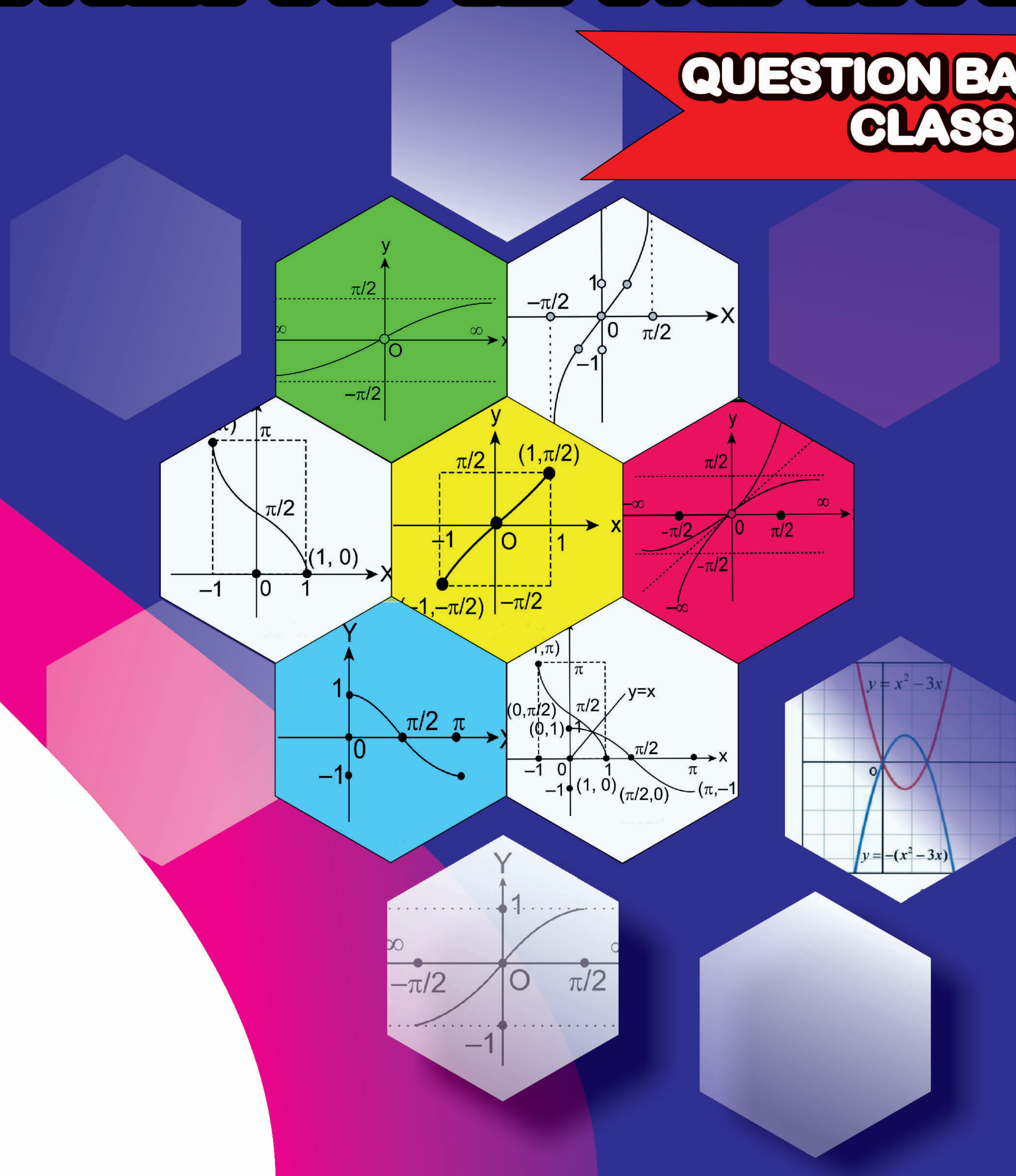
MENTAL MATHS

QUESTION BANK
CLASS XI



DIRECTORATE OF EDUCATION GOVT.OF N.C.T.OF DELHI

MENTAL MATHS QUESTION BANK

CLASS XI

2023-2024

DIRECTORATE OF EDUCATION
GOVT. OF NCT OF DELHI

HIMANSHU GUPTA, IAS
Director, Education & Sports
No. PS/DE/2023/158
Dated: 01/06/2023

सत्यमेव जयते

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MESSAGE

'Maths is not only seen as beautiful—**beauty is also mathematical**'. -**Dr. Thomas Britz**

Look at all the inventions, buildings, means of transport including starships and other technological developments around us, and you will find Maths at the heart of everything. Even nature exhibits symmetry through Maths. The famous Fibonacci Sequence or the Golden Ratio accounts for many patterns in the cosmos, such as flower petals, seed heads, fruits, vegetables, tree branches, our faces and even our bodies. It helps explain the way galaxies spiral, a seashell curves, patterns replicate and rivers bend. We can't deny the significance of Maths in our day to day lives.

Whether you look forward to solving a problem or becoming a scientist, Maths has got your back. Maths is a friend to those who play with numbers and shapes, and leads to the development of mathematical thinking and logical reasoning. Isn't it interesting to know that 'jiffy' is a unit of time for 1/100th of a second and '2520' is the smallest number that can be exactly divided by all the numbers from 1 to 10.

All competitive exams check mathematical ability without letting the students use a calculator. In order to enable students to do fast and accurate mental calculations and gain expertise in word problems, Mental Maths is being introduced in Classes XI and XII from the academic session 2023-2024.

It is imperative that students with mathematical aptitude are identified and trained to participate in the Mental Maths Competitions and Olympiads. These books have ample material for regular practice by such students, who not only become proficient in Maths, but also bring laurels for their school and motivate other students like them. I commend the Maths teachers who work dedicatedly to support these students in schools and provide regular feedback for improvement of the Mental Maths Books.

I also take this opportunity to appreciate the efforts of the Core Team, HOSs and coordinators who work round the clock to promote Mental Maths in DOE schools, and DBTB for timely publication of the Mental Maths books.

(HIMANSHU GUPTA)

विकास कालिया
क्षेत्रीय शिक्षा निदेशक
उत्तर एवं मध्य क्षेत्र,
पुरस्कार एवं कल्याण शाखाएँ,
पत्राचार विद्यालय एवं रा. मुक्त विद्यालयी शिक्षा शाखाएँ
परियोजना निदेशक: मेंटल मैथ्स
उपशिक्षा निदेशक: कार्मिक

सत्यमेव जयते

VIKAS KALIA
Regional Director of Education
North & Central Regions
Awards & Welfare Branches
Patrachar Vidyalaya & NIOS (Branch)
Project Director: Mental Maths
DDE Personnel

Maths: My Friend!

I am a student of language. So, I view Maths also from a linguist's perspective.

For example, 'Thousand' is the only word from numbers 0 to 1000 which contains the vowel 'a'. Or for example, the units that come after Million $(10)^6$, Billion $(10)^9$ & Trillion (10^{12}) are called: Quadrillon (10^{15}), Quintillion (10^{18}), Sextillion (10^{21}), Septillion (10^{24}), Octillion (10^{27}), Nonillion (10^{30}) & Decillion (10^{33}).

But do you know that world's highest paying jobs include Data Scientist, Quantitative Analyst, Acturial Analyst, Statistician, Data Modeler etc. And Mathematics is the backbone of all these fields!

Then, there is hardly any competition – for Entrance into some prestigious course or college or for any job opportunity – which does not include Maths, directly or indirectly. So, no matter, whether you love Maths or abhor it – if you are a sensible person, you cannot ignore it!

And I know, all of you are most sensible guys out there because you have chosen to opt for Maths!

Now that you have made your choice, why not also make your mark! Why not master Maths – which many of your friends, consider as an 'Untamable horse'!

Believe me, investment of time & brains made in Maths will give you highest returns!

As for 'Mental Maths', it is the same as normal Maths with the only difference that in Mental Maths you can not use your calculator or even your pen/paper to solve mathematical problems. It is a matter of practice. The more you practice its techniques, the more confidence you will gain.

This Question Bank will give you ample practice to succeed not only in Mental Maths Quiz Competitions that are being held for you (i.e. for the students of XI & XII classes) for the first time, but also in your entire career – whichever career you pick!

Take my word, there is no such book available in the market! Our best teachers have burnt midnight oils in writing this and our best experts have critically analysed and edited it. Still if – despite the best efforts of my team – there remain some typographical errors/misprint, I am solely responsible for the same.

As this is the first print, I sincerely request you to send your suggestions/corrections to me so that we can serve our students better, next time.

01/06/2023

(VIKAS KALIA)
PROJECT DIRECTOR (MENTAL MATHS)

ACKNOWLEDGEMENT

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MENTAL MATHS QUIZ COMPETITIONS SCHEDULE

SESSION 2023-2024

DIRECTORATE OF EDUCATION

GOVT OF NCT OF DELHI

• Practice to students from Question Bank	01.04.2023 to 16.10.2023
• School level Quiz Competition	17.10.2023 to 27.10.2023
• Cluster level Quiz Competition	02.11.2023 to 09.11.2023
• Zonal level Quiz Competition	20.11.2023 to 28.11.2023
• District level Quiz Competition	05.12.2023 to 13.12.2023
• Regional level Quiz Competition	26.12.2023 to 29.12.2023 & 16.01.2024 to 20.01.2024
• State level Quiz Competition	25.01.2024 to 12.02.2024

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CLASS XI

CHAPTER – 1
SETS

POINTS TO REMEMBER

- Set: A well-defined collection of distinct objects.

Sets are represented in (a)Roster Form for example $\{1, 2, 3, 4\}$

(b) Set Builder Form for example $\{x : x \leq 4, x \in N\}$.

- Types of Sets:

(a) Empty set: A set which does not have any element.

It is also called Null Or Void set and is represented by Phi ϕ or $\{\ \}$.

(b) Finite set: A set which has finite number of elements.

(c) Infinite set: A set which has infinite number of elements.

- Subset: A set A is called subset of set B if every element of set A is also an element of set B. It is denoted as $A \subseteq B$.

- Types of Intervals:

(a) Open interval:

A subset of all real numbers between $a \,\&\, b$ denoted as (a, b).

(b) Closed interval:

A subset of all real numbers from a *to* b denoted as $[a, b]$.

- Union of Two Sets:

A Set of all the elements which are either present in one or both the sets.

- If n(A) $= p$,then number of subsets of A are 2^p & number of proper subsets of A are $2^p - 1$

Properties of Union : $(a)\ A\cup B = B\cup A$	$(b)\ (A\cup B)\cup C = A\cup(B\cup C)$
$(c)\ A\cup\phi = A$	$(d)\ U\cup A = U$

- Intersection of Two Sets: Intersection of two sets is a set of all the elements present in both the sets.It is denoted by the symbol $\cap$.

Properties of Intersection : $(a)\ A\cap B = B\cap A$	$(b)\ (A\cap B)\cap C = A\cap(B\cap C)$
$(c)\ A\cap\phi = \phi$	$(d)\ U\cap A = A$

- Difference of two sets:Set of all the elements of set A which are not in B. It is denoted as $A - B$ Or $A\cap B'$.
- Complement of a set: Let A be any set, then complement of set A is denoted as A^c or A' is $U - A$ i.e. set of all those elements of U which are not in A.

Properties of Complement : $(a)\ A\cap A' = \phi$	$(b)\ A\cup A' = U$
$(c)\ (A\cup B)' = A'\cap B'$	$(d)\ (A\cap B)' = A'\cup B'$
$(e)\ \phi' = U$	$(f)\ U' = \phi$

- Disjoint Sets: Two sets A & B are disjoint if they have no common element in them. $i.e. A\cap B = \phi$

- If A, B and C are finite sets and U is the finite universal set, then

$(a)\ n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$(b)\ n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(A\cap C) - n(B\cap C) + n(A\cap B\cap C)$

$(c)\ n(A - B) = n(A) - n(A\cap B)$

- Cardinal Number: A number associated with a set that is equal to the number of elements present in the set.

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. If $a \& b$ represents the number of elements in a Null set & a Singleton Set respectively,then find the value of $(b^b + a^b + b^a)$.
2. Find the cardinal number of set A, where $A = \{a,\ e,\ i,\ o,\ u\}$
3. Find the cardinal number of set A, where $A = \left\{x : x^2 < 17, x \in I\right\}$.
4. Find the cardinal number of set A, where $A = \{0, 3, 6, 9, ..., 93\}$
5. Find the roster form of set, $A = \{x : x \in W,\ 2x + 11 < 15\}$.
6. Find the roster form of set $A = \{x :\ x \text{ is a positive factor of a prime number } p\}$
7. Find the roster form of set $A = \left\{x : x \in R,\ x^3 = x\right\}$.
8. Find the roster form of set $A = \{x : x \in R, x^4 - 5x^2 + 6 = 0\}$.
9. Find the set-builder form of set $B = \{1, 3, 5, ..., 99\}$.
10. Find the set-builder form of set $C = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ..., \frac{9}{10}\}$.
11. If numbers of non-empty subsets of a set A are 255, then find the number of elements of set A.
12. What is the total number of subsets of a singleton set?
13. If a set has 5 elements, then what is the number of its non-empty subsets?
14. Find the smallest subset of $A = \{a,\ b,\ c\}$.
15. If $A = \{0, 1, 5, 9\}$, then find number of elements present in power set of A.
16. If $A = \{x : |x - 2| = 5, x \in R\}$, then find number of subsets of set A.
17. Find the number of subsets of set A, where $A = \{x : x \in I, |3 - x| < 1\}$.
18. If $A = \{a,\ b,\ 2,\ 3,\ 4\}$ & $B = \{1,\ 2,\ 3,\ e,\ f\}$, then find $n(A \cup B)$.
19. If $A = \{2, 4, 6, ...90\}$ & $B = \{3, 6, 9, 12, ..., 90\}$, then find $n(A \cap B)$.
20. If $A = \{4, 8, 12, ..., 100\}$ & $B = \{6, 12, 18, ..., 96\}$, then find $n(A - B)$.

21. If $A=\{x:x^2-2x+4<0, x\in R\}$ & $B=\{x:x>1 \& x\in R\}$, then find $n(A\cap B)$.

22. Let $A=\{x:x=3n,\ n\in N\}$, $B=\{x:x<6, x\in N\}$. Find $n(A\cap B)$.

23. Let $A=\{x:x\in N, 1\le x\le 6\}$, $B=\{x:x\in N,\ 6<x<10\}$. Find $A\cap B$.

24. If R is a set of real numbers and Q is a set of rational numbers, then What will be $R-Q$.

25. If $n(A)=5$ & $n(B)=7$, then what can be the Maximum value of $n(A\cap B)$.

26. If $n(A)=10$ & $n(B)=6$, then what can be the Maximum value of $n(A\cup B)$.

27. Let $A=\{x:x^2-5|x|+6=0\}$ & $n(B)=3$, then find the value of $n_{Minimum}(A\cup B)$.

28. If $n(A)=p,\ n(B)=q$ where $q-p<0$, then find the value of $\frac{n_{Maximum}(A\cup B)-n(A)}{n_{Maximum}(A\cap B)}$.

29. Let $n(A)=4,\ B=\{x:x^2=1, x\in R\}$, then find the value of $n_{Minimum}(A\cap B)$.

30. For $L=\{1,\ 2,\ 3,\ 4\}$, $M=\{3,\ 4,\ 5,\ 6\}$ & $N=\{1,\ 3,\ 5\}$. Find $n(L\cap(M\cup N))$.

31. Let $A=\{1,\ 2,\ 3, \ldots 10\}$ & $B=\{0,4,8,12\}$, then find the number of Subsets of set $A\cap B$.

32. Given $A=\{a,\ e,\ i,\ o,\ u\}$, $B=\{a,\ b,\ c,\ d,\ e,\ f\}$ & $C=\{e,\ f,\ l,\ m,\ n\}$. Find $n[(A-B)\cup C]$.

33. For $A=\{1,\ 2,\ 3, \ldots, 10\}$, $B=\{2,\ 4,\ 6,\ 8, \ldots, 20\}$ & $C=\{1,\ 3,\ 5,\ \ldots, 19\}$. Find $n[A-(B\cup C)]$.

34. Let $U=\{1,2,3,4,5,6,7,8,9\}$ & $P=\{2,4,6,8\}$ then find the value of $n(P')$.

35. Let $U=\{1,2,3,4,5,6,7,8,9\}$, $P=\{1,3,5,8\}$ & $Q=\{3,7,8,9\}$. Find the number of non-empty subsets of $(P\cup Q)^c$.

36. Find the cardinal number of a set which has exactly 16 subsets.

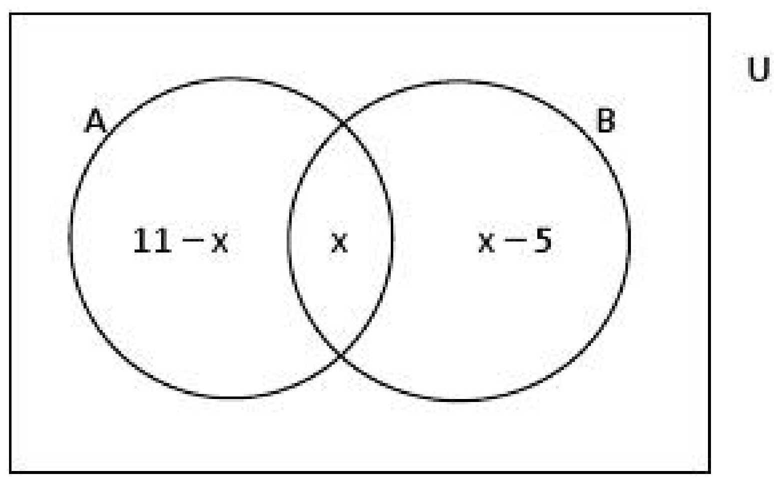
37. Let n(A) = m & n(B) = p. If the total number of subsets of the set A is 48 more than the total number of subsets of the set B, then find the value of $(m+p)$.

38. Let n(A) = m & n(B) = p. If the total number of subsets of the set A is 56 less than the total number of subsets of the set B,then find the value of $|m-p|$.

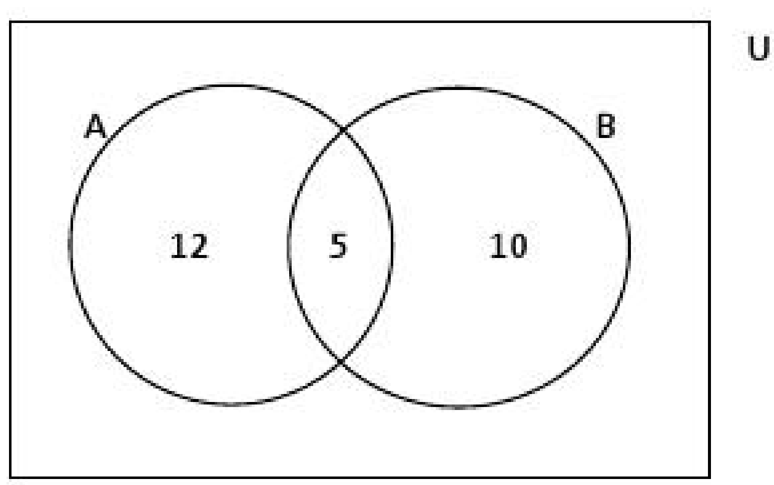
39. Let n(A) = m & n(B) = p. If the total number of subsets of the set A is 112 more than the total number of subsets of the set B,then find the value of $\frac{8m}{p}$.

40. If set A has 6 elements, then what is the number of non-empty proper subsets of A?

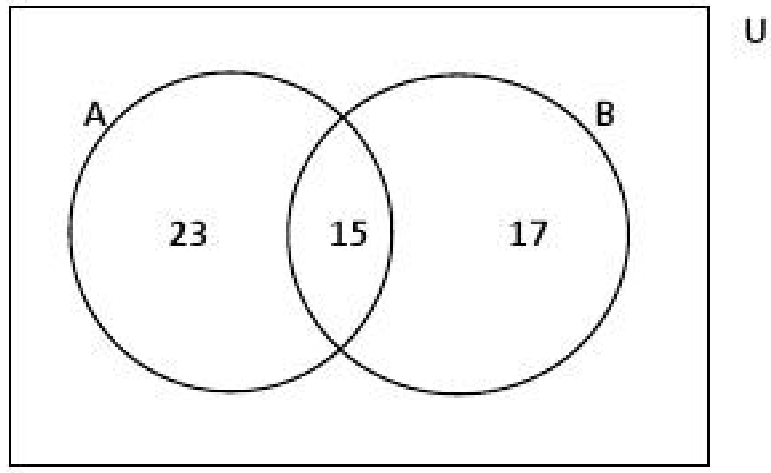
41. If $n(A) = n(B)$, then find the value of $n(A-B)$.



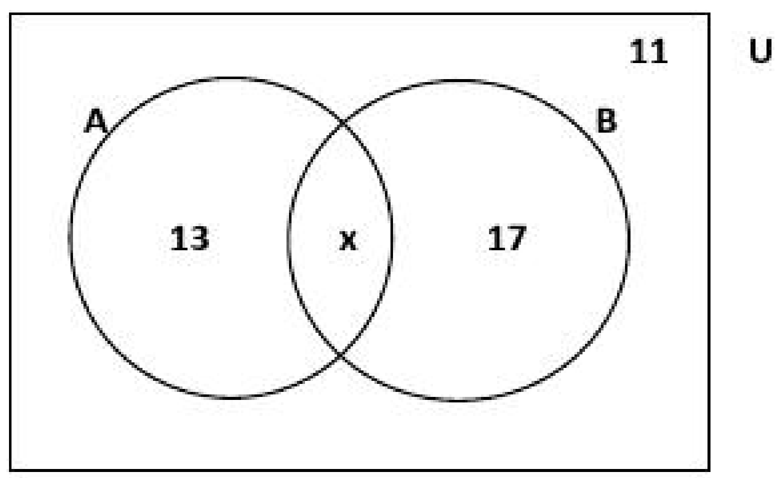
42. If number written in venn diagram shows the number of elements present in that region, then find $n(A \cup B)$.



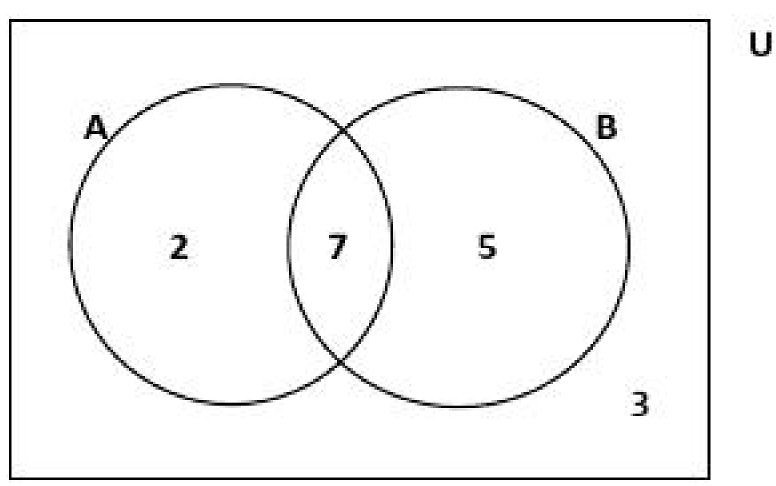
43. If number written in venn diagram shows the number of elements present in that region, then find the value of $n(A' \cap B')$ if n(U) = 100.



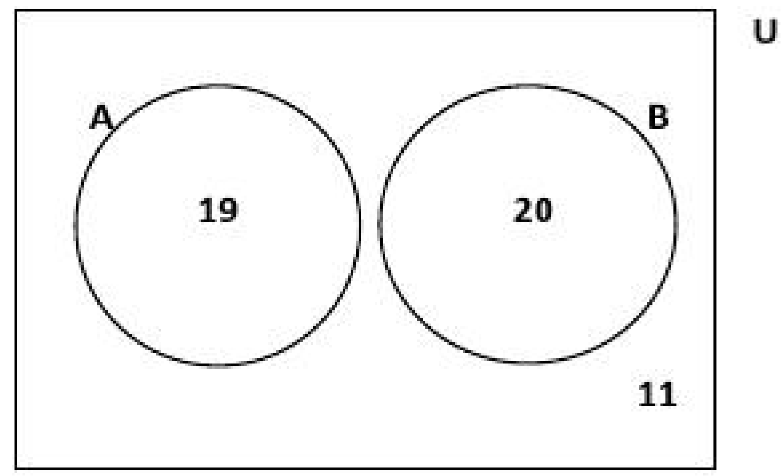
44. If number written in venn diagram shows number of elements present in that region, then find the value of $\dfrac{n(A)}{n(A' \cap B')}$ if n(U) = 50.



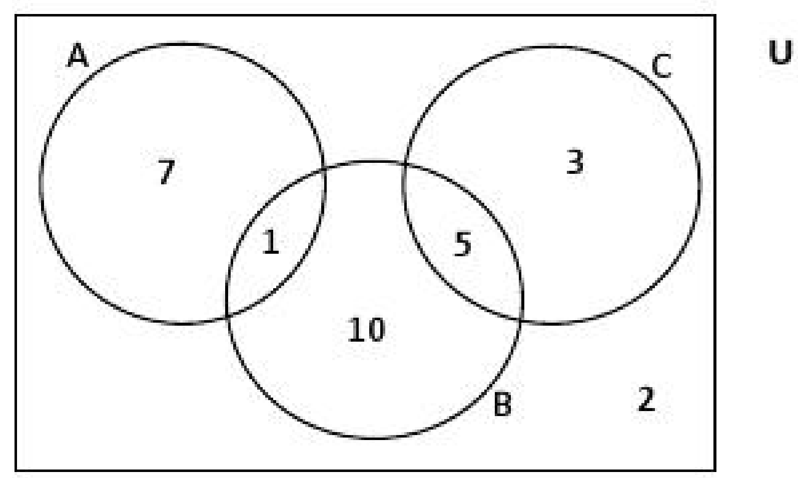
45. If number written in venn diagram shows number of elements present in that region, then find the value of $\dfrac{n(U) - n(A-B)}{n(B-A)}$.



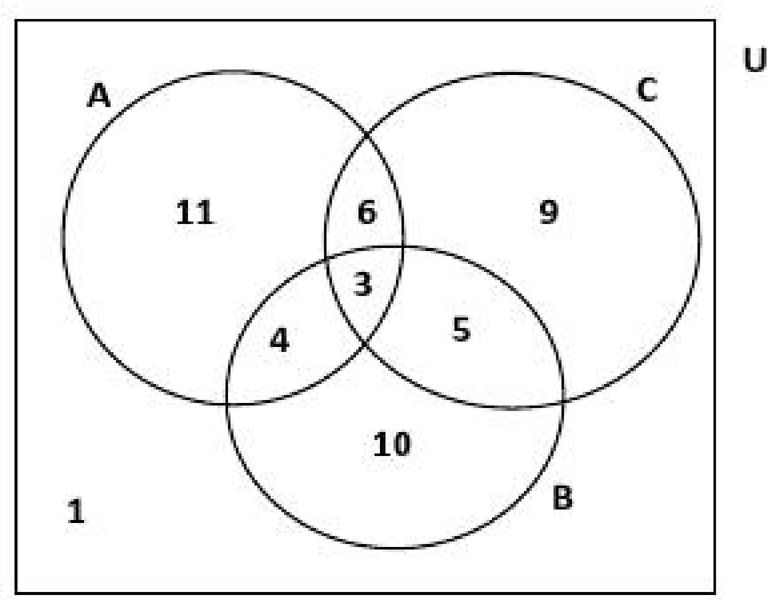
46. If number written in venn diagram shows number of elements present in that region, then find the value of $n(A)+n(B)-n(A \cup B)$.



47. If number written in venn diagram shows number of elements present in that region, then find the value of $n(A \cup B \cup C)-n(A \cap C)$.

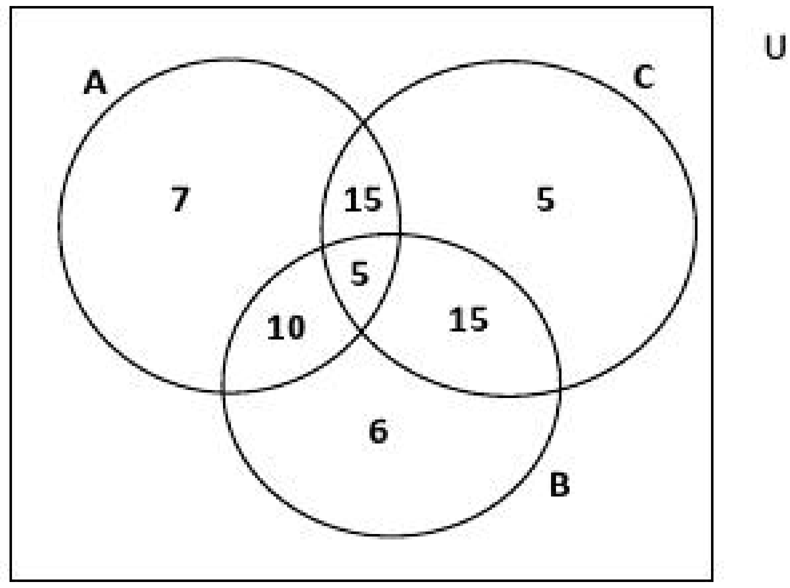


48. If number written in venn diagram shows number of elements present in that region, then find the value of $n(A \cap B)+n(A \cap C)+n(B \cap C)$.

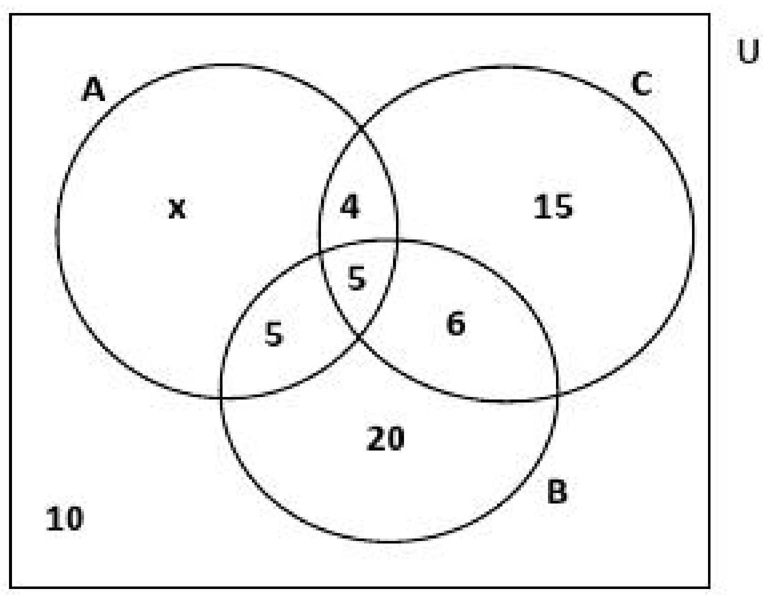


49. If number written in venn diagram shows number of elements present in that region, then find the value of $n(A' \cap B' \cap C')$ if $n(U) = 100$.

[HINT: $n(A' \cap B' \cap C') = n(U) - n(A \cup B \cup C)$]



50. If number written in venn diagram shows number of elements present in that region, then find the value of n(Exactly one of A, B or C), if $n(U) = 100$.



ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	2	26	16
2	5	27	4
3	9	28	1
4	32	29	Zero
5	$A=\{0,1\}$	30	3
6	$A=\{1,p\}$	31	4
7	$A=\{-1,0,1\}$	32	8
8	$A=\{-\sqrt{3},-\sqrt{2},\sqrt{2},\sqrt{3}\}$	33	0
9	$B=\{x:x=2n-1,n\in N,n\le 50\}$	34	5
10	$C=\{x:x=\frac{n}{n+1},n\in N,n\le 9\}$	35	$2^3-1=7$
11	8	36	4
12	2	37	10
13	31	38	3
14	*Null set or* ϕ	39	14
15	16	40	62
16	4	41	3
17	2	42	27
18	8	43	45
19	15	44	2
20	17	45	3
21	0 (Zero)	46	Zero
22	1	47	26
23	$A\cap B=\phi$	48	24
24	*set of Irrational numbers* $(R-Q)$	49	37
25	5	50	70

CHAPTER – 2

RELATIONS & FUNCTIONS

POINTS TO REMEMBER

- Ordered pair: If a pair of elements are written in a specific order, then such a pair is called an ordered pair.
- Cartesian product of Two Sets: Set of all ordered pairs $(a,\ b)$, *where* $a \in A$ & $b \in B$ *is* called Cartesian product of set A and B and is denoted by $A \times B$ *OR*

  $\boxed{A \times B = \{(a,b) : a \in A,\ b \in B\}}$

For example, $A = \{2,\ 3\}$, $B = \{a,\ b,\ c\}$, $A \times B = \{(2,a),(2,b),(2,c),(3,a),(3,b),(3,c)\}$

- Relation: Let A *and* B be two non-empty finite sets, then a relation R from set A to set B is a subset of $A \times B$ *i.e.* $R \subseteq A \times B$.
- Domain: Let R be a relation from A *to* B, then the set of all the first elements of ordered pairs of R is called domain of R. *i.e.* Domain of $R = \{a : (a,b) \in R\}$.
- Range: Let R be a relation from A *to* B, then the set of all the second elements of ordered pairs of R is called range of R. *i.e.* Range of R $= \{b : (a,b) \in R\}$.

- Empty Relation: ϕ is the relation of A, since $\phi \subseteq A \times A$.
- Universal Relation: The relation $R = A \times B$ is a universal relation.

- Function: A function f from a non-empty finite set A to a non-empty finite set B is a relation such that " All elements of set A are associated with unique elements of set B". *i.e.* $\{(x, f(x));\ x \in A,\ f(x) \in B\}$

  Set A is called Domain of the function and set B is called Codomain of the function.
- Image: If $x \in A$ corresponds $y \in B$ under the function f, then we say y is the image of x under f and x is called pre image of y under function f.
- Range: In a function f from A *to* B, set of all images is called range of a function.

  Range $\subseteq$ *Codomain*

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. If $(2a+b,\ a-b)=(8,\ 3)$, then find $(5a+b)$.

2. If $(\frac{x}{4}-1, 12-2y)=(0, 6+y)$, then find $(x-y)$.

3. Given $A=\{1,-1\}$ *and* $B=\{-2, 0,\ 2\}$, then find the value of $n(A \times B)$.

4. For $A=\{a,\ b\}$, $B=\{c,\ d\}$ *and* $C=\{d,\ e\}$, find the value of $n(A \times (B \cup C))$.

5. For $A=\{1,\ 2,\ 3\}$ *and* $B=\{3,\ 8\}$, find the value of $n[(A \cap B) \times (A \cup B)]$.

6. A set A has 2 elements and a set B has 4 elements, what is the total number of subsets of $A \times B$?

7. A set A has 5 elements and a set B has 2 elements, what is the total number of relations from set A to set B?

8. For $A=\{d,\ o,\ e\}$ & $B=\{23,\ 24\}$, find the number of Relation R *on* $A \times B$.

9. If $A=\{m, a, t, h, s\}$ & $B=\{d, o, e\}$, then find the value of
$\frac{\text{number of Relation } R \text{ on } A \times B}{\text{number of Relation } R \text{ on } B \times A}$.

10. For $A=\{a,\ b,\ c\}$ & $B=\{1,\ 2,\ 3\}$, find the number of non-empty Relation R on $A \times B$.

11. If numbers of non-empty subsets of a set A are 7, then find the number of non-empty Relation R *on* $A \times A$.

12. Let $A=\{1,\ 2,\ 3,\ 4\}$ and let R be a relation on A, given by $R=\{(a,b): 2a+3b=20\}$, then find the n(R).

13. For $A=\{1,\ 2,\ 3,\ 4\}$ *andB* $=\{1,\ 3,\ 4,\ 8\}$, find the number of elements satisfying Relation $R=\{(x,y):\ x \in A,\ y \in B$ *and* x is divisible by $y\}$.

14. For $A=\{1,\ 2,\ 3,\ 4\}$ *and* $B=\{1,\ 3,\ 5,\ 8\}$, find the number of elements satisfying Relation $R=\{(x,y):\ x \in A,\ y \in B$ *and* $|x-y|$ is even$\}$.

15. If $f(x)=\text{sgn}(x)$, *where* sgn(.) represents signum function, then find the value of $\frac{f(0)-f(\sqrt{53})}{f(-7.239)}$.

16. If A be the Range of Signum function, then find n(A).

17. For $A=\{x : x\in R, |x-3|<-1\}$, find the number of Relation R on $A\times A$.

18. If $f(x)=[x]$, where [.] represents Greatest Integer Function, find the value of $\frac{f(-\sqrt{3})}{f(\sqrt{5})}$.

19. Find the value of $\left[\frac{1}{8}\right]+\left[\frac{2}{8}\right]+\left[\frac{3}{8}\right]+\dots\dots\dots+\left[\frac{17}{8}\right]$, where [.] denotes Greatest Integer Function.

20. If $x=\left[\frac{1}{2023}\right]+\left[\frac{2}{2023}\right]+\left[\frac{3}{2023}\right]+\dots\dots\dots+\left[\frac{2024}{2023}\right]$, where [.] denotes Greatest Integer Function,then find the value of x^x.

21. For $A=\left\{x : x\in W, [x]<\sqrt{5.0235}\right\}$, [.] denotes Greatest Integer Function, Find the number of Relation R on $A\times A$.

22. For $A=\{x : x\in R, |x-1|=-5\}$, find the number of non-empty Relation R on $A\times A$.

23. If $f(x)=ax+b$ such that $f(0)=3$ *and* $f(3)=0$, then find the value of $(a+b)$.

24. If $f(x)=\frac{9}{5}x+32$, then find the value of $f(-10)$.

25. If $f(x)=x^3$, then find the value of $\frac{(f(5)-f(1))}{4}$.

26. If Range of a function $f(x)$ *is* $[a,\infty)$, where $f(x)=3x^2-6x+10$, then find the value of $(a-5)$.

27. If Range of a function $f(x)$ is $(-\infty,b]$, where $f(x)=-x^2-4x+10$, then find the value of $\sqrt{(b+2)}$.

28. If Range of $f(x)$ *is* $[a,b)$, where $f(x)=\dfrac{x^2}{1+x^2}$, then find the value of $ab+a+b$.

29. Find the number of integers in the domain of $f(x)$, where $f(x)=\sqrt{x-2023}+\sqrt{2024-x}$.

30. If $f(x)=x^2+7$ & $g(x)=3x+5$, then find the value of $f(-2)+g(3)$.

31. If $f(x)=3x^2-8$ & $g(x)=5x-3$, then find the value of $f(0)-g(-2)$.

32. If $f(x)=[x]$ & $g(x)=|x|$, then find the value of $f(\sqrt{19.180059})\times g(-3)$, where [.] denotes Greatest Integer Function & |.| denotes Absolute Value Function.

33. If $f(x)=|x|$ & $g(x)=[x]$, then find the value of $\dfrac{g(-\sqrt[3]{342})}{-f(-3.5)}$, where [.] denotes Greatest Integer Function & |.| denotes Absolute Value Function.

34. If the solution set for $f(x)=0$ *is* $[a,b)$ such that $f(x)=[x]^2-5[x]+6$, then find the value of $(a+b)$ *where* [.] denotes Greatest Integer Function

35. If $f(x)=x^2$, then find the value of $\dfrac{f(2.17)-f(1.83)}{4}$.

36. For $P=\{1,\ 2\}$, find $n(P\times P\times P)$.

37. For $A=\{a,b,c\}$, find $n(A\times A\times A)-n(A\times A)$.

38. For $A=\{a,b\}$ *and* $B=\{2,3,5\}$, find the value of $\sqrt{9\left(\dfrac{n(A\times B)-n(A)}{n(B\times A)+n(B)}\right)}$.

39. Let $S=\{1,2,3,4,5,6,7,8\}$, A function $f:S\rightarrow S$ is defined as

$$f(x)=\begin{cases} 2x, \text{if } x=1,2,3,4 \\ 2x-9, \text{if } x=5,6,7,8 \end{cases}$$

then find the value of $f(2)+f(3)-f(5)$.

40. If Domain of $f(x)$ is $(-\infty,\infty)-\{a,b\}$, where $f(x)=\dfrac{x^2-1}{x^2-7x+12}$, then find the value $|a-b|$.

41. How many subsets of $A\times B$ are possible such that it has atleast 2 elements and not more than 9 elements if $n(A)=2$ and $n(B)=5$.
(Hint: ${}^nC_0+{}^nC_1+{}^nC_2+{}^nC_3+....+{}^nC_{n-1}+{}^nC_n=2^n$)

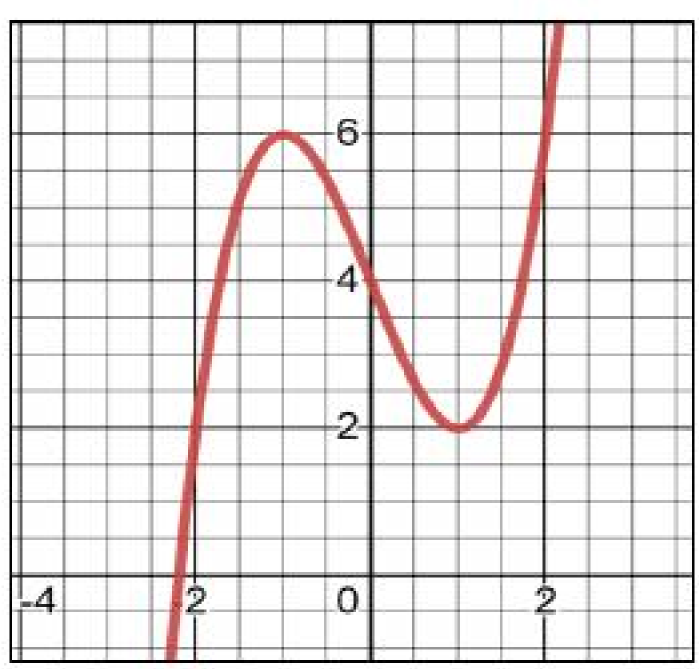
42. How many subsets of $P\times P$ are possible such that it has atleast 1 element and not more than 8 elements if $n(P)=3$.
(Hint: ${}^nC_1+{}^nC_2+{}^nC_3+....+{}^nC_{n-1}=2^n-2$)

43. Find the number of functions that can be defined from set A={23, 24} to B ={m, a, t, h, s}.

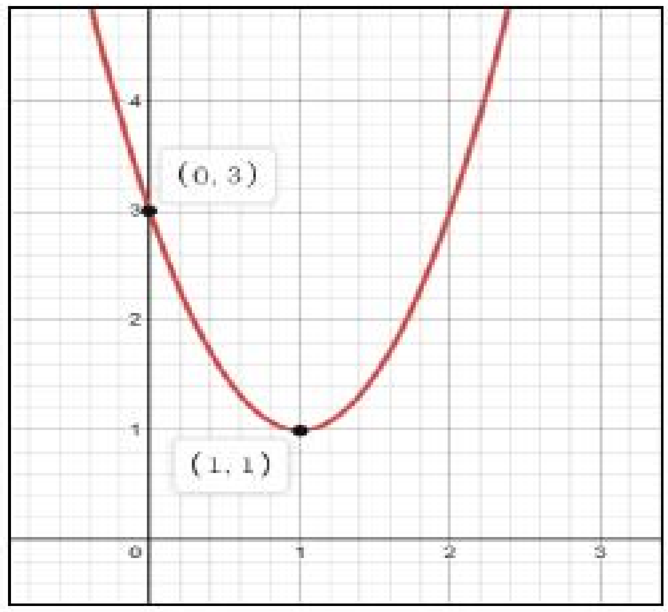
44. Find the number of functions that can be defined from set P ={1, 2, 3} to Q ={a, b, c, d}.

45. If Domain of $f(x)$ is $[a,b]$ where $f(x)=\sqrt{16-x^2}$, then find the value $b-a-ab$.

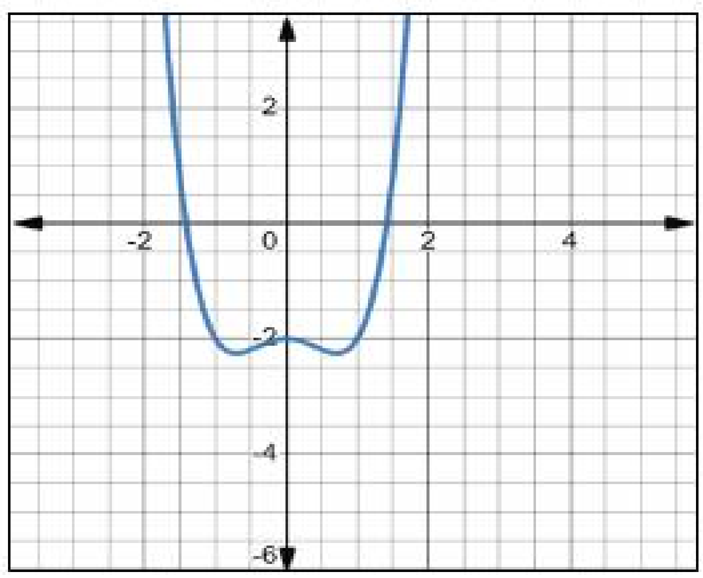
46. If the given graph represents the sketch of a function $f(x)$, then find the value of $\dfrac{f(-1)+f(1)}{f(0)}$.



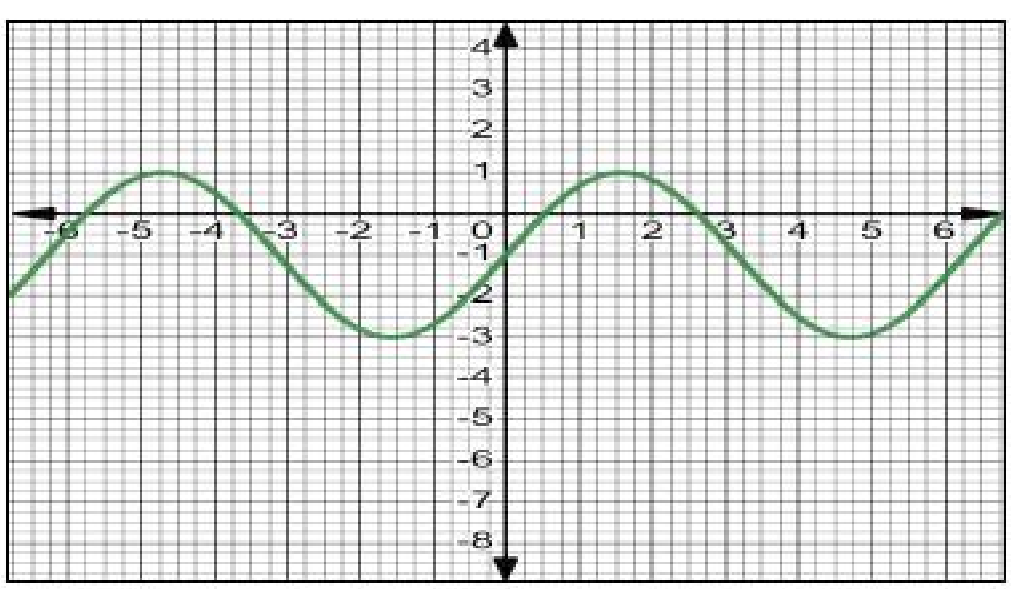
47. If the given figure represents the graph of a function $f(x)=2x^2+ax+b$, then find the value of $|3a+2b|$.



48. If the given figure represents the graph of a function $f(x)=x^4-x^2+a$, then find the value of (a^2-3).

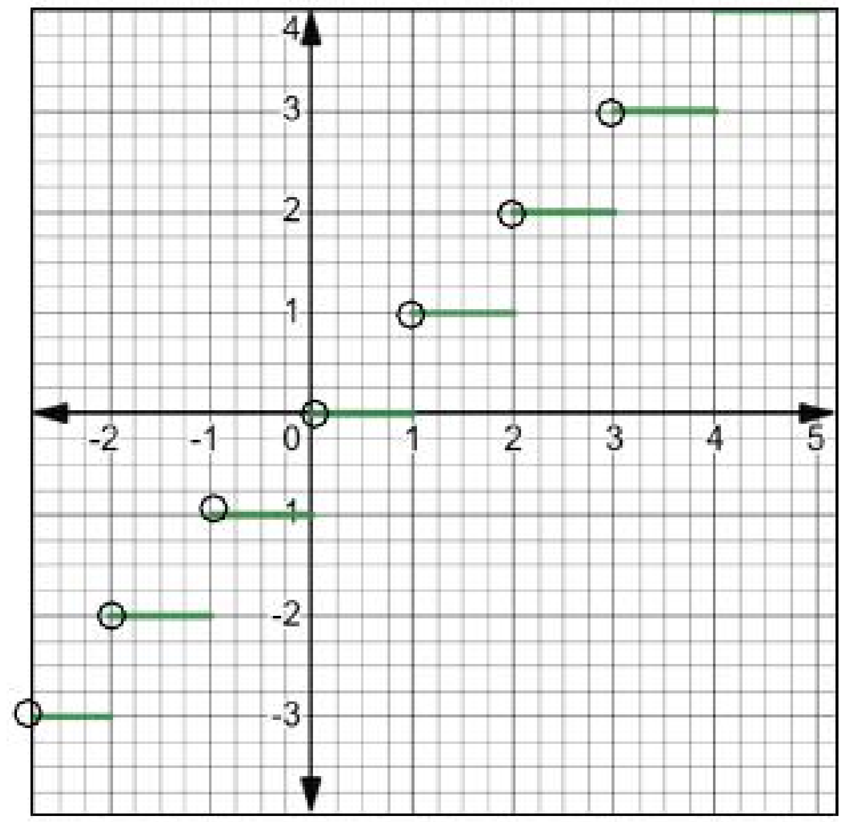


49. If Range of the function f(x) represented by the given graph is $[a,b]$, then find the value of $(b-a)$.



50. From the given graph of $f(x)$, find the value of

$$\frac{f(0.5)+f(1.5)+f(3.5)}{f(-2.5)+f(-1.5)+f(2)}$$



ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	19	26	2
2	2	27	4
3	6	28	1
4	6	29	2
5	4	30	25
6	$2^8 = 256$	31	5
7	$2^{10} = 1024$	32	12
8	64	33	2
9	1	34	6
10	511	35	0.34
11	511	36	8
12	1	37	18
13	6	38	2
14	8	39	9
15	1	40	1
16	3	41	$1012\ (= 2^{10} - 1 - 10 - 1)$
17	1	42	$510\ (= 2^9 - 1 - 1)$
18	-1	43	25
19	12	44	64
20	4	45	24
21	512	46	2
22	0 (Zero)	47	6
23	2	48	1
24	14	49	4
25	31	50	-1

CHAPTER – 3
TRIGONOMETRIC FUNCTIONS

POINTS TO REMEMBER

- Angle: An angle is a measure of rotation of given ray about its initial point.
- Vertex: Point of rotation is called vertex of the angle.
- Positive angle: If the direction of rotation is anticlockwise, then the angle formed is called positive angle.
- Negative angle: If the direction of rotation is clockwise, then the angle formed is called negative angle.

Measurement of Angles

- Sexagesimal System: Angles are measured in degrees $(^0)$, minutes$(')$ & seconds$('')$.

$$\text{One right angle} = 90^0,\ 1^0 = 60',\ 1' = 60''$$

- Circular System: The angle subtended by an arc of a circle whose length is equal to the radius of circle at the centre is called 1 Radian.
- The number of radians in the angle subtended by an arc of circle at the centre is length of the arc divided by the radius of the circle i.e.

$$\theta = \frac{l}{r} \Rightarrow l = r\theta$$

- Relation between Degree Measure and Radian Measure:

$$\pi\, Radians = 180^0$$

FUNCTION	DOMAIN	RANGE
$y=\sin x$, $y=\cos x$	R	$[-1,1]$
$y=\tan x$	$R-\{(2n+1)\frac{\pi}{2}\}$	$(-\infty,\infty)$
$y=\cot x$	$R-\{n\pi\}$	$(-\infty,\infty)$
$y=\operatorname{cosec} x$	$R-\{n\pi\}$	$(-\infty,\infty)-(-1,1)$
$y=\sec x$	$R-\{(2n+1)\frac{\pi}{2}\}$	$(-\infty,\infty)-(-1,1)$

- $\sin(A \pm B)=\sin A\cos B\pm\cos A\sin B$
- $\cos(A \pm B)=\cos A\cos B\mp\sin A\sin B$
- $\tan(A\pm B)=\dfrac{\tan A\pm\tan B}{1\mp\tan A.\tan B}$, • $\cot(A\pm B)=\dfrac{\cot A.\cot B\mp 1}{\cot B\pm\cot A}$

- $\sin C+\sin D=2\sin(\frac{C+D}{2})\cos(\frac{C-D}{2})$
- $\sin C-\sin D=2\cos(\frac{C+D}{2})\sin(\frac{C-D}{2})$
- $\cos C+\cos D=2\cos(\frac{C+D}{2})\cos(\frac{C-D}{2})$
- $\cos C-\cos D=-2\sin(\frac{C+D}{2})\sin(\frac{C-D}{2})$

$\sin(-x)=-\sin x, \cos(-x)=\cos x$

$\tan(-x)=-\tan x$

- $\sin 3A=3\sin A-4\sin^3 A$
- $\cos 3A=4\cos^3 A-3\cos A$
- $\tan 3A=\dfrac{3\tan A-\tan^3 A}{1-3\tan^2 A}$

- $\sin 2A=2\sin A.\cos A=\dfrac{2\tan A}{1+\tan^2 A}$, • $\tan 2A=\dfrac{2\tan A}{1-\tan^2 A}$
- $\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A=\dfrac{1-\tan^2 A}{1+\tan^2 A}$

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. If $135^0 = \frac{a\pi}{b}$, where a & b are coprime numbers,then find the value of $(a+b)$.

2. If $330^0 = \frac{a\pi}{b}$, where a & b are coprime numbers,then find the value of $(a-b)$.

3. If $22^0 30' = \frac{a\pi}{b}$, where a & b are coprime numbers,then find the value of (ab).

4. If $\frac{7\pi}{6} = x^0$, *then* find the value of $\sqrt{(x-14)}$.

5. If $\frac{3\pi}{5} = x^0$ & $\frac{\pi}{12} = y^0$, *then* find the value of $\sqrt{x+y-2}$.

6. If the length of the arc of a circle of radius 5 cm subtending an angle 15^0 at the centre is $\frac{a\pi}{b}cm$ (where a & b are coprime numbers), then find $(a+b)$.

7. Find x, If the angle subtended by the hands of clock at 7:20 pm in Degree measure is x^0.

8. Find $(a+b)$, If the angle subtended by the hands of clock at 1:15 AM in Radian measure is $\frac{b\pi}{a}$.

9. Find the value of $(6\sin 10^0 - 8\sin^3 10^0)$.

10. Find the value of $(8\cos^3 20^0 - 6\cos 20^0)$.

11. If $P = \frac{3\tan 15^0 - \tan^3 15^0}{1 - 3\tan^2 15^0}$, then find the value of P^P.

12. Find the value of $(4\sin 15^0 \cos 15^0)$.

13. If $P = 2(\cos^2 15^0 - \sin^2 15^0)$, then find the value of P^2.

14. If $\tan A + \cot A = \sqrt{8}$, then find the Positive value of $(\tan A - \cot A)$.

15. Find the value of $\cos 1^0 + \cos 2^0 + \cos 3^0 + \ldots + \cos 179^0 + \cos 180^0$.

16. Find the value of $\cos 1^0 . \cos 2^0 . \cos 3^0 ..... \cos 179^0 . \cos 180^0$.

17. Find the value of $\tan 16^0 . \tan 17^0 . \tan 18^0 . \cot 18^0 . \cot 17^0 . \cot 16^0$.

18. If $\sin\ \theta + \cos ec\ \theta = 2$, then find the value of $(\sin^{2024}\theta + \cos ec^{2024}\theta)$.

19. If $\sin^2 a + \sin^2 b + \sin^2 c + \sin^2 d = 0$, then find the minimum value of $\cos a + \cos b + \cos c + \cos d$.

20. If $\left(1+\tan 1^0\right)\left(1+\tan 2^0\right)\left(1+\tan 3^0\right)......\left(1+\tan\ 44^0\right) = 2^x$, then find the value of $\sqrt{x+3}$.

$[HINT : (1+\tan A)(1+\tan B) = 2, if\ A+B = 45^0]$

21. Find the value of $\sin(45^0 + x) - \cos(45^0 - x)$.

22. If $\sin x + \cos x = 1$, then find the value of $\sin 2x$.

23. Find the value of $\left(\dfrac{2\tan 15^0}{1+\tan^2 15^0} - \dfrac{1}{2}\right)$.

24. Find the maximum value of $8\sin x.\cos x + 3\cos\ 2x$.

25. Find the maximum value of $(7\cos x - 24\sin x) + 5$.

26. Find the minimum value of $(5\cos x + 12\sin x) + 17$.

27. If $\sin\dfrac{x}{2} = \dfrac{3}{5}$, find the value of $25\cos x$.

28. Find the value of $16\cos 20^0 . \cos 40^0 . \cos 80^0$.

$(HINT : \cos x \cos(60^0 - x)\cos(60^0 + x) = \dfrac{\cos 3x}{4})$

29. If the range of $f(x) = \sin x + \cos x$ is $[a,b]$, then find the value of $(a^2 + b^2)$.

30. If $P = \sin 330^0, Q = \cos 150^0$, then find the value of $P^2 + Q^2$.

31. If $P = \sin 22^0 30', Q = \cos 22^0 30'$, then find the value of $2(Q^2 - P^2)^2$.

32. If $x = 22^0 30'$, then find the value of $2\cos^2 2x + \sin^3 4x$.

33. Find the maximum value of $16\sin\frac{x}{2023}\cos\frac{x}{2023}$.

34. If $A = \tan\frac{\pi}{4}, B = \tan\frac{3\pi}{4}, C = \tan\frac{5\pi}{4} \,\&\, D = \tan\frac{7\pi}{4}$, then find the value of $(A + B + C + D)$.

35. If $\sin x = \frac{1}{2}$, where $x = \frac{a\pi}{b} \in \left(\frac{\pi}{2}, \pi\right)$, such that $a \,\&\, b$ are coprime numbers then find $|a - b|$.

36. Find the value of $\left(\frac{\sin 135^0}{\cos 315^0} + \frac{\tan 330^0}{\cot 120^0}\right)^2$.

37. Find the value of $\frac{\sin 2\pi + \cos 3\pi}{\cos 4\pi + \sin 5\pi}$.

38. If $\sin 15^0 = \frac{\sqrt{a} - \sqrt{b}}{\sqrt{8}}$, then find the value of $(a + b)$.

39. If $\cos 15^0 = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{8}}$, then find the value of $|a - b|$.

40. If $\tan 15^0 = \sqrt{a} - \sqrt{b}$, then find the value of (ab).

41. If $\sin x = \frac{3}{5}$, $x \in (\frac{\pi}{2}, \pi)$, such that $\tan x = \frac{a}{b}$ (where $a \,\&\, b$ are coprime), then find the value of $|a + b|$.

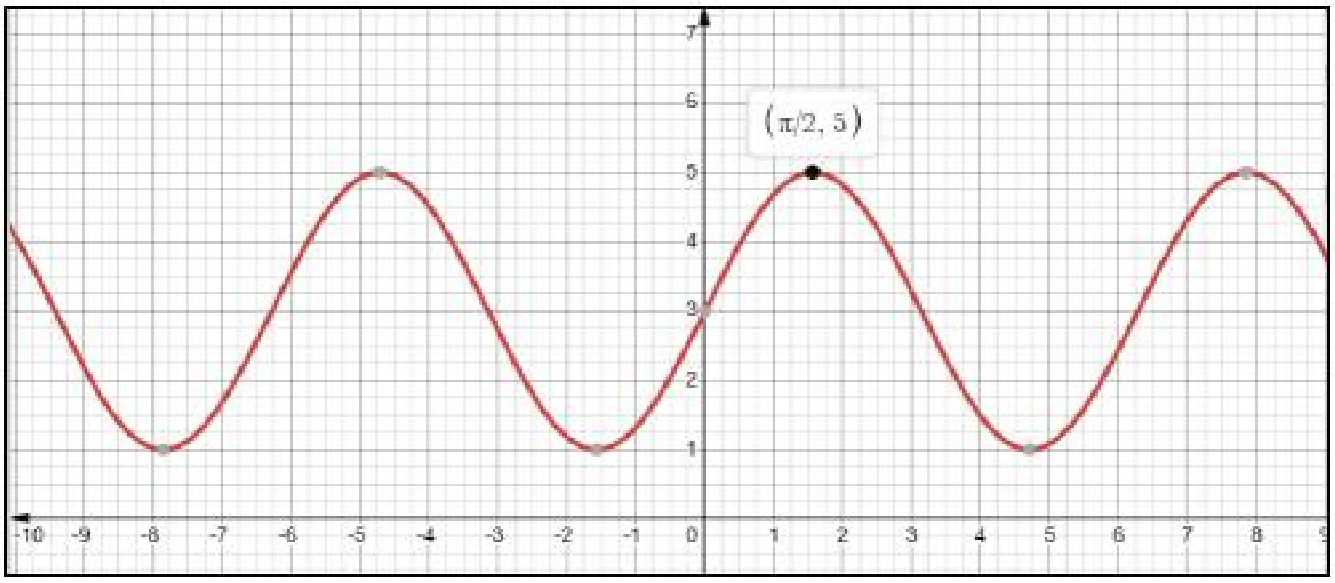
42. If $\cos x = \frac{5}{13}$, $x \in (\frac{3\pi}{2}, 2\pi)$ such that $\sin x = \frac{a}{b}$ (where $a \,\&\, b$ are coprime), then find the value of $|a + b|$.

43. Find the value of $[\sin^2 10^0 + \sin^2 20^0 + \sin^2 30^0 + ... + \sin^2 90^0]$, *where* [.] represents greatest integer function.

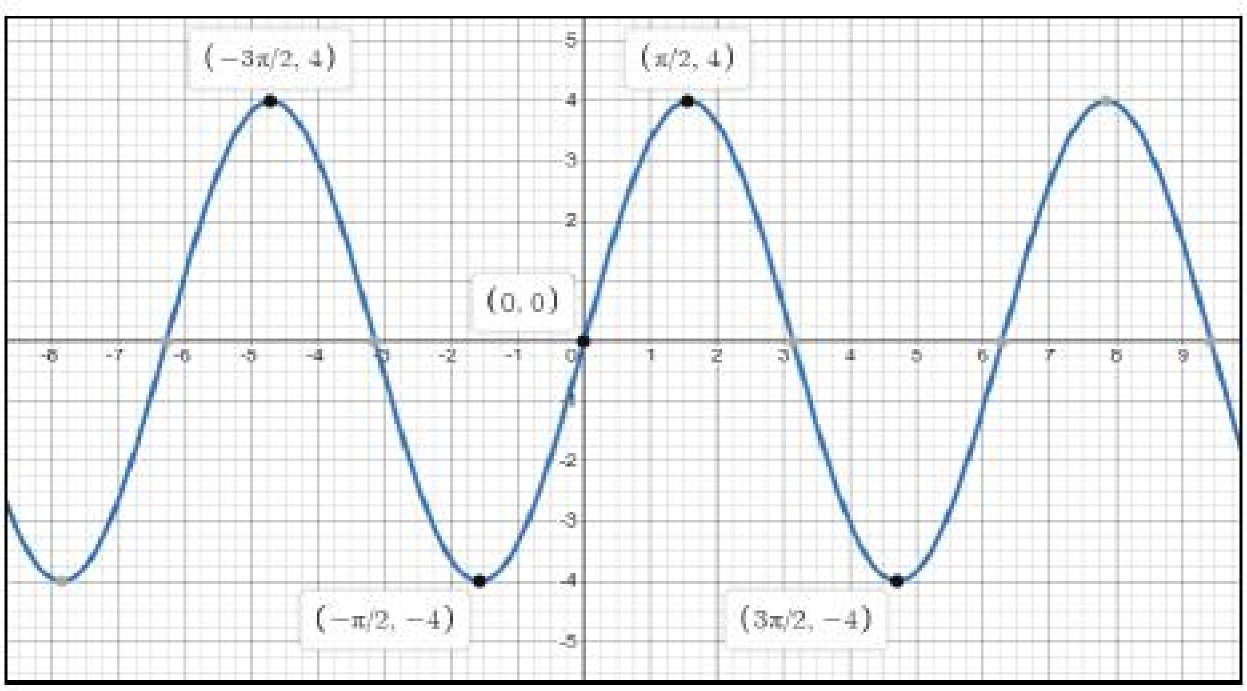
44. Find the value of $(\cos^2 10^0 + \cos^2 20^0 + \cos^2 30^0 + ... + \cos^2 90^0)$.

45. Find the value of $\tan\left(\frac{\pi}{4} - x\right).\tan\left(\frac{\pi}{4} + x\right)$.

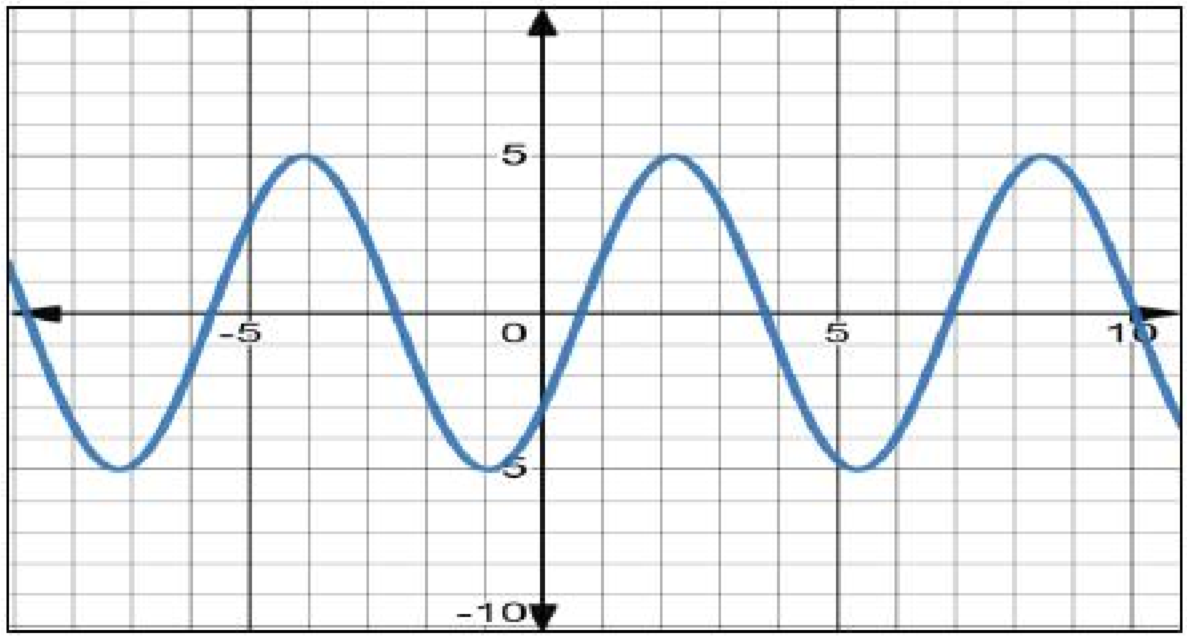
46. If graph of the function $f(x) = a(\sin x) + b$ is given below, then find the value of $(a+b)^2$.



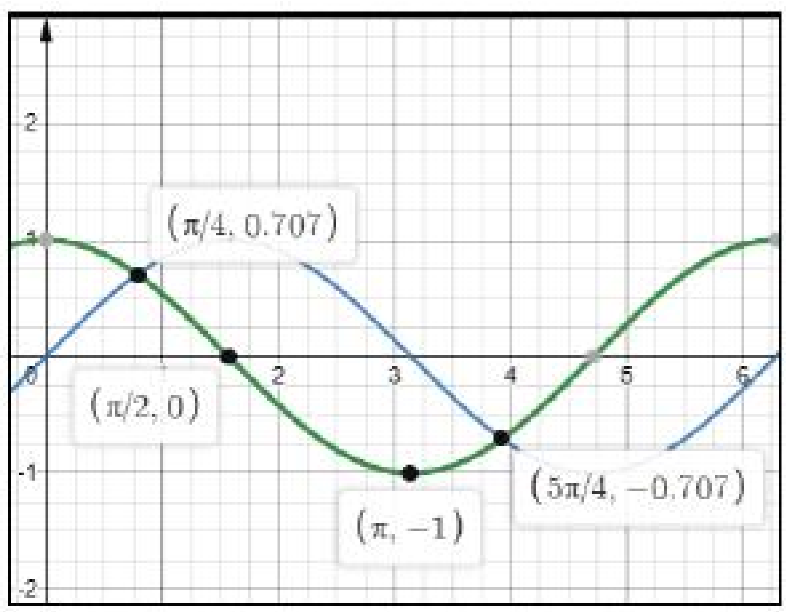
47. If graph of the function $f(x) = a(\sin x)$ is given below, then find the value of $f(\frac{5\pi}{6})$.



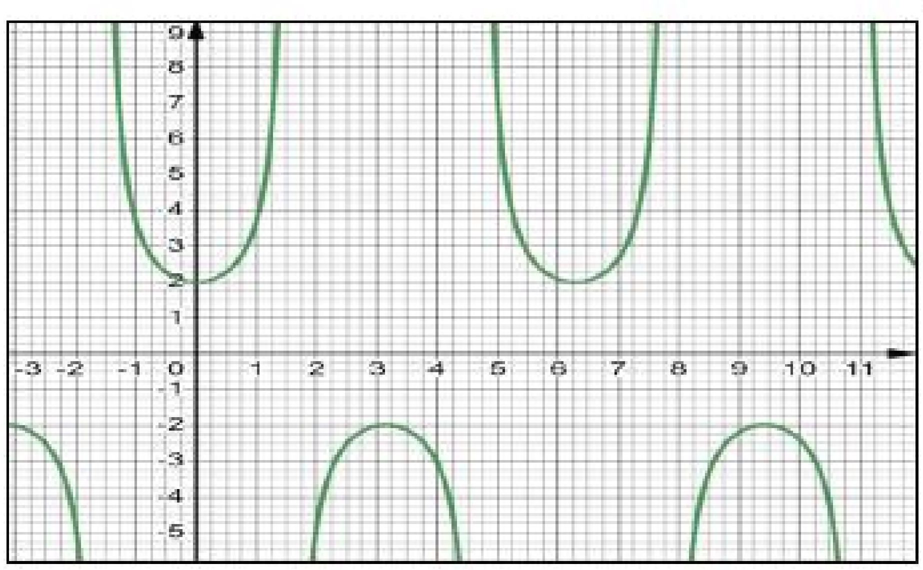
48. If range of the function $f(x)$ represented by the given graph is $[a,b]$, then find the value of $(b-a)$.



49. If two curves $y=f(x)$ *and* $y=g(x)$ intersect at (a,b) *and* (c,d) as shown in the graph given below, find $(a+c)$ in degree measure.



50. If graph of $f(x)=k\sec x$ is shown below, then find $f\left(\dfrac{7\pi}{3}\right)$.



ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	7	26	4
2	5	27	7
3	8	28	2
4	14	29	4
5	11	30	1
6	17	31	1
7	100	32	2
8	31	33	8
9	1	34	0
10	1	35	1
11	1	36	4
12	1	37	-1
13	3	38	4
14	2	39	2
15	-1	40	12
16	0	41	1
17	1	42	1
18	2	43	5
19	-4	44	0
20	5	45	1
21	0 (Zero)	46	25
22	0 (Zero)	47	2
23	0 (Zero)	48	10
24	5	49	270°
25	30	50	4

CHAPTER – 4
COMPLEX NUMBERS & QUADRATIC EQUATIONS

POINTS TO REMEMBER

- Iota: $i = \sqrt{-1} \Rightarrow \boxed{i^2 = -1}$
- For any integer k, $\boxed{i^{4k} = 1}$, $\boxed{i^{(4k+1)} = i}$, $\boxed{i^{(4k+2)} = -1}$, $\boxed{i^{(4k+3)} = -i}$
- Equality of two complex numbers: $a + ib = c + id \Rightarrow \boxed{a = c \text{ and } b = d}$
- Given $z = a + ib$, Additive inverse of z is $-z$ i.e. $-a - ib$.
- Given $z = a + ib$, $\bar{z}$ is called conjugate of z and is denoted as $\bar{z} = a - ib$.
- Given $z = a + ib$, $|z|$ is called modulus of z and is denoted as $|z| = \sqrt{a^2 + b^2}$.
- Multiplicative Inverse of z is $z^{(-1)}$ or $\frac{1}{z} = \frac{\bar{z}}{|z|^2}$.
- $|Z_1 Z_2| = |Z_1| . |Z_2|$
- $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$
- $Z.\bar{Z} = |Z|^2$ & $|\overline{Z_1 - Z_2}| = |Z_1 - Z_2|$.
- Given $z = a + ib$, argument of z denoted as $arg(z) = \tan^{-1}\left(\frac{b}{a}\right)$
- $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0 + 0i, \ \forall \ n \in N$
- If $ax^2 + bx + c = 0$, then $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$, $a \neq 0$

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. Find the value of $(x+y)$, if $x+iy=i^{2021}+i^{2022}+i^{2023}+i^{2024}$.

2. If $\dfrac{(1+i)^{2024}}{2^{1012}}=x+iy$, then find the value of $(x-y)$.

3. If $\left(\dfrac{1+i}{1-i}+\dfrac{1-i}{1+i}\right)=x+iy$, then find the value of $x\times y$.

4. If $z=\cos 7^0+i\sin 7^0$, then find the value of $|z|$.

5. If $z=\dfrac{\cos 89^0-i\sin 89^0}{\cos 11^0+i\sin 11^0}$, then find the value of $|z|$.

6. If $z=(\cos 13^0+i\sin 13^0)(\sin 37^0-i\cos 37^0)$, then find the value of $|z|$.

7. If $z=(\sqrt{2}+i\sqrt{3})$, then find the value of $z.\bar{z}$.

8. If $z=\dfrac{\sqrt{2023}+i\sqrt{2024}}{\sqrt{2024}-i\sqrt{2023}}$, then find the value of $z.\bar{z}$.

9. Find the value of $(2x-3y)$, if $(1+i+i^2+...+i^{2024})=x+iy$.

10. Find the value of $(x+y)$, if $(i^{99}+i^{100}+i^{101}+...+i^{500})=x+iy$.

11. If $z_1=\dfrac{2+3i}{1-5i}$ & $z_2=\dfrac{5-i}{3-2i}$, then find the value of $|z_1z_2|$.

    $\left(HINT:\ |z_1z_2|=|z_1||z_2|\ \&\ \left|\dfrac{z_1}{z_2}\right|=\dfrac{|z_1|}{|z_2|}\right)$.

12. If $z_1=\cos 18^0+i\sin 18^0$ & $z_2=\sin 36^0-i\cos 36^0$, then find the value of $|z_1z_2|$.

    $(HINT:\ |z_1z_2|=|z_1||z_2|)$

13. Find the $\text{Re}(z)$, if $z=(1+2i)(2-3i)$.

14. Find the $\operatorname{Im}(z)$, if $z=(1-2i)(1-3i)(1-4i)$.

15. If $\theta=\dfrac{a\pi}{b}\in(0,2\pi)$ is the argument of the complex number $\sin 30^0+i\cos 30^0$ (where $a\,\&\,b$ are coprime numbers), then find $(a+b)$.

16. If $\theta=\dfrac{a\pi}{b}\in(0,2\pi)$ is the argument of the complex number $-\sin 45^0-i\cos 45^0$ (where $a\,\&\,b$ are coprime numbers), then find $(a+b)$.

17. If $x^0\in(0,360^0)$ is the argument of the complex number $-\sqrt{3}+i$, then find the value of x.

18. If $x^0\in(0,360^0)$ is the argument of the complex number $-1-i$, then find the value of $\sqrt{x}$.

19. Find the modulus of the complex number $0.01+i\sqrt{0.9999}$.

20. Find the modulus of the conjugate of complex number $z=\sqrt{0.2023}+i\sqrt{0.7977}$.

21. If $x^2+x+1=0$, then find the value of (x^6+x^3).

22. If $z=\dfrac{i-1}{\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}}$, then find the modulus of complex number z.

23. Find modulus of the complex number $z=\left(1+\cos\dfrac{2\pi}{3}\right)+i\sin\dfrac{2\pi}{3}$.

24. Find modulus of the complex number $z=-2\left(\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}\right)$.

25. Find the maximum value of $'a'$ for which $x^2-50x+25a=0$ has real roots.

26. Find the value of $(x+y)$, if $(2x-3)+i(y-4)=3+4i$.

27. Find the value of $(2x+y)$, if $(x+y)+i(y-x)=7+5i$.

28. Find the least positive value of n, such that $\left(\frac{1-i}{1+i}\right)^n = 1$.

29. Find the Greatest negative value of n, such that $\left(\frac{1+i}{1-i}\right)^{2n} = 1$.

30. If $z = x+iy$, then how many complex numbers z satisfy $z-2 = |z|$.

31. If $z = x+iy$, then find the modulus of z where x and y are sum and product of the roots of quadratic equation $p^2+5p+12=0$ respectively.

32. If $x=1+i$, then find the value of (x^2-2x+1).

33. If $x=\frac{1+i}{\sqrt{2}}$, then find the value of (x^8+x^4+2).

34. Find the value of $\sqrt{-1}\times\sqrt{-4}\times\sqrt{-9}\times\sqrt{-16}$.

35. Find the value of $i\ .i^2.i^3.......i^{100}$.

36. If $\theta\in(0,2\pi)$ is the argument of complex number $z=-\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}$, then find θ.

37. Find the absolute value of real part of complex number $z=\sqrt{5+12i}$.

38. Find the absolute value of Imaginary part of complex number $z=\sqrt{7-24i}$.

39. If $z=3-12i$, then find the value of $|z+2|$.

40. If $z=-6+13i$, then find the value of $|z-5i|$.

ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	0	21	2
2	1	22	$\sqrt{2}$
3	0	23	1
4	1	24	2
5	1	25	25
6	1	26	11
7	5	27	8
8	1	28	4
9	2	29	-2
10	0	30	0
11	1	31	13
12	1	32	-1
13	8	33	2
14	15	34	24
15	4	35	-1
16	9	36	$\frac{4\pi}{3}$
17	150	37	3
18	15	38	3
19	1	39	13
20	1	40	10

CHAPTER – 5

LINEAR INEQUALITIES

POINTS TO REMEMBER

- A Linear equation involving one of the symbols $<$, $>$, $\leq or \geq$ is called a linear inequality.
- $'<'$ & $'>'$ are called strict inequalities.
- $\leq or \geq$ are called slack inequalities.
- $ax+b>0,\ ax+b<0,\ ax+b\leq 0\ and\ ax+b\geq 0\ (a\neq 0)$ are called linear inequalities in one variable x.
- $ax+by+c>0,\ ax+by+c<0,\ ax+by+c\leq 0\ and\ ax+by+c\geq 0$ $(a,b\neq 0)$ are called linear inequalities in two variables x & y.

- Algebraic Solution of linear inequality in one variable:

⊙ Equal number may be added (or subtracted) to both sides without affecting the sign of inequality.

⊙ If both sides of inequality are multiplied (or divided) by same positive number then sign of inequality remains unchanged.

⊙ If both sides of inequality are multiplied (or divided) by same negative number then the sign of inequality gets reversed.

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. How many non-zero positive integral values of x satisfy $2x-7<4$?
2. How many non-zero negative integral values of x satisfy $-3x-17\leq 4$?

3. If (a,∞) is the solution set of the inequality $\frac{2}{x-3} > 0$, then find the value of a^a.

4. If $(-\infty, a)$ is the solution set of the inequality $\frac{-4}{x-25} \geq 0$, then find the value of $\sqrt{a-9}$.

5. How many integral values of x satisfy $|x-4| < 1$?

6. How many integral values of x satisfy $|x+2| \leq 3$?

7. How many integral values of x satisfy $|x-2023| \leq -2024$?

8. If A is the solution set of $\frac{x}{2} + \frac{x}{3} \leq \frac{1}{23}, x \in N$, then find $n(A)$.

9. If A is the solution set of $\frac{x}{2023} - \frac{x}{2024} \leq \frac{7}{(2023)(2024)}, x \in N$, then find $n(A)$.

10. If B is the solution set of $\frac{2024x}{2023} - 1929 \leq \frac{95x}{2023}, x \in W$, then find $n(B)$.

11. How many integral values of x satisfies $5x \leq 25$ and $7x - 28 \geq 0$.

12. If $[a,b]$ is the solution set of the inequality $x^2 - 11x + 28 \leq 0$, then find the value of $(2a+b)$.

13. If $[a,b]$ is the solution set of the inequality $(x-2)^2(x+3)(x-5) \leq 0$, then find the value of $(a+b)$.

14. Find the value of $(a+b-c)$, If $(a,b)-\{c\}$ is the solution set of the inequality $(x-3)^2(x+1)(x-6) < 0$.

15. If $[a,\infty)$ is the solution set of $-2x+3 \leq x$, then find the value of $\sqrt[3]{a+7}$.

16. If $(\infty, a]$ is solution set of $5x-43 \le 34-6x$, then find the value of a^3.

17. If A is the solution set of $[x]^2-[x]<0$, where [.] represents the Greatest Integer Function, then find $n(A)$.

18. Find the minimum value of x satisfying $7x-13 \ge 22$.

19. Find the maximum value of x satisfying $-3x+12 \ge 12$.

20. Find the Maximum Integral value of x satisfying $\frac{3-x}{5} \ge \frac{5}{2}$.

21. Find the Minimum Integral value of x satisfying $\frac{x-19}{9} \ge \frac{2}{5}$.

22. If A is the solution set of $\left|\frac{7-x}{2023}\right|+1<0$, then find $n(A)$.

23. If P is the solution set of $2024-x \ge x-2023, x \in W$, then find $n(P)$.

24. How many integral values of x satisfy $|2x-3|<7$?

25. How many integral values of x satisfy $|3-x| \le 0.999123$?

26. If $[a,b)$ is the solution set of the inequality $\frac{x}{x-1} \le 0$, then find the value of $(a-b+3)$.

27. If $(a,b]$ is the solution set of the inequality $-1<2x-3 \le 7$, then find the value of $\sqrt{a+b+3}$.

28. If $[a,b]$ is the solution set of the inequality $-1 \le -x \le 7$, then find the value of $(a+b+14)$.

29. If $-5 \le x \le 3$, then what will be the Maximum Value of $|x|$.

30. If $-17.5 \le y \le 9.39$, then what will be the Minimum Value of $|y|$.

ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	5	16	343
2	7	17	0
3	27	18	5
4	4	19	0
5	1	20	-10
6	7	21	23
7	0	22	0
8	0	23	2024
9	7	24	6
10	2024	25	1
11	2	26	2
12	15	27	3
13	2	28	8
14	2	29	5
15	2	30	0

CHAPTER – 6
PERMUTATIONS & COMBINATIONS

POINTS TO REMEMBER

- $n! = n(n-1)(n-2)...3.2.1 \Rightarrow \boxed{n! = n(n-1)!}$
- $0! = 1,\ 1! = 1,\ 2! = 2,\ 3! = 6,\ 4! = 24,\ 5! = 120$
- ${}^nP_r = \dfrac{n!}{(n-r)!}, n \geq r, {}^nC_r = \dfrac{n!}{(n-r)!r!}, n \geq r \Rightarrow \boxed{{}^nC_r.r! = {}^nP_r}$
- Number of circular arrangements of n different things taken all at a time is $(n-1)!\cdot$
- The number of permutations of n objects taken all at a time, if it is given that $'a'$ objects are similar and of first kind, $'b'$ objects are of second kind and remaining all are different is $\dfrac{n!}{a!b!}$.
- *IMPORTANT RESULTS*

(a) ${}^nC_r = {}^nC_{n-r}$

(b) ${}^nC_a = {}^nC_b \Rightarrow Either\ a = b\ Or\ a + b = n$

(c) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(d) ${}^nC_r = \left(\dfrac{n}{r}\right){}^{n-1}C_{r-1} = \dfrac{n(n-1)}{r(r-1)}.{}^{n-2}C_{r-2}$

(a) ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = \sum_{r=0}^{n} {}^nC_r = 2^n$

(b) $\underbrace{{}^nC_0 + {}^nC_2 + {}^nC_4 + {}^nC_6 + ...}_{\text{SUM OF EVEN COEEFIECIENTS}} = \underbrace{{}^nC_1 + {}^nC_3 + {}^nC_5 + {}^nC_7 + ...}_{\text{SUM OF ODD COEEFIECIENTS}} = 2^{n-1}$

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. Find the value of $5!-4!$.

2. Find the value of $\frac{6!-5!}{5!}+\frac{4!-3!}{3!}+2!-1!$.

3. Find the positive value of n, if $\frac{n(n-1)(n-2)(n-3)(n-4)}{5}=24$.

4. Find the positive value of n, if $\frac{(n+2)!}{(n-1)!}=6$.

5. Find the value of $\frac{{}^{2024}C_2}{2023}$.

6. Find the value of $({}^{2024}C_0+{}^{2023}C_0+....+{}^{2}C_0+{}^{1}C_0)$.

7. Find the value of $({}^{2023}C_{2023}+{}^{2022}C_{2022}+....+{}^{2}C_2+{}^{1}C_1)$.

8. If ${}^{2024}C_{2x+2}={}^{2024}C_{8-x}$, then find the sum of all possible values of x.

9. If ${}^{17}C_{x-3}={}^{17}C_{15}$, then find the sum of all possible values of x.

10. If ${}^{59}C_{32}=x+{}^{59}C_{27}$, then find the value of $(2x+3)$.

11. Find the value of $\frac{{}^{2024}C_{2022}}{{}^{2024}C_2}$.

12. If $\frac{{}^{17}C_{11}}{{}^{17}C_6}=x$, then find the value of $(14x-3)$.

13. If ${}^{17}C_{11}+{}^{17}C_{12}={}^{x}C_{12}$, then find the value of $\sqrt{2x}$.

14. If ${}^{15}C_9+{}^{15}C_{10}={}^{16}C_x$, then find the sum of all possible values of x.

15. Find the value of ${}^{8}C_8+{}^{8}C_7+....+{}^{8}C_1+{}^{8}C_0$.

16. Find the value of $\frac{{}^{9}C_8+{}^{9}C_7+....+{}^{9}C_2+{}^{9}C_1}{2}$.

17. Find the value of $({}^{10}C_{10} + {}^{10}C_{8} + {}^{10}C_{6} + {}^{10}C_{4} + {}^{10}C_{2})$.

18. Find the value of $\dfrac{({}^{2024}C_{2024} + {}^{2024}C_{2022} + {}^{2024}C_{2020} + \ldots .. + {}^{2024}C_{0})}{({}^{2023}C_{2023} + {}^{2023}C_{2021} + {}^{2023}C_{2019} + \ldots .. + {}^{2023}C_{1})}$.

19. Find the value of n, if $({}^{100}C_{0} + {}^{100}C_{1} + {}^{100}C_{2} + \ldots .. + {}^{100}C_{100}) = 2^{n}$

20. Find the value of r, if ${}^{n}P_{r} = 120 \times {}^{n}C_{r}$.

21. Find the non-zero value of r, if ${}^{n}P_{r} = {}^{n}C_{r}$.

22. Find the number of diagonals in an octagon.

    (HINT: Number of diagonals in an n - vertices polygon $= \dfrac{n(n-3)}{2}$)

23. Find the number of diagonals in a Hexagon.

24. A polygon has 14 diagonals, then find the number of its sides.

25. In how many ways can the letters of the word 'MATHS' be arranged?

26. If all the letters of the word 'EMPTY' are arranged as in a dictionary, then find the rank of the word 'EMPTY'.

27. Let m be the total number of words with or without meaning that can be formed using the letters of the word "EFFORT". If all such words are arranged as in a dictionary and n is the rank of the word 'EFFORT', then find $(m-n)$.

28. In how many ways 6 girls can be seated in a circle?

29. Find the different number of ways in which 4 boys can be seated in a row?

30. Find the number of ways of selecting one black and one red card from a pack of 52cards.

31. If all the letters of the word 'FIVE' are arranged as in a dictionary, then find the rank of the word 'FIVE'.

32. Find the number of ways of selecting 2 cards from a pack of 52 cards if both of them are of same color.

33. If n and r are the roots of the equation $x^2 - 7x + 12 = 0$, then find the Possible value of ${}^nC_r$.

34. If n and r are the roots of the equation $x^2 - 10x + 16 = 0$, then find the Possible value of ${}^nP_r$.

35. Find the number of ways in which letters of the word 'SQUARE' can be arranged if Word starts with A and end with E.

36. How many 3-digit numbers are formed using digits 0, 1, 3, 5, 7 if repetition of digits is allowed?

37. How many Even 4-digit numbers can be formed using the digits 1, 3, 5, 2 and 0,if repetition of digits is not allowed?

38. How many numbers lying between 500 and 1000 can be formed using digits 0, 1, 2, 3 and 8 if repetition is not allowed?

39. If all the letters of the word 'PAPA' are arranged as in a dictionary, then find the rank of the word 'PAPA'.

40. There are 8 points in a plane. Of these 8 points, four points are in a straight line, and except for these four points, no other three points are in a straight line. Find the number of straight lines formed using these 8 points.

ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	96	21	1
2	9	22	20
3	5	23	9
4	1	24	7
5	1012	25	120
6	2024	26	1
7	2023	27	360-1=359
8	2016	28	120
9	23	29	24
10	3	30	676
11	1	31	6+2+1+1=10
12	11	32	650
13	6	33	4
14	16	34	56
15	256	35	24
16	255	36	100
17	511	37	42
18	2	38	12
19	100	39	5
20	5	40	23

CHAPTER – 7
BINOMIAL THEOREM

POINTS TO REMEMBER

- An algebraic expression having two terms is called a Binomial Expression.
- The expansion $(a+b)^n = {}^nC_0 a^{n-0} b^0 + {}^nC_1 a^{n-1} b^1 + \ldots\ldots + {}^nC_n a^0 b^n$, where $n \in N$ is called Binomial Theorem.
- $$\boxed{(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r = {}^nC_0 a^{n-0} b^0 + {}^nC_1 a^{n-1} b^1 + \ldots\ldots + {}^nC_n a^0 b^n}$$
- The coefficients ${}^nC_r$ occurring in Binomial Theorem are known as Binomial Coefficients.
- The coefficients of expansion are arranged in an array.This array is called Pascal′s Triangle.
- There are $(n+1)$ terms in the expansion of $(a+b)^n$ i.e. one more than the index.
- Coefficient of terms $a^p b^q$ and $a^q b^p$ are equal.
- ${}^nC_a = {}^nC_b \Rightarrow Either\ a = b\ Or\ a + b = n$
- $(a-b)^n = {}^nC_0 a^{n-0} b^0 - {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 - \ldots\ldots + (-1)^n . {}^nC_n a^0 b^n$
- $(1+x)^n = {}^nC_0 x^0 + {}^nC_1 x^1 + \ldots\ldots + {}^nC_n x^n$
- $(a+b)^n + (a-b)^n = 2[{}^nC_0 a^{n-0} b^0 + {}^nC_2 a^{n-2} b^2 + \ldots\ldots]$
- $(a+b)^n - (a-b)^n = 2[{}^nC_1 a^{n-1} b^1 + {}^nC_3 a^{n-3} b^3 + \ldots\ldots]$
- Number of terms in the expansion $(a+b+c)^n$ *is* $\dfrac{(n+1)(n+2)}{2}$.

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. Find the number of terms in the expansion of $(a+b)^{15}$.
2. Find the number of terms in the expansion of $(a^2-b^3)^{27}$.
3. Find the number of terms in the expansion of $(x-\sqrt{y})^{11}(\sqrt{y}+x)^{11}$.
4. Find the number of terms in the expansion of $(p^2+2p+1)^{21}$.
5. Find the number of terms in the expansion of $(p^3-3p^2+3p-1)^{23}$.
6. Find the number of terms in the expansion of $(1-x)^6(1+x^2)^6(1+x)^6$.
7. Find the number of terms in the expansion of $(a+b+c)^{10}$.
8. If number of terms in the expansion of $(2x-3y)^{12}$ is 'a', then find the value of ${}^{a}C_{12}$.
9. If number of terms in the expansion of $(a+4b)^{21}$ is 'n', then find the value of ${}^{n}P_{2}$.
10. Find number of terms in the expansion of $(u+v)^{16}+(u-v)^{16}$.

    [HINT: If n is even, then $(a+b)^n+(a-b)^n$ has $\left(\frac{n}{2}+1\right)$ number of terms]
11. Find number of terms in the expansion of $(p+q)^{24}-(p-q)^{24}$.

    [HINT: If n is even, then $(a+b)^n-(a-b)^n$ has $\frac{n}{2}$ number of terms]
12. Find the number of terms in the expansion of $(x+y)^{29}-(x-y)^{29}$.

    [HINT: If n is odd, then $(a+b)^n+(a-b)^n \, and (a+b)^{(n)}-(a-b)^{(n)}$ both have $\frac{n+1}{2}$ number of terms].

13. Find the number of terms in the expansion of $(u+v)^{2023}+(u-v)^{2023}$.

14. Find sum of all the coefficients in the expansion of $(x+y)^8$.

15. Find sum of all the coefficients in the expansion of $(x^2+y)^5$.

16. Find sum of all the coefficients in the expansion of $(p-q)^{11}$.

17. If the sum of all coefficients in the expansion of $(x+y)^n$ is 256, then find the total number of terms in the expansion of $(x+y)^n$.

18. If a,b,c are distinct,then find the number of distinct terms in the expansion of $(a+b+c)^5+(a+b-c)^5$.
[Hint: For odd n, $(a+b+c)^n+(a+b-c)^n$ has $(n+1)+\ (n-1)+...+2$ number of distinct terms]

19. If a,b,c are distinct,then find the number of distinct terms in the expansion of $(a+b+c)^6+(a+b-c)^6$.
[Hint: For even n, $(a+b+c)^n+(a+b-c)^n$ has $(n+1)+(n-1)+...+3+1$ number of distinct terms]

20. If the largest coefficient in the expansion of $(1+x)^{30}$ is ${}^nC_r$, then find the value of $(n+r)$.

21. If the largest coefficient in the expansion of $(1+x)^{10}$ is x, then find the value of $\frac{5x}{{}^{10}C_4}$.

22. Find the smallest coefficient in the expansion of $(1+x)^{23}$.

23. If p^{th} term is the middle term in the expansion of $(2-3x)^{50}$, then find the value of $\sqrt{p-1}$.

24. If ${}^nC_0+{}^nC_1\times 4+{}^nC_2\times 16+.....+{}^nC_n\times 4^n=r^n$, then find the value of r^3.

25. If ${}^{20}C_0 + {}^{20}C_1 \times 7 + {}^{20}C_2 \times 49 + ... + {}^{20}C_{20} \times 7^{20} = r^{20}$, then find the value of $r^{\frac{1}{3}}$.

26. Find the remainder when $(8^n - 7n), n \in N$ is divided by 49.

27. Find the remainder when $(12^n - 11n), n \in N$ is divided by 121.

28. Find the remainder when $(101^n - 100n - 1), n \in N$ is divided by 10000.

29. Find sum of all the coefficients in the expansion of $(1 - 3x + x^2)^{2023}$.

30. If sum of all the coefficients in the expansion of $(1 + x + x^2)^{2024}$ is a^{1012}, then find the value of ${}^{a}C_8 + {}^{a}C_9$.

ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	16	16	0
2	28	17	9
3	12	18	6+4+2=12
4	43	19	7+5+3+1= 16
5	70	20	45
6	7	21	6
7	66	22	1
8	13	23	5
9	462	24	125
10	9	25	2
11	12	26	1
12	15	27	1
13	1012	28	0
14	256	29	-1
15	32	30	10

CHAPTER – 8
SEQUENCE & SERIES

POINTS TO REMEMBER

- The n[th] term of Arithmetic Progression is written as

  $$\boxed{a_n = a + (n - 1)d}$$

  Where a is the first term and d is the common difference
- If S_n denotes the sum of first n terms of an A.P then

  $$S_n = \frac{n}{2}[a + a_n] = \frac{n}{2}[2a + (n-1)d]$$
- The n^{th} term of the Geometric Progression is given by $a_n = a \times (r)^{n-1}$, where a is the first term & 'r' is the common ratio.
- *Sum* of n-terms of G.P. is given by $\boxed{S_n = \begin{cases} \dfrac{a(r^n - 1)}{r - 1}, When\ r \neq 1 \\ na,\ When\ r = 1 \end{cases}}$
- *Sum* of *Infinite* G.P. is given by $\boxed{S_\infty = \dfrac{a}{1-r}}$, $|r| < 1$
- Arithmetic Mean (A.M.) of two numbers a *and* b is given by $A = \dfrac{a+b}{2}$.
- Geometric Mean (G.M.) of two numbers a *and* b is given by $G = \sqrt{ab}$.
- The $AM - GM$ inequality is given by, $AM \geq GM \Rightarrow \boxed{\dfrac{a+b}{2} \geq \sqrt{ab}}$

  where a & b are two positive numbers

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. If $a, b, 2$ are in A.P. as well as in G.P then find the value of $a^2 + b^3$.
2. If $p, 1, q$ are in A.P. as well as in G.P then find the value of $p^{2023} + q^{2024}$.
3. If the common ratio of the G.P. 2, 8, 32, 128.... is r, then find the value of $\sqrt{4r^2 + 17}$.
4. If the common ratio of the G.P. $\sqrt{3} - 1,\ 1,\ \sqrt{3} + 1, \ldots\ldots$ is r, then find the value of $(r^2 - 2r)$.
5. Find the number of terms in the sequence $2, 7, 12, 17, \ldots\ldots, 202$.
   $\left(\text{HINT: } n = \dfrac{\text{Last term} - \text{First term}}{d} + 1\right)$
6. Find the Minimum value of $(3^{2023-x} + 3^{x-2023})$, where $x \in R$.
7. Find the Minimum value of $(25^{2024-x} + 25^{x-2023})$, where $x \in R$.
8. If $a + b = 2$, where $a, b > 0$ then find the Maximum value of ab.
9. If $0.001, x$ & 100000 are in G.P. then find the positive value of x.
10. Find the 9^{th} term of the sequence: $\frac{1}{9}, \frac{1}{3}, 1, 3, \ldots\ldots$
11. If the 10th term of the sequence 1, 4, 16, 64, ...is x, then find the value of $\sqrt[9]{x}$.
12. Find the value of S, where $S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\infty$.
13. Find the value of S, where $S = 2023^{(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\infty)}$.
14. Find the value of S, where $S = \sqrt{36\sqrt{36\sqrt{36\sqrt{36\ldots\ldots\infty}}}}$

15. If $x = 5 - 3\left(\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots.\infty\right)$, then find the value of $(x^2 + 2^x)$.

16. Find the sum S_{10}, where $S_n = 1 + 2 + 4 + \ \ldots\ldots n\ terms$.

17. If the sum $S_{23} = b(a^{23} - 1)$ such that $S_n = 3 + 9 + 27 + \ \ldots.n\ terms$, then Find the value of $(2b - a)$.

18. How many terms of G.P $4, -8, 16, \ldots$ gives sum 44?

19. If the 2[nd] term of a G.P is -4, then find the product of first 3 terms of the G.P.

20. If the 3[rd] term of a G.P is 2, then find the product of first 5 terms of the G.P.

21. If the n[th] term $(n \in N)$ of a G.P is 1, then find the product of first $(2n - 1)$ terms of G.P.

22. If a, b and c as well as $\log_e a, \log_e b$ and $\log_e c$ are in A.P, then find the value of $(bc - ac)$.

23. If the first term a_1 of G.P is 50 and common ratio is 2, then find the value $\frac{a_8 - a_7}{100}$. (Here, a_n denotes n[th] term of the G.P.)

24. Find the sum of an infinite G.P. with first term 10 and common ratio $\frac{1}{3}$.

25. The sides $a,\ b,\ c$ of a right angled triangle are in increasing A.P. where $HCF(a,b,c) = 1$. Find the value of $(a + b + c)$.

26. If the sides of a right angled triangle are in A.P. and product of sides is 60, then find the largest side.

27. If the sides of a right angled triangle are in A.P. and product of sides is 480, then find the smallest side.

28. Find x, if $(-40) + (-35) + (-30) + \ldots..Upto\ x$ terms $= 0$.

29. Find x, if $(-55)+(-45)+(-35)+\ldots\ldots Upto\ x$ terms $= 0$.

30. Find the value of $\dfrac{5(\text{A.M.})}{13(G.M.)}$ for two numbers 1 and 25.

31. Find the G.M. of 4 and 16.

32. If G.M and A.M. of $a,\ b\ \&\ 4$ are 4 and $\dfrac{14}{3}$ respectively, then find the value of $|b-a|$.

33. If $\dfrac{a}{b}=\dfrac{1}{2}+\dfrac{1}{6}+\dfrac{1}{12}+\dfrac{1}{20}+\ldots.+\dfrac{1}{90}$ such that $a\ \&\ b$ are co-prime numbers then find the value of $(a+b)$.

34. If Sum of n-terms of a series is given by $S_n=3+\dfrac{3}{2}+\dfrac{3}{4}+\ldots+a_n$, then find the value of $\dfrac{1024(S_{10}-S_9)}{3}$.

35. If $G_1:1,5,25,125,\ldots.\ \&\ G_2:1,\dfrac{1}{5},\dfrac{1}{25},\dfrac{1}{125},\ldots.$ be two G.P.'s with t_n $\&\ t'_n$ as the nth term of $G_1\ \&\ G_2$ respectively, then find the value of $t'_{2023}\times t_{2024}$.

36. If $G_1:1,3,9,27,\ldots.\ \&\ G_2:\dfrac{1}{81},\dfrac{1}{27},\dfrac{1}{9},\dfrac{1}{3},\ldots.$ be two G.P.'s with t_n $\&\ t'_n$ as the nth term of $G_1\ \&\ G_2$ respectively,then find the value of k such that $t'_{100}=t_k$.

37. *Find* the value of S, where $S=2^2-1^2+4^2-3^2+\ldots..+100^2-99^2$.

38. If $S=2^2-1^2+4^2-3^2+\ldots..+2024^2-2023^2$, then find the value of $\dfrac{S}{2025}$.

39. If $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$, then find the value of $1^2 + 2^2 + ... + 12^2$.

40. If $1^3 + 2^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2$, then find the value of $\frac{1^3 + 2^3 + ... + 20^3}{1 + 2 + ... + 20}$.

ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	12	21	1
2	2	22	0
3	9	23	32
4	2	24	15
5	41	25	12
6	2	26	5 units
7	10	27	6 units
8	1	28	17
9	10	29	12
10	729	30	1
11	4	31	8
12	2	32	6
13	2023	33	19
14	36	34	2
15	32	35	5
16	1023	36	96
17	0	37	5050
18	5	38	1012
19	-64	39	650
20	32	40	210

CHAPTER – 9
STRAIGHT LINE

POINTS TO REMEMBER

- General Equation of first degree straight line is $ax + by + c = 0$.
  (atleast a or b is non-zero)

- Slope of a line (m)
  - ⊙ passing through two points (x_1, y_1) and (x_2, y_2) is $\dfrac{y_2 - y_1}{x_2 - x_1}$
  - ⊙ making an angle α with positive direction of x-axis is $\tan\alpha$
  - ⊙ with equation $ax + by + c = 0$ is $\dfrac{-a}{b} = \dfrac{-(\text{coefficient of } x)}{(\text{coefficient of } y)}$
  - ⊙ parallel to x-axis is 0
  - ⊙ parallel to y-axis is not defined

- Angle (ϕ) between two lines is given by $\tan\phi = \left|\dfrac{m_1 - m_2}{1 + m_1.m_2}\right|$ *where* m_1 & m_2 are slopes of the lines.
- Two lines are parallel if their slopes are equal i.e. $m_1 = m_2$
- Two lines are perpendicular if product of their slopes is -1, i.e. $m_1.m_2 = -1$
- If three points are collinear, then the slope of the lines formed by any two points are equal. i.e. If A, B and C are collinear points then
  slope of AB = slope of BC = slope of AC.
- The perpendicular distance between a point (x_1, y_1) and line $Ax + By + C = 0$
  is $d = \left|\dfrac{(Ax_1 + By_1 + C)}{\sqrt{A^2 + B^2}}\right|$ *units*.

- Distance 'r' between two parallel lines $Ax+By+c=0$ and $Ax+By+d=0$ is

$$r=\left|\frac{c-d}{\sqrt{A^2+B^2}}\right| \text{ units.}$$

- <u>Various forms of lines:</u>

⊙ Slope Intercept Form:

$\boxed{y=mx+c}$, where m is the slope and c is y-intercept

$\boxed{y = m(x-d)}$, where m is the slope and d is x-intercept

⊙ Slope Point Form:

$y-y_1=m(x-x_1)$, where m is the slope & (x_1, y_1) is the point on the line.

⊙ Two Point Form:

$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$, where (x_1, y_1) & (x_2, y_2) are the points on the line.

⊙ Intercept Form:

$\frac{x}{a}+\frac{y}{b}=1$, where a and b are x-intercept & y-intercept respectively

⊙ Normal Form: $x\cos\alpha+y\sin\alpha=p$, where p is the perpendicular distance of line from the origin & α is the angle made by the perpendicular line with x-axis at origin.

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. Find the slope of a line passing through points (2, 9) and (9, 23).
2. Find the slope of the line $6x-3y+7=0$.
3. If (1, 2), (3, 4) and (a, 7) are collinear points, then find the value of a.

4. Find the slope of a line making an angle of 30^0 with the positive direction of $x-$axis.

5. Find k, if the points $(8, 1)$, $(k, -4)$ and $(2, -5)$ are collinear.

6. If the Lines $2x-y=3$ and $ax+by=9$ are parallel, then find the value of $4b^2+a^2+4ab$.

7. Find the distance between the lines $3x-4y=5$ and $3x-4y+5=0$.

8. Find the distance between the lines $5x+12y=24$ and $10x+24y+4=0$.

9. Find the Minimum distance of a point (1, 2) from the line $7x+24y=5$.

10. If perpendicular distance of $(2, k)$ from the line $x+y=2$ is 5 units, then find the sum of all possible values of k.

11. If perpendicular distance of $(k, 2)$ from the line $3x+4y-14=0$ is 3 units,then find the sum of all possible values of k.

12. If the point $(2,3)$ lies on the line $ax+y=6$, then find the value of $(3-2a)$.

13. Find the y-intercept of the line $4x-3y+15=0$.

14. Find the x-intercept of the line $-2x+3y+6=0$.

15. Find the sum of x-intercept and y-intercept of the line

$$\frac{x}{2}+\frac{y}{4}=3.$$

16. Find the acute angle between the lines $y-\sqrt{3}x=1$ and $\sqrt{3}y-x=5$. (HINT: Visual/ Graphical Thinking)

17. Find the obtuse angle between the lines $x-y=0$ and $x=0$. (HINT: Visual/ Graphical Thinking)

18. Find the value of y if the line passing through the points $(3, y)$ and $(2, 7)$ is parallel to the line passing through the points $(-1, 4)$ and $(0, 6)$.

19. If the line $6x-y+2+k(2x+3y+13)=0$ is parallel to x-axis, then find the value of k.

20. If the line $3x+6y+k(2x-y+19)=0$ is parallel to y-axis, then find the value of k.

21. If the medians AD and BE of a triangle with vertices $A\ (0,q), B\ (0,0)$ and $C\ (p,0)$ are perpendicular to each other, then find the value of $\frac{p^2}{q^2}$.

22. Find equation of the line passing through the point (2,2) and parallel to the line $x-y=5$.

23. Find the equation of a line passing through the point (3,3) and perpendicular to the line $x-y=4$.

24. Find equation of the line having y-intercept as -3 and tangent of the angle with positive direction of x-axis as 2.

25. Find equation of the straight line passing through $(-2, 0)$ and making a triangle of area 16 sq.units with coordinate axes in second quadrant.

26. If the equation of base of an equilateral triangle is $3x-4y=4$ and the vertex is $(-1, 2)$, then find the length of the side of the triangle.

27. If the equation of base of an equilateral triangle is $5x+12y+3=0$ and the vertex is $(0, 3)$, then find the Area of equilateral triangle.

28. The line $x-7y+42=0$ intersects y-axis at $P(a,b)$ and line $6x+y=12$ intersects x-axis at Q(c,d). Find the value of $(a+b+c+d)$.

29. Find the angle between positive direction of x - axis & the line joining $(2, 3)$&$(3,2)$ measured anticlockwise.

30. If C is the centroid of the triangle with vertices $(3,-1),(1,3)$&$(2, 4)$, then find the slope of a line passing through point C and origin.

31. Let $A(1,3)$, $B(2,5)$ and $C(3,1)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB & BPC have equal areas, then find the slope of the line PQ, where $Q(5,6)$.

32. If $L_1 : 3x+4y=7$ and $L_2 : x-y=0$ intersect at A, then find the equation of a line Passing through A and Perpendicular to L_1.

33. If $L_1 : 4x-3y=1$ and $L_2 : x+y=2$ intersect at A, then find the equation of a line Passing through A and Perpendicular to L_2.

34. A ray of light along $x+y=2$ gets reflected by x - axis, then find the equation of reflected ray.

35. A ray of light along $\sqrt{3}x+y=2\sqrt{3}$ gets reflected by x - axis, then find the equation of reflected ray.

36. Find the angle between the lines $x+y=2023$ and $x-y=2024$.

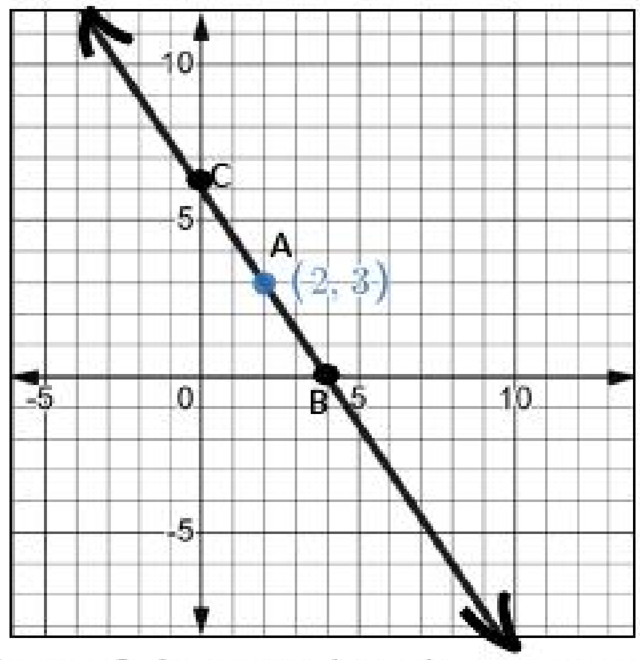
37. If three lines $x-3y=1$, $ax+2y=7$ and $ax+y=0$ forms a right-angled triangle then find the sum of all possible values of a.

38. If x - intercept of a Line L is double that of line $3x+12y+12=0$ and y - intercept of a Line L is half that of line $4x+3y=12$, then find the value of 4m where m is the slope of line L.

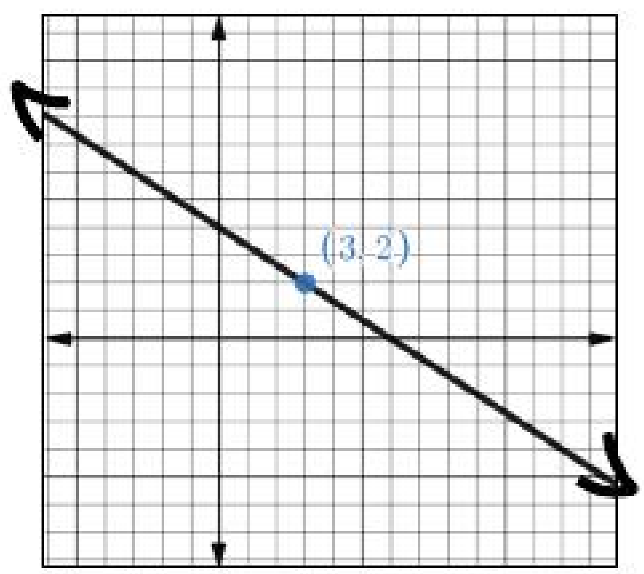
39. Find the Slope of a line which is perpendicular to line $x-3y=12$.

40. Find the distance of (0, 0) from $x+4y=5$ along the line $x-y=0$.

41. If $A(2,3)$ is the mid-point of the portion intercepted between coordinate axes, then find the Slope of the line passing through B and C.



42. If $A(3,2)$ is the mid-point of the portion intercepted between coordinate axes, then find the Area of the triangular region formed by the line and the coordinate axes.



(For Q. 43 - 44) Consider three lines L_1, L_2 and L_3 such that $L_1 \| L_2$ and $L_1 \perp L_3$.

43. Find the Equation of L_2.

44. If the Equation of L_3 is $ax + by = 1$, then find the value of $(a - b)$.

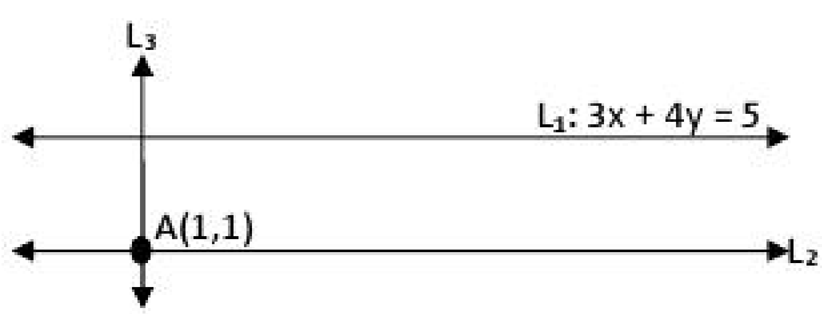


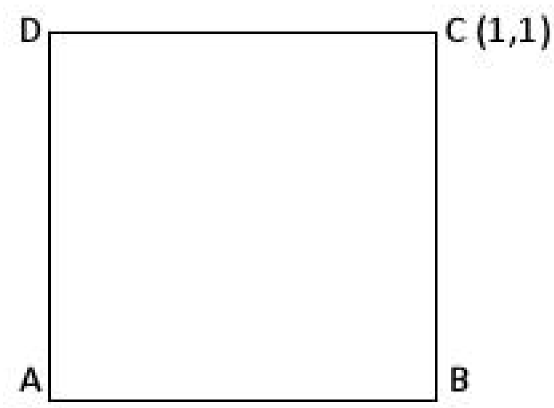
FIGURE FOR Q. 43 & 44

(For Q. 45 - 46)

$ABCD$ is a square where Equation of AB is $3x + 4y = 2$.

45. Find the Equation of CD.

46. Find the area of Square ABCD.



(For Q. 47 - 48)

$ABCD$ is a Rectangle where Equation of AB is $5x + 12y = 0$.

47. Find the Equation of AD.

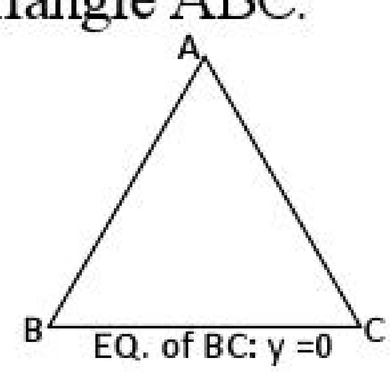
48. Find the Equation(s) of CD if Breadth of rectangle (AD) is 4 units.

(For Q. 49 - 50)

ABC is an Equilateral Triangle with Base BC & Vertex A $(2, 2\sqrt{3})$.

49. Find the length of altitude AD.

50. Find the Perimeter of the triangle ABC.



ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	2	26	$2\sqrt{3}$ *units*
2	2	27	$3\sqrt{3}$ *sq. units*
3	6	28	8
4	$\frac{1}{\sqrt{3}}$	29	135^0
5	3	30	1
6	0	31	1
7	2 units	32	$4x-3y=1$
8	2 units	33	$x-y=0$
9	2 units	34	$x-y=2$
10	0	35	$\sqrt{3}x-y=2\sqrt{3}$
11	4	36	90°
12	0	37	9
13	5	38	1
14	3	39	-3
15	18	40	$\sqrt{2}$ units
16	30°	41	$\frac{-3}{2}$
17	135°	42	12 sq. units
18	9	43	$3x+4y=7$
19	-3	44	7
20	6	45	$3x+4y=7$
21	2	46	1 sq. unit
22	$x-y=0$	47	$12x-5y=0$
23	$x+y=6$	48	$5x+12y\pm 52=0$
24	$2x-y=3$	49	$2\sqrt{3}$ units
25	$-8x+y=16$	50	12 units

CHAPTER – 10

CONIC SECTIONS

POINTS TO REMEMBER

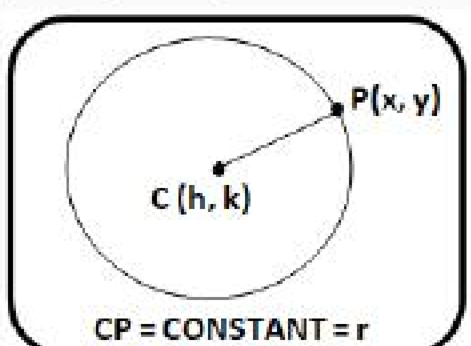
- The curves obtained by slicing the cone with a plane not passing through the vertex are called conic sections or simply conics.
- Circle, ellipse, parabola and hyperbola are curves which are obtained by intersection of a plane and a cone in different positions.
- A conic is the locus of a point which moves in a plane, so that its distance from a fixed point always bears a constant ratio to its distance from a fixed straight line.

  The fixed point is called focus, the fixed straight line is called directrix, and the constant ratio is called eccentricity, which is denoted by 'e'.

  For Circle, $e = 0$ | For Parabola, $e = 1$, | For Ellipse, $e < 1$, | For Hyperbola, $e > 1$

- Circle: It is the set of all points in a plane that are equidistant from a fixed point in that plane

  Equation of circle: $(x-h)^2+(y-k)^2=r^2$ where Centre $=(h, k)$, radius $= r$



- Parabola: It is the set of all points in a plane which are equidistant from a fixed point (focus) and a fixed line (directrix) in the plane. The fixed point does not lie on the fixed line. Latus Rectum of a parabola is a chord through focus perpendicular to axis of parabola.

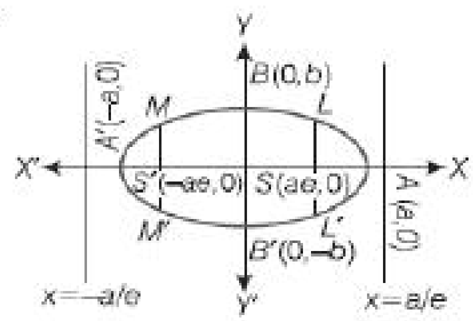
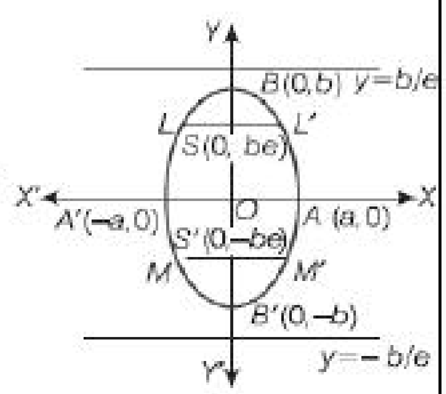
NOTE: In the standard equation of parabola, a > 0.

	$y^2 = 4ax$ Parabola towards right	$y^2 = -4ax$ Parabola towards left	$x^2 = 4ay$ Parabola opening upwards	$x^2 = -4ay$ Parabola opening downwards
Vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Equation of axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Equation of directrix	$x + a = 0$	$x - a = 0$	$y + a = 0$	$y - a = 0$
Length of latus rectum	$4a$	$4a$	$4a$	$4a$

Ellipse: It is the set of points in a plane the sum of whose distances from two fixed points in the plane is a constant and is always greater than the distances between the two fixed points.

Latus rectum of Ellipse: Chord through foci perpendicular to major axis is called latus rectum.

NOTE: If e = 0 for an ellipse then b = a and equation of ellipse reduces to equation of a circle. Its eq. will be $x^2 + y^2 = a^2$.

Standard equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ (a > b)$ (Horizontal form of an ellipse)	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ (a < b)$ (Vertical form of an ellipse)
Shape of the ellipse		
Centre	(0, 0)	(0, 0)
Equation of major axis	$y = 0$	$x = 0$
Equation of minor axis	$x = 0$	$y = 0$
Length of major axis	$2a$	$2b$
Length of minor axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Equation of directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{b}{e}$
Eccentricity	$e = \sqrt{\frac{a^2 - b^2}{a^2}}$	$e = \sqrt{\frac{b^2 - a^2}{b^2}}$
Length of latusrectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

Hyperbola: It is the set of all points in a plane, the difference of whose distances from two fixed points located in the plane is a constant.

	Hyperbola	**Conjugate hyperbola**
Standard equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{-x^2}{a^2}+\frac{y^2}{b^2}=1$ or $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$
Centre	(0, 0)	(0, 0)
Equation of transverse axis	$y = 0$	$x = 0$
Equation of conjugate axis	$x = 0$	$y = 0$
Length of transverse axis	$2a$	$2b$
Length of conjugate axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{b}{e}$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Eccentricity	$e = \sqrt{\frac{a^2+b^2}{a^2}}$	$e = \sqrt{\frac{a^2+b^2}{b^2}}$
Length of latusrectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

Latus Rectum:Chord through foci perpendicular to transverse axis is called latus rectum.

If e $=\sqrt{2}$ for a hyperbola, then hyperbola is called rectangular or equilateral hyperbola.

For e $=\sqrt{2} \Rightarrow b=a$ and eq. of this hyperbola will be $x^2-y^2=a^2 or\ y^2-x^2=a^2$.

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. Find radius of the circle $x^2+y^2+2x-4y-4=0$.
2. If the coordinates of centre the circle $x^2+y^2+2x-4y-4=0$ are (a,b), then find the value of $2a+b$.
3. Find the eccentricity of the parabola $y^2=16x$.
4. If the eccentricities of the parabolas $y^2=16x$ and $x^2=4y$ is p &q respectively,then find the value of p^q.
5. If radius of the circle $3x^2+3y^2+6x-12y\ -15=0$ be r units, then find the value of r^4.
6. If the equation of a point circle is $x^2+y^2+2px+8y+25=0$, then find the positive value of p.(A point circle is a circle with radius zero unit)
7. Let 'r' be the radius of a point circle. Find the value of $\sqrt{r^2+r+4}$.
8. Find centre of the circle $2x^2+2y^2+4x-8y-4=0$.
9. Find equation of a circle, if end points of one of its diameter are $(-2,3)$ and $(0,-1)$.
10. If parabola $y^2=px$ passes through the point $(3,6)$, then find the value of p.
11. If parabola $x^2=py$ passes through the point $(4,2)$, then find the length of latus rectum of the parabola.
12. Find equation of the circle which passes through the point $(3,2)$ and has its centre at $(2,3)$.
13. Find the value of 5k, if k denotes the length of latus rectum of an ellipse whose foci are $(0,\pm 3)$ and length of minor axis is 8 units.

14. If one of the foci of the ellipse $9x^2 + 16y^2 = 144$ is (a,b), then find the value of $2a^2 + 3b^2$.

15. If one of the foci of the ellipse $4x^2 + y^2 = 4$ is (a,b), then find the value of $3a^2 + 5b^2$.

16. If length of the latus rectum of the ellipse $2x^2 + y^2 = 4$ is 'p' units, then find the value of p^4.

17. Find equation of the circle which touches both the coordinate axes in first quadrant and has radius equal to 2 units.

18. Find equation of the circle which touches both the coordinate axes in Second quadrant and has radius equal to 1 units.

19. A circle C touches both the coordinate axes in Third quadrant and has radius equal to positive root of $x^2 - 2x - 24 = 0$. Find centre of the circle C.

20. Find equation(s) of the circle which touches the lines $x = 0$, $y = 0$ and $x = 10$.

21. If eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is 'e' then the value of $\sqrt{10e+1}$.

22. If eccentricity of the Hyperbola $x^2 - 25y^2 = 25$ is 'e' then the value of $\sqrt{25e^2 - 1}$.

23. If length of the latus rectum of parabola $3y^2 = kx$ is 8 units, then find the value of k.

24. Find area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 16y$ to the end points of its latus rectum and the latus rectum.

25. If eccentricity of the hyperbola passing through (4, 0) is $\sqrt{2}$,then find the equation of hyperbola.

26. If an ellipse has vertices $(0, \pm 5)$ and foci $(0, \pm 4)$,then find the value of $(5e-1)$ where 'e' denotes the eccentricity of the ellipse.

27. If an ellipse has vertices $(\pm 13, 0)$ and foci $(\pm 12, 0)$,then find the value of $\sqrt{\frac{13p}{2}}$ where 'p' denotes length of the latus rectum of the ellipse.

28. Find the distance between the directrices to the ellipse $5x^2+9y^2=180$.

(HINT: Distance between the directrices $=\frac{2a}{e}$)

29. Find coordinates of the points on parabola $y^2=16x$ whose focal distance is 8 units.

30. An equilateral Triangle ABC with side 12 cm is circumscribed by a circle.If area of the Circumcircle is $K\pi$ cm^2,then find the value of K.

31. If one end of a diameter of the circle $2x^2+2y^2-8x-12y+22=0$ is $(1, 2)$,then find coordinates of other end of the diameter.

32. If end points of a diameter of the circle $x^2+y^2+ax+by+c=0$ are $(1, 2)$ and (3,4) ,then find the value of $(a+b)$.

33. Find equation of the set of all points difference of whose distances from the points $(5,0)$ & $(-5,0)$ is always equal of 8 units.

34. Find the equation of the set of all points sum of whose distances from the points $(0, 2\sqrt{5})$ & $(0, -2\sqrt{5})$ is always equal of 12 units.

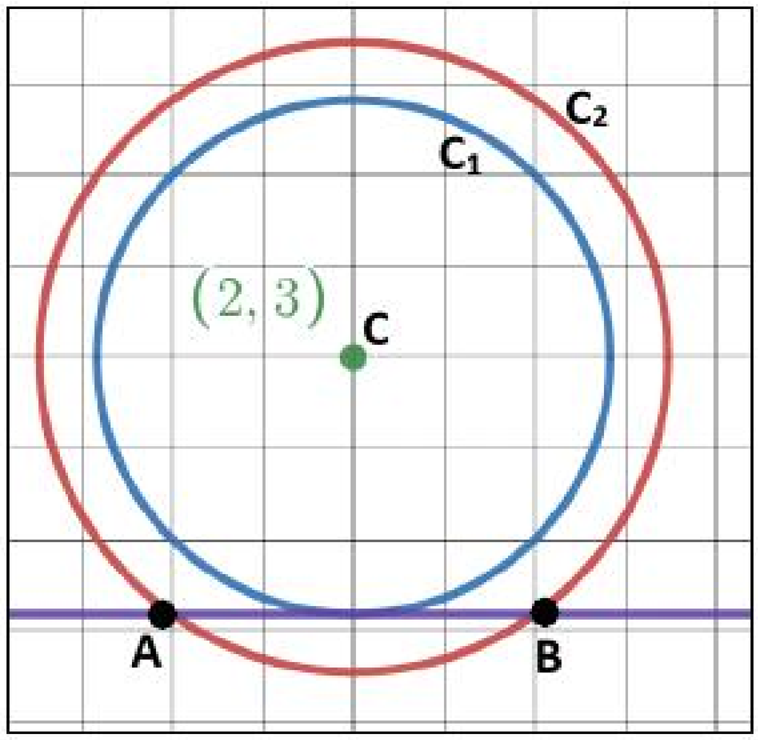
35. Find the equation of a circle with centre $(0, 0)$ and tangent $3x+4y=10$.

FOR Q.36 – 37

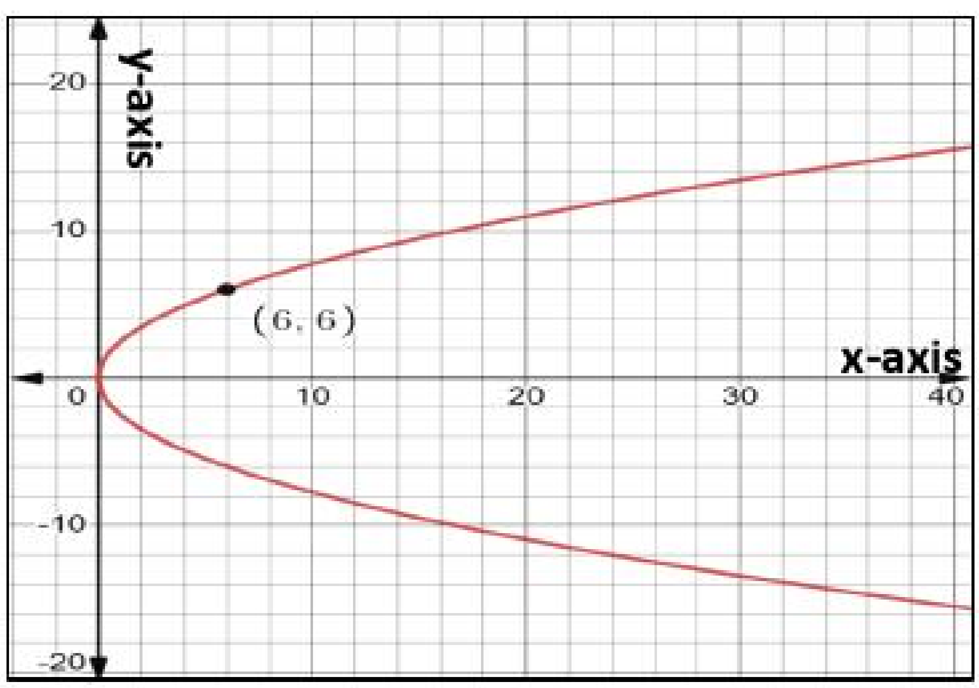
Let C_1 and C_2 be two concentric circles with centre at (2,3). If $5x + 12y = 7$ is tangent to circle C_1.

36. Find the equation of C_1.

37. Find the equation of C_2 if $AB = 2\sqrt{7}$ units.



38. In the given figure origin is the vertex of the parabola. If (6,6) lies on the standard parabola, then find length of the latus rectum.

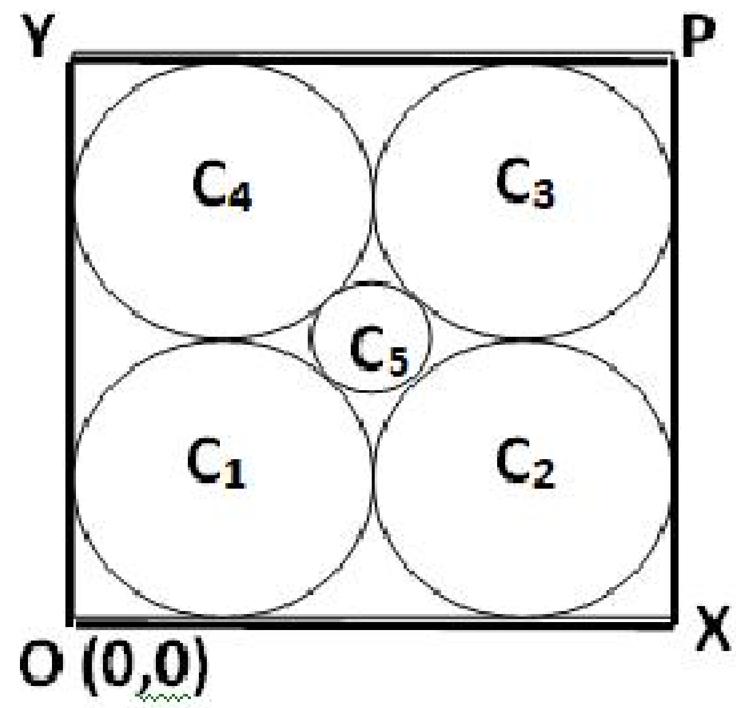


FOR Q.39-40

If OXPY is a square in first quadrant with side 8 cm, where O is the origin. (OY and OX represent y-axis & x-axis respectively).

39. If coordinates of centre of circle C_5 are (a,b), then find the value of $a \times b$.

40. Find the radius of circle C_5.



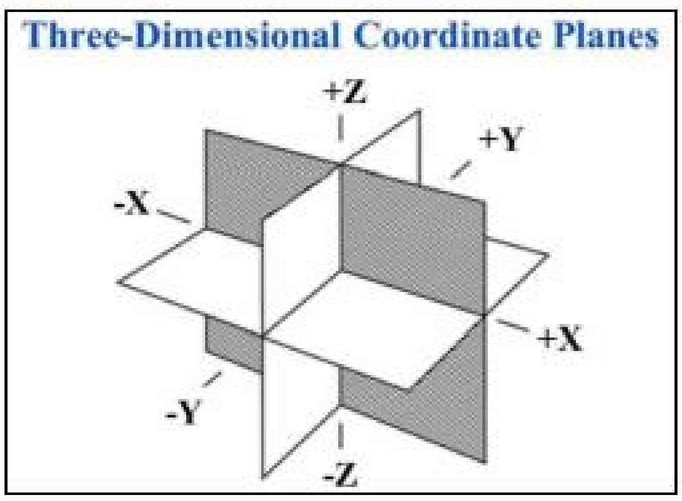
ANSWERS

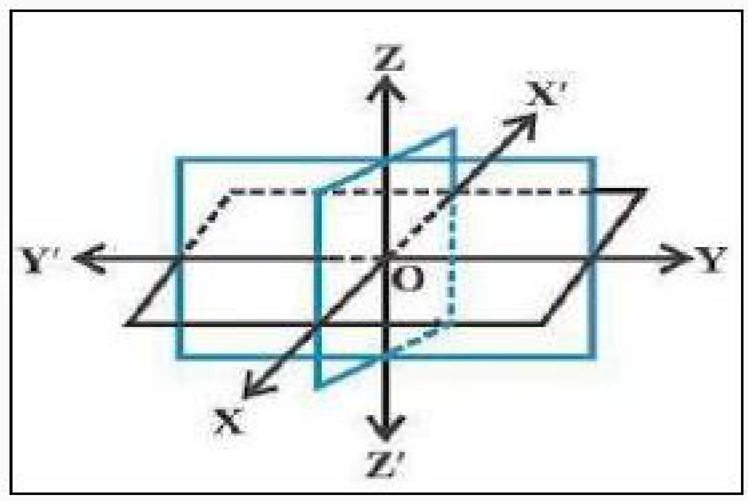
Q. No.	ANSWER	Q. No.	ANSWER
1	3 units	21	3
2	0	22	5
3	1	23	24
4	1	24	32 sq units
5	100	25	$x^2 - y^2 = 16$
6	3	26	3
7	2	27	5
8	$(-1,2)$	28	18
9	$x^2 + y^2 + 2x - 2y - 3 = 0$	29	$(4, \pm 8)$
10	12	30	48
11	8	31	(3, 4)
12	$x^2 + y^2 - 4x - 6y + 11 = 0$	32	-10
13	32	33	$\frac{x^2}{16} - \frac{y^2}{9} = 1$
14	14	34	$\frac{x^2}{16} + \frac{y^2}{36} = 1$
15	15	35	$x^2 + y^2 = 4$
16	16	36	$(x-2)^2 + (y-3)^2 = 9$
17	$(x-2)^2 + (y-2)^2 = 4$	37	$(x-2)^2 + (y-3)^2 = 16$
18	$(x+1)^2 + (y-1)^2 = 1$	38	6 units
19	$(-6,-6)$	39	16
20	$x^2 + y^2 - 10x \pm 10y + 25 = 0$	40	$2(\sqrt{2}-1)$ units

CHAPTER – 11
INTRODUCTION TO THREE-DIMENSIONAL GEOMETRY

POINTS TO REMEMBER

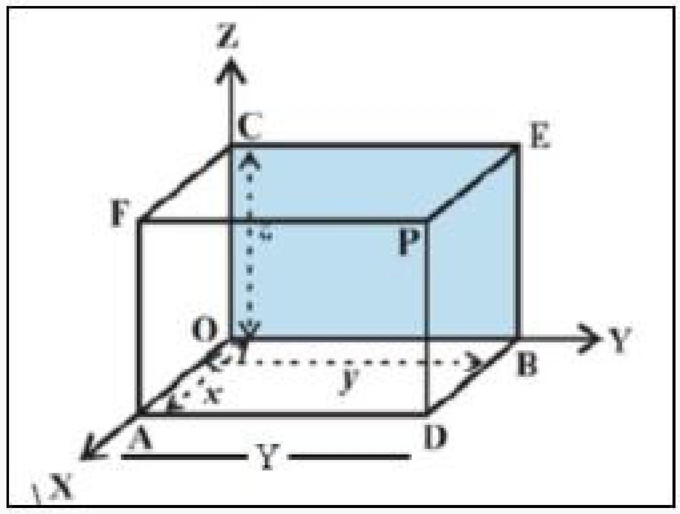
- Three mutually perpendicular lines in space define three mutually perpendicular planes, called Coordinate planes, which in turn divide the space into eight parts known as octants and the lines are known as Coordinate axes.





- Coordinate axes: XOX', YOY', ZOZ'
- Coordinate planes: XOY, YOZ, ZOX or XY, YZ, ZX planes
- Octants:
  OXYZ, OX'YZ, OXY'Z, OXYZ', OX'Y'Z, OXY'Z', OX'YZ', OX'Y'Z'
- Coordinates of a point lying on x-axis, y-axis & z-axis are of the form $(x,0,0)$, $(0,y,0)$ and $(0,0,z)$ respectively.
- Coordinates of a point lying on xy-plane, yz-plane, & xz-plane are of the form $(x, y, 0)$, $(0, y, z)$ and $(x, 0, z)$ respectively.
- The reflection of point (x, y, z) in xy-plane, yz-plane and xz-plane is $(x, y, -z)$, $(-x, y, z)$ and $(x, -y, z)$ respectively.

- Absolute value of the Coordinates of a point P (x, y, z) represents the perpendicular distances of point P from three coordinate planes YZ, ZX *and* XY respectively.



- The distance between the point $A(x_1, y_1, z_1)$ *and* $B(x_2, y_2, z_2)$ is given by $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ units

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. Find the perpendicular distance of the point $P(-5,12,7)$ from z-axis.
2. Find the perpendicular distance of the point $P(3,12,-4)$ from y-axis.
3. Find the perpendicular distance of the point $P(-5,-6,-8)$ from x-axis.
4. Find the perpendicular distance of the point $P(-5,-6,-8)$ from xy-Plane.
5. Find the perpendicular distance of the point $P(-1,2,-8)$ from yz-Plane.
6. Find the perpendicular distance of the point $P(-3,-6,-7)$ from xz-Plane.
7. If the point $P(1, 2, -3)$ lies in the p^{th} octant, then find the value of p.

8. If the points $P(5,-2,3)$ and $Q(-1,-4,-3)$ lies in the p^{th} and q^{th} octant respectively, then find the value of $(p+q)$.

9. Find the distance between the points A $(\cos 10^0, -\sin 10^0, 3)$ and $B(3\cos 10^0, \sin 10^0, 3)$.

10. Find the distance between the points P $(\sin 17^0, -\cos 17^0, 4\sqrt{14})$ and $B(\cos 17^0, \sin 17^0, 3\sqrt{14})$.

11. Find the value of $\sqrt{p^2+14}$, if the point $(3, 4, 5)$ is at a distance of 'p' unit from the origin $(0, 0, 0)$.

12. Find the coordinates of the image of point $A(3, 6, 4)$ in xz-Plane.

13. Find the coordinates of the image of point $B(-3, 6, 4)$ in yz-Plane.

14. Find the coordinates of the image of point $C(-1, -2, 5)$ in xy-Plane.

15. If the coordinates of the image of point $D(5, -1, -3)$ in xz-Plane is $P(a, b, c)$, then find the value of $(a+b+c)$.

16. Find the coordinates of the image of point $P(3, 4, 5)$ along *x-axis*.

17. Find the coordinates of the image of point $Q(5, -12, -13)$ along *y-axis*.

18. Find the coordinates of the image of point $R(-7, 24, -25)$ along *z-axis*.

19. If the coordinates of the image of point $S(-9, -2, 11)$ along *x-axis* is $P(a, b, c)$, then find the value of $|a+b+c|$.

20. Find the length of the foot of perpendicular drawn from the point $M(3, 4, -7)$ on z-axis.

21. If the distance between the points $(p, -2, 2023)$ and $(1, 1, 2023)$ is 5, then find the sum of all possible values of p.

22. Let A be the foot of perpendicular from point $P(5, -2, -3)$ on the xy-plane. Find the coordinates of A.

23. If the image of P$(a,\ b,\ c)$ in YZ-plane is Q $(1, 2, -3)$, then find the value of $(a+b-c)$.

24. Find the area of the square ABCD, where coordinates of its diagonal AC are A(2, 3, 4) and C$(-6, 9, 14)$.

25. Find the area of the square ABCD, where coordinates of its side AB are A(2, 3, 4) and B$(-6, 9, 4)$.

26. Find the point on x-axis which is equidistant from the points P $(3, -5, 2)$ and Q $(5, 5, -2)$.

27. Find the point on y-axis which is equidistant from the points A (3, 5, 5) and B (5, 9, 3).

28. Find the point on z-axis which is equidistant from the points M $(1, -2, 7)$ and N $(2, -1, 13)$.

29. Find the point on y-axis which is at a distance of 10 units from the point P $(-8, 2, -6)$.

30. Find the point on z-axis which is at a distance of 5 units from the point A $(-3, -4, 2024)$.

ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	13 units	16	(3,-4,-5)
2	5 units	17	(-5,-12,13)
3	10 units	18	(7,-24,-25)
4	8 units	19	18
5	1 unit	20	5 units
6	6 units	21	2
7	5	22	(5,-2,0)
8	11	23	4
9	2 units	24	100 sq units
10	4 units	25	100 sq units
11	8	26	(4,0,0)
12	(-3,-6,4)	27	(0,7,0)
13	(3,6,4)	28	(0,0,10)
14	(-1,-2,-5)	29	(0,2,0)
15	3	30	(0,0,2024)

CHAPTER – 12

LIMITS AND DERIVATIVES

POINTS TO REMEMBER

- For any two functions $f(x)$ and $g(x)$, the following properties hold:

$$(a)\lim_{x\to a}\left[f(x)\pm g(x)\right]=\lim_{x\to a}\left[f(x)\right]\pm\lim_{x\to a}\left[g(x)\right]$$

$$(b)\lim_{x\to a}\left[f(x).g(x)\right]=\lim_{x\to a}\left[f(x)\right].\lim_{x\to a}\left[g(x)\right]$$

$$(c)\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to a}\left[f(x)\right]}{\lim_{x\to a}\left[g(x)\right]},\lim_{x\to a}\left[g(x)\right]\neq 0$$

- Some standard limits

(a) $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1$	(b) $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1$	(c) $\lim_{x\to 0}\left(\cos x\right)=1$
(d) $\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)=1$	(e) $\lim_{x\to 0}\left(\frac{1-\cos x}{x}\right)=0$	(f) $\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right)=n.a^{n-1}$

- The value of derivative of $f(x)$ at a point $'a'$ is given by

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

- Some standard results:

(a) $\frac{d}{dx}\left(x^n\right)=n.x^{n-1}$	(b) $\frac{d}{dx}(\sin x)=\cos x$	(c) $\frac{d}{dx}(\cos x)=-\sin x$
(d) $\frac{d}{dx}(\tan x)=\sec^2 x$	(e) $\frac{d}{dx}(\cot x)=-\text{cosec}^2 x$	(f) $\frac{d}{dx}(\sec x)=\sec x.\tan x$
(g) $\frac{d}{dx}(\text{cosec}x)=-\text{cosec}x.\cot x$		(h) $\frac{d}{dx}(\log x)=\frac{1}{x}, x>0$

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. Find the value of L, where $L = \lim_{x\to 0}\left(\frac{\sin 10x}{\tan 5x}\right)$.

2. Find the value of L, where $L = \lim_{x\to 0}\left(\frac{2023x+\sin x}{\tan x}\right)$.

3. Find the value of L, where $L = \lim_{x\to 0}\left(\frac{1-\cos(2023x)}{x}\right)$.

4. Find the value of L, where $L = \lim_{x\to 0}\left(\frac{1-\cos 8x}{x^2}\right)$.

5. Find the value of L, where $L = \lim_{x\to 0}\left(\frac{1-\cos 8x}{1-\cos 4x}\right)$.

6. Find the value of L, where $L = \lim_{x\to 1}\left(\frac{x^{2024}-1}{x-1}\right)$.

7. Evaluate: $L = \lim_{x\to 1}\left(\frac{(x^{2024}+x^{2023}+....x^1)-2024}{2025(x-1)}\right)$.

8. Evaluate: $L = \lim_{x\to 1}\left(\frac{x^3-2x+1}{x-1}\right)$.

9. If $L = \lim_{x\to \frac{\pi}{4}}\left(\frac{\cos x-\sin x}{\frac{\pi}{4}-x}\right)$, then find the value of $\sqrt{L^2+2}$.

10. If $L = \lim_{x\to \frac{\pi}{6}}\left(\frac{\sqrt{3}\sin x-\cos x}{x-\frac{\pi}{6}}\right)$, then find the value of L^L.

11. Find the value of L, where $L = \lim_{x\to 0}(\sin 8x.\cos 4x.\cot 2x)$.

12. If $L = \lim_{x\to 0}\left(\frac{\sqrt{2024+x}-\sqrt{2024-x}}{x}\right)$, then find the value of $\frac{1}{L^2}$.

13. Find the value of L, where $L = \lim_{x\to 0}\left(\frac{12x}{x+\sin 2x+\tan 3x}\right)$.

14. Find the value of L, where $L = \lim_{x\to \infty}\left(x.\sin\left(\frac{2024}{x}\right)\right)$.

15. If $f(x) = \sin^2(\cos 5x) + [\cos(\cos 5x)]^2$, then find the value of $f'(x)$.
[Hint: $\sin^2\alpha + \cos^2\alpha = 1$]

16. If $f(x) = \tan^2(\sin(\cos x)) - [\sec(\sin(\cos x))]^2$, then find the value of $f'(x)$.
[Hint: $\sec^2\alpha - \tan^2\alpha = 1$]

17. If $y = \left(\frac{3tan\frac{x}{3} - tan^3\frac{x}{3}}{1-3tan^2\frac{x}{3}}\right)$, then find $\frac{dy}{dx}$ when $x = \frac{\pi}{4}$.

18. If $y = \left(\frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}\right)$, then find $\frac{dy}{dx}$ when $x = \frac{\pi}{2}$.

19. If $y = \left(\frac{\tan^2 5x - \tan^2 4x}{1-\tan^2 5x.\tan^2 4x}\right).\cot 9x$, then find $\frac{dy}{dx}$ when $x = \frac{\pi}{4}$.

20. If $y = (x^{32}+1)(x^{16}+1)(x^8+1)(x^4+1)(x^2+1)(x^2-1)$, then find $\frac{dy}{dx}$ at $x = 1$.

21. If $y = (x^{2023}+x^{2022}+x^{2021}+......+1)(x-1)$, then find $\frac{dy}{dx}$ at $x = 2^{\frac{1}{2023}}$.

22. If $f(x)=\dfrac{x+2022}{x+2024}$, then find $f'(x)$ at $x=-2023$.

23. If $f(x)={}^{99}C_x+{}^{99}C_{x+1}-{}^{100}C_{x+1}$, where $x\in(0,98)$ then find the value of $f'(x)$.

24. Find the value of L, where $L=\lim\limits_{x\to 0}\left(\dfrac{\sin 3x+2\sin 2x-\sin x}{x}\right)$.

25. Find the value of L, where $L=\lim\limits_{x\to 0}\left(\dfrac{\sin 3x-2\sin 2x+\sin x}{x^2}\right)$.

26. Find the value of L, where $L=\lim\limits_{x\to 0}\left(\dfrac{\sin 3x-2\sin 2x+\sin x}{x^3}\right)$.

27. If $\lim\limits_{x\to 2}\dfrac{x^n-2^n}{x-2}=80$, then find the value of $\sqrt[3]{5n+2}$.

28. If $L=\lim\limits_{x\to 0}\dfrac{(1+x)^{16}-1}{(1+x)^4-1}$, then find the value of $(L-3)$.

29. Find the value of $L=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{4\sec^2 x-8}{\tan x-1}$.

30. If $y=\dfrac{2024-\cos^2(x^3+x^2+x+1)-\sin^2(1+x+x^2+x^3)}{5}$, then find $\dfrac{dy}{dx}$.

ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	2	16	0
2	2024	17	2
3	0	18	-1
4	32	19	2
5	4	20	64
6	2024	21	4048
7	1012	22	2
8	1	23	0
9	2	24	6
10	4	25	0
11	4	26	-2
12	2024	27	3
13	2	28	1
14	2024	29	8
15	0	30	0

CHAPTER – 13

STATISTICS

POINTS TO REMEMBER

- Range of ungrouped data and discrete frequency distribution

  $\boxed{\text{Range} = \text{Largest observation} - \text{smallest observation}}$

- Range of Continuous Frequency Distribution

  $\boxed{\text{Range} = \text{Upper limit of highest class} - \text{lower limit of lowest class}}$

- Mean deviation for ungrouped data or raw data

  Mean deviation about mean $= \dfrac{\sum |x_i - \bar{x}|}{n}$, where $\bar{x}$ is the Mean

  Mean deviation about median $= \dfrac{\sum |x_i - M|}{n}$, where M is the Median

- Mean deviation for grouped data

  (Discrete frequency distribution and continuous frequency distribution)

  Mean deviation about mean $= \dfrac{\sum f_i |x_i - \bar{x}|}{N}$, where $\bar{x}$ is the Mean

  Mean deviation about median $= \dfrac{\sum f_i |x_i - M|}{N}$, where M is the Median

  Note: $N = \sum f_i$

- Variance is defined as the mean of the squares of the deviations of observations from mean.

- Standard Deviation 'σ' is positive square root of variance. $\Rightarrow$ $\boxed{\sigma = \sqrt{\text{Variance}}}$

- Variance and Standard Deviation (SD) for ungrouped data

  $$\sigma^2 = \frac{1}{n}\sum (x_i - \bar{x})^2 \Rightarrow S.D. = \sigma = \sqrt{\frac{1}{n}\sum (x_i - \bar{x})^2}$$

- Standard deviation of a discrete frequency distribution

$$\text{S.D.} = \sigma = \sqrt{\frac{1}{n}\sum f_i(x_i - \bar{x})^2} = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - \left(\sum f_i x_i\right)^2}$$

- Short cut method to find variance and standard deviation

$$\text{Variance} = \sigma^2 = \frac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right]$$

$$\text{S.D.} = \sigma = \frac{h}{N}\sqrt{N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2}, \text{ where: } y_i = \frac{x_i - A}{h}$$

- If each observation is multiplied by a positive constant k then variance of the resulting observations becomes k^2 times of the original value and standard deviation becomes k times of the original value.
- If each observation is increased by k, where k is positive or negative, then variance and standard deviation remains same.
- Standard deviation is independent of choice of origin but depends on the scale of measurement.
- The mean of first 'n' natural number is $\frac{n+1}{2}$.
- The mean of first 'n' even natural numbers is $(n+1)$.
- The mean of first 'n' odd natural numbers is n.

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. Find the mean of first nine natural number.
2. Find the mean of first 101 whole number.
3. If the mean of first 10 even natural numbers is 'm', then find the value of $\sqrt[3]{m-3}$.

4. If the variance of n observations is 42.25, then find the standard deviation.
5. If the standard deviation of n observations is 4.5, then find the variance.
6. If the mean of first 10 odd natural numbers is 'n', then find the value of $\sqrt{10n}$.
7. If the mean of a, b, c, d and e is 2024, then find the mean of $\frac{a}{4}, \frac{b}{4}, \frac{c}{4}, \frac{d}{4}$ and $\frac{e}{4}$.
8. If the variance of a, b, c, d and e is 16, then find the variance of $\frac{a}{4}, \frac{b}{4}, \frac{c}{4}, \frac{d}{4}$ and $\frac{e}{4}$.
9. If the variance of a, b, c, d, e and f is 25, then find the standard deviation of $3a, 3b, 3c, 3d$ and $3e$.
10. If the variance of a, b, c and d is 9, then find the value of $\frac{\text{standard deviation of } a-2024, b-2024, c-2024, d-2024, e-2024}{\text{standard deviation of } a+2023, b+2023, c+2023, d+2023, e+2023}$.
11. If the standard deviation of a, b, c, d and e is 5, then find the value of $\frac{\text{variance of } a+2024, b+2024, c+2024, d+2024, e+2024}{\text{variance of } a-2023, b-2023, c-2023, d-2023, e-2023}$.
12. If the standard deviation of a, b, c and d is $\frac{1}{\sqrt{2024}}$, then find the variance of $2024a+2023, 2024b+2023, 2024c+2023, 2024d+2023, 2024e+2023$.
13. Find range of the observations 1, 2, 5, 3, 0, 8, 10, 9.
14. Find the mean deviation about Mean for 5, 15, 25, 35, 45.
15. Find the mean deviation about Median for 5, 25, 15, 45, 35.
16. The sum of the squares of deviation about mean for 10 observations is 250. If the mean is 50, then find the Standard deviation.

17. The sum of the squares of deviation about mean for 16 observations is 64. If the mean is 4, then find the Variance.

18. If the variance of 1.4, 1.8, 2.2, 2.6 and 3 is 'k', then find the variance of 2.8, 3.6, 4.4, 5.2 and 6 in terms of 'k'.

19. If the S.D. of 0.01, 0.03, 0.006, 0.008 and 0.23 is 'k', then find the variance of 0.1, 0.3, 0.06, 0.08 and 2.3 in terms of 'k'.

20. Find the variance for the following data:

$$\sum_{i=1}^{10} x_i = 55, \sum_{i=1}^{10} x_i^2 = 385$$

21. Find the standard deviation for the following data:

$$\sum_{i=1}^{10} x_i = 60, \sum_{i=1}^{10} x_i^2 = 400$$

22. Find the standard deviation for the following data:

$$\sum_{i=1}^{10} x_i = 50, \sum_{i=1}^{10} (x_i - 5)^2 = 160$$

23. If the variance of 1, 2, 3, ...,10 is $\frac{p}{12}$, then find $\sqrt{p+1}$.

[Hint: Variance of first n natural numbers is $\frac{n^2-1}{12}$]

24. Find the variance of 6, 12, 18, ...,60.

[Hint: Variance of first n natural numbers is $\frac{n^2-1}{12}$].

25. If the standard deviation of 1, 2, 3, 4 and 5 is the p, then find the value of p^2+1.

26. The median of a set of 2023 distinct observations is 1204. If each of largest 1011 observations of the set is increased by 5, then find the median of the new set.

27. Find the mean of the following data:

x	1	2	3	4	5
f	${}^4C_0$	${}^4C_1$	${}^4C_2$	${}^4C_3$	${}^4C_4$

28. Find the mean of ${}^9C_0, {}^9C_1, {}^9C_2, \ldots, {}^9C_9$.

29. Find the mean deviation about median of the following data:

0, 1, 2, 2, 3, 3, 3, 3, 3, 6.

30. Find the mean deviation about mean of the following data:

1.5, 1.5, 2, 2.5, 2.5, 3, 3, 4, 3.5, 6.5.

ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	5	16	5
2	50	17	4
3	2	18	$4k$
4	6.5	19	$100k^2$
5	20.25	20	8.25
6	10	21	2
7	506	22	4
8	1	23	10
9	15	24	297
10	1	25	3
11	1	26	1204
12	2024	27	3
13	10	28	51.2
14	12	29	1
15	12	30	1

CHAPTER – 14

PROBABILITY

POINTS TO REMEMBER

- Random experiment: If an experiment has more than one possible outcomes and it is not possible to predict the outcome in advance then experiment is called random experiment.
- Sample space: The collection or set of all possible outcomes of a random experiment is called sample space. It is denoted by S.
  Each element of the sample space (set) is called a sample point.
- Event: A subset of the sample space associated with a random experiment is called an event.
- Elementary or Simple event: An event which has only one sample point is called a simple event.
  For example: when an unbiased die is thrown, then getting an even prime number on the die is an example of simple event i.e. $\{2\}$
- Compound event: An event which has more than one sample point is called a compound event.
  For example: when an unbiased die is thrown, then getting an even number on the die is an example of compound event i.e. $\{2, 4, 6\}$
- Sure event: If event is same as the sample space of the experiment, then event is called sure event.
  In other words, an event which is certain to happen is a sure event.
  For Example: when an unbiased die is thrown, then getting a number less than 7 on the die is an example of Sure event i.e. $\{1, 2, 3, 4, 5, 6\}$

- Impossible event:Let S be the sample space of the experiment, $\phi \subset S, \phi$ is called impossible event. In other words an event which is impossible to happen is the impossible event.
  For Example: when an unbiased die is thrown, then getting a number More than 6 on the die is an example of impossible event i.e. {} or ϕ
- Exhaustive and Mutually Exclusive Events:
  Events $E_1, E_2, E_3 \ldots\ldots E_n$ are such that
  (i) $E_1 \cup E_2 \cup E_3 \cup \ldots\ldots \cup E_n = S$, then Events $E_1, E_2, E_3 \ldots\ldots E_n$ are called exhaustive events.
  (ii) $E_i \cap E_j = \phi$ for every $i \neq j$ then Events $E_1, E_2, E_3 \ldots\ldots E_n$ are called mutually exclusive.
- Probability of an Event: For a finite sample space S with equally likely outcomes, probability of an event A is defined as $P(A) = \frac{n(A)}{n(S)}$
  where $n(A)$ is number of elements in A and $n(S)$ is number of elements in set S and $0 \leq P(A) \leq 1$
- If A and B are any two events then
  $$\boxed{P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$
  If A and B are mutually exclusive events, then $P(A \cap B) = 0$, so
  $$\boxed{P(A \cup B) = P(A) + P(B)}$$
- $P(A) + P(\bar{A}) = 1 \Rightarrow \boxed{P(A) + P(\text{not } A) = 1}$
- $\text{P(Sure event)} = P(S) = 1$, $\text{P(impossible event)} = P(\phi) = 0$
- $\text{P(only A)} = P(A) - P(A \cap B) = P(A \cap B')$
  $\text{P(only B)} = P(B) - P(A \cap B) = P(A' \cap B)$

LET'S PRACTICE THE FOLLOWING QUESTIONS

1. The probability of an event A is 0.7 and that of B is 0.2.If A and B are mutually exclusive events, then find the probability of either A or B.

2. If $P(A) = \frac{1}{3}, P(\overline{B}) = \frac{3}{4}$, then find the value of $\frac{8P(B)}{P(\overline{A})}$.

3. If $P(A) = a, P(B) = b$, such that $a, b \neq 0$, then find the value of $\frac{P(A-B)+P(B-A)}{P(A\cup B)-P(A\cap B)}$.

4. If $P(A) = a, P(B) = b$, such that $a, b \neq 0$, then find the value of k, when $P(A-B)+P(B-A)=P(A)+P(B)-k\times P(A\cap B)$.

5. If A and B are mutually exclusive events and $P(A) = 0.1, P(B) = 0.4$, then find the value of $10[P(\overline{A}\cap B)-3P(\overline{B}\cap A)]$.

6. A fair die is rolled. If the probability of getting a multiple of 2, 3 or 5 is 'p', then find the value of 6p.

7. Let $S=\{1,2,3,4,5,6\}$ and $E=\{2,\ 3,\ 6\}$, then find the value of $\frac{3-3P(E)}{2-P(\overline{E})}$.

8. If A and B are Mutually Exclusive & Exhaustive events,then find the value of $P(A\cap B)+P(A\cup B)$.

9. If a pair of fair dice is rolled, then find the probability of getting sum of the numbers as 9 or a multiple of 5..

10. A pair of fair dice is rolled. If the probability of getting different numbers on both the dice is 'p', then find the value of $\sqrt{36p-5}$.

11. If a pair of fair dice is rolled, then find the probability of getting the sum as neither an even number nor a multiple of 3.

12. If a is an integer belonging to $[1, 20]$, then find the probability that the graph $y = x^2 - 2(a-3)\ x + (5-a)$ is strictly above the x-axis.
[HINT: For graph to be above x-axis, $D = b^2 - 4ac < 0$]

13. If a is an integer lying in $[-9, 10]$, then find the probability that the graph $y = -x^2 + (a-1)\ x - 9$ is strictly below the x-axis.
[HINT: For graph to be below x-axis, $D = b^2 - 4ac < 0$]

14. Three letters are written to three different persons and addresses are written on three envelopes. Find the probability that without looking at the addresses, exactly one letter is placed in the right envelope.

15. Three letters are written to three different persons and addresses are written on three envelopes. Find the probability that without looking at the addresses, all three letters are placed in the right envelope.

16. Four digit numbers are formed using the digits 0, 7, 9 and 5 without repetition of the digit. If 'p' is the probability of a number being divisible by 5,then find the value of 9p.

17. Four digit numbers are formed using the digits 0, 7, 9 and 5 without repetition of the digit. If 'p' is the probability of a number being divisible by 4,then find the value of 11p.

18. Three digit numbers are formed using the digits 0, 1, 2, 5, 7. A number is chosen at random out of these numbers.If 'p' is the probability that this number has the same digits, then find the value of $10\sqrt{p}$.

19. Three dice are rolled once. If the probability of getting different numbers on the three dice is $\frac{m}{n}$, where m and n are co-prime, then find the value of $mn+m+n$.

20. Three unbiased coins are tossed once. If the probability of getting atleast two heads is p, then find the value of $(2p)^{2p}$.

21. Four unbiased coins are tossed once. If the probability of getting even numbers of heads is p, then find the value of 8p.

22. Let N be the sum of numbers appeared when two fair dice are rolled. If the probability that $N-2, N$ and N + 6 are in geometric progression is $\frac{m}{36}$, then find the value of m.

23. Let N be the sum of the numbers appeared when two fair dice are rolled. If the probability that $N-4, N$ and $2N-3$ are in Arithmetic progression be $\frac{m}{6}$, then find the value of m.

24. Let A and B be two mutually exclusive events such that $P(A) = 0.4$ and $P(A \cup B) = 0.9$, then find the value of $10P(B)$.

25. Let S={1,2,3,…,100}.If the probability that a randomly chosen number n from the set S such that HCF(n, 17) = 1 is p,then find the value of $20p$.

26. Let S={1,2,3,…,100}.If the probability that a randomly chosen number n from the set S such that HCF(n, 97) = 1 is p,then find the value of $100p$.

27. In a leap year, If p is the probability of having 53 Mondays or 53 Tuesdays and q is the probability of having 53 Mondays and 53 Tuesdays, then find the value of $7(p+q)$.

28. In a leap year, If p is the probability of having 53 Mondays or 53 Fridays and q is the probability of having 53 Mondays and 53 Fridays, then find the value of $7(p-q)$.

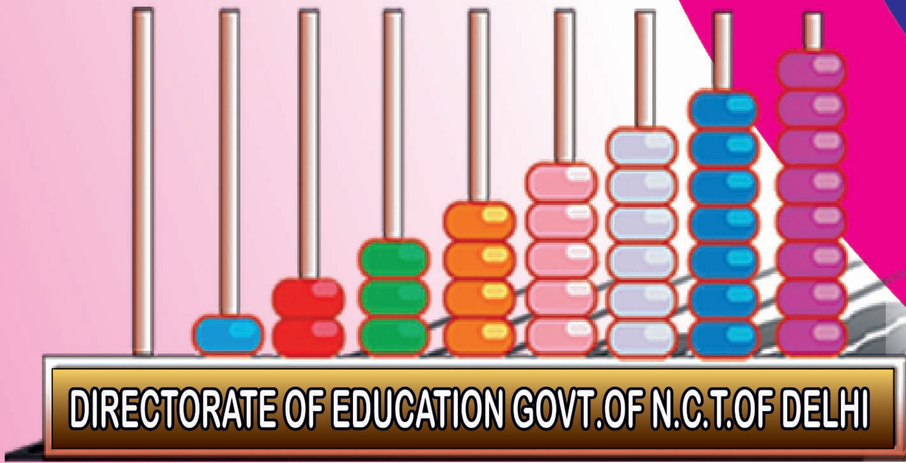
29. If $\mathrm{P}(A-B)=\dfrac{P(B-A)}{3}=0.1$, then find the value of $\mathrm{P}(A\cup B)-P(A\cap B)$.

30. If $\mathrm{P}(B-A)=\dfrac{P(A-B)}{3}=0.1$, then find the value of $\mathrm{P}(A\cup B)$ such that $P(A\cap B)=0.2$.

ANSWERS

Q. No.	ANSWER	Q. No.	ANSWER
1	0.9	16	5
2	3	17	0
3	1	18	2
4	2	19	59
5	1	20	1
6	5	21	4
7	1	22	2
8	1	23	1
9	$\frac{11}{36}$	24	5
10	5	25	19
11	$\frac{1}{3}$	26	99
12	$\frac{1}{10}$	27	4
13	$\frac{11}{20}$	28	4
14	$\frac{1}{2}$	29	0.4
15	$\frac{1}{6}$	30	0.6

मैंटल मैथ्स
MENTAL MATHS



पढ़े चलो बढ़े चलो