> (d) Perpendicular <br> (e) Collinear |
| 11. | One | 24. | (a) (iii) a Postulate <br> (b) (i) Theorem <br> (c) (ii) Surfaces <br> (d) (iv) curves or straight lines |
| 12. | Whole > Part | 25. | (a) (i) 465 <br> (b) (ii) $4: 2: 1$ <br> (c) (iv) any polygon <br> (d) (i) Triangles |
| 13. | Equal |  |  |

## CHAPTER - 6 LINES AND ANGLES

## POINTS TO REMEMBER

- If a ray stands on a line, then the sum of the two adjacent angles so formed is $180^{\circ}$ \& vice-versa. This property is known as the Linear Pair Axiom.
- If two lines intersect each other, then vertically opposite angles are equal.
- If a transversal intersects two parallel lines, then (refer figure)

(a) each pair of corresponding angles is equal e.g. $\angle 1=\angle 5, \angle 4=\angle 8$.
(b) each pair of alternate interior angles is equal. e.g $\angle 4=\angle 6, \angle 3=\angle 5$
(c) each pair of interior angles on the same side of the transversal is supplementary. e.g. $\angle 4+\angle 5=180^{\circ}, \angle 3+\angle 6=180^{\circ}$
- If a transversal intersects two lines such that, either any one pair of corresponding angles is equal, or any one pair of alternate interior angles is equal, or any one pair of interior angles on the same side of the transversal is supplementary, then the lines are parallel.
- Two intersecting lines cannot be parallel to the same line.
- Lines which are parallel to a given line are parallel to each other.
- The sum of three interior angles of a triangle is $180^{\circ}$.
- If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two Interior opposite angles.


## QUESTIONS:

1. Two adjacent angles on a straight line are in ratio $5: 4$. Find the angles.
2. Two angles measure $25^{\circ}-x$ and $135^{\circ}+2 x$. If each one is the supplement of the other, find the value of $x$.
3. An angle is equal to one-third of its supplement. Find its measure.
4. In the given figure, $l_{1} \| l_{2}$, find the value of $x$.

5. In the given figure, if $A B \| C D, \angle A P Q=40^{\circ}$ and $\angle P R D=118^{\circ}$, find $x$ and $y$.

6. In $\triangle A B C, \angle A-\angle B=30^{\circ}, \angle B-\angle C=15^{\circ}$, Find the measure of $\angle B$.
7. In $\triangle A B C, \angle A=60^{\circ}, \angle B=80^{\circ}$, the bisector of $\angle B$ and $\angle C$ meet at $O$, Find $\angle B O C$.
8. If $x, y$ and $z$ are the interior angles of a triangle and $\frac{x}{2}=\frac{y}{4}=\frac{z}{3}$, then find the greatest angle of the triangle.
9. If a wheel has six spokes equally spaced, then find the measure of the angle between two adjacent spokes.
10. One angle is twice the other in a supplementary pair. Find the angles.
11. If angle is half of its complement, find the measure of that angle.
12. An angle is $20^{\circ}$ more than its complement. Find their measures.
13. Find the angle which is 5 times its supplement.
14. The ratio of the complement and supplement of an angle is $2: 5$. Find the angle.
15. The three angles of a triangle are $\frac{x}{2}+10^{\circ}, \frac{x}{3}+30^{\circ}$ and $x-25^{\circ}$, find the value of $x$.
16. If side $B C$ of $\triangle A B C$ is produced to $D$ such that $\angle A C D=120^{\circ}$ and $\angle B=\frac{1}{2} \angle A$, then find $\angle A$.
17. The exterior angle of a triangle is $108^{\circ}$. If the interior opposite angles of the triangle is in the ratio 4: 5, find the interior angles of the triangle.
18. The exterior angle of a triangle is $93^{\circ}$. If one of the interior opposite angles is $19^{\circ}$ more than the other, find the smallest angle of the triangle.
19. If $\frac{x}{2}-19^{\circ}$ and $\frac{x}{3}+14^{\circ}$ are complement to each other, find the value of $x$.
20. In $\triangle A B C, 2 \angle A=3 \angle B=6 \angle C$, find the smallest angle of the triangle.
21. If $\mathrm{OD} \perp \mathrm{AB}$ and $\angle \mathrm{DOC}=30^{\circ}$. Find $\angle \mathrm{BOA}-\angle \mathrm{BOC}$.

22. Find the value of $q$ in the given figure.

23. In the given figure, find the value of $\frac{1}{2} z$ if $x=58^{\circ}$ and $y=42^{\circ}$.

24. Find the value of $a+b$ in the given figure.

25. If $\angle A D B$ is a right angle, find the value of $x$.

26. Find the value of $x$ and $y$.

27. Angles of a triangle are in the ratio 2: 4: 3. Find the smallest angle of the triangle.
28. If $(5 y+62)^{\circ}$ and $(22+y)^{\circ}$ are supplementary angles. Find $\mathbf{y}$.
29. In the figure $\mathrm{AB} \| \mathrm{CD}, \angle \mathrm{ABE}=130^{\circ}, \angle \mathrm{BED}=20^{\circ}$ then find $\angle \mathrm{EDC}$.

30. In the given figure $\mathrm{AB} \| \mathrm{DE}$. Find $a+b-c$.

31. In given figure, find the value of $x$ in terms of $y$, if $l \| m$.

32. Find the value of $x$, if $B Q \| O P$.

33. If $A B$ and $C D$ are parallel, find the value of $x+y$.

34. Find the value of angle $z$.

35. Find the value of $x, y, z$.

36. Find the value of $x$ and $y$ if $A B \| C D$.

37. If $A C \| E F$, find $\angle A D B$.

38. If $A B \| C D$, find the value of $x$.

39. If $\mathbf{m} \| \mathrm{n}$ and $\angle 1=110^{\circ}$, then find $\angle 7$.

40. In given figure, lines $X Y$ and MN intersect at O , if $\angle \mathrm{POY}=90^{\circ}$, $\angle P O M: \angle M O X=2: 3$. Find $c$.

41. The complement of an angle is equal to one third of its supplement, find the angle.
42. If $A B \| D E$, find the value of $\angle A C B$.

43. If $A B \| P Q$, find the angle $x$.

44. If $P Q \| T S$, find the value of $x$.

45. If AD and BD are bisectors of $\angle \mathrm{CAB}$ and $\angle \mathrm{CBA}$ respectively, $\angle \mathrm{ACB}=90^{\circ}$. Find the value of $\angle A D B$.

46. In the given figure, find the value of $c$.

47. In given figure $B D$ and $C D$ are bisectors of external angles at $B$ and $C$ respectively. If $\angle B A C=80^{\circ}$, then find $\angle B D C$.

48. In the given figure, find the value of $x$, if $B C \| D E$ and $\angle A B C=150^{\circ}$ and $\angle \mathrm{BAD}=30^{\circ}$.

49. In the given figure, if $\angle A=40^{\circ}$ and $\angle B=70^{\circ}$, then find $\angle B C E$.

50. In the given figure, PS is the bisector of $\angle \mathrm{QPR}$ and $\mathrm{PT} \perp \mathrm{QR} . \angle \mathrm{PRT}=60^{\circ}$, $\angle P Q S=40^{\circ}$, find $\angle S P T$.


## ANSWERS

| Question No. | Answer | Question No. | Answer |
| :---: | :---: | :---: | :---: |
| 1 | $100^{\circ}, 80^{\circ}$ | 26 | $\mathrm{x}=50^{\circ}, \mathrm{y}=\mathbf{8 0}{ }^{\circ}$ |
| 2 | $20^{\circ}$ | 27 | $40^{\circ}$ |
| 3 | $45^{\circ}$ | 28 | $y=16$ |
| 4 | $\mathrm{x}=75^{\circ}$ | 29 | $110^{\circ}$ |
| 5 | $\mathrm{x}=40^{\circ}, y=78^{\circ}$ | 30 | $180^{\circ}$ |
| 6 | $55^{\circ}$ | 31 | $180^{\circ}-y^{\circ}$ |
| 7 | $120^{\circ}$ | 32 | $130^{\circ}$ |
| 8 | $80^{\circ}$ | 33 | $100^{\circ}$ |
| 9 | $60^{\circ}$ | 34 | $63^{\circ}$ |
| 10 | $120^{\circ}, 6{ }^{\circ}$ | 35 | $\begin{gathered} \mathrm{x}=115^{\circ}, y=65^{\circ}, \mathrm{z}= \\ 115^{\circ} \end{gathered}$ |
| 11 | $30^{\circ}$ | 36 | $\mathrm{x}=130^{\circ}$ and $\mathrm{y}=130^{\circ}$ |
| 12 | 35 ${ }^{\circ}$ 55 ${ }^{\circ}$ | 37 | $85^{\circ}$ |
| 13 | $150{ }^{\circ}$ | 38 | $21^{\circ}$ |
| 14 | $30^{\circ}$ | 39 | $70^{\circ}$ |
| 15 | $90^{\circ}$ | 40 | $216^{\circ}$ |
| 16 | $80^{\circ}$ | 41 | $45^{\circ}$ |
| 17 | $48^{\circ}, 60^{\circ}, 7{ }^{\circ}$ | 42 | $91^{\circ}$ |
| 18 | $37^{\circ}$ | 43 | $111^{\circ}$ |
| 19 | $114^{\circ}$ | 44 | $50^{\circ}$ |
| 20 | $30^{\circ}$ | 45 | $135^{\circ}$ |
| 21 | $60^{\circ}$ | 46 | C $=95^{\circ}$ |
| 22 | $\mathrm{q}=9$ | 47 | $50^{\circ}$ |
| 23 | $40^{\circ}$ | 48 | $120^{\circ}$ |
| 24 | $310{ }^{\circ}$ | 49 | $110^{\circ}$ |
| 25 | $26^{\circ}$ | 50 | $10^{\circ}$ |

## CHAPTER 7 TRIANGLES

## POINTS TO REMEMBER

- Two figures are congruent, if they are of same shape and same size.
- If two triangles $A B C$ and $X Y Z$ are congruent under the correspondence $A \leftrightarrow X$, $B \leftrightarrow Y$ and $C \leftrightarrow Z$, then symbolically, $\triangle A B C \cong \Delta X Y Z$.
- SAS Congruence Rule: If two sides and the included angle of one triangle are equal to corresponding two sides and the included angle of the other triangle, then the two triangles are congruent.
- ASA Congruence Rule: If two angles and the included side of one triangle are equal to corresponding two angles and the side of the other triangle, then the two triangles are congruent.
- AAS Congruence Rule: If two angles and one side of one triangle are equal to corresponding two angles and the corresponding side of the other triangle, then the two triangles are congruent.
- RHS congruence Rule: If in two right triangles, hypotenuse and one side of a triangle are equal to the hypotenuse and corresponding one side of other triangle, then the two triangles are congruent.
- SSS Congruence rule: If three sides of one triangle are equal to the corresponding three sides of another triangle, then the two triangles are congruent.
- Angles opposite to equal sides of a triangle are equal.
- Sides opposite to equal angles of a triangle are equal.
- Each angle of an equilateral triangle is $60^{\circ}$.
- Of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.
- In a triangle, angle opposite to the longer side is greater.
- In a triangle, side opposite to the greater angle is longer.
- Sum of any two sides of a triangle is greater than the third side.
- Difference between any two sides of a triangle is less than its third side


## QUESTIONS:

1. Which congruence rule is used for the congruency of the following triangles?
(i)


(ii)

(iii)

2. If $T$ is the mid-point of $S U, R$ is the mid-point of $S V$ and $R T=x \mathrm{~cm}$, where $x^{2}-2 x+1=0$ then find the value of $U V$.

3. In triangle $A B C$, the measure of angle $A$ is $25^{\circ}$ and the measure of angle $B$ is greater than $90^{\circ}$. Then angle $C$ must be less than $x^{\circ}$, find the value of $x$ ?
4. If $\triangle \mathrm{ABC} \cong \triangle P Q R$, then find $(x-y)^{2}+(x+y)^{2}$.

5. In the given figure, CP is the bisector of $\angle C$ and $\mathrm{CA}=\mathrm{CB}$. Then $\triangle C A P$ and $\triangle C B P$ are congruent by which criterion?

6. In figure, $P Q=P R=P S$. Find $(3 \angle Q R S-2 \angle S Q R-\angle Q S R)$.

7. If the length of the three sides of a triangle is $5 \mathrm{~cm}, 12 \mathrm{~cm}$ and 13 cm respectively, then calculate the length (in cm ) of the median of the hypotenuse of the given triangle.
8. In the given figure, $P Q=P S=S R$ and $\angle Q P S=40^{\circ}$, then what is the value of $\angle Q P R$.

9. In the figure, find $x$ if $B C=A D$ and $A B=C D$

10. In the figure, $P Q \| S R$ and $P Q=Q R$. Find $2 x+y$.

11. In a triangle $P Q R$, internal angular bisectors of $\angle Q$ and $\angle R$ intersect at a point $O$. If $\angle \mathrm{P}=110^{\circ}$ then what is the value of $\angle \mathrm{QOR}$ ?
12. In a triangle $X Y Z, X A$ is the angle bisector onto $Y Z$. If the semi-perimeter of the triangle is 12 cm and $X Y=12 \mathrm{~cm}, Y Z=6 \mathrm{~cm}$, then what is the ratio of $Y A: A Z$ ?
13. In a triangle $A B C, A X$ is the angle bisector onto $B C$. If the semi-perimeter of the

14. In the figure, $\mathrm{LM}=\mathrm{MN}, \mathrm{QM}=\mathrm{MR}, \mathrm{ML} \perp \mathrm{PQ}, \mathrm{MN} \perp \mathrm{PR}, \angle \mathrm{Q}=50^{\circ}$, Find x .

15. In the figure, $A B=B C$ and $\angle A=\angle C$. Find $x$.

16. In $\triangle P Q R, \angle P-\angle Q=16^{\circ}, \angle Q-\angle R=28^{\circ}$, Find the value of $\angle Q$.
17. In $\triangle \mathrm{ABC}, \mathrm{AB}=\mathrm{AC}$ and $\angle 1=\angle 2, \angle A=40^{\circ}$. Find $\angle \mathrm{PBC}$ and $\angle P C A$.

18. In the given figure, find $\angle A B D: \angle A C D$.

19. In $\triangle \mathrm{ABC}$ right angled at $\mathrm{C}, \mathrm{AC}=\mathbf{1 2} \mathrm{cm}, \mathrm{BC}=\mathbf{5} \mathrm{cm}$ then find AB .
20. In $\triangle P Q R$ right angled at $R, P R=7 \mathrm{~cm}, Q R=24 \mathrm{~cm}$ then find $P Q$.
21. In figure, find the value of $(x+y)$.

22.In the figure, $A B C D$ is a square. Sides $A B$ and $B C$ are produced to points $P$ and $Q$ such that $B P=C Q$. If $D P=7 \mathrm{~cm}$, find $A Q$.

22. In the figure, find $x$.

23. In the given figure, $A B=A C$ and $\angle A C D=110^{\circ}$, then find $x$.

24. Find the value of $x$ in the given triangle.


B
C
26. In $\triangle A B C, \angle B=65^{\circ}, \angle C=45^{\circ}$ and the bisectors of $\angle B A C$ meets $B C$ at $D$, then find $\angle D A C: \angle C$.

27. In the given figure, $\angle A=\angle D=30^{\circ}$, find $(x+y)$.

28. In the given figure, find the value of $(x+y+z)$.

29. In the figure, $\angle P=24^{\circ}, \angle Q=46^{\circ}$ and $\angle S=40^{\circ}$ then find $x$.

30. In the figure, $A C$ bisects $\angle A$ and $\angle C$. If $A B=\mathbf{3} \mathbf{~ c m}$ then find $A D$.

31. In the figure, $L \| M, A B=D C$, find $A C+B D$, if $O B=3.5 \mathrm{~cm}, O C=3.5 \mathrm{~cm}$, $A B=6 \mathrm{~cm}$.

32. In the figure, $\triangle D B C \cong \triangle E A F$. If $A B=4 \mathrm{~cm}$ find $F C$.

33. In the figure, $\triangle \mathrm{ABC} \& \triangle \mathrm{DEF}$ are equilateral triangles, then find $(3 \angle \mathrm{~A}-\mathbf{2} \angle \mathrm{E})$.

34. In the figure, $A B=D B$, If $\angle A B D=48^{\circ}$, find $x$.

35. In the figure, if $\angle D E C=90^{\circ}$, find $x$.

36. If one angle of a triangle is equal to the sum of other two angles, then what type of triangle is it?
37. $\triangle \mathrm{PQR}$ is right angled at Q , then name the longest side of the triangle and find the relation between $P Q$ and $P R$.
38. In $\triangle A B C, \angle A=40^{\circ}, \angle B=50^{\circ}$, name the longest side of the triangle.
39. What is the ratio in which Centroid of the triangle divides each median?
40. The ratio between the base angle and vertical angle of an isosceles triangle is 5:2. Find all the angles of the triangle.
41. The length of the base of a triangle is $\mathbf{4} \mathbf{~ c m}$ smaller than the length of its altitude. The area of the triangle is $\mathbf{9 6} \mathbf{~ s q . c m}$. Find the length of the base.
42. The perimeter of an isosceles triangle is 100 cm . If the base is 36 cm , find the length of the equal sides.
43. A triangle has an area of 615 sq . $\mathbf{c m}$. If one of its sides is given as $\mathbf{1 2 3} \mathbf{~ c m}$, then what is the length of the perpendicular that is dropped on that particular side from the opposite vertex.
44. In $\triangle X Y Z, M$ and $N$ are points on $X Z$ and $Y Z$ respectively such that $M N$ is the perpendicular bisector of $Y Z$. If $X M=X Y$ and $\angle X Y Z=90^{\circ}$, what is $\angle X Z Y$ ?
45. In $\triangle L M N$, medians MX and NY are perpendicular to each other and intersect at $Z$. If $M X=\mathbf{8} \mathbf{~ c m}, N Y=12 \mathrm{~cm}$, what is the area of $\Delta L M N$ ?
46. The sum of all the three sides of an equilateral triangle is $72 \sqrt{3} \mathrm{~cm}$. Find the height of this triangle.
47. In triangle $P Q R, S$ and $T$ are midpoints of sides $P R$ and $P Q$ respectively. The medians QS and RT intersect at $U$. Find the area (PQU): area (PQR).
48. The length of the three sides of a triangle are in the ratio of $\mathbf{5 : 1 2 : 1 3}$. The difference between largest side of this triangle and the smallest side of this triangle is $1.6 \mathbf{~ c m}$. Find the area of the triangle.
49. If the shaded area is one half the area of triangle ABC and angle $\angle \mathrm{ABC}$ is right angle, then find the length of line segment $A D$ in terms of $y$ and $w$ only.

50. What is the area of the largest triangle that can be fitted into a rectangle of length ' $I$ ' units and width ' $w$ ' units?

## ANSWERS

| 1. (i)ASA (ii) Triangles may not be congruent (iii) SSS |
| :---: |
| 2.2 cm |
| 3. $x=65$ |
| 4. 148 |
| 5. SAS |
| 6. $135{ }^{\circ}$ |
| 7.6 .5 cm |
| 8. $75{ }^{\circ}$ |
| 9.75 ${ }^{\circ}$ |
| 10.180 ${ }^{\circ}$ |
| 11. $145{ }^{\circ}$ |
| 12. 2:1 |
| 13.1:2 |
| 14. $40^{\circ}$ |
| 15.50 ${ }^{\circ}$ |
| 16.64 ${ }^{\circ}$ |
| 17.15 ${ }^{\circ}$, 55 ${ }^{\circ}$ |
| 18.1:1 |
| 19.13 cm |
| 20.25 cm |
| 21.120 ${ }^{\circ}$ |
| 22.7 cm |
| 23.50 ${ }^{\circ}$ |
| 24.40 ${ }^{\circ}$ |
| 25.70 ${ }^{\circ}$ |


| 26. 7:9 |
| :---: |
| 27.120 ${ }^{\circ}$ |
| 28.360 ${ }^{\circ}$ |
| 29.110 ${ }^{\circ}$ |
| 30.3 cm |
| 31.14 cm |
| 32.4 cm |
| 33. $60{ }^{\circ}$ |
| 34. $114{ }^{\circ}$ |
| 35.45 ${ }^{\circ}$ |
| 36. Right angled triangle |
| 37. PR, PR > PQ |
| 38. AB |
| 39. 2:1 |
| 40. Base angles $-75^{\circ}, 75^{\circ}$ Vertical angle $30^{\circ}$ |
| 41.12 cm |
| 42.32 cm |
| 43.10 cm |
| 44. $30^{\circ}$ |
| $45.48 \mathrm{~cm}^{2}$ |
| 46.36 cm |
| 47. 1 : 3 |
| 48. $1.2 \mathrm{~cm}^{2}$ |
| 49. $\mathrm{AD}=\sqrt{w^{2}-3 y^{2}}$ |
| 50. $\frac{1}{2} \mathrm{lw}$ square unit |

## CHAPTER 8 QUADRILATERALS

## POINTS TO REMEMBER

- Sum of the angles of a quadrilateral is $360^{\circ}$.
- A diagonal of a parallelogram divides it into two congruent triangles.
- In a parallelogram, (i) both pair of opposite angles are equal (ii) both pair of opposite sides are equal (iii) diagonals bisect each other.
- A quadrilateral is a parallelogram, if any one of the following statement is true
(i) both pair of opposite angles are equal .
(ii) both pair of opposite sides are equal.
(iii) diagonals bisect each other .
(iv) a pair of opposite sides are equal and parallel.
- Diagonals of a rectangle bisect each other and are equal and vice versa.
- Diagonals of a rhombus bisect each other at right angles and vice versa.
- Diagonals of a square bisect each other at right angles and are equal, and vice versa.
- The line-segment joining the midpoints of any two sides of a triangle is parallel to the third side and is half of it.
- A line through the midpoint of a side of a triangle parallel to another side bisects the third side.
- The quadrilateral formed by joining the midpoints of the sides of a quadrilateral, in order, is a parallelogram.
- Two congruent figures have equal areas but the converse may not to be true.
- Parallelograms on the same base (or equal bases) and between the same parallels are equal in area.
- Area of a parallelogram is the product of its base and the corresponding altitude.
- Parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels.
- If a parallelogram and a triangle are on the same base (or equal bases) and between the same parallels then area of triangle is half the area of parallelogram.
- Triangles on the same base (or equal bases) and between the same parallels are equal in area.
- Area of a triangle is half the product of its base and the corresponding altitude.
- Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.
- A median of a triangle divides it into two triangles of equal areas.


## QUESTIONS:

1. The angles of a quadrilateral are in the ratio $3: 6: 8: 13$. Find the largest angle.
2. The angles of a quadrilateral are $x^{\circ},(x-10)^{\circ},(x+52)^{\circ}$ and $3 x^{\circ}$, find the smallest angle.
3. In the given figure, ABCD is a quadrilateral and AP and DP are the bisectors of $\angle A$ and $\angle D$. Find the value of $x$.

4. Three angles of a quadrilateral are $60^{\circ}, 110^{\circ}$ and $86^{\circ}$, find the fourth angle.
5. In a quadrilateral, if two angles are complimentary and other two angles are in the ratio of $\mathbf{2 : 7}$, then find the measure of these two angles.
6. In a quadrilateral, if one angle is of measure $100^{\circ}$ and other three angles are in the ratio 1:5:7, then find the measure of these three angles.
7. In the given figure, $P Q R S$ is a rectangle if $\angle R P Q=30^{\circ}$, then find the value of $(x+y)$.

8. In the figure, ABCD is a parallelogram. Find $y$.

9. Find $\left(\frac{x+y}{3}\right)$ in the given parallelogram PQRS.

10. In a parallelogram $A B C D$, if $\angle A=(2 x+15)^{\circ}, \angle B=(3 x-25)^{\circ}$, then find the value of $\boldsymbol{x}$.
11. $P Q R S$ is a rhombus with the $\angle Q P S=50^{\circ}$. Find the angles $a^{\circ}$ and $b^{\circ}$ from the figure.

12. ABCD is a rhombus such that $\angle A C B=40^{\circ}$. Find the measure of $\angle A D C$.
13. Diagonals of a quadrilateral $A B C D$ bisect each other. If $\angle A=35^{\circ}$, find $\angle B$.
14. $P Q R S$ is a rectangle with $\angle Q P R=35^{\circ}$. Find the sum of $\angle S Q R$ and $\angle S P R$.
15. Two opposite angles of a parallelogram are, $(3 x-2)^{\circ}$ and $(63-2 x)^{\circ}$. Find the measure of each angle of the parallelogram.
16. In the given figure, ABCD is a rhombus with $\angle A=60^{\circ}$. If $\triangle \mathrm{DEC}$ is an equilateral triangle, then find $\angle D B E$.

17. If the ratio of numerical value of altitude and area of the parallelogram is $2: 11$. Find the length of its base.
18. If the ratio of numeral values of base and area of parallelogram is $1: 8$, then find the length of its altitude.
19. $D, E, F$ are midpoints of sides $B C, C A$ and $A B$ of $\triangle A B C$. If perimeter of $\triangle A B C$ is 12.8 cm , then find the perimeter of $\triangle D E F$.
20. In a quadrilateral STAR, if $\angle S=120^{\circ}$ and $\angle T: \angle A: \angle R=5: 3: 7$, then find measure of $\angle R$.
21. The perimeter of a parallelogram is 30 cm , if the shorter side measures $\mathbf{6 c m}$, then find the measure of the longer side.
22. If one angle of a parallelogram is $36^{\circ}$ less than twice its adjacent angle, then find the angles of parallelogram.
23. Find the angles of the Parallelogram, in the given figure, if $\mathrm{CM} \perp \mathrm{AD}, \mathrm{AL} \perp \mathrm{DC}$ and $\angle \mathrm{MPA}=\mathbf{5 0}^{\circ}$

24. Find the sum of length of diagonals in rhombus PQRS.

25. Perimeter of a parallelogram is $\mathbf{1 2 8 m}$. Find the value of $\boldsymbol{x}$.

26. In the given figure, $A B C D$ is a parallelogram, find the value of $x$ and $y$.

27. In the figure, $X$ is a point in the interior of square $A B C D$. $A X Y Z$ is also a square. If $D Y=3 \mathbf{c m}$ and $A Z=2 \mathbf{c m}$, find $B Y$.

28. In Quadrilateral $\mathrm{ABCD}, \mathrm{BM}$ and DN are drawn perpendiculars to AC such that $B M=D N$. If $B R=8 \mathrm{~cm}$ then find $B D$.

29. In the given figure, find $\angle A$ and $\angle C$, if $A D=D C$ and $A B=B C$.

30. The length and breadth of a rectangle are in the ratio $4: 3$. If the diagonal measures 25 cm , then find the perimeter of rectangle.
31. Area of rectangle $A B C D$ and parallelogram $A B E F$ are equal in area. If base $A B$ $=8 \mathrm{~cm}$ and height $\mathrm{BC}=3 \mathrm{~cm}$. Find the perimeter of parallelogram ABEF if C is a midpoint of EF.

32. ABCD is a parallelogram, if the two diagonals are equal find the measure of $\angle A B C$.
33. In a parallelogram $\mathrm{ABCD}, \mathrm{DL} \perp \mathrm{AB}$ and $\mathrm{BM} \perp \mathrm{AD}$. If $\mathrm{AB}=\mathbf{2 0} \mathbf{c m}$, and $\mathrm{DL}=15$ cm and $B M=25 \mathrm{~cm}$, then find $A D$.
34. In the given figure, if ABCD is a square, then find the value of x .

35. A diagonal of a rectangle is inclined at $25^{0}$ on one side of the rectangle. Find the acute angle between the diagonals.
36. The two parallel lines $l$ and $m$ intersect with the transversal line' ${ }^{\prime}$ '. Explain what type of quadrilateral will be formed by bisectors of internal angles.
37. In a quadrilateral, if $B C=A D$ and $D C \| A B$. Find the sum of $\angle B$ and $\angle D$.
38. In a triangle $\triangle \mathrm{ABC}, \mathrm{AD}$ is the median of BC and E is the midpoint of AD . If $D G \| B F$ and $A C=x \cdot A F$, then find the value of $x$.
39. Find the length of the longest diagonal in the parallelogram FAST.

40. In the given rhombus RHOM, what will be the value of $\angle \mathrm{ROH}$ ?


## ANSWERS

| Q. No. | Answers | Q. No. | Answers |
| :---: | :---: | :---: | :---: |
| 1 | $15{ }^{\circ}$ | 21 | 9 cm |
| 2 | $43^{\circ}$ | 22 | $72^{\circ}, 108^{\circ}, 72^{\circ}, 108{ }^{\circ}$ |
| 3 | $x=95^{\circ}$ | 23 | $50^{\circ}, \mathbf{1 3 0}^{\circ}, 50^{\circ}, 130^{\circ}$ |
| 4 | $104{ }^{\circ}$ | 24 | 62 cm |
| 5 | $60^{\circ}, 210{ }^{\circ}$ | 25 | $\mathrm{x}=8 \mathrm{~m}$ |
| 6 | $20^{\circ}, 100^{\circ}, 140^{\circ}$ | 26 | 29, 73 |
| 7 | $180^{\circ}$ | 27 | 7 cm |
| 8 | $y=44^{\circ}$ | 28 | 16 cm |
| 9 | $35^{\circ}$ | 29 | $\angle A=130^{\circ}, \angle C=130^{\circ}$ |
| 10 | $x=38{ }^{\circ}$ | 30 | 70 cm |
| 11 | $\mathrm{a}=65^{\circ}, \mathrm{b}=25^{\circ}$ | 31 | 26 cm |
| 12 | $\angle \mathrm{ADC}=100^{\circ}$ | 32 | $90^{\circ}$ |
| 13 | $145{ }^{\circ}$ | 33 | 12 cm |
| 14 | $110^{\circ}$ | 34 | $60^{\circ}$ |
| 15 | $37^{\circ}, 37^{\circ}, 143{ }^{\circ}, 143^{\circ}$ | 35 | $50^{\circ}$ |
| 16 | $\angle D B E=30^{\circ}$ | 36 | RECTANGLE |
| 17 | 5.5 units | 37 | $180^{\circ}$ |
| 18 | 8 units | 38 | 3 |
| 19 | 6.4 cm | 39 | 46 units |
| 20 | $\angle R=112^{\circ}$ | 40 | $55^{\circ}$ |

## CHAPTER - 9

## AREAS OF PARALLELOGRAMS

## AND TRIANGLES

## POINTS TO REMEMBER

- Two figures are said to be on the same base and between the same parallels, if they have a common side (base) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.
- Area of a figure is a number (in some unit) associated with the part of the plane enclosed by that figure.
- Two congruent figures have equal areas but the converse need not to be true.
- If there are two or more parallel lines and the intercepts made by these on one transversal are equal, the corresponding intercepts of any transversal are equal.
- If a plane region formed by the figure ( $F$ ) is made up two non-overlapping plane regions formed by figures (F1) and (F2), then Area (F) = Area (F1) + Area (F2)
- A diagonal of parallelogram divides it into two triangles of equal area.
- Parallelograms on the same base and between the same parallels are equal in area.
- The area of a parallelogram is the product of its base and the corresponding altitude.
- Triangles on the same bases and between the same parallels are equal in area.
- The area of the triangle is half the product of its sides and the corresponding altitude.
- If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to the half of the parallelogram.
- The area of the trapezium is half the product of its height and sum of parallel sides.
- Triangles having equal areas and having one side of one of the triangles, equal to one side of the other, have their corresponding altitudes equal.
- If each diagonal of the quadrilateral divides it into two triangles of equal area, then the quadrilateral is a parallelogram.
- The area of a rhombus is half the product of the lengths of its diagonals.
- Diagonals of a parallelogram divide it into four triangles of equal areas.
- If the diagonals AC and BD of a quadrilateral ABCD intersect at O and divide the quadrilateral into four triangles of equal areas, then the quadrilateral ABCD is a parallelogram.
- A median of a triangle divides it into two triangles of equal areas.


## QUESTIONS:

1. Area of a parallelogram is 48 sq cm .Its base and altitude are in the ratio of 1:3. Find the length of its base.
2. Area of a parallelogram is $1000 \mathrm{sq} \mathbf{~ m}$. Its base and altitude are in the ratio of 5: 2. Find the length of its altitude.
3. Find the area of the rhombus whose diagonals are 12 cm and 5 cm .
4. Find the perimeter of the rhombus if the lengths of its diagonals are 6 cm and 8 cm .
5. Two sides of a parallelogram are in the ratio 7: 3 and its perimeter is $\mathbf{1 2 0} \mathbf{c m}$. Find its longest side.
6. Two diagonals of a rhombus are in the ratio 4: 3 and its perimeter is 20 m . Find length of each side of rhombus.
7. Altitude and area of a parallelogram are in the ratio 3: 7. Find the length of the base of the parallelogram.
8. Find the area of rectangle with perimeter 300 cm and length is 90 cm more than its breadth.
9. Parallel side of the trapezium are 2 cm and 5 cm and height of the trapezium is 7 cm , Find its area.
10. Base of a triangle is 9 units and height is 5 units. Base of another triangle is 10 units and height is $\mathbf{6}$ units. Find the ratio of areas of these triangles.
11. Two triangles, $\triangle \mathrm{ABC}$ and $\triangle \mathrm{PBC}$ are on the same base BC and $\operatorname{ar}(\triangle \mathrm{ABC})=$ $49 \mathrm{sq} . \mathrm{cm}$ and $\operatorname{ar}(\triangle \mathrm{PBC})=49 \mathrm{sq} . \mathrm{cm}$, then find the ratio of their altitudes.
12. AD is the median of the $\triangle \mathrm{ABC}$ if ar $(\triangle \mathrm{ABD})=13.5 \mathrm{sq} . \mathrm{cm}$, then find the area of $\triangle \mathrm{ABC}$.
13. $\triangle \mathrm{ABC}$ and $\triangle \mathrm{PQR}$ have same area. If the lengths of their altitudes are 5 cm and 4 cm , find the ratio of their base.
14. One diagonal of a rhombus is equal to its side. Find the ratio of areas of an equilateral triangle formed at one side of rhombus to the area of rhombus.
15. If a triangle $\&$ a parallelogram are on the same base and between the same parallels, then find the ratio of the area of triangle to the area of parallelogram.
16. In the given figure, $P$ is a point on side $B C$ of triangle $A B C$ such that $B P: P C=1: 2$ and $Q$ is a point on $A P$ such that $P Q: Q A=2: 3$. Find ar $(\triangle A Q C):$ ar $(\triangle A B C)$.

17. In the figure, the perimeter of the parallelogram $A B C D$ is 70 cm . Find the length of the sides of parallelogram and area of the parallelogram.

18. ABCD is a trapezium with parallel sides $\mathrm{AB}=\mathbf{a c m}$ and $\mathrm{DC}=\mathrm{bcm} . E$ and $F$ are mid-points of non-parallel sides. Find the ratio of ar (ABFE) : ar (EFCD).

19. In figure, $A B C D$ is a parallelogram. If the area of $\triangle A E B$ is $16 \mathrm{sq} . \mathrm{cm}$ then find the area of $\triangle B F C$.

20. In figure, $E$ is the mid-point of side $B C$ of parallelogram $A B C D$, then find the ratio of the ar ( $\triangle \mathrm{ADE})$ to ar ( $\triangle \mathrm{ABE}$ ).

21. In figure, ABCD is a parallelogram, $\mathrm{AB}=6 \mathrm{~cm}$ and perpendicular DF on AB is of length 3 cm , then find the area of $\triangle \mathrm{ADE}$.

22. If $P, Q, R$ and $S$ are respectively the mid-points of the sides of the parallelogram ABCD , If area of parallelogram $\mathrm{ABCD}=48 \mathrm{sq} . \mathrm{cm}$, then find the area of quadrilateral PQRS.

23. In parallelogram $P Q R S, S R=15 \mathrm{~cm}, Q N \perp P S$ and $Q M \perp S R$. If $Q N=10 \mathrm{~cm}$ and $Q M=7.6 \mathrm{~cm}$, then find the length of PS.

24. If ar $(\| \mathrm{gm} A B C D)=\mathbf{1 3 4} \mathrm{sq} . \mathrm{cm}$ and side AB is produce to E . Find the area of $\Delta$ CDE.

25. In figure, the side AB of $\| \mathrm{gm} \mathrm{ABCD}$ is produce to a point E such that $\mathrm{AB}=\mathrm{BE}$. Join $A C$ and EC. If ar $(\| g m ~ A B C D)=64$ sq. cm. Find the area $(\triangle B C E)$.

26. In figure, rectangle $A B C D$ and $\| g m$ ABEF are on the same base $A B$ of length 12 cm . If $\mathrm{BC}=\mathbf{8 \mathrm { cm }}$, find the perimeter of ABEF , if $F$ is the mid-point of $C D$.

27. In $\triangle A B C, D$ is the midpoint of $B C, E$ is the midpoint of $A D$ and $F$ is the midpoint of $B E$. Find the ratio of ar $(\triangle B D E)$ to the area of remaining part of the triangle.

28. Trapezium ABCD and parallelogram ABEF are on same base and between same parallels. If side CD of trapezium is half of EF. Find the ratio of area of trapezium to the area of parallelogram.

29. In $\triangle A B C, D, E, F$ are mid points of side $B C, C A$ and $A B$ respectively. If $\operatorname{ar}(\triangle \mathrm{ABC})=\mathbf{1 6} \mathbf{~ s q . ~ c m}$, then find the ar (trapezium FBCE).

30. Find the area of the figure formed by joining the mid points of the adjacent sides of the rhombus with diagonals $B D=16 \mathrm{~cm}$ and $A C=12 \mathrm{~cm}$.

31. In the given fig., $A D$ is the median of $\triangle A B C$ and $P$ is the point on $A C$.

If $\operatorname{ar}(\triangle \mathrm{ADP}): \operatorname{ar}(\triangle \mathrm{ABD})=2: 3$ then find the $\operatorname{ar}(\triangle \mathrm{PDC}): \operatorname{ar}(\triangle \mathrm{ABC})$.

32. In figure, medians of $\triangle A B C$ intersect at $G$. if ar $(\triangle A B C)=27 \mathrm{sq} . \mathrm{cm}$ then find the area of $\triangle \mathrm{BGC}$.

33. In $\triangle A B C$, if $D$ and $E$ are mid points of $B C$ and $A D$ respectively such that ar $(\triangle \mathrm{AEC})=3.5 \mathrm{sq} . \mathrm{cm}$, then find the $\operatorname{ar}(\triangle \mathrm{BEC})$.
34. In the given figure $P Q R S$ is a $\| \mathrm{gm}$. If $X$ and $Y$ are the midpoint of $P Q$ and $S R$ respectively and diagonal SQ is joined, then find ar (\|gm XQRY): ar ( $\triangle$ QSR).

35. In the given figure $A D$ is the median of $\triangle A B C$ and is being trisected at $E$ and $F$. If $\operatorname{ar}(\triangle \mathrm{BEF})=4 \mathrm{sq} . \mathrm{cm}$, then find the $\operatorname{ar}(\triangle \mathrm{ABC})$.

36. In triangle $A B C$, base $B C$ is trisected at $D$ and $E$. If ar $(\triangle A B C)=27 \mathrm{sq} . \mathrm{cm}$, then find the ar ( $\triangle \mathrm{ADE}$ ).

37. $P$ is the point in the interior of $\mathbf{a} \| \mathrm{gm} A B C D$. If ar $(\triangle A P B)=14 \mathrm{sq} . \mathrm{cm}$ and $\operatorname{ar}(\triangle D C P)=15 s q . c m$, then find ar $(\| g m ~ A B C D)$

38. Two parallelogram PQRS and PQMN have common base PQ as shown in the figure. If $P Q=9 \mathrm{~cm}, S M=\mathbf{~ c m}$ and $S T=5 \mathrm{~cm}$ find the area of $P Q R N$.

39. In the given figure, $D$ is the midpoint of $B C$ and $L$ is the midpoint of $A D$ if $\operatorname{ar}(\triangle A B L)=x \operatorname{ar}(\triangle A B C)$, then find the value of $x$.

40. In the given figure $S$ and $T$ are the midpoints of $P R$ and $P Q$ respectively of $\triangle P Q R$. If
$\operatorname{ar}(\triangle P Q R)=64$ sq. cm , then find the difference between $\operatorname{ar}(\triangle P Q R)$ and $\operatorname{ar}(\Delta T S Q)$.

41. In the given figure, area of parallelogram PQRS is $60 \mathrm{sq} . \mathrm{cm}$. Find the area of $\operatorname{ar}(\Delta \mathrm{QPL})$ if $\operatorname{ar}(\Delta \mathrm{SLR})=7.5$ sq. $\mathbf{c m}$.

42. ABCD is a rectangle with ' $O$ ' as any point in its interior. If $\operatorname{ar}(\triangle A O D)=$ $4 \mathrm{sq} . \mathrm{cm}$. and $\operatorname{ar}(\triangle B O C)=8 \mathrm{sq} . \mathrm{cm}$, then find the area of rectangle $A B C D$.
43. In the figure, AD is the median of $\triangle A B C$. $E$ is the midpoint of $A D$ and $F$ is the midpoint of $A E$. If $\operatorname{ar}(\triangle A B F)=6 \mathrm{sq} . \mathrm{cm}$. , then find the $\operatorname{ar}(\triangle A B C)$.

44. In the figure, if $P Q=16 \mathrm{~cm}, Q R=17 \mathrm{~cm}$ and $R S=8 \mathrm{~cm}$, then find the $\operatorname{ar}($ trapezium $P Q R S)$.

45. Name the figure obtained by joining the mid points of the adjacent sides of a rectangle of sides 8 cm and 6 cm , also find its area.

| ANSWERS |  |  |  |
| :---: | :---: | :---: | :---: |
| Q.No. | ANSWERS | Q.No. | ANSWERS |
| 1. | 4 cm | 24. | 67 sq. cm |
| 2. | 20 m | 25. | 32 sq. cm |
| 3. | $30 \mathrm{sq.cm}$ | 26. | 44 cm |
| 4. | 20 cm | 27. | 1:3 |
| 5. | 42 cm | 28. | 3:4 |
| 6. | 5 m | 29. | 12 sq. cm |
| 7. | $\frac{7}{3} \text { unit }$ | 30. | 48 sq. cm |
| 8. | 3600 sq.cm | 31. | 1:6 |
| 9. | 24.5 sq. cm | 32. | 9 sq. cm |
| 10. | 3:4 | 33. | 7 sq. cm |
| 11. | 1: 1 | 34. | 1: 1 |
| 12. | 27 sq.cm | 35. | 24 sq. cm |
| 13. | 4: 5 | 36. | 9 sq. cm |
| 14. | 1:2 | 37. | 58 sq. cm |
| 15. | 1:2 | 38. | 60 sq. cm |
| 16. | 2:5 | 39. | $\frac{1}{4}$ |
| 17. | 15 cm and $20 \mathrm{~cm}, 180 \mathrm{sq} . \mathrm{cm}$ | 40. | 48 sq. cm |
| 18. | $(3 \mathrm{a}+\mathrm{b}):(\mathrm{a}+3 \mathrm{~b})$ | 41. | 22.5 sq. cm |
| 19. | 16 sq. cm | 42. | 24 sq. cm |
| 20. | 2:1 | 43. | 48 sq. cm |
| 21. | 9 sq. cm | 44. | 180 sq. cm |
| 22. | 24 sq. cm | 45. | Rhombus, 24 sq. cm |
| 23. | 11.4 cm |  |  |

## CHAPTER 10 CIRCLE

## POINTS TO REMEMBER

- A circle is a collection of all the points in the plane, which are equidistant from a fixed point in the plane.
- Equal chords of a circle (or of congruent circles) subtend equal angles at the centre.
- If the angles subtended by two chords of a circle (or of congruent circles) at the centre (corresponding centre) are equal, the chords are equal.
- The perpendicular from the centre of the circle to a chord bisects the chord.
- The line drawn through the centre of the circle to bisect a chord is perpendicular to the chord.
- There is one and only one circle passing through three non-collinear points.
- Equal chords of a circle (or of congruent circles) are equidistant from the centre (or corresponding centres).
- Chords equidistant from the centre (or corresponding centres) of a circle (or of congruent circles) are equal.
- If two arcs of a circle are congruent, then their corresponding chords are equal and conversely, if two chords of a circle are equal, then their corresponding arcs (minor, major) are congruent.
- Congruent arcs of the circle subtend equal angles at the centre.
- The angle subtended by an arc at the centre is double the angle subtend by it at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.
- Angle in a semicircle is a right angle.


## - If a line segment joining two points subtends equal angles at two other points

 lying on the same side of the line containing the line segment, the four points lie on the circle.- The sum of the either pair of opposite angles of a cyclic quadrilateral is $180^{\circ}$.
- If the sum of the pair of opposite angles of a quadrilateral is $180^{\circ}$, then the quadrilateral is cyclic.


## QUESTIONS:

1. How many circles can be drawn through three non collinear points?
2. In figure, $P$ is the midpoint of chord MN. Find the complement of $\angle O P N$ ?

3. In figure, $C$ is the midpoint of chord $A B$. Find the supplement of $\angle O C A$ ?

4. PQRS is a cyclic quadrilateral. If $\angle P=95^{\circ}$ then find the measure of $\angle R$.
5. In the given figure, $O$ is the centre of the circle with chords AP and BP being produced to $R$ and $Q$ respectively. If $\angle Q P R=35^{\circ}$, find the measure of $\angle A O B$.

6. If the perpendicular distance between the centre of a circle and a chord of length 8m
is 3 m , what is the radius of the circle?
7. In the given figure, what is the measure of angle $x$ ?

8. In figure, $\angle M O N=120^{\circ}$, Find $\angle M Q N$.

9. In a circle, chord $A B$ is at a distance of 8 cm from the centre $O$ of the circle. If the radius of the circle is 10 cm . Find the length of chord $A B$.
10. In the given figure, if $O$ is the centre of circle and $\angle P O Q=110^{\circ}$, then find $\angle P R Q$.

11. If $A B$ is the diameter of the circle with centre $O$. $C$ is any point on the circle.

Find $\angle A C B$.
12. $A B C D$ is a cyclic quadrilateral. If $\angle A=60^{\circ}$ and $\angle B=110^{\circ}$. Find the sum of $\angle B$ and $\angle C$.
13. A one rupee coin is placed on a piece of paper. How many more coins of the same size may be placed such that each touches the central coin and the two adjacent coins?
14. In the given figure, $\triangle \mathrm{ABC}$ is an equilateral triangle and ABDC is a cyclic quadrilateral, then find the measure of $\angle B D C$.

15. In the given figure, $O$ is the centre of the circle. $P Q$ is a chord of the circle and $R$ is any point on the circle. If $\angle \mathrm{PRQ}=\boldsymbol{l}$ and $\angle \mathrm{OPQ}=\mathrm{m}$, then find $\boldsymbol{l}+\mathrm{m}$.

16. Line segment $P Q$ is 6 meters in length and is tangent to the inner circle of the two concentric circles at point $R$. It is known that the radii of the two circles are integers. Find the radius of the outer circle.
18. Find length of the common chord of two circles of radii 15 cm and 20 cm , distance between the centers being 25 cm .
18. In figure, ABCD is a cyclic quadrilateral. Side DC of the quadrilateral is produced to point $P$. If $\angle B C P=110^{\circ}$, find $\angle A$.

19.In figure, ABCD is a cyclic quadrilateral. Side CD is produced to both sides so that $\angle B C P=110^{\circ}$ and $\angle A D Q=95^{\circ}$. What is the value of $(\angle A+\angle B)$ ?

20. In figure, ABCD is a cyclic quadrilateral. Side CD is produced to both sides so that $\angle B C P=110^{\circ}$ and $\angle A D Q=95^{\circ}$. What is the value of $(x+y-z)$ ?

21. The given figure shows a circle with centre $O$ in which a diameter $A B$ bisects the chord $P Q$ at the point $R$. If $P R=R Q=8 \mathrm{~cm}$ and $R B=4 \mathrm{~cm}$, then find the radius of the circle.

22. Two circles with centres $O$ and $O^{\prime}$ intersects at $A$ and $B$. Find $\angle P B Q$.

23. AB and AC are two equal chords of a circle. $\mathrm{OD} \perp \mathrm{AB}$ and $\mathrm{OE} \perp \mathrm{AC}$. Find the value of $(x+y)$.

24. In figure, two concentric circles with centre $O$ have a common secant AD cutting the inner circle at $B$ and $C$. If $O M \perp A D, A D=18 \mathrm{~cm}$ and $B M=8 \mathrm{~cm}$. Find $C D$.

25. In figure, $A B$ is equal to radius of a circle. Find $\angle O A B$.

26. In the figure, ' $O$ ' is the centre of the circle, $\angle \mathrm{ABO}=20^{\circ}$ and $\angle \mathrm{ACO}=30^{\circ}$, where $\mathrm{A}, \mathrm{B}, \mathrm{C}$ are points on the circle. What is the value of $x$ ?

27. In the given figure, $O$ is the centre of the circle. If $\angle B O C=120^{\circ}$, then find the value of $x$.

28. In the figure, find ( $\angle R T Q+\angle R Q T)$.

29. In figure, find the value of $(x+y+z)$. If $O$ is the centre and $\angle A O B=90^{\circ}$ and $\angle A O E=80^{\circ}$.

30. In figure, $\mathrm{AC}=\mathrm{BD}, \angle \mathrm{OAC}=48^{\circ}$. Find $\angle \mathrm{B}$.

31. In figure, ABCDE is a pentagon in the semicircle. Find ( $\angle \mathrm{ABC}+\angle \mathrm{CDE}$ ) .

32. In the given figure, if $\angle \mathrm{DAB}=60^{\circ}, \angle \mathrm{ABD}=50^{\circ}$, then find the value of $(\angle A C B+\angle A D B)$.

33. In figure, $O D \perp A B$. If $O$ is the centre of the circle and $O D=3 \mathrm{~cm}$. Find $A C$.

34. In figure, $O$ is the centre of the circle. Find the reflex of $\angle A O C$, if $A B=B C$.

35. In figure, ABCD is a cyclic quadrilateral in which $\mathrm{AB} \| \mathrm{CD}$ and $\angle \mathrm{D}=\mathbf{8 0 ^ { \circ }}$. Find $\angle C$.

36. Two diameters of a circle are perpendicular to one another. What is the type of Quadrilateral formed by joining the four points?
37. In figure, diagonal of a cyclic quadrilateral ABCD passes through the centre of the circle. Find the value of $(\angle B+\angle D)$.

38. Two angles $\angle P$ and $\angle R$ of a cyclic quadrilateral $P Q R S$ are in ratio 1:2. Find the value of $(\angle P+\angle R)$.
39. In figure, $A B C D$ is a cyclic quadrilateral in which $A B \| C D$ and $A D \| B C$. Find $\angle D A B$.

40.The length of two parallel chords of a circle of radius 5 cm are 6 cm and 8 cm on the same side of the center. Find the distance between them.
41. AB is the diameter of a circle with center O . P be a point on it. If $\angle \mathrm{POA}=120^{\circ}$, then find $\angle \mathbf{P B O}$.
42. In a circle, central angle is $120^{\circ}$. Find the ratio of major angle and minor angle?
43. In a circle with centre $O$ and diameter $A C$, $A B$ is chord of length of $3 \sqrt{2} \mathrm{~cm}$ and $\angle A C B=45^{\circ}$. Find area of the circle.
44. In figure, $O$ is the centre of the circle. Find $\angle B O C$.

45. In figure, if length of the chord of a circle of radius 25 cm is 48 cm . Find the distance of the chord from the centre of the circle.


| ANSWERS |  |  |  |
| :---: | :---: | :---: | :---: |
| Q.NO. | ANSWERS | Q.NO. | ANSWERS |
| 1. | ONE | 24. | 1 cm |
| 2. | $0^{\circ}$ | 25. | $60^{00}$ |
| 3. | $90^{\circ}$ | 26. | $100^{\circ}$ |
| 4. | $85^{\circ}$ | 27. | $30^{\circ}$ |
| 5. | $70^{\circ}$ | 28. | $155{ }^{\circ}$ |
| 6. | 5 cm | 29. | $190^{\circ}$ |
| 7. | $120^{\circ}$ | 30. | $48^{\circ}$ |
| 8. | $120^{\circ}$ | 31. | $270{ }^{0}$ |
| 9. | 12 cm | 32. | $140{ }^{\circ}$ |
| 10. | $55^{\circ}$ | 33. | 6 cm |
| 11. | $90^{\circ}$ | 34. | $240{ }^{0}$ |
| 12. | $230{ }^{\circ}$ | 35. | $80^{\circ}$ |
| 13. | Six | 36. | Square |
| 14. | $120{ }^{\circ}$ | 37. | $180^{\circ}$ |
| 15. | $90^{\circ}$ | 38. | $180^{\circ}$ |
| 16. | 5 m | 39. | $90^{\circ}$ |
| 17. | 24 cm | 40. | 1 cm |
| 18. | $110^{\circ}$ | 41. | $60^{\circ}$ |
| 19. | $205{ }^{\circ}$ | 42. | 2:1 |
| 20. | $110^{\circ}$ | 43. | $9 \pi \mathrm{~cm}$ |
| 21. | 10 cm | 44. | $160^{\circ}$ |
| 22. | $180^{\circ}$ | 45. | 7 cm |
| 23. | $180^{\circ}$ |  |  |

## CHAPTER - 11 HERONS FORMULA

## POINTS TO REMEMBER

- Herons Formula:


Consider a $\triangle \mathrm{ABC}$ in which $\mathrm{BC}=\mathrm{a}$ units, $\mathrm{CA}=\mathrm{b}$ units and $\mathrm{AB}=\mathrm{c}$ units
Let semi-perimeter $=s=\frac{1}{2}(a+b+c)$
By Herons Formula, the area of the $\triangle \mathrm{ABC}$ is given by
$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$ sq. units

- It's Application:

The area of the Quadrilateral whose sides and one diagonal are given, can be calculated by dividing the quadrilateral into two triangles and using Herons formula.

## QUESTIONS:

1. The difference between the semi-perimeter and the sides of a $\triangle \mathrm{ABC}$ are 8 $\mathrm{cm}, 7 \mathrm{~cm}$ and 5 cm respectively. Find the perimeter of the $\triangle A B C$.
2. The difference between the semi-perimeter and the sides of a $\triangle P Q R$ are 3 $\mathrm{cm}, 2 \mathrm{~cm}$ and 1 cm respectively. Find the area of the $\triangle P Q R$.
3. The perimeter of a triangle is 540 m and its sides are in the ratio 25: 17: 12 . Find its area.
4. The perimeter of a triangle is 24 m and its sides are in the ratio 3: 4: 5. Find its area.
5. The area of an isosceles right angled triangle is $8 \mathrm{sq} . \mathrm{cm}$. Find its semiperimeter.
6. The area of an equilateral triangle is $4 \sqrt{3}$ sq. m . Find its semi-perimeter .
7. If $s-a=12 \mathrm{~cm}, \mathrm{~s}-\mathrm{b}=19 \mathrm{~cm}, \mathrm{~s}-\mathrm{c}=\mathbf{4} \mathrm{cm}$, then find s .
8. If $\mathrm{s}-\mathrm{a}=\mathbf{1 7} \mathrm{cm}, \mathrm{s}-\mathrm{b}=\mathbf{2 8} \mathrm{cm}, \mathrm{s}-\mathrm{c}=\mathbf{3 5} \mathrm{cm}$, then find b .
9. The perimeter of an isosceles triangle is 16 cm . The ratio of one of the equal sides to its base is $3: 2$. Find the area of the triangle.
10. If each side of the rhombus is 40 m and its longer diagonal is 48 m , then find the area of the rhombus.
11. In $\triangle A B C, a=3 b=6 c$ and $s=\alpha * a$, then find the value of $\alpha$.
12. If $\mathbf{a}=\mathbf{3 5} \mathrm{cm}, \mathrm{b}=\mathbf{2 5} \mathrm{cm}, \mathrm{c}=50 \mathrm{~cm}$, then find the value of $\mathrm{s}-\mathrm{b}$.
13. In a triangle, $a=10 \mathrm{~cm}, b=7 \mathrm{~cm}$ and $s=15 \mathrm{~cm}$, then find the value of $c$.
14. Find the area of an isosceles triangle, each of whose equal sides is 13 cm and its base is 24 cm .
15. Find the area of an isosceles triangle, each of whose equal side measures 20 cm and base is 32 cm .
16. Find the area of an equilateral triangle of side 2 acm .
17. If the length of the median of an equilateral triangle is ' $a$ ' units, then find its area.
18. A square and an equilateral triangle have same perimeter. If the length of the diagonal of the square is $12 \sqrt{2} \mathrm{~cm}$, find the area of the equilateral triangle.
19. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are $26 \mathrm{~cm}, 28 \mathrm{~cm}$ and 30 cm , and the parallelogram stands on the base 28 cm , find the height of the parallelogram.
20. The area of a parallelogram ABCD in which $\mathrm{AB}=12 \mathrm{~cm}, \mathrm{BC}=9 \mathrm{~cm}$ and diagonal $A C=15 \mathrm{~cm}$ is $k$ sq. $\mathbf{c m}$. Find the value of $(k-100)$.
21. The area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm , is $k \sqrt{30} \mathrm{sq} . \mathrm{cm}$. Find the value of $k$.
22. In given figure $A D: D C=3: 2$. If the area of $\triangle A B C=40 \mathrm{sq} . \mathrm{cm}$, find the area of the triangle $\triangle \mathrm{BDC}$.

23. In $\triangle \mathrm{ABC}, \mathrm{AB}=6 \mathrm{~cm}, \mathrm{BC}=7 \mathrm{~cm}, C A=5 \mathrm{~cm}$, Find the area of $\triangle \mathrm{ABC}$.
24. In a triangle, the sides are $9 \mathrm{~cm}, 28 \mathrm{~cm}$ and 35 cm , find its area.
25. The sides of the triangle are $\mathbf{2 0} \mathbf{~ c m}, 50 \mathrm{~cm}$ and 50 cm , find its area.
26. The sides of a triangle are $\mathrm{AB}=17 \mathrm{~cm}, \mathrm{BC}=25 \mathrm{~cm}$, and $\mathrm{CA}=\mathbf{2 8} \mathrm{cm}$. Find the length of the altitude drawn from $B$.
27. A field is in the shape of a trapezium whose parallel sides are $\mathbf{2 5} \mathbf{~ m}$ and 10 m . The non-parallel sides are $\mathbf{1 4} \mathbf{m}$ and 13 m . Find the area of the field.
28. Find the cost of laying grass in a triangular field of sides $50 \mathrm{~m}, 65 \mathrm{~m}$ and 65 m at the rate of ₹7 per sq.m.
29. If each side of the triangle is doubled, find the percentage increase in its area.
30. The sides of the triangle are in ratio $\mathbf{2 5 : 1 4 : 1 2}$ and its perimeter is $\mathbf{2 5 5 m}$. Find the greatest side of the triangle.
31. The edges of the triangular board are $6 \mathrm{~cm}, 8 \mathrm{~cm}$ and 10 cm respectively. Find the cost of painting it at the rate of ₹ 9 per sq. cm .
32. The base of an isosceles triangle measures 24 cm and its area is192 sq. cm. Find its perimeter.
33. The perimeter of a field, in the form of an equilateral triangle is 36 cm , find its area.
34. $\triangle \mathrm{ABC}$ is an equilateral triangle of side $4 \sqrt{3} \mathrm{~cm} . P, Q$ and $R$ are mid-points of $A B$, $C A$ and $B C$ respectively. Find the area of $\triangle P Q R$.
35. A triangle has perimeter 32 cm , one side is 11 cm and difference of other two sides is $\mathbf{5} \mathbf{~ c m}$. determine its area.
36. Perimeter of the rhombus is 100 m and one of its diagonal is $\mathbf{4 0 m}$. Find the area of rhombus.
37. Find the area of quadrilateral ABCD in which $\mathrm{AB}=9 \mathrm{~cm}, \mathrm{BC}=40 \mathrm{~cm}, \mathrm{CD}=28$ $\mathrm{cm}, \mathrm{DA}=15 \mathrm{~cm}$ and $\angle \mathrm{ABC}=90^{\circ}$.
38. The difference between the sides containing a right angle in a right angled triangle is 14 cm . The area of a triangle is $120 \mathrm{sq} . \mathrm{cm}$. Calculate the semiperimeter of the triangle.
39. The sides of a triangular plot are in the ratio of $3: 5: 7$ and its perimeter is 900 m. Find its area.
40. Find the area of a triangle with sides of length $9 \mathrm{~cm}, 10 \mathrm{~cm}$ and 11 cm .

## ANSWERS

| Q.NO. | ANSWERS | Q.NO. | ANSWERS |
| :---: | :---: | :---: | :---: |
| 1. | 40 cm | 21. | 8 |
| 2. | 6 sq. cm | 22. | 16 sq. cm |
| 3. | 9000 sq. m | 23. | $6 \sqrt{6}$ sq. cm. |
| 4. | 24 sq. m | 24. | $36 \sqrt{6}$ sq. cm. |
| 5. | $(4+2 \sqrt{2}) \mathrm{cm}$ | 25. | $200 \sqrt{6}$ sq. cm. |
| 6. | 6 m | 26. | 15 cm |
| 7. | 35 cm | 27. | 196 sq. cm. |
| 8. | 52 cm | 28. | ₹ 10,500 |
| 9. | $8 \sqrt{2}$ sq. cm | 29. | 300\% |
| 10. | 1536 sq. m | 30. | 125 m |
| 11. | $\frac{3}{4}$ | 31. | ₹ 216 |
| 12. | 30 cm | 32. | 64 cm |
| 13. | 13 cm | 33. | $36 \sqrt{3}$ sq. cm |
| 14. | 60 sq. cm | 34. | $3 \sqrt{3}$ sq. cm |
| 15. | 192 sq. cm | 35. | $8 \sqrt{30}$ sq. cm |
| 16. | $\sqrt{3} a^{2}$ sq. unit | 36. | 600 sq. m |
| 17. | $\frac{\sqrt{3} a^{2}}{3} \text { sq. unit }$ | 37. | 306 sq. cm |
| 18. | $64 \sqrt{3} \mathrm{sq}$. | 38. | 30 cm |
| 19. | 12 cm | 39. | $13500 \sqrt{3}$ sq. m |
| 20. | 8 | 40. | $30 \sqrt{2}$ sq. cm |

## CHAPTER - 12

## SURFACE AREA AND VOLUME

## POINTS TO REMEMBER

- Units of Measurement of Length, Area \& Volume

| LENGTH |  |  |
| :---: | :---: | :---: |
| $1 \mathrm{~cm}=10 \mathrm{~mm}$ | $1 \mathrm{dm}=10 \mathrm{~cm}$ | $1 \mathrm{~m}=10 \mathrm{dm}$ |
| $1 \mathrm{~m}=100 \mathrm{~cm}$ | $1 \mathrm{~mm}=1000 \mathrm{~mm}$ | 1 dam $=10 \mathrm{~m}$ |
| $\begin{gathered} 1 \mathrm{hm}=10 \mathrm{dam}=100 \\ \mathrm{~m} \end{gathered}$ | $\begin{gathered} 1 \mathrm{~km}=1000 \mathrm{~m}=100 \text { dam }=10 \\ \mathrm{hm} \end{gathered}$ | $\begin{aligned} & 1 \text { Myriametre = } \\ & 10 \mathrm{~km} \end{aligned}$ |


| AREA |  |  |  |
| :---: | :---: | :---: | :---: |
| $1 \mathrm{~cm}^{2}=100 \mathrm{~mm}^{2}$ | $1 \mathrm{dm}^{2}=100 \mathrm{~cm}^{2}$ |  | $1 \mathrm{~m}^{2}=100 \mathrm{dm}^{2}$ |
| $1 \mathrm{hm}^{2}=1$ Hectare $=100 \mathrm{dam}^{2}=10000 \mathrm{~m}^{2}$ |  | $1 \mathrm{~km}^{2}=100 \mathrm{hm}^{2}=100$ hectare |  |
| VOLUME |  |  |  |
| $1 \mathrm{~cm}^{3}=1 \mathrm{ml}=1000 \mathrm{~mm}^{3}$ | 1 litre $=1000 \mathrm{ml}=1000 \mathrm{~cm}^{3}$ |  | $1 \mathrm{~m}^{3}=10^{6} \mathrm{~cm}^{3}=1000 \mathrm{l}=1$ <br> kl |
| $1 \mathrm{dm}^{3}=1000 \mathrm{~cm}^{3}$ | $1 \mathrm{~m}^{\mathbf{3}}=1000 \mathrm{dm}^{3}$ |  | $1 \mathrm{~km}^{\mathbf{3}}=10^{9} \mathrm{~m}^{\mathbf{3}}$ |

- Cuboid

Volume of Cuboid $=\boldsymbol{l} \times \boldsymbol{b} \times \mathbf{h}$
Total Surface Area of Cuboid $=\mathbf{2}(\boldsymbol{l b}+b h+h l)$
Curved Surface Area of Cuboid $=\mathbf{2 h}(l+b)$


I

Diagonal of a cuboid $=\sqrt{l^{2}+b^{2}+h^{2}}$
Area of four walls of a room $=2 h(l+b)=($ Perimeter of the floor $) \times$ Height

- Cube

Volume of Cube $=\boldsymbol{a}^{\mathbf{3}}$
Total Surface Area of Cube $=\mathbf{6} \boldsymbol{a}^{\mathbf{2}}$
Curved Surface Area of Cube $=\mathbf{4} \boldsymbol{a}^{\mathbf{2}}$


Diagonal of a cube $=\sqrt{3} a$

- Cylinder

Volume of Cylinder $=\boldsymbol{\pi} r^{\mathbf{2}} \boldsymbol{h}$
Total Surface Area of Cylinder $=\mathbf{2 \pi r}(\boldsymbol{r}+\boldsymbol{h})$


Area of each end $($ Base Area $)=\boldsymbol{\pi} \boldsymbol{r}^{\mathbf{2}}$
Curved Surface Area of Cylinder $=\mathbf{2 \pi r} \boldsymbol{h}=($ Perimeter of the base $) \times$ Height

- Cone

Volume of Cone $=\frac{1}{3} \boldsymbol{\pi} r^{2} h$
Total Surface Area of Cone $=\pi r(r+l)$
Curved Surface Area of Cone $=\pi r l$
Slant Height of the cone $(l)=\sqrt{h^{2}+r^{2}}$


- Sphere

Volume of Sphere $=\frac{4}{3} \boldsymbol{\pi} \boldsymbol{r}^{\mathbf{3}}$
Total Surface Area of Sphere $=4 \pi r^{2}$


- Hemi-Sphere

Volume of Hemi-Sphere $=\frac{2}{3} \pi r^{3}$
Total Surface Area of Hemi-Sphere $=\mathbf{3 \pi} \boldsymbol{r}^{\mathbf{2}}$
Curved Surface Area of Hemi-Sphere $=\mathbf{2 \pi} \boldsymbol{r}^{\mathbf{2}}$


## QUESTIONS:

1. If the volume of cube is $2.197 \mathrm{cu} . \mathrm{cm}$, then find the length of its edge.
2. Find the length of the longest rod that can be kept in the room of dimensions 5 m , $4 m$ and $3 m$.
3. The length of the diagonal of a cube is $2 \sqrt{3} \mathrm{~cm}$, then find its volume.
4. A cube of side 7 m is dug in a field, what is the volume of the earth removed?
5. Four walls of a room with dimension $8 \mathrm{~m} \times 6 \mathrm{~m} \times 5 \mathrm{~m}$ are to be painted. Find the area to be painted.
6. If volume of a cubical room is $512 \mathrm{cu} . \mathrm{m}$, then find the length of its diagonal.
7. How many cubes whose edges measures 3 cm can be formed by melting a cubical block of metal of edge 15 cm .
8. The total surface area of a cube is $96 s q . c m$, find its volume.
9. What is the difference between the total surface area and lateral surface area of a cube whose edge measures 5.5 cm ?
10. Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the resulting cuboid to that of the sum of the surface areas of three cubes.
11. If the volumes of two cubes are in ratio $8: 1$, then find the ratio of their edges.
12. The length, width and height of a rectangular solid box are in the ratio $3: 2$ : 1 . If the volume of the box is $\mathbf{6} \mathbf{c u} . \mathrm{cm}$, then find the total surface area of the box.
13. If each edge of a cube of volume ' $V$ ' is doubled, then find the volume of the new cube.
14. If each edge of the cuboid of surface area ' $S$ ' is doubled, then find the surface area of the new cuboid.
15. Find the number of cubes of side 3 cm that can be cut from a cuboid of dimensions $30 \mathrm{~cm} \times 30 \mathrm{~cm} \times 32.4 \mathrm{~cm}$.
16. On a particular day, the rainfall recorded on a terrace $\mathbf{6 m}$ long and 5 m broad is 15 cm , then find the quantity of water collected on the terrace.
17. If $A_{1}, A_{2}, A_{3}$ denote the areas of three adjacent faces of the cuboid, then find its volume.
18. If ' $L$ ' is the length of a diagonal of a cube of volume ' $V$ ' then find the relation between ' $L$ ' and ' $V$ '.
19. If ' $V$ ' is the volume of a cuboid of dimension $x, y, z$ and $A$ is its surface area, then find $\frac{A}{V}$.
20. The sum of the length, breadth and depth of a cuboid is $\mathbf{2 0} \mathbf{~ c m}$ and its diagonal is $5 \sqrt{5} \mathrm{~cm}$, then find its surface area.
21. If each edge of a cube is increased by $\mathbf{5 0 \%}$ then find the percentage increase in its surface area?
22. A cuboidal water tank is 8 m long, 4 m wide and 2 m deep. How many litres of water can it hold?
23. A cubical vessel is $\mathbf{1 5} \mathbf{m}$ long, $\mathbf{6 m}$ wide. How high must it be made to hold 36000 litres of a liquid?
24. The length of a cold storage is double of its breadth. If height is $\mathbf{3}$ meters and the area of four walls is $\mathbf{1 8 0} \mathbf{~ s q . m . ~ F i n d ~ i t s ~ v o l u m e . ~}$
25. The total surface area of a rectangular block is $\mathbf{2 2 0 0} \mathbf{s q} . \mathrm{cm}$. Find its volume if the dimensions are in ratio 1:2:3.
26. The radii of two right circular cylinders are in ratio 3: 2 and their heights are in ratio 4: 5. Calculate the ratio of their curved surface areas.
27. The radius and height of a cylinder are in ratio $1: 2$ and its volume is $16 \pi$ cu.cm. Find its radius.
28. A right circular tunnel of diameter 2 m and height 35 m is to be constructed from a sheet of iron. Find the area of the iron sheet required. (Answer in $\pi$ )
29. Two right circular cylinders of equal volume have their radii in the ratio $\sqrt{\mathbf{2}}: 1$. Find the ratio of their heights.
30. The radii of the two cylinders are in the ratio of 5: 3 and their heights are in the ratio $2: 3$. Find the ratio of their volumes.
31. If the radius of the cylinder is doubled and the height remains same, then what is the change in the volume?
32. If the radius of the base of the right circular cylinder is halved, keeping the same height, then find the ratio of the volume of the new cylinder to the volume of the original cylinder.
33. A copper sphere of radius $\mathbf{3 ~ c m}$ is beaten and drawn into wire of diameter 0.6 cm . Find the length of the wire.
34. The circumference of the base of 24 m high wooden solid cone is $\mathbf{4 4} \mathbf{~ m}$. Find the slant height of the cone.
35. If the diameter of the base of a closed right circular cylinder whose radius is ' $\mathbf{r}$ ', is equal to its height ' $h$ ' then find its curved surface area in the terms of ' $r$ '.
36. The radius of the wire is decreased to one - third. Then how many times the length increases, if the volume remains same?
37. The volume of the cylinder of radius ' $r$ ' is equal to the volume of rectangular box with a square base of side ' $x$ ' units. If the cylinder and the box have equal heights, then find the ' $r$ ' in terms of ' $x$ '.
38. A solid cylinder is melted and cast into a cone of same radius, then what is the ratio of the heights of the cone and cylinder?
39. If the radius of the base of the right circular cylinder is ' $3 r$ ' and its height is equal to the radius of the base then find its volume. (in the terms of $r$ )
40. If the volume of two cones are in ratio 1:4 and their diameters are in the ratio 4: 5 then find the ratio of their heights.
41. The curved surface area of one cone is twice that of the other, while the slant height of the later is twice that of the former. Find the ratio of their radii.
42. The diameters of two cones are equal. If their slants heights are in the ratio $5: 4$, then find the ratio of their curved surface areas.
43. A right circular cylinder and right circular cone have the same radius and the same volume, then find the ratio of the height of the cylinder to that of the cone.
44. If the slant height and the radius of the base of a right circular cone are ' $L$ ' and ' $R$ ' respectively, then find the ratio of the areas of the lateral surface and the base.
45. The circumference of the base of a 12 m high wooden solid cone is 22 m . Find the slant height of the cone.
46. What will be the percentage increase/ decrease in the volume of a sphere if its radius is doubled?
47. Find the ratio of the volume of cylinder, volume of cone and volume of the hemisphere of same radius and same height.
48. If the curved surface area of a cylinder and a cone are equal and the radius of their bases is also same then find the ratio of slant height of the cone to the height of the cylinder.
49. A cone, a hemisphere and a cylinder stand on equal bases and have same height. What is the ratio of their volumes?
50. A sphere is placed inside a right circular cylinder so as to touch to the top, base and curved surface of the cylinder. If the radius of the sphere is ' $r$ ', then find the volume of the cylinder.

## ANSWER

| Q.NO. | ANSWER | Q. NO. | ANSWER |
| :---: | :---: | :---: | :---: |
| 1 | 1.3 cm | 26 | 6:5 |
| 2 | $5 \sqrt{ } 2 \mathrm{~m}$ | 27 | 2 cm |
| 3 | $8 \mathrm{cu} . \mathrm{cm}$ | 28 | 70\% sq. m |
| 4 | 343cu.m | 29 | 1:2 |
| 5 | 140sq.m | 30 | 50:27 |
| 6 | $8 \sqrt{ } 3 \mathrm{~m}$ | 31 | 4 times the original volume |
| 7 | 125 | 32 | 1:4 |
| 8 | $64 \mathrm{cu} . \mathrm{cm}$ | 33 | 400 cm or 4 m |
| 9 | 60.50 sq.cm | 34 | 25 m |
| 10 | 79 | 35 | $4 \pi r^{2}$ sq. unit |
| 11 | 2 :1 | 36 | 9 times |
| 12 | 22 sq.cm | 37 | $\frac{x}{\sqrt{\pi}}$ |
| 13 | 8 times the volume of the original cube | 38 | 3:1 |
| 14 | 4 times the surface area of original Cuboid | 39 | $27 \pi r^{3}$ cubic units |
| 15 | 1080 | 40 | 25: 64 |
| 16 | 4.5 cu.m or 4500 L | 41 | 4:1 |
| 17 | $\sqrt{A_{1} A_{2} A_{3}}$ cubic units | 42 | 5:4 |
| 18 | $3 \sqrt{3} V=L^{3}$ | 43 | 1:3 |
| 19 | $2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ | 44 | L: R |
| 20 | 275 sq.cm | 45 | 12.5 m |
| 21 | 125\% | 46 | 700\% increase |
| 22 | 64000 liters | 47 | 3:1:2 |
| 23 | 0.4 m | 48 | 2:1 |
| 24 | $20 \times 10 \times 3=600 \mathrm{cu} . \mathrm{m}$ | 49 | 1:2:3 |
| 25 | $10 \times 20 \times 30=6000$ cu.cm | 50 | $2 \pi r^{3}$ cubic unit |

## CHAPTER - 13 STATISTICS

## POINTS TO REMEMBER

Measures of Central Tendency for ungrouped data

- Mean: Mean is calculated by adding all the values of the observations and dividing it by the total number of observations. It is denoted by $\overline{\boldsymbol{x}}$ and also known as average.
- If $x_{1}, x_{2}, x_{3}, \ldots \ldots \ldots, x_{n}$, are the observations, then

Mean $=\overline{\mathcal{X}}=\frac{\sum_{i=0}^{n} x_{i}}{n}, \quad$ where $n=$ number of observations
For Ungrouped frequency Distribution
Mean $=\bar{X}=\frac{\sum_{i=1}^{n} f_{i} x_{i}}{\sum_{i=0}^{n} f_{i}}$

- Median: Median is that observation which divides given data (arranged in ascending or descending order) into exactly two parts
Let $\mathbf{n}$ be the number of observations in the given data:

If $\mathbf{n}$ is odd, then Median $=$ Value of $\left(\frac{n+1}{2}\right)^{\text {th }}$ Observation
If $n$ is even, then Median $=\frac{\left(\frac{n}{2}\right)^{\text {th }} \text { observation }+\left(\frac{n}{2}+1\right)^{\text {th }} \text { observation }}{2}$

- Mode: Mode is that value of the observation which occurs most frequently.


## QUESTIONS:

1. Find the mean of all the factors of 8.
2. Find the mean of prime numbers lying between 20 to $\mathbf{5 0}$.
3. Find the mean of first 10 whole numbers.
4. Find the mean of the following data:

| Name of the Country | No. of peoples who <br> like football |
| :--- | :--- |
| India | 120 |
| China | 150 |
| UK | 100 |
| Italy | 130 |

5. Find the mean of all odd numbers between 90 and 100.
6. Find the median of all prime numbers from 1 to 11.
7. Find the mean of first six prime numbers.
8. If the mean of data $x, x+1, x+3, x+6$ is $\frac{15}{2}$, then find the value of $x$.
9. Find the mean of first twelve odd natural numbers.
10.The mean of eight numbers is 25 . If each number is multiplied by 4 , then what will be the new mean?
10. Find the mean of all the factors of 24.
11. For what value of $x$, the mode for the following data is 17 ?

$$
15,16,17,14,17,16,13,15, \frac{(x-1)}{2}, 17,16,15,15
$$

13. For what value of $x$, the mode for the following data is 7 ?

$$
3,5,6,7,5,4,7,5,6, \frac{(2 x-21)}{3}, 8,7
$$

14. In a class the average score of girls in an examination is 80 and that of boys is $\mathbf{7 0}$. The average score of the whole class is 74 . Find the ratio of girls and boys in the class.
15. Find the range of the data: $25.7,16.3,2.8,21.7,24.3,22.7,24.9$
16. Find the mean of the following: V, VI, IX, X, II, IV.
17. Find the median of $23,30,37,27,47,46,24,31$.
18. The mean of five numbers is 18 . If one number is excluded, their mean is 16. Find the excluded number.
19. Find the median of all prime numbers less than 25.
20. A data is such that its maximum value is 29 and range is 20 , then what will be its minimum value?
21. The class marks of frequency distribution are given as $15,20,25,30, \ldots \ldots$. .Find the class interval corresponding to the class mark 40.
22. If the mean of the observations: $x, x+3, x+5, x+7$ and $x+10$ is 9 , then find the mean of the last three observation.
23. Find the arithmetic mean of the squares of first six natural numbers.
24. Find the arithmetic mean of the cubes of first five natural numbers.
25. Find the difference of mean and median of first five multiple of 5.
26. If the mean of $a, b$ and $c$ is 6 and mean of $a, b, c$ and $d$ is 7 , then find $d$.
27. If the median of $x, x+2, x+4, x+6, x+8$ is 24 , find $2 x-3$.
28. If the mean of 5 observations is 30 and each observations is divided by 6 , then find the new mean.
29. Find the mode of following data: $15,14,19,20,14,15,16,14,15,18,14,19,15$, 17, 15
30. Find the median of $34,32, x, x-1,19,15,11$ where $x$ is the mean of $10,20,30$, 40, 50.
31. If the mean of the data $6,8,10,3,7$ and $m$ is 7 , then find the value of $\frac{m}{2}$.
32. A cricketer has a mean score of 60 runs in eight innings. Find out how many runs are to be scored by him in ninth innings to raise his mean score to 65.
33. The mean of 20 numbers is 18 . If $\mathbf{3}$ is added to each of the first ten numbers, find the mean of new set of twenty numbers.
34. The mean of five numbers is $\mathbf{3 0}$. If one number is excluded their mean becomes 28. Find the excluded number.
35. If the mean of ten observations is 20 and that of other fifteen observations is 16. Find the mean of all twenty-five observations.
36. The median of following observations, arranged in ascending order is 25 . Find $x$.

$$
11,13,15,19, x+2, x+4,30,35,39,46
$$

37. Out of total of 20 observations arranged in ascending order $10^{\text {th }}$ and $11^{\text {th }}$ observations are 41 and 43 . What is the median?
38. If mean of $5,9, x, 17,21$ is 13 , then find the value of $x$.
39. The mean of 80 items was found to be 25 . If at the time of calculations two items were mistaken as 60 and 40 instead of 16 and 4 , find the correct mean.
40. The mean 100 observations is 50 . If one of the observation which was 50 is replaced by 150 , then find the value of resulting mean.
41. Ten observation $6,14,15,17, x+1,2 x-13,30,32,34,43$ are written in an ascending order. The median of the data is 24 . Find the value of $x$.
42. If mean of set of $n$ observations is $\bar{x}$, then evaluate $\sum f_{i}\left(x_{i}-\bar{x}\right)$.
43. Find the mode of the following data: V, VI, V, VII, V, VIII, V, IX, V, X.
44. In a frequency distribution, the mid-value of a class is 10 and the width of the class is 6 . Find the lower limit of the class.
45. The two consecutive class marks of a distribution are 52 and 57 . Find the class limits of the two intervals.
46. Find the mean of all the factors of 10.
47. Find the median alphabet of the following data :I, N, D, I, A. (Hint: First arrange the alphabets in ascending/ descending order)
48. Find the mode of the following data: $\mathrm{C}, \mathrm{O}, \mathrm{V}, \mathrm{A}, \mathrm{C}, \mathrm{C}, \mathrm{I}, \mathrm{N}, \mathrm{E}$.
49. If $A=1, B=2, C=3, D=4$. $\qquad$ find the mean of first five alphabets.
50. Find the median of the following data : 41, 43, 127, 99, 61, 92, 71, 58, 57. If 58 is replaced by 85 , what will be the new median?

## ANSWERS

| QUESTION NO. | ANSWER | QUESTION NO. | ANSWER |
| :---: | :---: | :---: | :---: |
| 1 | 3.75 | 26 | 10 |
| 2 | 35.85 (Approx) | 27 | 37 |
| 3 | 4.5 | 28 | 5 |
| 4 | 125 | 29 | 15 |
| 5 | 95 | 30 | 29 |
| 6 | 5 | 31 | 4 |
| 7 | 6.83 (Approx) | 32 | 105 |
| 8 | 5 | 33 | 19.5 |
| 9 | 12 | 34 | 38 |
| 10 | 100 | 35 | 17.6 |
| 11 | 7.5 | 36 | 22 |
| 12 | $\mathrm{x}=35$ | 37 | 42 |
| 13 | $\mathrm{x}=21$ | 38 | $\mathrm{x}=13$ |
| 14 | 2:3 | 39 | 24 |
| 15 | 22.9 | 40 | 51 |
| 16 | VI | 41 | 20 |
| 17 | 30.5 | 42 | 0 |
| 18 | 26 | 43 | V |
| 19 | 11 | 44 | 7 |
| 20 | 9 | 45 | $\begin{gathered} 49.5-54.5 \\ \text { and 54.5-59.5 } \end{gathered}$ |
| 21 | 37.5-42.5 | 46 | 4.5 |
| 22 | 11.33 (Approx) | 47 | I |
| 23 | 15.16 (Approx) | 48 | C |
| 24 | 45 | 49 | C |
| 25 | 0 | 50 | 61 and 71 |

## CHAPTER - 14 PROBABILITY

## POINTS TO REMEMBER

- Trial - Trial is an action which results in one or more outcomes.

Example:- (i) Tossing a coin
(ii) Rolling a die

- Probability of an event $E$ is given by.
$\mathbf{P}(\mathbf{E})=\frac{\text { Number of favourable outcomes }}{\text { Total number of outcomes }}$
- The probability of an event always lies from 0 to 1 .

$$
\mathbf{0} \leq \mathrm{P}(\mathrm{E}) \leq \mathbf{1}
$$

- If $P(E)=1$, then $E$ is a sure event.
- If $P(E)=0$, then $E$ is an impossible event.
- $\mathbf{P}(\mathbf{E})+\mathbf{P}($ Not $E)=1$

Or

$$
\mathbf{P}(\mathbf{E})+\mathbf{P}(\overline{\mathbf{E}})=\mathbf{1}
$$

## QUESTIONS:

1. Find the probability of selecting a number which is the range of the data:
25.7, 16.3, 2.8, 21.7, 24.3, 22.9, 24.9.
2. A coin is tossed once, find the probability of getting a head.
3. In a pack of $\mathbf{5 2}$ cards, what is the probability of getting a face card?
4. A die is tossed once, find the probability of getting a prime number.
5. A die is tossed once, find the probability of getting a composite number.
6. Between 2 to $\mathbf{7 0}$, what is the probability of numbers which are divisible by $\mathbf{7 ?}$
7. In the word KASHMIR, what is the probability of selecting a vowel?
8. In a class of 47 students, 29 students study Mathematics and rest of the students study English. Find the probability of students who study English.
9. In the word INDIA, what is the probability of getting the letter $S$ ?
10. A survey of 250 students was conducted about the subject 'Mathematics', in which 143 students like Mathematics. Find the probability of students who do not like Mathematics.
11. In a locality, there are 67 vehicles, out of which 39 are black. What is the probability of a vehicle which is not black?
12. In a pack of 52 playing cards, what is the probability of getting a green card?
13. In a Hockey match, probability of winning the match is 0.04 . What is the probability of losing the match?
14. In a class of 50 students, $\mathbf{7 0 \%}$ were passed. What is the probability of failed students?
15. In a bag there are 5 white, 6 black and 3 green balls. One ball is drawn at random. What is the probability that the ball drawn is not green?
16. A die is tossed once. What is the probability of getting the number 7 ?
17. In a bag of 56 apples, 19 are rotten. One apple is chosen at random. Find the probability of getting a fresh apple.
18. In the word MISSISSIPPI, what is the probability of getting the letter M?
19. In 1500 families, 814 families have 2 children and rest of families have $\mathbf{1}$ child. Find the probability of families having one child.
20. Between 5 and 15, one number is selected. Find the probability that the selected number is an odd number.
21. In class IX total students are 36, out of which 20 students are boys. Find the probability of girls in the class.
22. In a cricket match, a batsman hits a boundary 5 times out of 30 balls he played. Find the probability that he did not hit boundary.
23. What is the probability of getting 5 Sundays and 5 Mondays in the month of January 2021?
24. In a pack of 52 playing cards, find the probability of getting a black king.
25. A die is tossed once find the probability of getting a number less than 5.
26. In a lottery there are 15 prizes and 25 blanks. Find the probability of getting a prize.
27. If $P(E)=0.023$, find $P($ not $E)$.
28. A pair of coins is thrown once. Find the probability of getting at least two heads.
29. In a throw of a die, find the probability of not getting 4 or 5.
30. A bag contains slips with numbers between 3 and 32. What is the probability that a slip chosen at random contains multiple of 5 ?
31. If the difference between the probabilities of happening and non-happening of an event $E$ is $\frac{3}{7}$. Find the probability of happening of the event $E$.
32. If the probability of getting a prize is $55 \%$ then what is the probability of not getting a prize?
33. A number is selected at random from first 30 whole numbers. What is the probability that the selected number is a multiple of 3 ?
34. The number of books issued from a school library in a week are :

$$
\text { 1029, 1130, 1538, 1600, 1236, } 1585
$$

Find the probability that the number of books issued on a day is less than 1500.
35. In the word TRIANGLE, what is the probability of getting the letter N?
36. What is the probability that Daksh and his friend Manish have same birthday?
37. What is the probability of choosing a consonant from the word 'MATHEMATICS'?
38. If the probability of non-occurrence of an event is $45 \%$ then what is the probability of occurrence of the event?
39. A bag contains 3 red balls, 5 black balls and 6 green balls. Find the probability of drawing a non red ball?
40. A card is chosen at random from a pack of 52 playing cards. What is the probability of getting a black card?
41. A die is rolled once. Find the probability of getting a multiple of 2.
42. A coin is tossed once. What is the probability of getting Head or Tail?
43. A die is rolled once. What is the probability of getting 8 on the die?
44. Neha and Pawan are playing a game. The probability that Pawan wins the game is 0.99 then what is the probability that Neha wins the game?
45. Jairam is attempting to solve a science problem. If the probability of the problem not being solved by Jairam is $1 / 2$, what is the probability that Jairam will solve it?
46. Mr. Naveen, class teacher of class IX, organizes an online class. Out of 35 students 28 students attended the class. What is the probability of absent students in the class?
47. In an online class, 12 students use their phones to join the class, $\mathbf{1 6}$ students use their tablets to join the class, and the remaining 7 students use their laptops to join the class. What is the probability of students using the tablet to join?
48. A total of 21 teachers are working on Mental Math books for students in grades $V$ to $X$. If four teachers are working on a book for class $X$, what is the probability of the teachers working on a book for classes $V$ to IX?
49. In a question paper, $\mathbf{3 0 \%}$ of the questions are MCQs, $\mathbf{4 0 \%}$ are True-False questions, and the rest are 'Match the following' questions. What is the probability of "Match the following" questions?
50. A survey of families with two-children is being conducted.

| No. of Family | No. of Girls |
| :---: | :---: |
| 25 | 0 |
| 50 | 1 |
| 75 | 2 |

Find the probability of a family with only one boy being chosen.

## ANSWERS

| QUESTI ON No. | ANSUER | QUESTI ON NO. | ANSUER |
| :---: | :---: | :---: | :---: |
| 1 | $\frac{1}{7}$ | 26 | $\frac{3}{8}$ |
| 2 | $\frac{1}{2}$ | 27 | 0.977 |
| 3 | $\frac{3}{13}$ | 28 | $\frac{1}{4}$ |
| 4 | $\frac{1}{2}$ | 29 | $\frac{2}{3}$ |
| 5 | $\frac{1}{3}$ | 30 | $\frac{3}{14}$ |
| 6 | $\frac{9}{67}$ | 31 | $\frac{5}{7}$ |
| 7 | $\frac{2}{7}$ | 32 | 45\% |
| 8 | $\frac{18}{47}$ | 33 | $\frac{3}{10}$ |
| 9 | 0 | 34 | $\frac{1}{2}$ |
| 10 | $\frac{107}{250}$ | 35 | $\frac{1}{8}$ |
| 11 | $\frac{28}{67}$ | 36 | $\frac{1}{365}$ |
| 12 | 0 | 37 | $\frac{7}{11}$ |
| 13 | 0.96 | 38 | 55\% |
| 14 | $\frac{3}{10}$ | 39 | $\frac{11}{14}$ |
| 15 | $\frac{11}{14}$ | 40 | $\frac{1}{2}$ |
| 16 | 0 | 41 | $\frac{1}{2}$ |
| 17 | $\frac{37}{56}$ | 42 | 1 |
| 18 | $\frac{1}{11}$ | 43 | 0 |
| 19 | $\frac{343}{750}$ | 44 | 0.01 |
| 20 | $\frac{4}{9}$ | 45 | $\frac{1}{2}$ |
| 21 | $\frac{4}{9}$ | 46 | $\frac{1}{5}$ |
| 22 | $\frac{5}{6}$ | 47 | $\frac{16}{35}$ |
| 23 | $\frac{2}{7}$ | 48 | $\frac{17}{21}$ |
| 24 | $\frac{1}{26}$ | 49 | 30\% |
| 25 | $\frac{2}{3}$ | 50 | $\frac{1}{3}$ |



