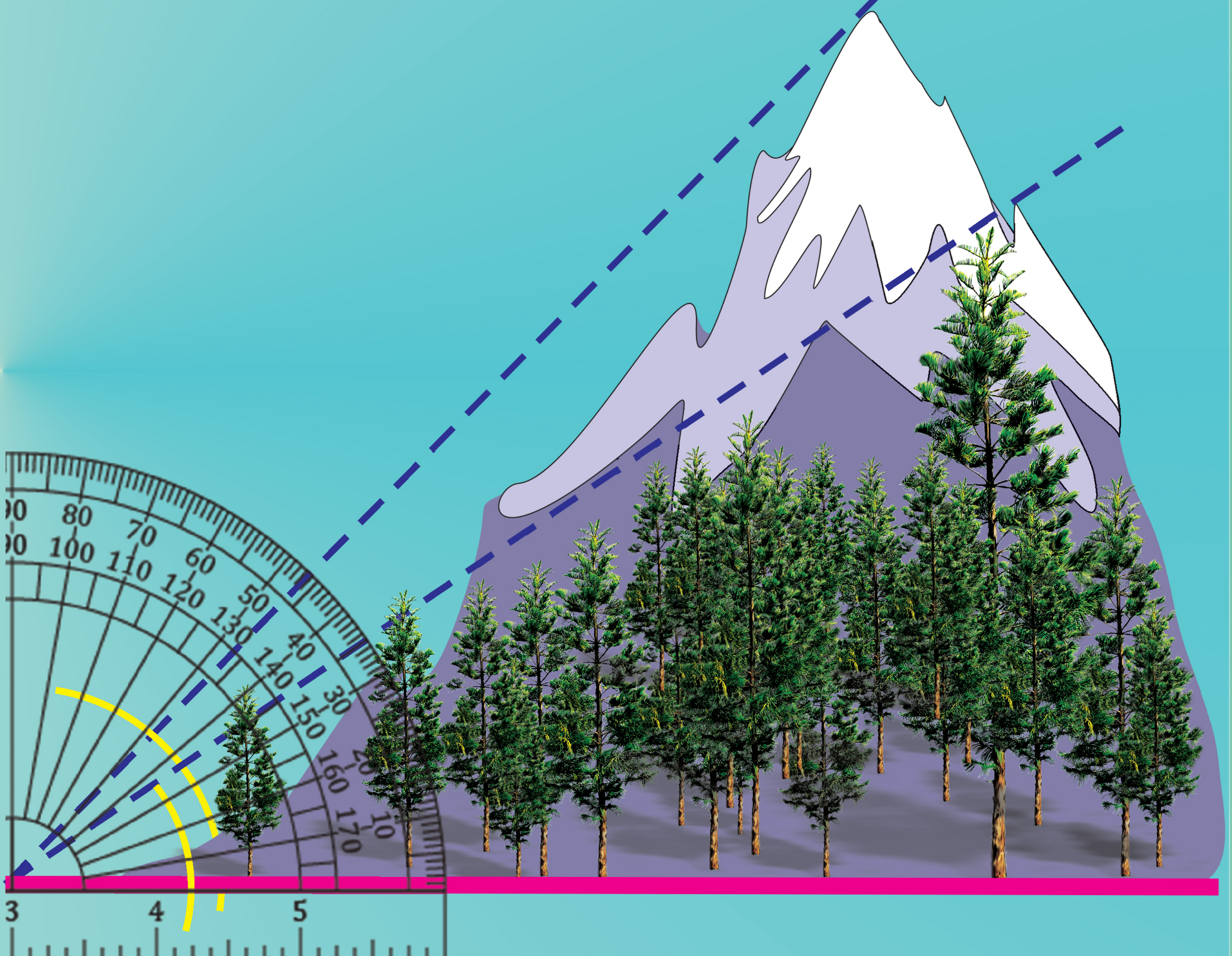
Mental Maths

QUESTION BANK
CLASS 10



DIRECTORATE OF EDUCATION Govt.of N.C.T.of DELHI

MENTAL MATHS

CLASS X

2022-23

DIRECTORATE OF EDUCATION
GOVT. OF NCT OF DELHI

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Director, Education & Sports

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No. PS/DE/2022/194
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MESSAGE

They say, 'Numbers are not just symbols on paper; numbers have life!'

It is not an exaggeration to place on record that numbers have played a pivotal role in the development and growth of human civilisation.

Numerical skills are very useful for students in their future life, especially when they appear in competitive exams.

Our Mental Maths Project aims at gradually developing and nurturing foundational numerical skills among our budding mathematicians. It started nearly two decades ago, and is striding, each passing year, on the path of progress.

Incidentally, I had an opportunity to witness the State Level Mental Maths Quiz Competition recently and I was spellbound by the speed, confidence and enthusiasm exhibited by the students. Indeed, it was to be seen to be believed!

I appreciate the dedication and hard work put in by the State Core Committee members and the Subject Experts under the able guidance of the Project Director (Mental Maths) in preparing the Question Banks and carrying this project forward with great zeal & fervour.

(HIMANSHU GUPTA)

विकास कालिया
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VIKAS KALIA
PROJECT DIRECTOR (MENTAL MATHS)
REGIONAL DIRECTOR OF EDUCATION
(NORTH & CENTRAL)

No. PD|MMP|609

Dated 02/01/2023

'A Few Interesting Facts About Maths'

The word 'Mathematics' has its origin in the Greek word "Mathema' which means 'something that is learnt' or 'something that one gets to know'. In the same country (Greece), an ancient scholar Archimedes is considered to be the 'Father of Mathema' as he discovered methods to measures the areas of different shapes.

However, in our own country, we consider Aryabhatta as Father of Mathematics because of his original contributions made in Spherical Trigonometry. Some people believe that Aryabhatta invented Zero also, while some others credit another Vedic scholar Brahmgupta for this landmark discovery. The Western Scholars believe that Zero was first invented by the 'Mayans' (Mesopotamia) and a little later, by the Indians from which places, Zero travelled gradually to Cambodia, China and to the Arab world.

By the way, 'Arab' reminds me of an important branch of Maths named 'Algebra' which has its roots in the Arabian word 'Al-jabr' which means 'reunion of broken parts' (also used for reuniting broken bones)!

Algebra seeks to find out 'the missing values' and restoring them, just like restoring broken bones by providing missing links. In Algebra, we first 'imagine' values in the form of symbols like 'x' or 'y' and then, manipulate them to find out the 'actual' values. This is how even today, we find the 'missing' values or links through Algebra.

In short, we can conclude that unlike the 'inventions' of bulb, printing press or pen which were made by certain individuals, Mathematics is not an invention made by one person or by one civilisation. Its various branches were cultivated and nurtured by various individuals across various continents & civilizations and through different millennia.

As for Mental Maths, one can master Mental Maths through rigorous practice. Apart from learning Tables and Formulae by heart, one needs to learn various tricks for breaking longer calculations into smaller parts and making numbers 'round'. I am sure, our Maths Teachers will be able to identify students who have aptitude for numbers and groom them for Mental Maths Quiz Competitions.

I take this opportunity to thank all our Maths Teachers who devote so much of their extra time to prepare our students to sit for these competitions. I am also indebted to our Maths Teachers who have 'written' and 'reviewed' these question banks.

I thank my HoSs, Coordinators and the Core Team who, I think, are devoted much more than their Project Director to promoting Mental Maths among students!

Finally, I thank DBTB for the efforts they made for successful publication of these Question Banks.

Above all, I am indebted to my superior, the Director of Education, for his consistent support & guidance.

02/01/23

(VIKAS KALIA)
PROJECT DIRECTOR
(MENTAL MATHS)

ACKNOWLEDGEMENT

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LEVEL-3
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REGION WEST (2ND RUNNER UP)

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3	X	SUNDRAM	SUNIL CHAUDHARY	20170064352	16.02.2007	RPVV CIVIL LINES	1207113	MANEETA SHARMA

SCHEDULE OF MENTAL MATHS QUIZ COMPETITIONS

FOR THE YEAR 2022-2023

DIRECTORATE OF EDUCATION

GOVT OF NCT OF DELHI

- **Practice to students from Question Bank** **01.04.2022 to 15.10.2022**
- **School level Quiz Competition** **17.10.2022 to 07.11.2022**
- **Cluster level Quiz Competition** **08.11.2022 to 14.11.2022**
- **Zonal level Quiz Competition** **21.11.2022 to 30.11.2022**
- **District level Quiz Competition** **07.12.2022 to 14.12.2022**
- **Regional level Quiz Competition** **26.12.2022 to 31.12.2022**
- **State level Quiz Competition** **18.01.2023 to 31.01.2023**

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CHAPTER -1

REAL NUMBERS

POINTS TO REMEMBER

- **Euclid's Division Lemma: Given two positive integers a and b($a \geq b$), there exists unique integers q and r satisfying $a = bq + r$, where $0 \leq r < b$**
- **If p is a prime and p divides a^n, then p divides a where a is any positive integer.**
- **If p is a prime number, then $\sqrt{p}$ is an irrational number.**
- **If decimal expansion of a rational number terminates, then we can express that rational number in the form of $\frac{p}{q}$ ($q \neq 0$), where p and q are coprime, and the prime factorization of q is of the form $2^m \times 5^n$, where m and n are non-negative integers.**
- **If $x = \frac{p}{q}$ is a rational number, such that the prime factorization of q is of the form $2^m \times 5^n$, where m and n are non-negative integers, then x has a decimal expansion which terminates.**
- **If the denominator of a rational number is of the form $2^m \times 5^n$, then it will terminate after m places if $m > n$ or after n places if $n > m$.**
- **Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form $2^m \times 5^n$, where m and n are non-negative integers, then x has non terminating repeating decimal expansion.**
- **If a and b are two positive numbers, then HCF (a, b) × LCM (a, b) = a × b.**

QUESTIONS

1. **Find the digit at the unit place of the number $7^{2019} \times 3^{2019}$.**
2. **Find the digit at the unit place of the number $7^{219} \times 3^{522}$.**
3. **Find the digit at the unit place of the number $12345^{6789} + 6788^{12345}$.**
4. **Find the digit at the unit place of the number $44^{11} \times 66^{11} \times 99^{11} + 11^{11}$.**

5. Find the digit at the unit place of the number

$4^1 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 \ldots \times 4^{99} \times 9^{100}$.

6. What is the number of zeros in the usual form of the following :

i) 200 + 1000 + 80000 + 12500000

ii) 200 × 5000 × 80000 × 12500000

7. Find the number of zeroes in $2^2 \times 5^4 \times 4^6 \times 10^8 \times 6^{12} \times 15^{14}$.

8. What is the remainder when $11^{11} + 22^{22} + 33^{33}$ is divided by 10 ?

9. What is the difference between the largest two digit prime number and the least 3 digit prime number?

10. For $p^n = (a \times 5)^n$ to end with the digit 0 what will be the value of a?

11. If $\frac{1}{7}$= 0.142857142857… then find the value of $\left[\frac{1}{2} + \frac{3}{14}\right]$ in decimal expansion.

12. What will be the smallest rational number by which $\frac{1}{3}$ should be multiplied so that its decimal expansion terminates after one place of decimal?

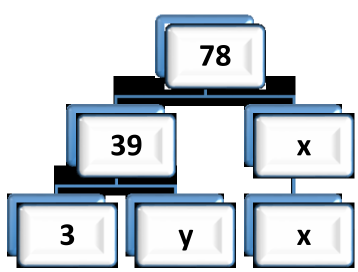
13. After how many places the decimal expansion of $\left[\frac{116}{2^5 \times 5^2}\right]$ will terminate?

14. If $(\sqrt{3} - \sqrt{2} - \sqrt{1})(\sqrt{3} + \sqrt{2} + \sqrt{1}) = a\sqrt{3} + b\sqrt{2} + c\sqrt{1}$, then find the value of (a + b + c).

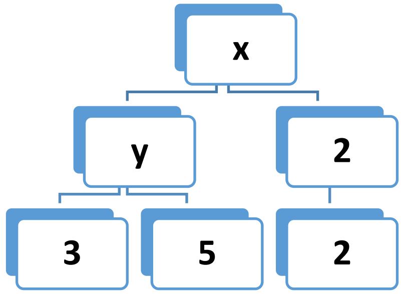
15. Find the value of $\sqrt{(\sqrt{2} - \sqrt{3})^2} + \sqrt{(\sqrt{2} + \sqrt{3})^2}$

16. Find the value of $\left(\sqrt{7} - \sqrt{9}\right)^2 . \left(\sqrt{7} + \sqrt{9}\right)^2$

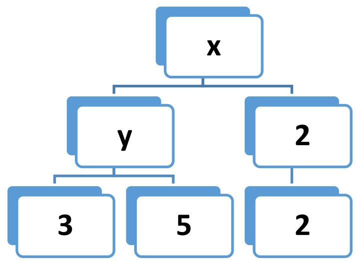
17. Find the value of (x + y) using factor tree.



18. Find the value of 2xy using factor tree.



19. Find x ÷ y using factor tree.



20. If $7560 = 2^3 \times 3^n \times q \times 7$, then what is the value of n + q.

21. What is the smallest prime factor of 11 × 13 × 19 × 23 + 23?

22. If $\left(\frac{15}{2^3} \times 5^2 \times 3^\beta \times 7^n\right)$ is a terminating decimal, then what are the least possible values of n and β?

23. Two equilateral triangles have sides of lengths 51 cm and 85 cm respectively. Find the greatest length of tape that can measure both of them exactly.

24. Two numbers are in the ratio 17 : 13. If their HCF is 15, then what is the sum of the numbers?

25. The HCF and LCM of two numbers are 33 and 264 respectively. When the first number is divided by 2, the quotient is 33, find the other number.

26. Find the HCF of $(2^{125} - 1)$ and $(2^{15} - 1)$.

27. The LCM of two numbers is 1890 and their HCF is 30. If one of them is 270, then find the other number.

28. The HCF of two numbers is 11 and their LCM is 616. If one of the numbers is 88, find the other.

29. Given that HCF (2730, 4400) = 110 and LCM (2730, 4400) = 273 k. Find the value of k.

30. In a seminar the number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required where in each room, the same number of participants are to be seated and all of them being of the same subject.

31. Six bells commence tolling together. They toll at the intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. In 30 minutes how many times do they toll together?

32. Three numbers are in the ratio of 3: 4: 5 and their LCM is 2400. Find their HCF.

33. If the adjacent sides 'a' and 'b' of a rectangle are in the ratio 3 : 5 such that HCF (a, b) = 11, then find the perimeter of the rectangle.

34. What is the HCF of smallest 3 digit number obtained using three different digits and greatest two digit composite number?

35. The length of a rectangle is LCM (a, b) and breadth of the rectangle is HCF (a, b), then what is its area?

36. If HCF (a, b) = LCM (a, b), then what is the relation between a and b?

37. If HCF (20, p) = 2 and LCM (20, p) = 60, then what is the value of p?

38. How much is $\left(\sqrt{180}+\sqrt{108}\right)$ greater than $\left(\sqrt{5}+\sqrt{3}\right)$?

39. A number which when divided by a divisor leaves remainder 23 .When twice the number is divided by the same divisor, the remainder is 11, what is the divisor?

40. If $\left(\frac{7^3+7^3+7^3+7^3+7^3+7^3+7^3}{7^x}\right)$ = 7, then what is the value of x?

41. The LCM of two numbers is 45 times their HCF. If one of the numbers is 125 and sum of HCF and LCM is 1150, then what is the other number?

42. If a is an odd number, b is not divisible by 3 and LCM of a and b is p, then what is the LCM of 3a and 2b?

43. What is the smallest number by which $\sqrt[3]{81}$ should be multiplied so as to get a rational number?

44. What is the total number of factors of an even prime number?

45. If HCF (144,180) = 13m – 3, then what is the value of m?

46. If r is the remainder when (5m +1) (5m+3) (5m+4) is divided by 5, then what are the possible values of r, if r is a natural number?

47. Find the least positive integer which is divisible by first five natural numbers.

48. The HCF (a, b) = 29, where a, b > 29 and LCM= 4147. What is the value of $|a - b|$?

49. When a = b q + r, then what are the possible factors of (a – r)?

50. If $(-1)^{n} + (-1)^{8n} = 0$, then what is the least positive value of n?

ANSWERS

1	1	26	31
2	7	27	210
3	3	28	77
4	5	29	400
5	6	30	21
6	i) 2 ii) 18	31	16
7	26	32	40
8	8	33	176 cm
9	4	34	3
10	2	35	ab sq.units
11	0.714285714285…	36	a = b
12	$\frac{3}{10}$	37	6
13	3 places	38	$5(\sqrt{5}+\sqrt{3})$
14	-2	39	35
15	$2\sqrt{2}$	40	3
16	4	41	225
17	15	42	6p
18	900	43	$\sqrt[3]{9}$
19	2	44	2
20	8	45	3
21	23	46	2
22	n=0, $\beta = 0$	47	60
23	17 cm	48	58
24	450	49	b and q
25	132	50	1

CHAPTER 2

POLYNOMIALS

POINTS TO REMEMBER

- Algebraic expressions in which power of the variable of each term is a whole number are called polynomials i.e. $2x + 3$, $5t^2 + 7t + 8$
- Degree of the polynomial in one variable: The highest power of the variable of any term in a polynomial is its degree.
- Following are the forms of various degree polynomials.

Examples	Name of the polynomial	Degree
5	Constant polynomial	0
2x+3	Linear polynomial	1
$5x^2 + 7x + 8$	Quadratic polynomial	2
$3x^3 + 2x^2 + 5x + 7$	Cubic polynomial	3
$t^4 + 8t^3 + 7t^2 + 4t + 5$	Biquadratic polynomial	4
0	Zero polynomial	Not defined

- If for a polynomial p(x), p $(\alpha) = 0$, then α is called a zero of the polynomial p(x).
- A polynomial of degree 'n' has atmost 'n' zeroes.
- If α and β are the zeroes of the polynomial $ax^2 + bx + c$, $a \neq 0$ then

  Sum of the zeroes $(\alpha + \beta) = -\frac{\text{coefficient of x}}{\text{coefficient of } x^2} = -\frac{b}{a}$

  Product of zeroes $(\alpha . \beta) = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{c}{a}$

- If α and β are the zeroes of the quadratic polynomial p(x) then

  $p(x) = k[\,x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$

  i.e. $p(x) = k[\,x^2 - (\alpha + \beta)x + \alpha\beta]$, where k is any real number.

- If α, β and γ are the zeroes of the cubic polynomial $f(x)=ax^3 + bx^2 + cx + d$ then

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

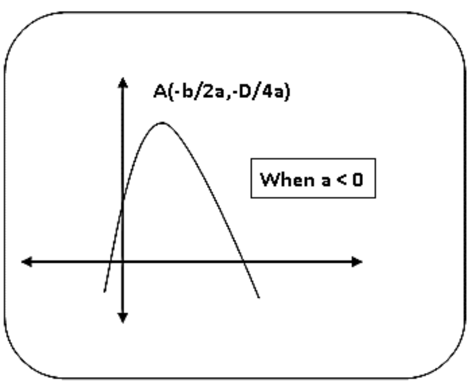
$$\alpha\beta\gamma = -\frac{d}{a}$$

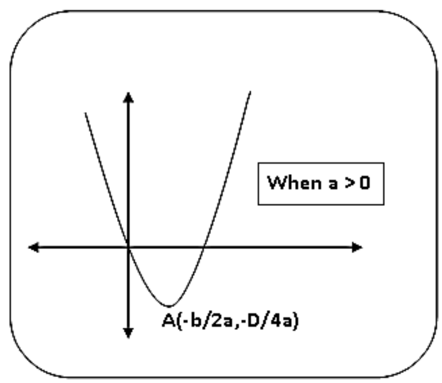
- **If α, β and γ are the zeroes of the cubic polynomial p(x) = $ax^3 + bx^2 + cx + d$ then p(x) =k[$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$], where k is any real number**
- **Geometrically zeroes of the polynomial f(x) are x – coordinates of the point where the graph y = f(x) intersects x – axis.**
- **Coordinates of vertex 'A' of graph of y = $ax^2 + bx + c$ is $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ i. e $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$, where**

  D = $b^2 - 4ac$

  e.g.: if $y = x^2 - 2x + 4$, then coordinates of its vertex are $-\frac{b}{2a} = 1$ and

  $-\frac{D}{4a} = \frac{12}{4} = 3$





- **The division algorithm states that given any polynomial p(x) and any non-zero polynomial g(x),**

  deg p(x)≥ deg g(x) there are polynomials q(x) and r(x) such that

  p(x) = g(x) q(x) + r(x), where r(x) = 0 or degree r(x) < degree g(x).
- **If (x+a) is a factor of polynomials $x^2 + px + q$ and $x^2 + mx + n$ then a = $\frac{n-q}{m-p}$.**

QUESTIONS

1. What will be the number of zeroes of the polynomial whose graph is parallel to y axis?
2. At how many points the graph of the quadratic polynomial intersect x-axis?
3. Find the sum of the zeroes of the quadratic polynomial $3x^2 + 15x + 7$.
4. Find the product of the zeroes of the quadratic polynomial $2x^2 - 7$.
5. Find the sum of the zeroes of the polynomial $x^2 - 64$.
6. Find a quadratic polynomial whose sum and product of the zeroes are - 3 and 2 respectively.
7. Find the quadratic polynomial whose zeroes are -9 and $-\frac{1}{9}$.
8. Form a quadratic polynomial, if product and sum of its zeroes are $-\frac{3}{5}$ and 0. Find zeroes also.
9. Find a quadratic polynomial whose zeroes are $(5 + \sqrt{2})$ and $(5 - \sqrt{2})$.
10. If α and β are the zeroes of the quadratic polynomial $2x^2 + 5x + 1$, then find the value of $\alpha + \beta + \alpha\beta$.
11. Find the cubic polynomial whose zeroes are 0, 5 and -5.
12. If α and β are the zeroes of the quadratic polynomial $2x^2 - 5x + 8$, then find the value of $\alpha^2 + \beta^2$.
13. If α and β are the zeroes of the quadratic polynomial $2x^2 - 5x + 8$, then find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.
14. Find the quadratic polynomial whose sum of the zeroes is 0 and one zero is 5.
15. Form a quadratic polynomial whose one of the zeroes is $2 + \sqrt{5}$ and sum of the zeroes is 4.
16. If 1 is a zero of the polynomial $ax^2 + bx + c$, then find the value of $\frac{b+c}{a}$.
17. If sum of the zeroes of $(a + 1)x^2 + (2a + 3)x + (3a + 4)$ is -1, then find a.
18. If the sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find k.
19. Form a quadratic polynomial whose zeroes are reciprocal of the zeroes of $ax^2 + bx + c$.

20. If α and β are the zeroes of x^2-3x+2, form a quadratic polynomial whose zeroes are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$.
21. If one zero of the polynomial $z^2+13z-p$ is reciprocal of the other, then find the value of p.
22. For what value of p, (-4) is a zero of the polynomial $x^2-2x-(7p+3)$.
23. If 1 is a zero of the polynomial $ax^2-3(a-1)x-1$, then find the value of a.
24. If sum and product of the zeroes of ax^2-5x+c is equal to 10 each, find a and c.
25. If α and β are the zeroes of the polynomial x^2-3x+p and $\alpha-\beta=1$, then what is the value of p?
26. If x+2 is a factor of $x^2+ax+2b$ and $a+b=4$, then find the value of a and b.
27. Which is the common factor in x^2+x-12 and $x^2+9x+20$?
28. If a polynomial of degree 5 is divided by a quadratic polynomial, then find the degree of the remainder polynomial.
29. Find the quotient when $x^2-9x+20$ is divided by $x-5$.
30. If $x+a$ is a common factor of the polynomials $x^2-3x-10$ and $x^2-8x+15$, then find a.
31. What is the common factor in x^2-1, x^4-1 and $(x-1)^2$?
32. Find the common zero of x^2+2x+1, x^2-1 and x^3+1.
33. Find the quotient when $f(x)=16x^3+13x^2+x-2$ is divided by $g(x)=(2x+1)^3$.
34. If x^3+x^2-ax+b is completely divisible by x^2-x, then find the values of a and b.
35. For what value of x both the polynomials $2x^2+8x+8$ and $x^2-3x-10$ becomes zero?
36. What should be added to the polynomial x^2-8x+6 so that 4 becomes a zero of the polynomial?
37. What should be subtracted from $x^3-3x^2+6x-15$ so that it is completely divisible by $(x-3)$?

38. If the sum of zeroes of $5x^2 + (p + q + r)x + pqr$ is 0, then what is the value of $p^3 + q^3 + r^3$?

39. If one of the zeroes of the polynomial $x^2 - 9x + (7k + 4)$ is double of the other, then find the value of k.

40. If two zeroes of the polynomial $x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find its third zero.

41. Find the zeroes of the polynomial $x^3 - 5x^2 - 16x + 80$, if its two zeroes are equal in magnitude but opposite in sign.

42. If α, β and γ are the zeroes of the polynomial $x^3 + bx^2 + cx + d$, then find the value of

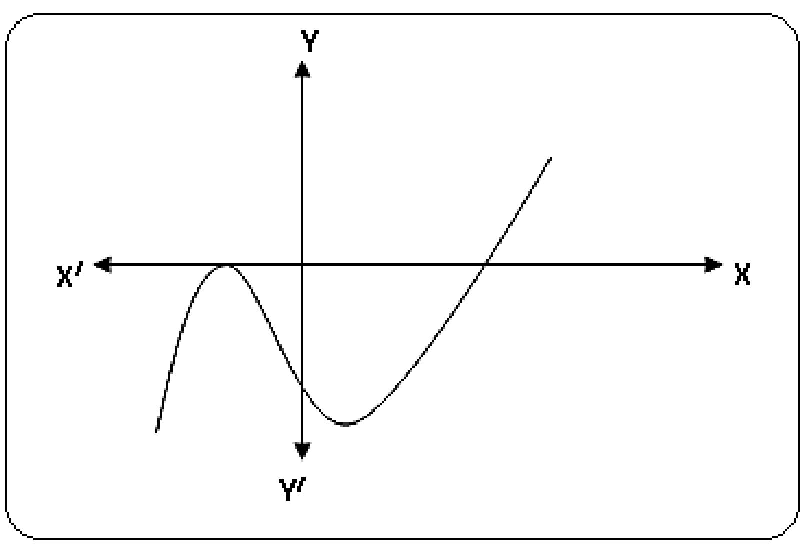
$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$

43. If α, β and γ are the zeroes of the polynomial $x^3 - px^2 + qx - r$, then find the value of

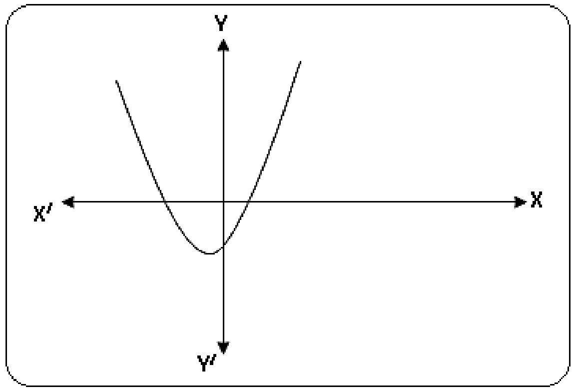
$$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$$

44. Find the coordinates of the vertex of the figure obtained by drawing the graph of $2x^2 - 4x + 5$.

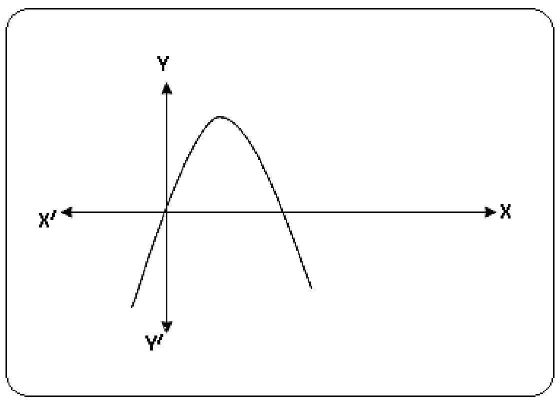
45. Find the degree of the polynomial p(x) representing the given graph.



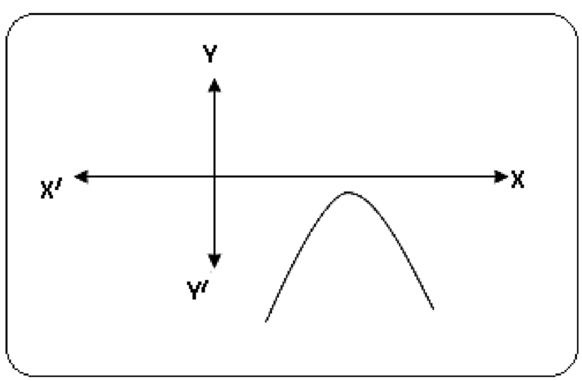
46. **Find the number of zeroes of the polynomial p(x) represented in the given graph.**



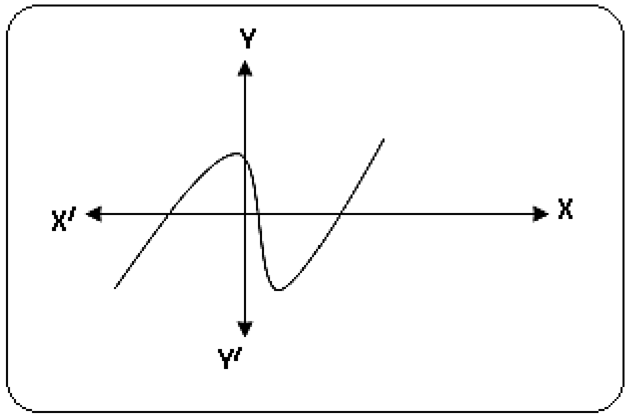
47. **Find the number of zeroes of the polynomial p(x) represented in the given graph.**



48. **Find the number of zeroes of the polynomial p(x) represented in the given graph.**



49. Find the number of zeroes of the polynomial p(x) represented in the given graph.



50. Find a cubic polynomial whose zeroes are 0, 4 and -4.

ANSWERS

1	1	26	a = 3 , b = 1
2	Atmost 2 points	27	x + 4
3	-5	28	1 or 0 or not defined
4	$-\frac{7}{2}$	29	$x - 4$
5	0	30	-5
6	$k(x^2 + 3x + 2)$	31	$x - 1$
7	$k(9x^2 + 82x + 9)$	32	-1
8	$k(5x^2 - 3)$	33	2
9	$x^2 - 10x + 23$	34	a = 2, b = 0
10	-2	35	-2
11	$x^3 - 25x$	36	10
12	$-\frac{7}{4}$	37	3
13	$-\frac{7}{16}$	38	3pqr
14	$x^2 - 25$	39	k = 2
15	$x^2 - 4x - 1$	40	4
16	-1	41	4, -4 and 5
17	-2	42	$-\frac{c}{d}$
18	9	43	$\frac{p}{r}$
19	$k\left(x^2 + \frac{b}{c}x + \frac{a}{c}\right)$	44	(1, 3)
20	$x^2 - 10x + 9$	45	greater than or equal to 3
21	-1	46	2
22	3	47	2
23	1	48	0
24	$a = \frac{1}{2}, c = 5$	49	3
25	2	50	$x^3 - 16x$

CHAPTER 3

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

POINTS TO REMEMBER

- **General form of pair of linear equations is**

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Where $a_1, b_1, a_2, b_2, c_1, c_2$ are real numbers such that $(a_1)^2 + (b_1)^2 \neq 0$ and

$$(a_2)^2 + (b_2)^2 \neq 0$$

- **In above equations if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then**

(i) The pair of linear equations is consistent.

(ii) The pair of linear equations represent intersecting lines.

(iii) The pair of linear equations has a unique solution.

- **In above equations if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then**

(i) The pair of linear equations is dependent and consistent.

(ii) The pair of linear equations represents coincident lines.

(iii) The pair of linear equations have infinitely many solutions.

- **In above equations if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then**

(i) The pair of linear equations is inconsistent.

(ii) The pair of linear equations represents parallel lines.

(iii) The pair of linear equations has no solution.

- **Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$**

- **If area of triangle is zero, then the points are collinear and vice versa.**

Special case: When coefficient of x and y are interchanged in two equations i.e.

$ax + by = c$, $bx + ay = d$, then $x + y = \frac{c + d}{a + b}$ and $x - y = \frac{c - d}{a - b}$.

QUESTIONS

1. **For what value of k, (6, k) is a solution of the equation 3 x + y = 22.**
2. **Form a linear equation whose solution is (−2, 3).**
3. **If one equation of a pair of dependent linear equation is −5 x + 7 y = 2, then what may be the second equation?**
4. **If x = a, y = b, is a solution of the equations x + y = 8, x – y = 2, then what are the values of a and b?**
5. **Find the point of intersection of y = 2 and 2 x + 3 y = 5.**
6. **What are the values of x and y when x + 2y = 9, x − 2y = 1?**
7. **What is the least value of p for which x and y have same values in 2x + py = 8?**
8. **Find the point of intersection of the lines represented by 3x−2y= 6 and y axis.**
9. **If x = 4 and y = 3p − 1 is a solution of x + y = 6 then what is the value of p and y?**
10. **Find the area of triangle formed by the lines x = y, y = 4 and y -axis.**
11. **Find the area of the triangle formed by the co-ordinate axes and the lines**
    i. **x + y = 6**
    ii. $\frac{x}{a}+\frac{y}{b}=1$
12. **If 2x + 3y = 0 and 4x −3y = 0, then find the value of x + y.**
13. **Solve for x and y:** $\sqrt{5}x+\sqrt{7}y=0, \sqrt{3}x-\sqrt{2}y=0$
14. **Solve for x and y**

    $2^x+5^y=33$
15. **If** $\sqrt{a}x-\sqrt{b}y=0, \sqrt{b}x-\sqrt{a}y=0$**, then find the value of xy.**
16. **For what value of k the given system of equation has no solution?**

    k x + 2 y – 1 = 0, 5 x – 3 y + 2 = 0
17. **For what value of k the given system of equations have infinitely many solutions?**

    2x – 3 y = 7, (k+2) x – (2k+1) y = 3(2k–1)
18. **For what value of m the given system of equations has unique solution?**

    2x + 3 y – 5 = 0, mx - 6 y = 8

19. For what value of p the given system of equations represents coincident lines?

$3x - y + 8 = 0$

$6x - py = -16$

20. For what value of c the given system of equations represent parallel lines?

$3x + 2cy = 2$

$2x + 5y + 1 = 0$

21. If $2^x = 8^{y-1}, 9^y = 3^{x-6}$, then find the value of x and y.

22. Solve for x and y : $31x + 29y = 89$, $29x + 31y = 91$

23. If $47x + 31y = 63$ and $31x + 47y = 15$, then find the value of $x - y$.

24. What is the value of $x + y$ for the following pair of linear equations

$152x - 378y = -74$

$-378x + 152y = -604$

25. Find $x - y$ for the following:

$217x + 131y = 913$

$131x + 217y = 827$

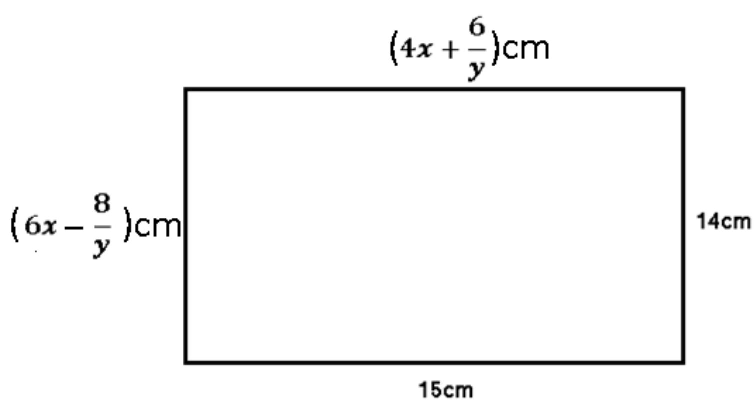
26. For what value of x and y: $\sqrt{x} + \sqrt{y} = 7, \sqrt{x} - \sqrt{y} = 1$

27. Solve for x and y: $\frac{2x}{a} + \frac{y}{b} = 2, \frac{x}{a} - \frac{y}{b} = 4.$

28. Solve for x and y : $x - y = 0.9$, $\frac{11}{2(x+y)} = 1.$

29. Solve for x and y: $\frac{3}{(x+5)} - \frac{4}{(y+1)} = 0, \frac{6}{(x+5)} - \frac{8}{(y+1)} = 0$

30. From the following figure find the values of x and y.



31. In triangle ABC, $\angle A = x$, $\angle B = y$, $\angle C = y + 20°$. If $y - x = 50°$, what type of triangle is ABC ?
32. Megha has only one rupee and two rupee coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹75, then find the number of ₹1 and ₹2 coins.
33. The sum of digits of a two digit number is 9. If 27 is added to it, then digits of the number get reversed, then find the number.
34. At what point the linear equation $2x + 3y = -7$ intersect x axis?
35. At what point the linear equation $3x - 7y = 5$ intersect y axis?
36. If linear equation $3x + 2y = 5$ intersects x and y axis, then find the sum of intercepts on x and y axis.
37. Find the area of the triangle if its vertices are (3, 2), (5, 2) and (-7, 2).
38. Find the area of the triangle if its vertices are (3, 5), (3,-7) and (3, 0).
39. Find the area of the triangle if its vertices are (0, 0), (2, 2) and (4, 0).
40. Find the area of the triangle if its vertices are (2, 2), (4, 4) and (6, 2).
41. Sum of two numbers is 35 and their difference is 13. Find the numbers.
42. If one number is twice the other and their sum is 117, then find the numbers.
43. The sum of two numbers is 20 and their product is 75. Find the sum of their reciprocals.
44. The sum of two numbers is 20 and their product is 19. Find their difference.
45. The sum of numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Find the fraction.
46. Half the perimeter of a garden whose length is 12m more than its width is 60m. Find the length of the garden.
47. Cost of 3 books and 4 pens together is ₹257 and the cost of 4 books and 3 pens together is ₹324. Find the total cost of two books and two pens.
48. A father is three times as old as his son. After 12 years his age will be twice as that of the age of his son. Find their present ages.
49. Two numbers are in the ratio 3: 4. If 8 is added to each of the number, the ratio becomes 4:5. Find the numbers.

50. The monthly income of A and B are in the ratio 9:7 and their monthly expenditure are in the ratio 4:3. If each of them saves ₹1600 per month, find the monthly income of each.

51. Meena went to the bank to withdraw ₹2000. She asks the cashier to give her ₹50 and ₹100 note only. She receives 25 notes in all, find how many notes of ₹50 and ₹100 did she received?

52. The angles of a triangle are x, y and 40°. The difference between the angles x and y is 30°. Find x and y.

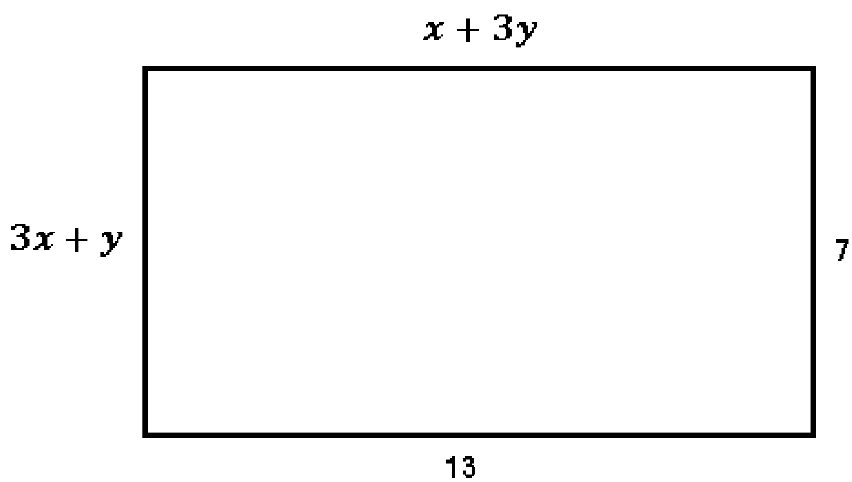
53. Find x and y where the angles of a cyclic quadrilateral ABCD are $\angle A = (6x + 10)°$, $\angle B = (5x)°$, $\angle C = (x + y)°$, $\angle D = (3y - 10)°$.

54. When we draw the graph of the lines x = −2 and y = 3, what will be the coordinates of the vertices the figure formed by the co-ordinate axes and above lines?

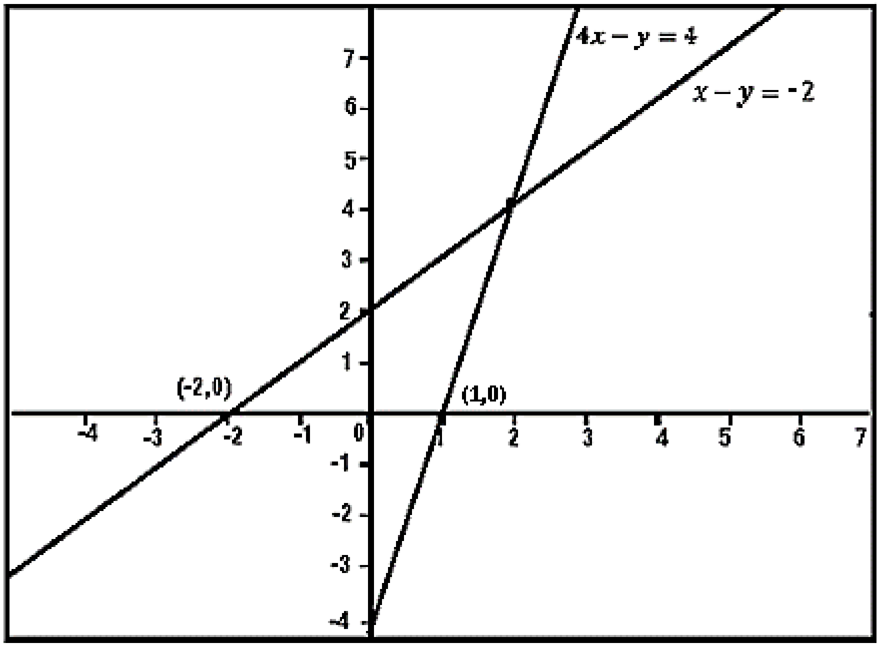
55. (a)The larger the two supplementary angles exceeds the smaller by 18°. Find the angles.

(b) Find the area of the triangle formed by three lines y = x, x = a and y = b.

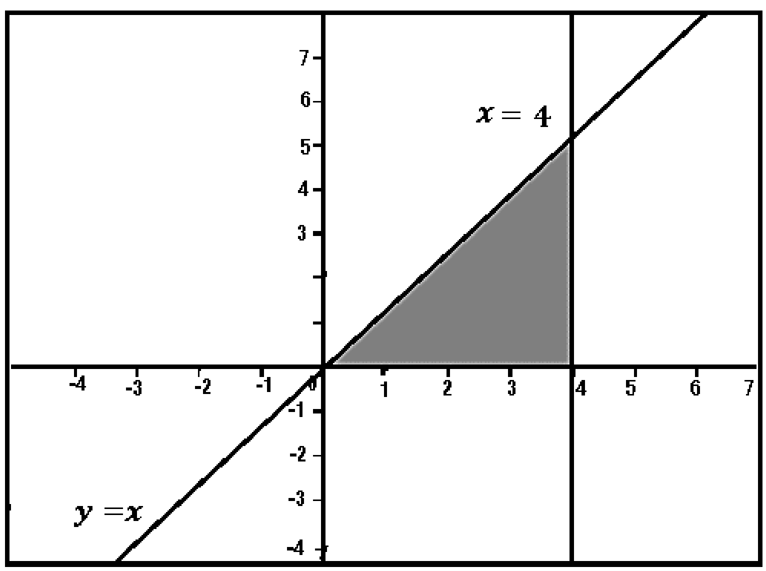
56. Find the value of x and y in the following figure:



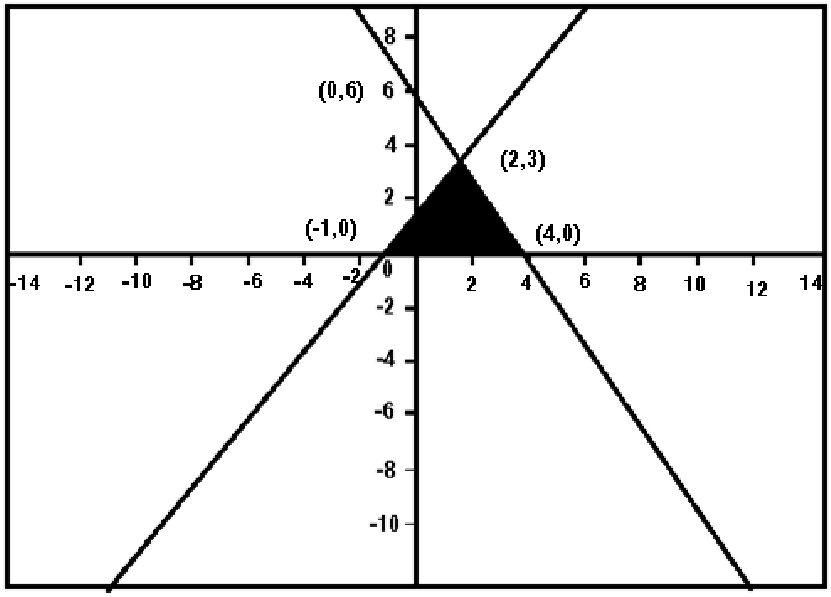
57. **From the figure find the area of the triangle formed by the pair of linear equations : $x - y + 2 = 0$, $4x - y - 4 = 0$ and x axis.**



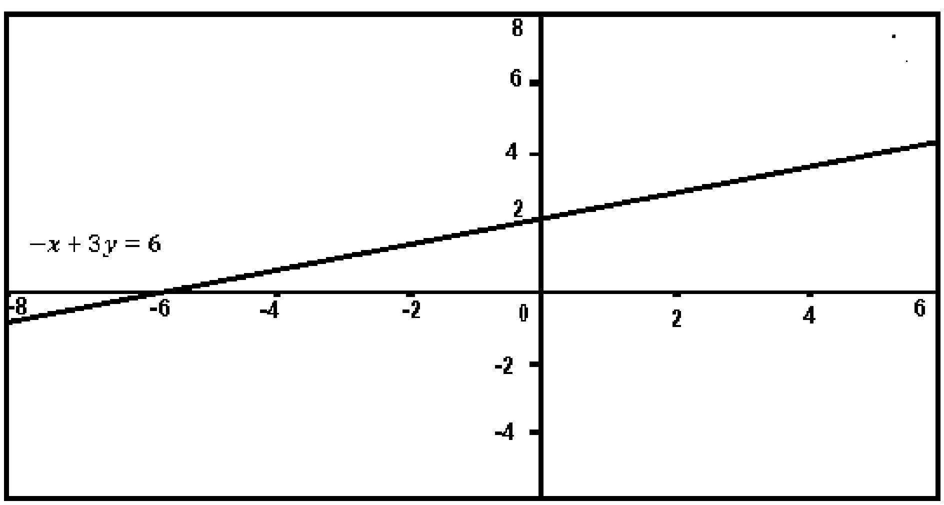
58. **From the figure find the area of the shaded triangle.**



59. Find the ratio of the area of the triangle formed by given lines with x - axis and y- axis in the given figure.



60. In the given figure find the coordinates of points where – x + 3 y = 6 meets x axis and y axis.



ANSWERS

1	k = 4	31	Right angled triangle
2	x + y = 1	32	25, 25
3	-10x + 14y = 4 or any other suitable answer	33	36
4	x = 5, y = 3	34	$(-\frac{7}{2}, 0)$
5	$(-\frac{1}{2}, 2)$	35	$(0, -\frac{5}{7})$
6	x = 5, y = 2	36	$4\frac{1}{6}$
7	p = 2, x = y = 2	37	0
8	(0, -3)	38	0
9	p = 1, y = 2	39	4 sq. units
10	8 sq. units	40	4 sq. units
11	i) 18 sq. units ii) $\frac{1}{2}ab$ sq. units	41	24 and 11
12	0	42	39, 78
13	x = 0, y = 0	43	$\frac{4}{15}$
14	x = 3, y = 2	44	18
15	$\sqrt{ab}$	45	$\frac{5}{7}$
16	$k = -\frac{10}{3}$	46	36 cm
17	k = 4	47	₹ 166
18	$m \neq -4$	48	36 years, 12 years
19	p = 2	49	24,32
20	$c = \frac{15}{4}$	50	A = ₹14400, B= ₹11200
21	x = 24, y = 9	51	10 notes of ₹50 , 15 notes of ₹100
22	x = 1, y = 2	52	x = 85° , y = 55°
23	x – y = 3	53	x = 20, y = 30
24	x + y = 3	54	(0,0),(0,3),(-2,3),(-2,0)
25	x – y = 1	55	a)99°, 81° b)$\frac{1}{2}(a-b)^2$
26	x = 16, y = 9	56	x = 1 unit , y = 4 units
27	x = 2a, y = - 2b	57	6 sq. units
28	x = 3.2, y = 2.3	58	8 sq. units
29	x = -2, y = 3	59	3:2
30	x = 3, y = 2	60	(-6,0), (0,2)

CHAPTER – 4

QUADRATIC EQUATIONS

POINTS TO REMEMBER

- **Quadratic Equation :**

  An equation of degree 2 is called a quadratic equation. The general form of a quadratic equation in one variable x is $ax^2 + bx + c = 0$ where a, b and c are real numbers and $a \neq 0$.

- **Methods for solving quadratic equations are**
  - **Factorization method**
  - **Completing the square method**
  - **Quadratic formula**

- **Discriminant: For the quadratic equation $ax^2 + bx + c = 0, a \neq 0$**

  $D = b^2 - 4ac$ is called discriminant.

- **Nature of roots**

  If D = 0, Real and equal roots

  If D > 0, Real and distinct roots

  If D < 0, No real roots

- **If $D \geq 0$, then real roots α, β of the quadratic equation $ax^2 + bx + c = 0$ are given by $\alpha = \frac{-b+\sqrt{D}}{2a}$ and $\beta = \frac{-b-\sqrt{D}}{2a}$ $(a \neq 0)$**

- **Relationship between roots and coefficients :**

  If α and β are two roots of $ax^2 + bx + c = 0$, then

  Sum of the roots = $\alpha + \beta = -\frac{b}{a}$

  Product of the roots = $\alpha\beta = \frac{c}{a}$

- **Quadratic Equation : $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$**

QUESTIONS

1. Find the discriminant of the quadratic equation $3x^2 + 8x + 2 = 0$.
2. Find the value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots.
3. Find the value of k for which x = 2 is the root of the equation $kx^2 + 2x - 3 = 0$.
4. Find the value of x in the equation $(2x - 4)^2 = 64$
5. Find the value of k for which roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other.
6. Find the value(s) of z if $z^2 + \frac{1}{z^2} = 2, z \neq 0$.
7. If the value of discriminant for equation $3x^2 + rx + 4 = 0$ is 1. Find the value(s) of r.
8. Find the value of x which satisfies the equation $\frac{x}{17} + 1 = \sqrt{1 + \frac{35}{289}}$.
9. The roots of the equation $x^2 - 12x + p = 0$ are in the ratio 1:2, find the value of p.
10. If sum and product of roots of equation $kx^2 + 6x + 4k = 0$ are equal, then find the value of k.
11. Form a quadratic equation whose roots are $5 + \sqrt{3}$ and $5 - \sqrt{3}$.
12. What is the coefficient of x in the equation whose roots are 5 and -1?
13. If x = 1 is a common root of the equations $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$, then find the value of ab.
14. If the sum of the roots of the equation $x^2 - x = \lambda(2x - 1)$ is zero, then find the value of λ.
15. Find the quadratic equation whose one root is 2 and sum of the roots is zero.
16. Form a quadratic equation whose one root is $2 + \sqrt{7}$.
17. Solve for x if $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}}$.
18. Solve for x if $x = \sqrt{72 - \sqrt{72 - \sqrt{72 - \cdots}}}$.

19. If α and β are roots of the equation $x^2 - 3x + 2 = 0$,then find value of

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$ (ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

20. If one root is negative of the other then what is the coefficient of the middle term of the quadratic equation.

21. For what value of 'p' the equation $9x^2 - 12x + p = 0$ will be in the form of a perfect square?

22. Find the value of x which satisfy the equation $x + \frac{4}{x} = -4, x \neq 0$.

23. If the roots of $2x^2 + (4m + 1)x + 2(2m - 1) = 0$ are reciprocals of each other, find the value of m.

24. What is the ratio of the product and sum of the roots of the equation $5x^2 - 18x + 12 = 0$?

25. What is the sum of reciprocal of the roots of the equation $x^2 - 7x + 12 = 0$?

26. If roots of the equation $ax^2 + bx + c = 0$ are$5 \pm \sqrt{5}$, then find the value of a: c.

27. Find the value of p for which the product of roots of the quadratic equation $px^2 + 6x + 4p = 0$ is equal to sum of the roots.

28. If quadratic equation $x^2 - 5x - 6 = 0$ is expressed as (x+a) (x+b) =0, then find the value of a and b.

29. Find the positive value of k for which the equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will both have real roots.

30. If -4 is the root of the equation $x^2 + px - 4 = 0$ and the quadratic equation $x^2 + px + k = 0$ has equal roots, find the value of k.

31. If one root of the equation $ax^2 + bx + c = 0$ is three times the other, then find $b^2: ac$.

32. If one root of the equation $kx^2 - 14x + 8 = 0$ is six times the other, then find the value of k.

33. If α and β are roots of the equation $x^2 - 4x + 3 = 0$, then find value of $\alpha^4\beta^2 + \alpha^2\beta^4$.

34. Find the values of k for which $x^2 + 5kx - 16 = 0$ has no real roots.

35. If one root of the quadratic equation $2x^2 + kx + 4 = 0$ is 2, find the other root.

36. Find the quadratic equation whose roots are twice the roots of equation $3x^2 - 7x + 4 = 0$.

37. If the sum of first n natural numbers is given by S = $\frac{n(n+1)}{2}$ = 66, find the value of n.

38. If α and β are roots of the equation $x^2 - 3x - 2 = 0$, find a quadratic equation whose roots are $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$.

39. Find the value of k, if the difference of roots of quadratic equation $x^2 - 5x + (3k - 3) = 0$ is 11.

40. Solve for x, if $x = \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cdots\infty}}}$

41. The sum of a natural number and its reciprocal is $\frac{37}{6}$. Find the number.

42. Divide 29 into two parts such that their product is 198.

43. The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}$, then find the number.

44. Find two consecutive even integers whose squares have the sum 340.

45. A two digit number is 4 times the sum of digits and twice the product of digits. Find the number.

46. If the sum of first n even natural numbers is 420, then find n.

47. If an integer is added to its square, the sum is 90, then find the integer.

48. What is the condition to be satisfied for which quadratic equations $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ac}x + b = 0$ have equal roots?

49. Solve for x : $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

50. What must be the value of k so as to solve the quadratic equation $9x^2 + \frac{3}{4}x + k = 0$ by method of completing the square?

ANSWERS

1	40	26	1:20
2	0, 8	27	$-\frac{3}{2}$
3	$k = -\frac{1}{4}$	28	-6,1
4	6, -2	29	16
5	k=3	30	$\frac{9}{4}$
6	1, -1	31	16:3
7	7, -7	32	3
8	1	33	90
9	32	34	$-\frac{8}{5} < k < \frac{8}{5}$
10	$-\frac{3}{2}$	35	1
11	$x^2 - 10x + 22 = 0$	36	$3x^2 - 14x + 16 = 0$
12	-4	37	11
13	3	38	$16x^2 - 9x + 1 = 0$
14	$-\frac{1}{2}$	39	-7
15	$x^2 - 4 = 0$	40	$\sqrt{2} - 1$
16	$x^2 - 4x - 3 = 0$	41	$6, \frac{1}{6}$
17	3	42	11, 18
18	8	43	5, 10
19	(i) $\frac{3}{2}$ (ii) $\frac{5}{2}$	44	12, 14
20	0	45	36
21	4	46	20
22	-2	47	-10, 9
23	1	48	$b^2 = ac$
24	2:3	49	$-\frac{2b}{3a}, \frac{3a}{4b}$
25	$\frac{7}{12}$	50	$k = \frac{1}{64}$

CHAPTER – 5

ARITHMETIC PROGRESSION

POINTS TO REMEMBER

- General A P with n terms is $a, a+d, a+2d, \ldots,\ a+(n-1)\,d$ where a is the first term and d is the common difference.
- n^{th} term or last term of an A P is

$$a_n \text{ or } t_n \text{ or } l = a + (n-1)d$$

- r^{th} term or general term of an A P

$$a_r \text{ or } t_r = a + (r-1)d$$

- Sum of n terms of an A P = $S_n = \frac{n}{2}[2a + (n-1)d]$

Or

$$S_n = \frac{n}{2}[a + l]$$

- r^{th} term from the end of an A P = $(n-r+1)^{th}$ term from the beginning

$$= a + (n - r + 1 - 1)d$$

$$= a + (n - r)d$$

- r^{th} term of an A P from the end is $T_r = a_n - (r-1)d$ where a_n is the last term.
- If a, b and c are in A.P then $2b = a + c$
- If sum of first three terms in A P is given then we take the first three terms as

$$a - d, a, a + d$$

- If sum of first four terms in A P is given then we take the first four terms as

$$a - 3d, a - d, a + d, a + 3d$$

- If sum of first five terms in A P is given then we take the first three terms as

$$a - 2d, a - d, a, a + d, a + 2d$$

- To find a_n when S_n given : $a_n = S_n - S_{n-1}$
- Common difference $d = a_{n+1} - a_n$

QUESTIONS

1. If $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}$ are in A.P, then find next three terms.
2. What is the next term of the A.P
$$\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$$
3. For what value of k, the terms 2k, k+10 and 3k+2 are in A.P.
4. If $a_n = 5 - 11n$, then find the common difference.
5. Find the value of x if $8x + 9, 6x - 2, 2x - 7$ are three consecutive terms of an A.P.
6. Find the common difference of an A.P where n^{th} term is 2n+5.
7. If sum of first n terms of an A.P is $S_n = an^2 + bn$, find its common difference.
8. If the sum of first n terms of an A.P is $5n^2 + 2n$, then find its 2^{nd} term.
9. Find the common difference of the A.P
$$\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$$
10. What is the n^{th} term of the A.P
$$\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, \ldots$$
11. If $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ are in A.P, then find the value of x.
12. If x, 13, y, 3 are in A.P, then find the value of x and y.
13. What is the sum of first n natural numbers?
14. What is the sum of first n odd natural numbers?
15. What is the sum of first n even natural numbers?
16. If the sum of first n even natural numbers is equal to k times the sum of first n odd natural numbers, then find the value of k.
17. If 18, a, b, -3 are in A.P, then find a+b.
18. If $4, a_2, a_3, a_4, 28$ are in A.P, then find a_4.
19. If the sum of n terms of an A.P is $3n^2 - n$ and common difference is 6, then find its first term.
20. If the numbers a, b, c, d, e form an A.P, then find the value of a-4b+6c-4d+e.

21. If three consecutive terms of an A.P are a-d, a, a+d. Their sum is 33 and d is 5, then find the terms.
22. If a, b, c are in A.P, then find the value of (a +2b –c) (2b+ c -a) (c+a-b).
23. If the sides of a right triangle are in A.P, then find the ratio of its sides.
24. If sum of three consecutive terms of an A.P is 24, then find its middle term.
25. If sum of five consecutive terms of an A.P is 115, then find its third term.
26. Angles of a triangle are in A.P. If smallest angle is $40°$, then find the largest angle.
27. The angles of a quadrilateral are in A.P whose common difference is $10°$, find the angles.
28. Find the sum of n terms of the series

$$\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\cdots$$

29. Find a, b and c such that the following numbers are in A.P: a, 7, b, 23, c.
30. Divide 16 into 4 parts which are in A.P such that the product of extremes is one less than the sum of means.
31. Which term of the AP. 72, 63, 54... is 0?
32. If the first three terms of an A.P are b, c and 2b, then find the ratio of b and c.
33. If 7 times the 7^{th} term of an A.P is equal to 11 times its 11^{th} term, then find its 18^{th} term.
34. Find the 20^{th} term from the end of the A.P 3, 8, 13 …,253.
35. How many two digit natural numbers are there, which when divided by 3 yield 1 as reminder?
36. If sum of m terms of an A.P is same as the sum of its n terms, then find the sum of its $(m+n)^{th}$ term.
37. Find the sum of $1-6+2-7+3-8+\cdots$ to 100 terms.
38. Which term of the A.P $52, 48, 44, \ldots$ will be the first negative term?
39. From your pocket money you save ₹1 on day 1, ₹ 2 on day 2, ₹3 on day 3 and so on. How much money will you save in the month of February 2024?
40. Find the sum of all 11 terms of an A.P whose middle most term is 30.

41. If the p^{th} term of an A.P is q and q^{th} term is p, then find its n^{th} term.
42. If the m^{th} term of an A.P is $\frac{1}{n}$ and n^{th} term of an A.P is $\frac{1}{m}$, then find its mn^{th} term.
43. Find the sum of first 20 odd natural numbers.
44. The 9^{th} term of an A.P is 449 and 449^{th} term is 9. Find which term is equal to 0?
45. For the A.P: $-3, -7, -11, \ldots$ Find $a_{30} - a_{20}$.
46. The first and last term of an A.P are 5 and 45 respectively. Find the number of terms if sum of all the terms is 500.
47. If 8^{th} term of an A.P is zero, then what is the relation between 28^{th} and 18^{th} term?
48. If 15^{th} term of an A.P exceeds its 10^{th} term by 20, then find the common difference.
49. If 3^{rd} and 9^{th} term of an A.P are 4 and -8 respectively, then which term of the A.P is 0?
50. A man got a job with monthly salary of ₹ 7000 with an annual increment of ₹ 500. What will be his salary after 10 years?

ANSWERS

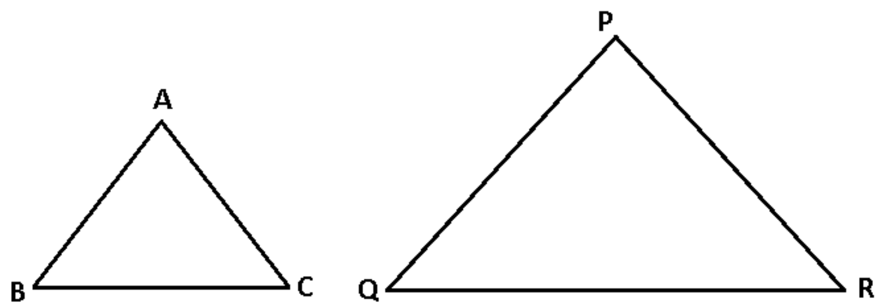
1	$5\sqrt{3}, 6\sqrt{3}, 7\sqrt{3}$	26	80°
2	$\sqrt{112}$	27	$75°, 85°, 95°, 105°$
3	6	28	$\frac{1}{2}(7n-1)$
4	-11	29	a = -1, b= 15, c = 31
5	3	30	1, 3, 5, 7
6	2	31	9
7	2a	32	b:c = 2:3
8	17	33	0
9	-1	34	158
10	$\frac{1+mn-m}{m}$	35	30
11	1	36	0
12	18,8	37	-250
13	$\frac{n(n+1)}{2}$	38	15th term
14	n^2	39	₹ 435
15	$n(n+1)$	40	330
16	$\frac{n+1}{n}$	41	p+q-n
17	15	42	1
18	22	43	400
19	2	44	458th term
20	0	45	-40
21	6,11,16	46	20
22	4abc	47	twice
23	3:4:5	48	4
24	8	49	5th term
25	23	50	₹ 12,000

CHAPTER – 6

TRIANGLES

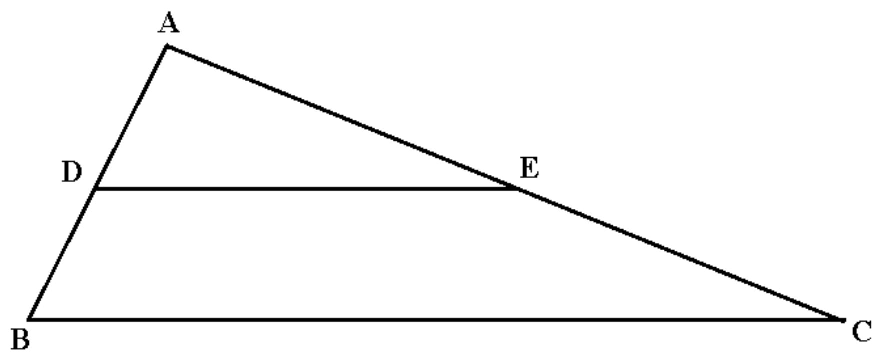
POINTS TO REMEMBER

- **Two triangles are said to be similar if their corresponding angles are equal and their corresponding sides are proportional(in the same ratio)**



$$\Delta ABC \sim \Delta PQR \Rightarrow \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \ \& \ \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

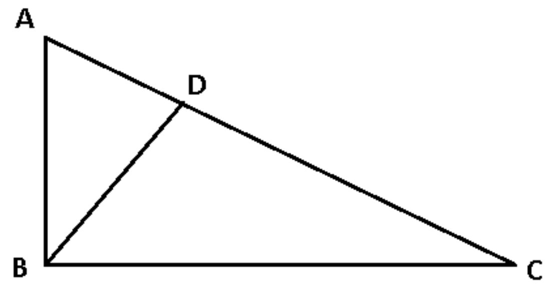
- **Criteria of Similarity: (a) AAA (b) SSS (c) SAS**
- **If a line is drawn parallel to one side of triangle to intersect the other two sides in two distinct points, then the other two sides are divided in same ratio.**



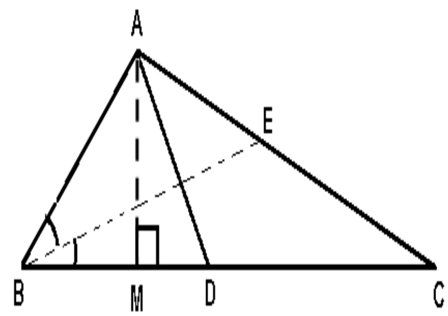
$$\frac{AD}{DB} = \frac{AE}{EC} \text{ OR } \frac{AB}{DB} = \frac{AC}{EC} \text{ OR } \frac{AB}{AD} = \frac{AC}{AE}$$

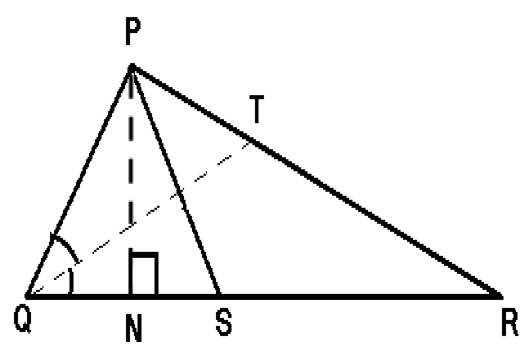
- **If** $\Delta ABC \sim \Delta PQR \Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

- **In $\Delta ABC, \angle B = 90^\circ$ then $AC^2 = AB^2 + BC^2$(Pythagoras theorem)**



- **In $\Delta ABC, \angle B = 90^\circ$ and $BD \perp AC$ then $\Delta ABD \sim \Delta ACB \sim \Delta BCD$.**
- **If two triangles are similar then their perimeters, medians, altitudes and angle bisectors are in the same ratio.**
- **The areas of two similar triangles are in the ratio of squares of their corresponding sides, altitudes, medians, perimeter and angle bisectors.**



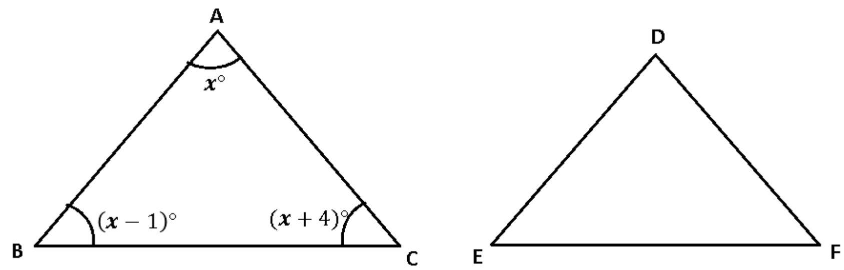


- **E.g. If $\Delta ABC \sim \Delta PQR \Rightarrow \frac{\text{perimeter}(\Delta ABC)}{\text{perimeter}(\Delta PQR)} = \frac{AD}{PS} = \frac{AM}{PN} = \frac{BE}{QT}$ and**

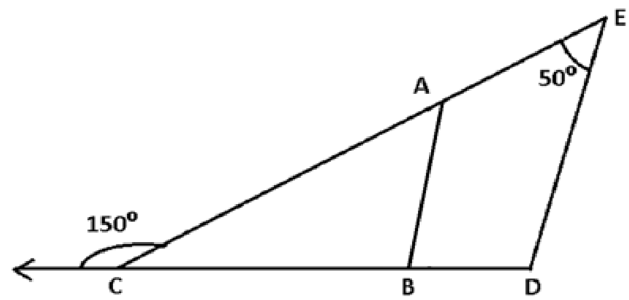
$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{AD^2}{PS^2} = \frac{AM^2}{PN^2} = \frac{(\text{perimeter of } \Delta ABC)^2}{(\text{perimeter of } \Delta PQR)^2} = \frac{BE^2}{QT^2}$$

QUESTIONS

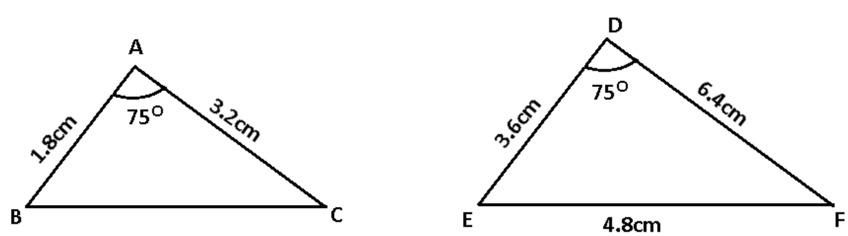
1. **In fig. $\Delta ABC \sim \Delta DEF$, find $\angle D$.**



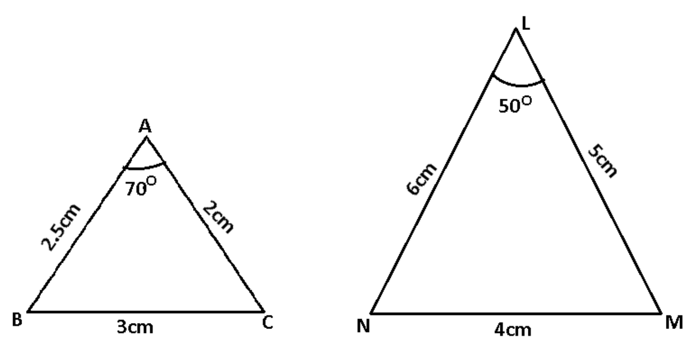
2. **In fig. $\Delta ACB \sim \Delta ECD$, find $\angle ABC$.**



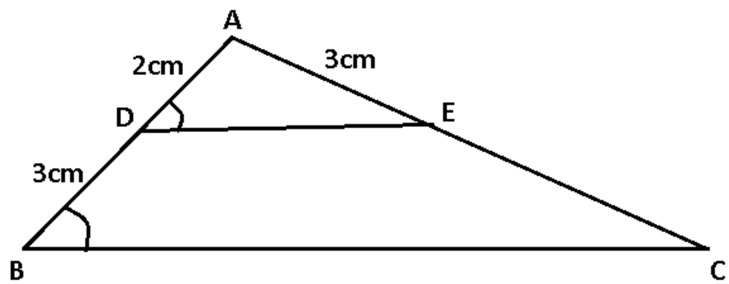
3. **In fig. find BC**



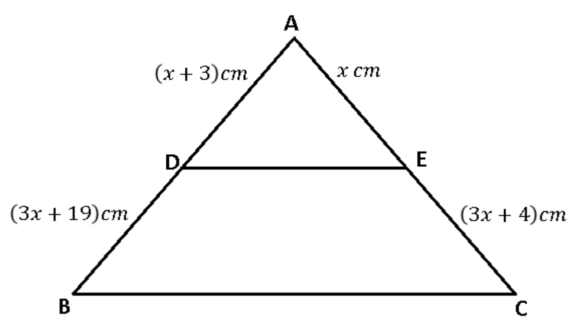
4. **In fig. find $\angle M$ and $\angle N$.**



5. In fig., if $\angle ADE = \angle ABC$, then find CE.

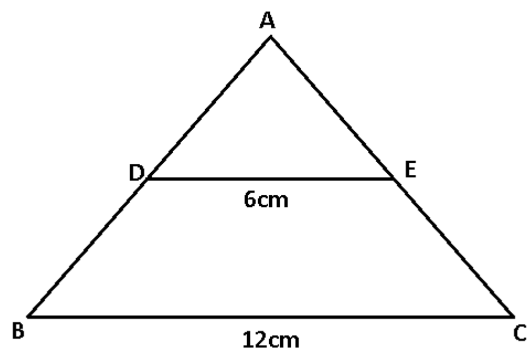


6. ΔABC and ΔBDE are two equilateral triangles such that D is the mid-point of BC. Find the ratio of the areas of triangles ABC and BDE.
7. If $\Delta ABC \sim \Delta DEF$, $ar(\Delta DEF) = 100$ sq. cm and $\frac{AB}{DE} = \frac{1}{2}$, find $ar(\Delta ABC)$.
8. $\Delta ABC \sim \Delta DEF$, AB = 4cm, DE = 6cm, EF = 9cm and FD = 12cm, find the perimeter of ΔABC.
9. Find the value of x for which DE//BC.

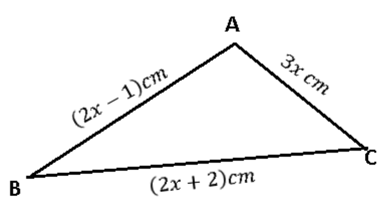


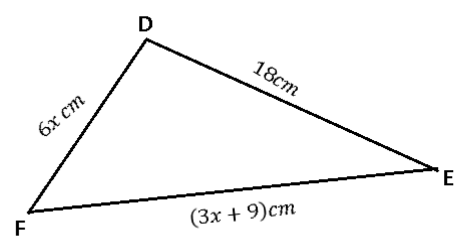
10. Corresponding sides of two similar triangles are in the ratio 2:3. If the area of the smaller triangle is 48 sq.cm, then find the area of larger triangle.
11. Areas of two similar triangles are 36sq.cm and 100sq.cm. If the length of a side of the larger triangle is 20cm, then find the length of the corresponding side of smaller triangle.
12. ΔABC and ΔPQR are similar triangles such that $\angle A = 32°$ and $\angle R = 65°$. Find $\angle B$.
13. In ΔABC, AB=$6\sqrt{3}$cm, AC=12cm and BC= 6cm. Find $\angle B$.

14. A man goes 15m due East and then 8m due North. How much distance is he away from the starting point?
15. $\Delta PQR \sim \Delta ABC$, if PQ:AB=3:4 and $ar(\Delta PQR) = 216 sq. units$, find the area of ΔABC.
16. In the given fig., if DE//BC, then find the ratio of $ar(\Delta ADE)$ and $ar(\blacksquare DECB)$.



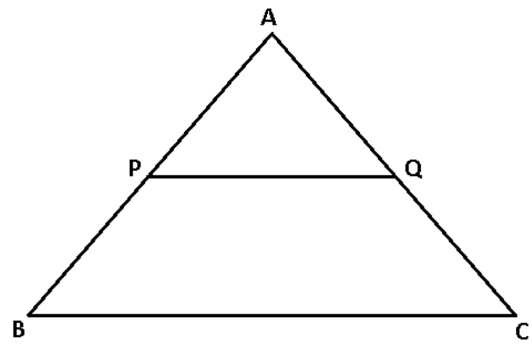
17. An Aeroplane leaves an airport and flies due north at 300km/hr. At the same time, another aeroplane leaves the same airport and flies due west at 400km/hr. How far apart the two aeroplanes would be after $1\frac{1}{2}$ hours?
18. If$\Delta ABC \sim \Delta DEF$, find the value of x.





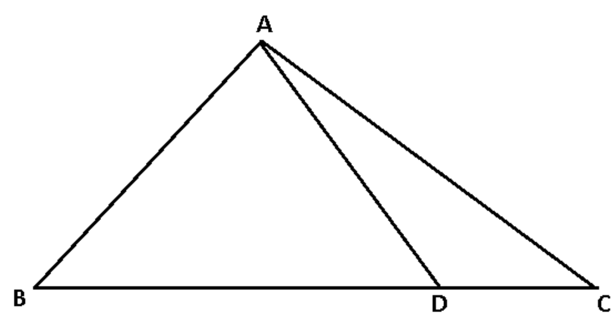
19. In an isosceles right triangle, if the hypotenuse is $5\sqrt{2}$cm, then find the length of each equal side of the triangle.
20. If$\Delta ABC \sim \Delta DEF$, and AB = 10cm, EF = 6cm, DE = 8cm then find $\frac{ar(\Delta ABC)}{ar(\Delta DEF)}$.

21. In fig. PQ//BC and AP: PB=1:2, find $\frac{\text{perimeter of } \Delta APQ}{\text{perimeter of } \Delta ABC}$.

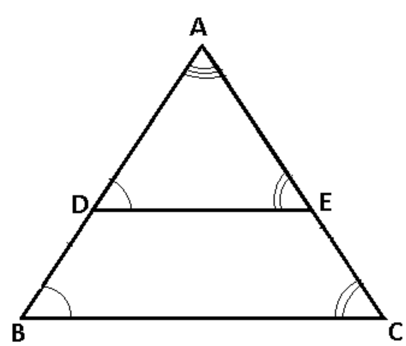


22. Two poles of heights 10 m and 15 m stand vertically on a plane ground. If the distance between their feet is $5\sqrt{3}$ m, find the distance between their tops.

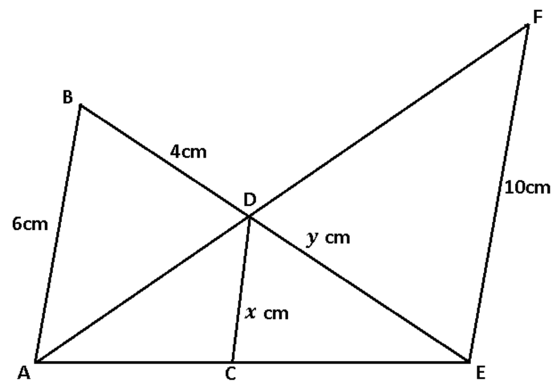
23. In ΔABC, it is given that $\frac{AB}{AC} = \frac{BD}{DC}$, if $\angle B = 70°$ and $\angle C = 50°$, find $\angle BAD$.



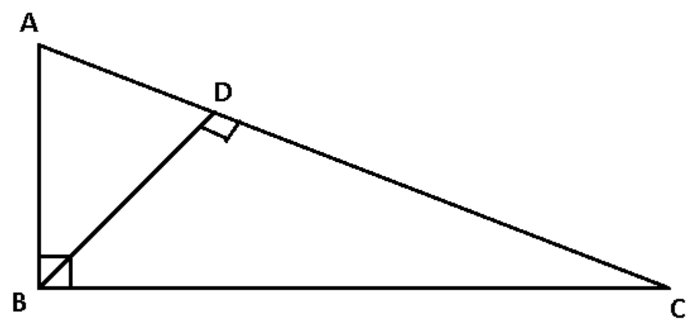
24. In ΔABC, DE//BC, If BC=8 cm, DE= 6 cm and $ar(\Delta ADE) = 45$sq.cm, then find $ar(\Delta ABC)$.



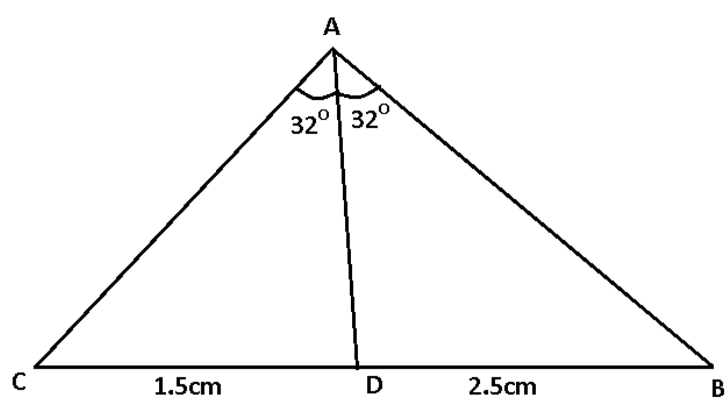
25. **In fig. AB//CD//EF, if AB= 6cm, CD= x cm, EF = 10cm, BD = 4cm and DE = y cm, find the value of x and y.**



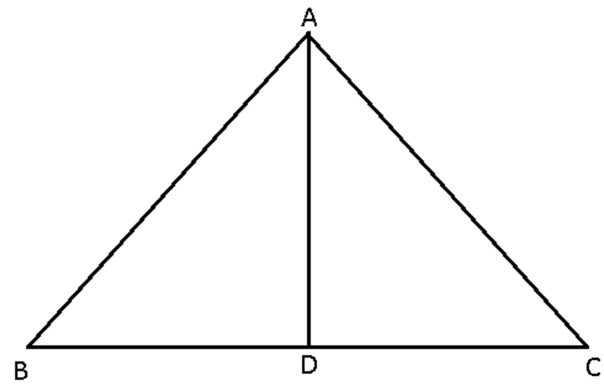
26. **In fig. ABC is a right angled triangle at B andBD $\perp$ AC, if AD = 4cm and CD= 5cm. Find BD.**



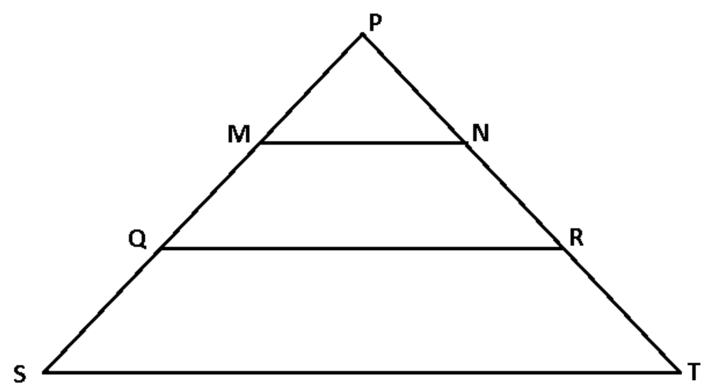
27. **In fig. find $\frac{AB}{AC}$**



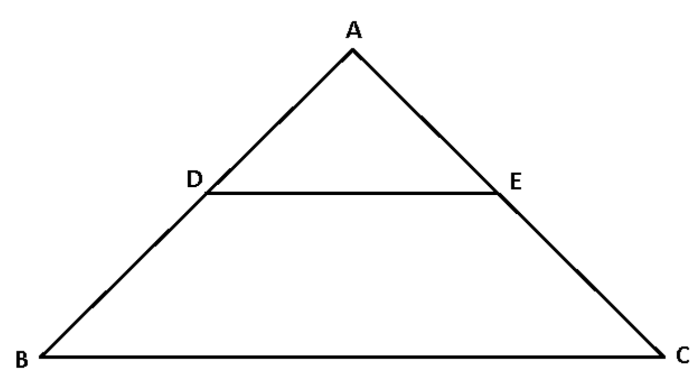
28. In fig. ΔBAC is an isosceles triangle with AB=AC. If AD is the bisector of $\angle BAC$ then find $\frac{BD}{DC}$.



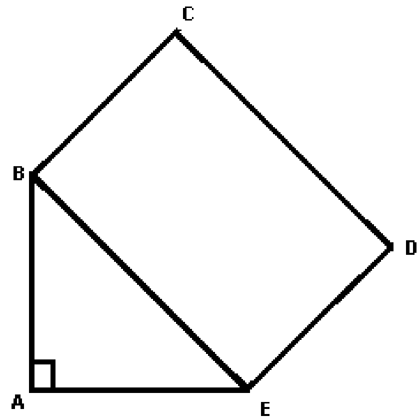
29. In fig.$\angle S = \angle PRQ$, MN//QR, if $\frac{PS}{PT} = \frac{2}{3}$, then find $\frac{PN}{PM}$.



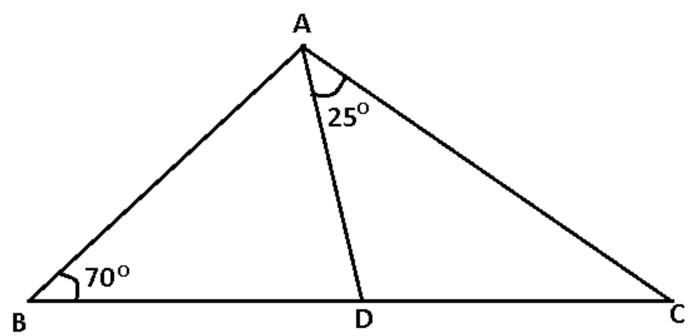
30. In fig. DE∥BC and AD: DB= 2:3. find $ar(\Delta ADE): ar(\Delta ABC)$



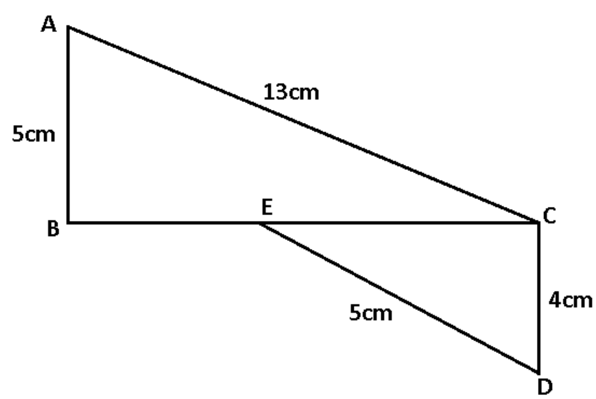
31. In the given fig. AB=BC=12cm and AE=5cm. Find the perimeter of rectangle BCDE.



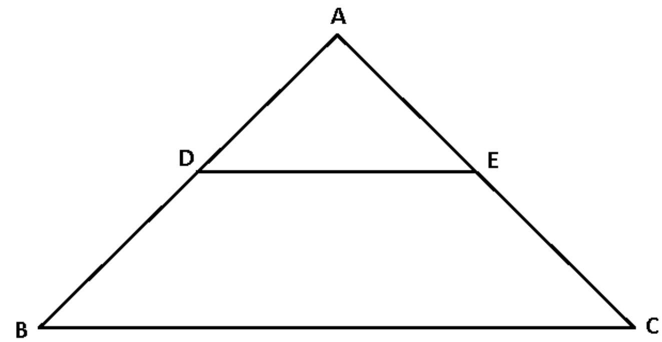
32. In the given fig. if $\frac{AB}{AC} = \frac{BD}{CD}$, then find $\angle ACD$.



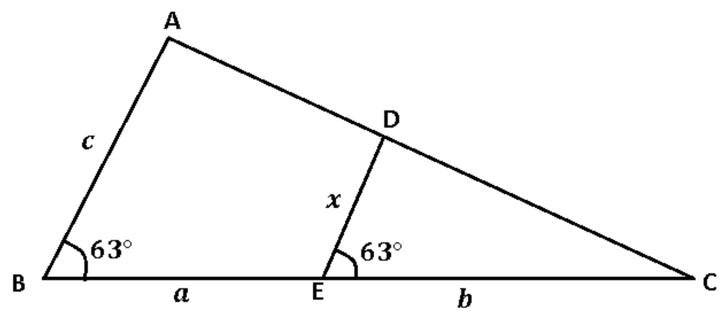
33. ΔABC And ΔDCE are right angled triangles in which $\angle ABC = . \angle DCE = 90°$, find BE



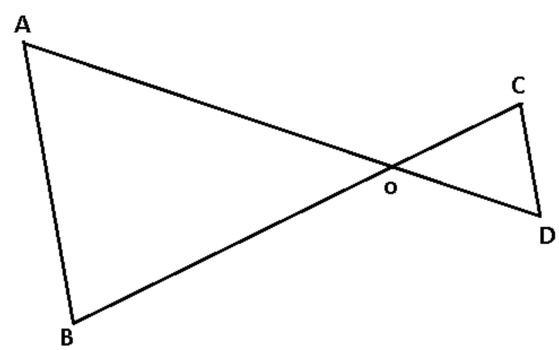
34. In the given fig., if DE∥BC, AD=4BD and AC=1.6 cm then find AE.



35. Express x in terms of a, b and c from the given figure.

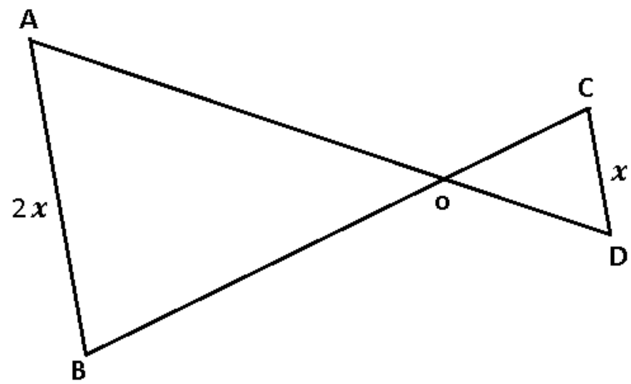


36. In the given figure if AB = 3 cm, CD = 2 cm, OD = 2.2 cm and $\angle B = \angle C$, then find OA.

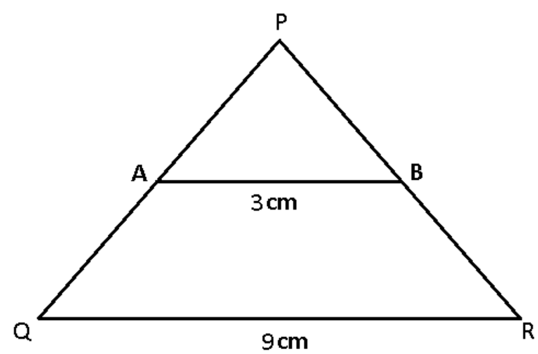


37. The length of sides of triangle are 12cm, 16cm and 21cm. The bisector of the greatest angle divides the opposite sides into 2 parts. Find the ratio of these two parts.

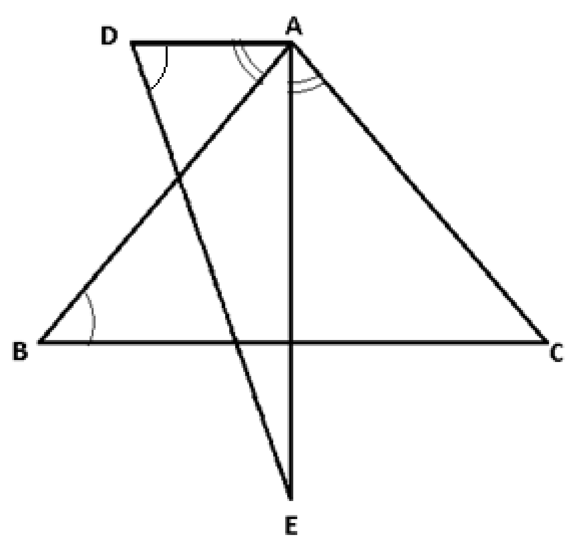
38. In fig. AB//CD, if the area of ΔAOB is 84sq.cm then find the ar(ΔCOD)



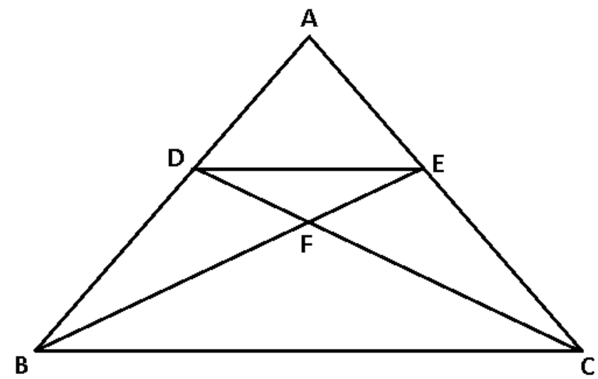
39. In the given fig. if $\angle PBA = \angle PRQ$, find BP if PR=6cm.



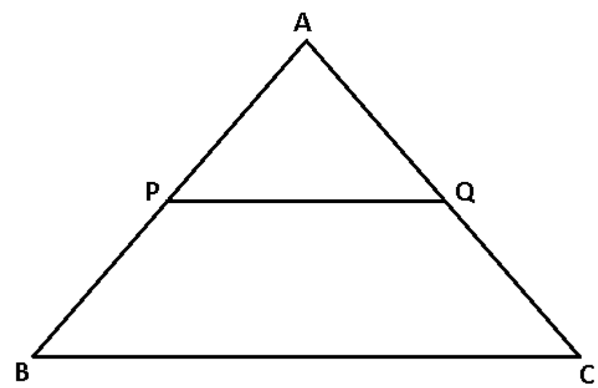
40. In fig. $\angle BAD = \angle CAE$ And$\angle ADE = \angle ABC$. If AC: BC=3:2, then find the ratio DE: AE.



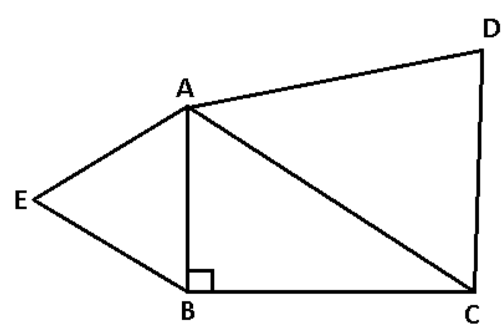
41. In fig. DE//BC and AD:DB = 5:4, then find $\frac{ar(\Delta DFE)}{ar(\Delta CFB)}$



42. In the given figure, PQ∥BC and AP: PB=3:7. If ar(ΔABC) is 100 sq.cm, then find ar(trapezium PQCB)



43. ΔABC is an isosceles right triangle right angled at B. Two equilateral triangles are constructed on sides AB and AC. Find $\frac{ar(\Delta ABE)}{ar(\Delta ACD)}$.



44. **If $\Delta ABC \sim \Delta DEF$ such that DE=3cm, EF=2cm, DF=2.5cm, BC=4cm then find the perimeter of ΔABC.**
45. **If $\Delta ABC \sim \Delta DEF$ such that AB=9.1cm and DE=6.5cm. If the perimeter of ΔDEF is 25cm, then find the perimeter of ΔABC.**
46. **If $\Delta ABC \sim \Delta DEF$ and 2AB=DE and BC=8cm then find EF.**
47. **$\Delta ABC \sim \Delta PQR$ such that $ar(\Delta ABC) = 4ar(\Delta PQR)$. If BC=12cm then find QR.**
48. **The foot of the ladder is 6m away from the wall and its top reaches a window 8m above the ground. If the ladder is shifted in such a way that its foot is 8m away from the wall, to what height does it tip reaches?**
49. **In an isosceles ΔABC, if AB=AC=25cm and BC=14cm, then find the measure of altitude from A on BC.**
50. **In ΔABC, $\angle A = 90°$,AB=5cm and AC=12cm. if AD$\perp$BC, then find AD.**

ANSWERS

1	59°	26	BD=$2\sqrt{5}$cm
2	100°	27	$\frac{5}{3}$
3	2.4cm	28	1
4	∠M=70°,∠N=60°	29	$\frac{2}{3}$
5	4.5cm	30	4:25
6	4:1	31	50cm
7	25sq.cm	32	60°
8	18cm	33	9cm
9	x=2	34	1.28cm
10	108sq.cm	35	$\frac{bc}{a+b}$ units
11	12 cm	36	3.3cm
12	83°	37	3:4
13	90°	38	21sq.cm
14	17m	39	2cm
15	384sq.units	40	2:3
16	1:3	41	$\frac{25}{81}$
17	750km	42	91 sq.cm
18	x=5	43	1:2
19	5cm	44	15cm
20	$1\frac{9}{16}$	45	35cm
21	1:3	46	16cm
22	10m	47	6cm
23	30°	48	6 m
24	80sq.cm	49	24 cm
25	x=3.75cm , y= 6.67cm	50	$4\frac{8}{13}$cm

CHAPTER -7

COORDINATE GEOMETRY

POINTS TO REMEMBER

- To locate the position of a point in a plane, we require a pair of coordinates.
- Coordinate axes divide the plane into four quadrants.
- The perpendicular distance of a point from the y-axis measured along the x-axis is called its x coordinate (abscissa).
- The perpendicular distance of a point from the x – axis measured along the y axis is called its y coordinate (ordinate).
- The coordinate of a general point on x axis is of the form (x, 0).
- The coordinate of a general point on y axis is of the form (0, y).
- The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

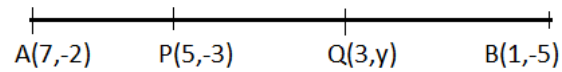
- The distance of a point P(x, y) from the origin O (0, 0) is given by

$$OP = \sqrt{x^2 + y^2}$$

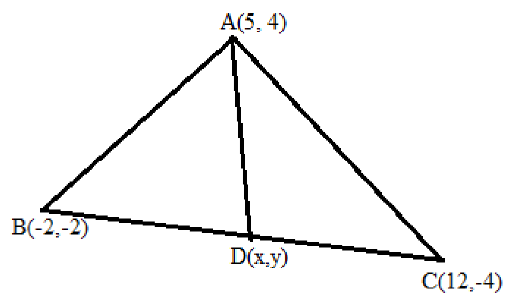
- Coordinates of the point P(x, y) which divides the line segment joining the points $A(x_1, y_1)$, and $B(x_2, y_2)$ internally in the ratio m_1: m_2 are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$. It is called section formula.
- The coordinates of the midpoint of the line segment joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
- The coordinates of the centroid of the triangle formed by points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
- The area of the triangle formed by points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$is given by $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$ sq.units.
- If points A (x_1, y_1), B (x_2, y_2) and C $(x_3, y_3)$are collinear, then the Area of triangle formed by these three points is 0 and vice versa.

QUESTIONS

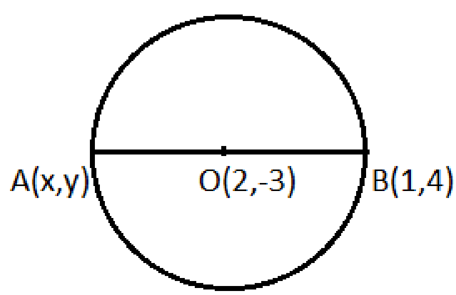
1. Find the distance between the points (a cos 55°, 0) and (0, a cos 35°)
2. Find the distance between the points (cosθ, sinθ) and (- sinθ, cosθ).
3. In the given figure find y, if P (5,-3) and Q (3,y) are the points of trisection of the line segment joining A (7,-2) and B (1,-5).



4. In the given figure, ABC is a triangle and D is the midpoint of BC. Find the co-ordinates of D.

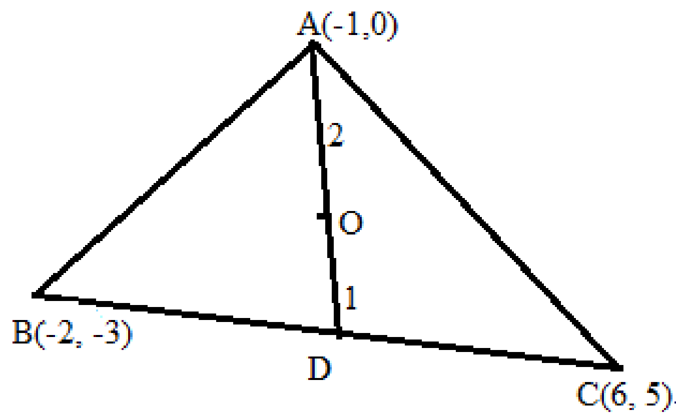


5. In figure as given in question no. 4, find the length of AD.
6. Find the distance between the points (a cosb, a sinb) from the origin.
7. Find the coordinates of the point, which divides the line joining the points A (3, -6) and B (-2, 7) in the ratio 2:3.
8. In the given figure, find the coordinates of A.

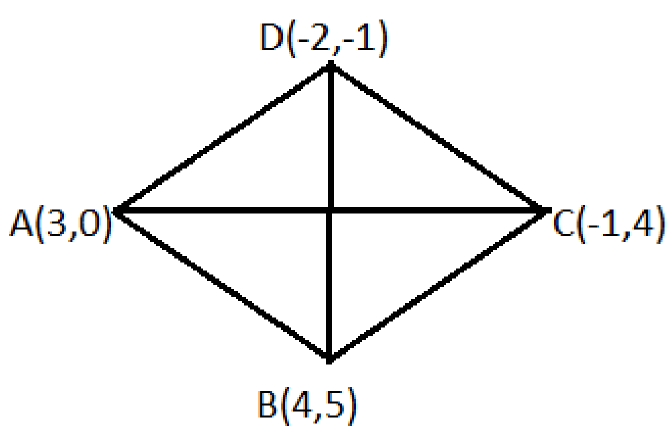


9. Find the distance of the point (0, 2) from the midpoint of the line segment joining (4, 10) and (2, 2).
10. Find the value of y, if P(x, y) divides the line segment joining points (-3, 3) and (1, -2) in the ratio 2:3.

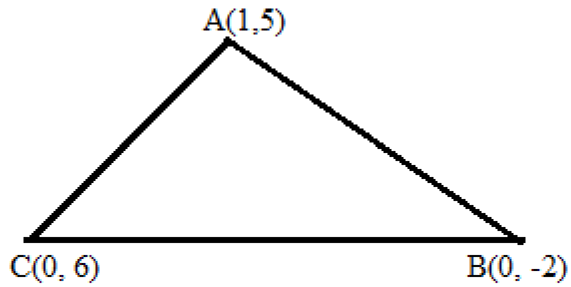
11. **Find the value of x, if P(x, y) divides the line segment joining points A (7, -5) and B (2, -1) in the ratio 4:1.**
12. **Find the value of k, if point (0, 4) is equidistant from the points (10, k) and (k, 8)**
13. **Find the value of k, if x axis divides the line joining the points (-4, -6) and (5, 2) in the ratio k: 1.**
14. **In the given figure, A (-1, 0), B (-2, -3) and C (6, 5) are the coordinates of ΔABC. If D is the mid-point of BC, then find the coordinates of O which divides AD in the ratio 2:1.**



15. **In the given figure, find the area of rhombus.**



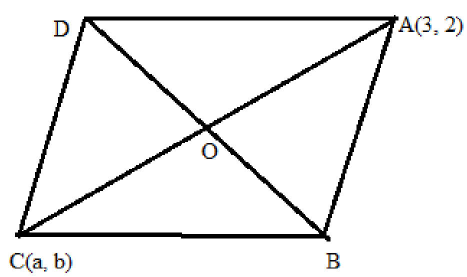
16. Find the area of triangle whose coordinates are A (1, 5), B (0, -2) and C (0, 6).



17. Find the value of p for which the points (-1, 3), (2, p), and (5, -1) are collinear.

18. Find x and y, if O (0, 0), A (0, 2), B(x, y) and C (3, 0) form a rectangle OABC.

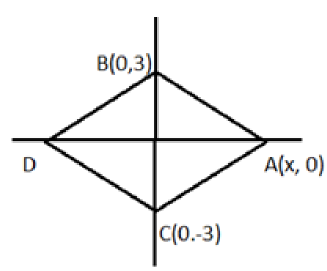
19. In the parallelogram ABCD, coordinates of A and C are (3, 2) and (a, b) respectively. If AC and BD intersects at O (0, 0), then find the values of 'a' and 'b'.



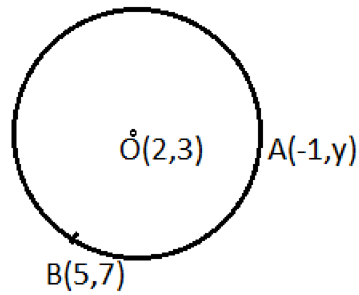
20. Find the area of triangle ABC with A (1,-4) and mid points of sides through A being (2,-1) and (0,-1).

21. Find the value of y such that points A (5, y), B (5, 2), C (2, 2) and D (2, 5) form a square ABCD.

22. In the given figure, find the coordinates of the points A and D, if BACD is a rhombus and the base BC of an equilateral ΔABC lies on y axis.



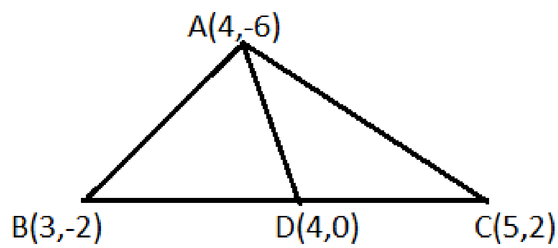
23. In the given figure, O is the Centre of circle and A and B are any points on circle, find y.



24. Q is the midpoint of the line segment PR where coordinates of P, Q and R are (6, -2), (1, 3) and (x, 8) respectively. Find 'x'.

25. Find coordinates of point P, if P and Q trisect the line segment joining the points (5, -3) and (-1, 3)

26. In figure, find the value of Area of (ΔABC): Area of (ΔABD).



27. Find the area of the triangle formed by joining the mid points of the sides of a triangle, whose vertices are (3, 2), (5, 4), and (3, 6).

28. Find the values of p and q, if the line segment joining the points (3, -4) and (1, 2) is trisected at the point (p, -2) and $\left(\frac{5}{3}, q\right)$.

29. The line joining the points (2, 1) and (5, -8) is trisected at points P and Q. If point P lies on the line $2x - y + k = 0$, then find k.

30. Find the coordinates of vertex C, if length of one of the sides of an equilateral triangle is 'a' and base BC lies on x-axis with B at the origin.

31. Find the coordinates of P, if the distance of the point P from the point (3, 4) is $\sqrt{10}$ units and abscissa of P is double of its ordinate.

32. If the area of the triangle ABC formed by A(x, y), B (1, 2) and C (2, 1) is 6 square units, then find the value of x + y.

33. If ($\frac{a}{3}$, 4) is the midpoint of the segment joining the points P (-6, 5) and R (-2, 3), then find the value of a.

34. Find the value of x, if the distance of the point (0, x) from (3, 5) is 5 units.

35. Find the area of triangle formed by (a, b + c), (b, c +a) and (c, a + b).

36. If points (a, 0), (0, b) and (1, 1) are collinear, then find the value of $\left(\frac{1}{a} + \frac{1}{b}\right)$.

37. If the centroid of the triangle formed by the points (a, b), (b, c) and (c, a) is at the origin, then find the value of $a^3 + b^3 + c^3$.

38. If the centroid of a triangle is (1, 4) and two of its vertices are (4, -3) and (- 9, 7), then find the area of triangle.

39. If the centroid of the triangle formed by (7, x), (y, -6) and (9, 10) is at (6, 3), then find the value of x and y.

40. Find the value of y, if the points A (5, y), B (5, 5), C (1, 5) and D (1, 2) are the vertices of rectangle.

41. Find the area of triangle formed by joining the points (0, 0), (0, 2) and (2, 0).

42. Find the coordinates of point P which lies on x axis and equidistant from (-2, 5) and (2, -3).

43. Find the value of p + q, if A (p, q) is the midpoint of the line segment joining the points (5, 3) and (-2, 4).

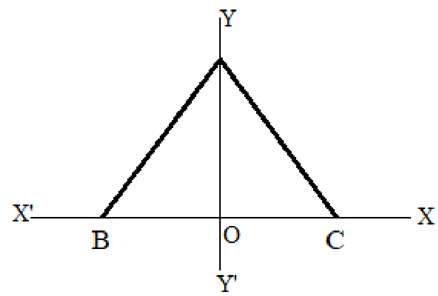
44. Find the coordinates of point p that lies on y axis and equidistant from (3,4) and (-2, 5)

45. The points $(0, -1)$, (2, 1), (0, 3) and $(-2, 1)$ are the vertices of a square. Find the sum of the length of all sides and diagonals.

46. Find the ratio in which the line joining the points A (-4, 4) and B (8, 8) is divided by (-1, 5).

47. Find the value of p and q, if the midpoints of the line segment joining (3p, 4) and (-2, 2q) is (2, 6).

48. **The base BC of an equilateral ΔABC with side 24cm lies along the x-axis such that the midpoint of the base is at origin. Find the coordinates of B.**



49. **The three vertices of a rhombus taken in order are (-2, -1), (3, 0) and (4, 5). Find the coordinates of the fourth vertex.**
50. **Find the value of 'a' for which the points (0, 0), (1, 1) and (2, a) will be collinear.**

ANSWERS

1	**a units**	26	**2:1**
2	$\sqrt{2}$ **units**	27	**1 square unit**
3	**y = -4**	28	**p =** $\frac{7}{3}$ **and q =0**
4	**(5, -3)**	29	**-8**
5	**7 units**	30	$(a, 0)$ **or** $(-a, 0)$
6	**a units**	31	**(6, 3), (2, 1)**
7	$(1, -\frac{4}{5})$	32	**15**
8	**A(3, -10)**	33	**-12**
9	**5 units**	34	**x = 1, 9**
10	**y = 1**	35	**0 square unit**
11	**x = 3**	36	**1**
12	$k = \frac{25}{2}$	37	**3abc**
13	**k = 3**	38	**91.5 square units**
14	$(1, \frac{2}{3})$	39	**x = 5, y = 2**
15	**24 square units**	40	**y = 2**
16	**4 square units**	41	**2 square units**
17	**p = 1**	42	$(-2, 0)$
18	**x = 3, y = 2 or (3,2)**	43	**p + q = 5**
19	**a = -3 and b = -2**	44	$P(0, 2)$
20	**12 square units**	45	$8(\sqrt{2} + 1)$ **units**
21	**y = 5**	46	**1 : 3**
22	$A = (3\sqrt{3}, 0)$, $D\ (-3\sqrt{3}, 0)$	47	**p = 2, q = 4**
23	**y = -1 or 7**	48	$(-12, 0)$
24	**x = -4**	49	$(-1, 4)$
25	$(3, -1)$	50	**a = 2**

CHAPTER – 8

INTRODUCTION TO TRIGONOMETRY

POINTS TO REMEMBER

1) $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{\text{cosec}\theta}$

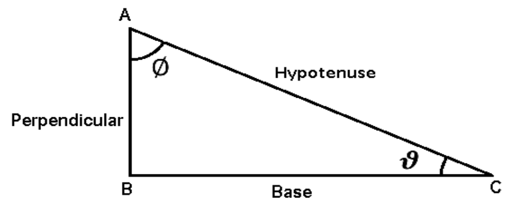
2) $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{1}{\sec\theta}$

3) $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{1}{\cot\theta}$

4) $\text{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Perendicular}} = \frac{1}{\sin\theta}$

5) $\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{1}{\cos\theta}$

6) $\cot\theta = \frac{\text{Base}}{\text{Perendicular}} = \frac{1}{\tan\theta}$



$$\emptyset = 90° - \vartheta$$

$$\sin\emptyset = \sin(90° - \vartheta) = \frac{BC}{AC} = \cos\vartheta$$

Trigonometric ratios of complementary angles

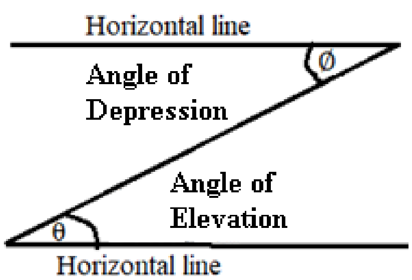
1) $\sin(90^\circ-\theta) = \cos\theta$ 2) $\cos(90^\circ-\theta) = \sin\theta$ 3) $\tan(90^\circ-\theta) = \cot\theta$

4) $\cot(90^\circ-\theta) = \tan\theta$ 5) $\sec(90^\circ-\theta) = \text{cosec}\theta$ 6) $\text{cosec}(90^\circ-\theta) = \sec\theta$

Trigonometric Identities

1) $\sin^2\theta + \cos^2\theta = 1$ 2)) $\sec^2\theta - \tan^2\theta = 1$ 3) $\text{cosec}^2\theta - \cot^2\theta = 1$

Angles of elevation and angles of depression: -



1) **Elevation is the angle up from the Horizontal.**
2) **Depression is the angle down from the Horizontal.**
3) **Angle of Elevation = Angle of Depression**

QUESTIONS

1. If $A + B = 90^\circ$, then find the value of tanA tanB.
2. If $\alpha + \beta = 90^\circ$ and $\sec\alpha = \frac{19}{7}$, then find cosec β.
3. If $\theta = 45^\circ$, then find the value of $2\sin\theta\cos\theta$.
4. Find the value of $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$.
5. If $5\cos\theta = 3$, then find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}$
6. If $\tan\theta = \frac{12}{5}$, then find the value of $\frac{13\sin\theta}{3}$.
7. Find the value of $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2$.
8. If $\sin A = \frac{1}{2}$, then find value of $3\cos A - 4\cos^3 A$.
9. Find the value of $(\sec^2\theta - 1)(1 - \mathrm{cosec}^2\theta)$.
10. Find the value of $3\tan^2 26^\circ - 3\mathrm{cosec}^2 64^\circ$.
11. If $\tan\theta + \cot\theta = 2$, then find the value of $\tan^2\theta + \cot^2\theta$.
12. If $\cos\theta - \cos(90^\circ - \theta) = 0$, then find the value of θ.
13. Evaluate $\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ$
14. Find the value of acute angle θ if $\sin(\theta + 26^\circ) = \cos\theta$.
15. Find the value of $\sin^2 1^\circ + \sin^2 5^\circ + \sin^2 9^\circ + \ldots + \sin^2 89^\circ$
16. If $a = 3\sec^2\theta - 1$ and $b = 3\tan^2\theta + 2$, then find the value of $(a - b)$.
17. If $\sec\theta - \tan\theta = k$, then what is the value of $\sec\theta + \tan\theta$.
18. If $x = 15^\circ$, then find the value of $4\sin 2x \cos 4x \sin 6x$.
19. If $\sin x + \sin^2 x = 1$, then find the value of $\cos^2 x(1 + \cos^2 x)$.
20. If $6x = \sec\theta$ and $\frac{6}{x} = \tan\theta$, find the value of $9\left(x^2 - \frac{1}{x^2}\right)$.
21. If $k - 2 = \sec^2 A(1 + \sin A)(1 - \sin A)$, then find the value of k.
22. Evaluate $5\tan^2 A - 5\sec^2 A + 1$
23. If $\sin A - \cos A = 0$, then find the value of $(\sin^2 A)^2 + (\cos^2 A)^2$
24. Find the value of $\sin^2 10^\circ + \sin^2 20^\circ + \sin^2 30^\circ + \ldots + \sin^2 80^\circ$
25. If $\tan A + \cot A = 4$, then find the value of $\frac{\tan^2 A + \cot^2 A}{\tan^2 A + \cot^2 A + 20\tan A\cot A}$
26. Find the value of $\cot^4 A - \mathrm{cosec}^4 A + \cot^2 A + \mathrm{cosec}^2 A$
27. If $7\sin^2 A + 3\cos^2 A = 4$ and $0^\circ < A < 90^\circ$, then find the value of tanA.
28. If $\cos\theta + \sec\theta = 2$, find the value of $\cos^{68}\theta + \sec^{68}\theta$.

29. If $x = a\cos^3\theta$, $y = b\sin^3\theta$, then find the value of $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}}$.

30. Find the value of $3(\sec^2\theta + \tan^2\theta)$, if $\sec^2\theta = \frac{4}{3}$.

31. If $\cos\theta = \frac{3}{5}$, then find the value of $2\sec^2\theta + \tan^2\theta + 1$.

32. If $1 + \tan^2\theta + 4k\tan^2\theta\sec^2\theta = \sec^2\theta + 2\tan^2\theta\sec^2\theta$, then find the value of k.

33. If $x = a(\sin\theta + \cos\theta)$, $y = b(\sin\theta - \cos\theta)$, then find the value of $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2$.

34. If $x\sin45^\circ = y\text{cosec}30^\circ$, then find the value of $\left(\frac{x}{y}\right)^4$.

35. If $\cos x + \cos y = 2$, then find the value of $\sin x + \sin y$.

36. What is the value of 5θ, if $\tan2\theta = \cot3\theta$.

37. Find the value of $\tan\theta \times \frac{\sin\theta - \sin^3\theta}{\cos\theta - \cos^3\theta}$.

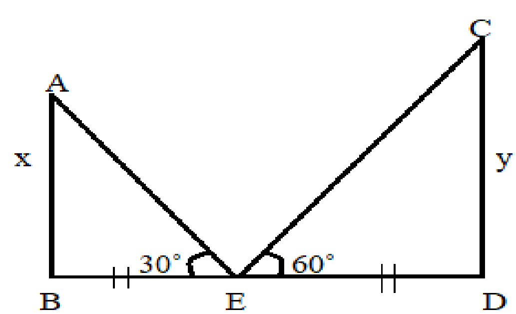
38. Find the angle of elevation of the sun at an instant when the length of the shadow of a pole is $\sqrt{3}$ times its height.

39. A ladder was placed against a wall in such a way that it makes an angle of 30° with the ground. If its top is 10m above the ground, find the distance between wall and foot of the ladder.

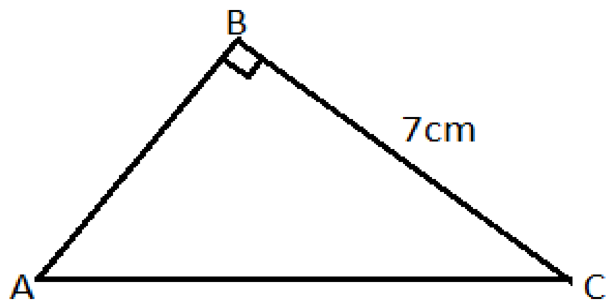
40. Two posts are 'k' meter apart and the height of the one is double that of the other. If from the middle point of the line joining their feet, an observer finds the angular elevation of their tops to be complementary, then find the height (in m) of shorter post.

41. If a tower of 6 meter height casts a shadow of $2\sqrt{3}$ meter along the ground, then what is the angle of elevation of the sun at that time?

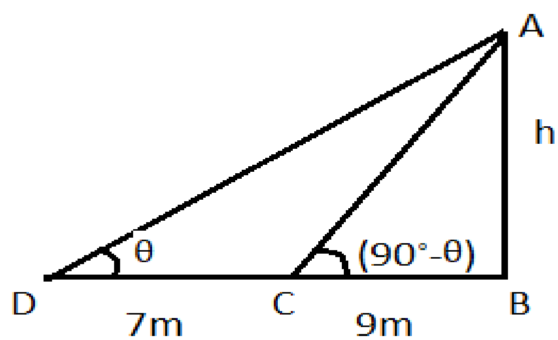
42. In the given figure, if BE = ED then find x:y



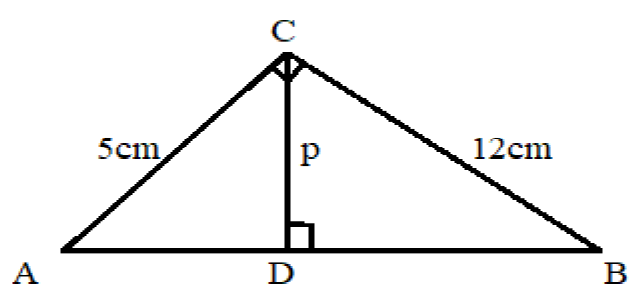
43. In right angle ΔABC, ∠B = 90° and AC – AB = 1, then find the value of cosA + cosB + cosC



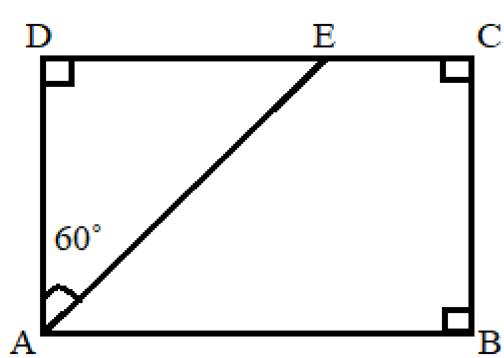
44. In the given figure, find the height 'h'



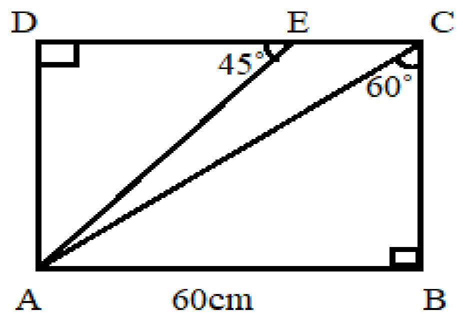
45. In the given figure, find the value of 'p'.



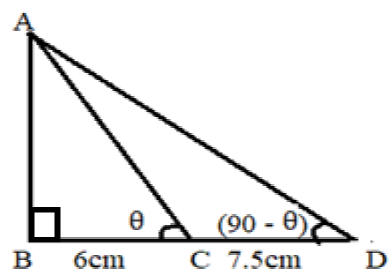
46. In the given figure, find AE



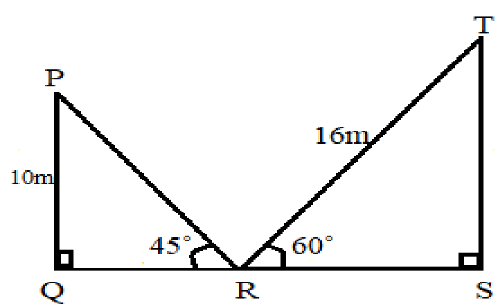
47. In the given figure, find AC + AD



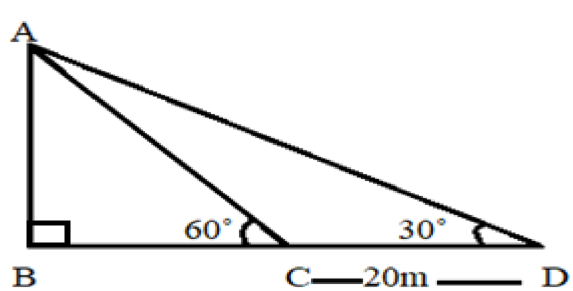
48. In the given figure, find the height of the tower AB (in m).



49. In the given figure, find QS.



50. In the given figure, $\angle B = 90^\circ$, find the height of the tower AB (in m)



ANSWERS

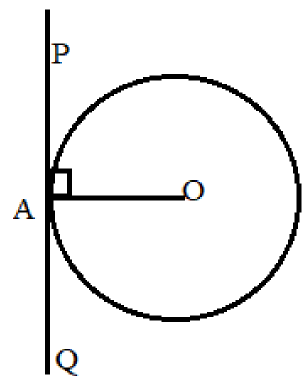
1	1	26	0
2	$2\frac{5}{7}$	27	$\frac{1}{\sqrt{3}}$
3	1	28	2
4	1	29	1
5	$\frac{11}{29}$	30	5
6	4	31	$8\frac{1}{3}$
7	2	32	$\frac{1}{2}$
8	0	33	2
9	-1	34	64
10	-3	35	0
11	2	36	90°
12	45°	37	1
13	1	38	30°
14	32°	39	$10\sqrt{3}$m
15	$11\frac{1}{2}$	40	$\frac{k}{2\sqrt{2}}$m
16	0	41	60°
17	$\frac{1}{k}$	42	1:3
18	1	43	$1\frac{6}{25}$
19	1	44	12 m
20	$\frac{1}{4}$	45	$4\frac{8}{13}$ cm
21	3	46	80 cm
22	-4	47	$60\sqrt{3}$
23	$\frac{1}{2}$	48	0.09 m
24	4	49	18 m
25	$\frac{7}{17}$	50	$10\sqrt{3}$ m

CHAPTER – 9

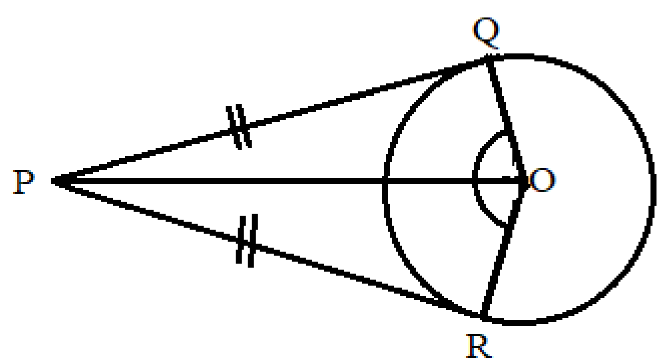
CIRCLES

POINTS TO REMEMBER

- **A tangent to a circle is a line that touches the circle at only one point.**



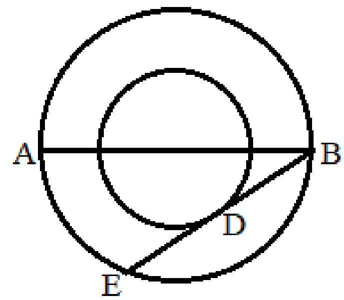
- **The tangent at any point of a circle is perpendicular to the radius through the point of contact. OA $\perp$ PA. i.e. $\angle$OAP = 90°.**
- **There are exactly two tangents to a circle through a point lying outside the circle. PQ and PR are exactly two tangents from the external point P.**
- **The length of tangents drawn from an external point to a circle are equal PQ = PR**



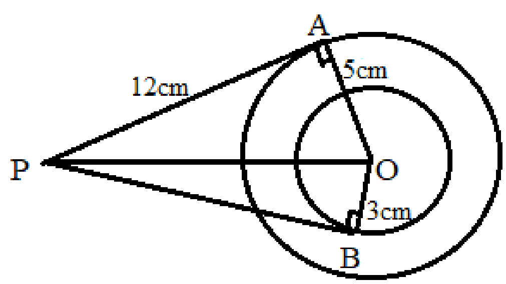
- **In the above figure, the sum of opposite angles of a Quadrilateral OQPR is 180°.**

QUESTIONS

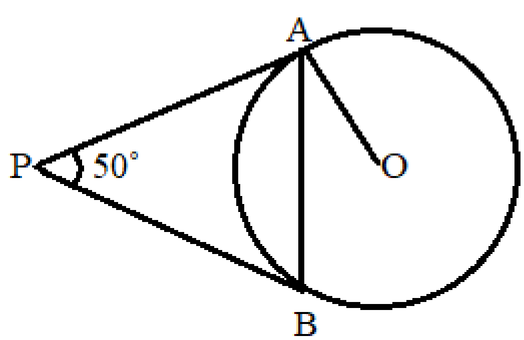
1. The radii of two concentric circles are 5cm and 3cm. AB is a diameter of the bigger circle and BD is tangent to the smaller circle touching it at D and the bigger circle at E. Find the length of AD.



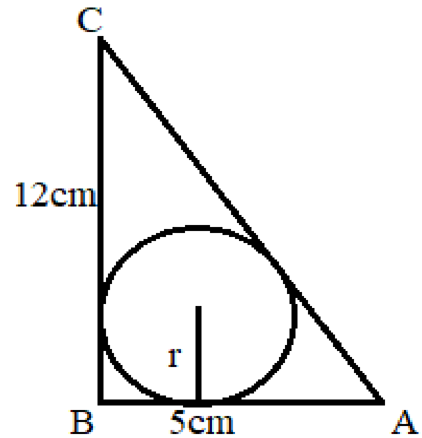
2. Find the value of (5p + 7), where p is the distance between two parallel tangents to a circle whose radius is 12.5cm.
3. In the given figure, find BP.



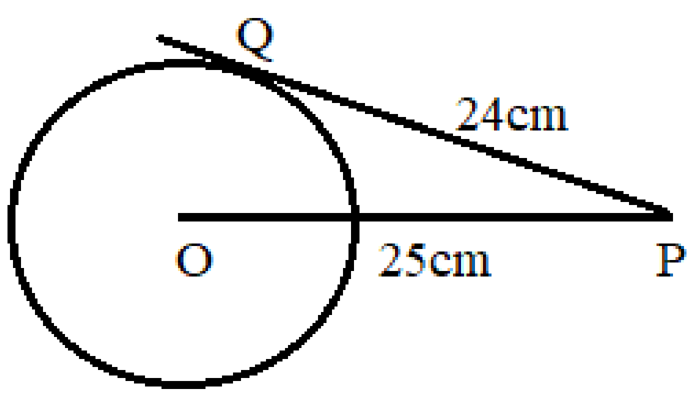
4. Find the radius of the circle passing through the vertices of a right angled triangle, when lengths of perpendicular sides are 6cm and 8cm.
5. In the given figure, PA and PB are tangents to the circles with Centre O such that $\angle APB = 50^{\circ}$, what is the value of $\angle OAB$.



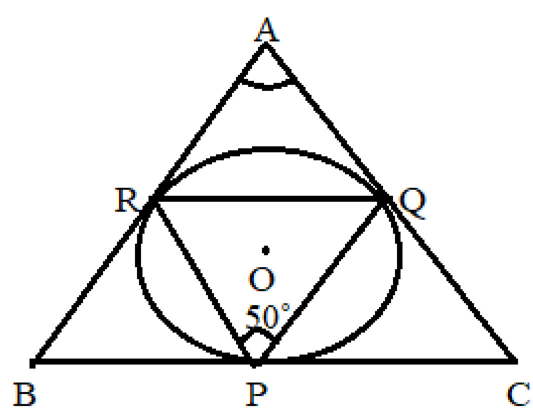
6. **In the given figure, ∠ABC is right angled at B. Find the radius of circle, if AB = 5cm and BC = 12cm.**



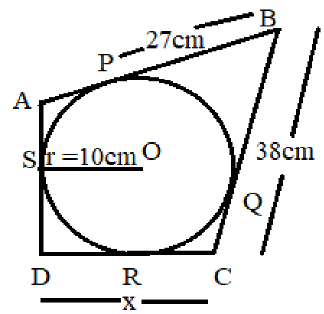
7. **In the given figure, find the value of OQ.**



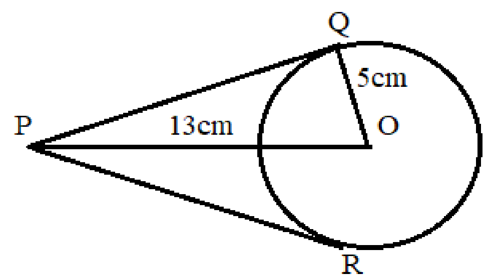
8. **In the given figure, ∠RPQ = 50°, and O is the center of circle, then find ∠BAC.**



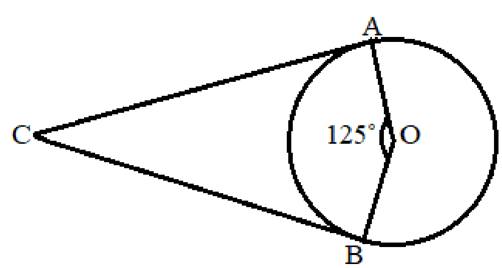
9. **In the given figure, if O is the Centre of circle, find the value of x.**



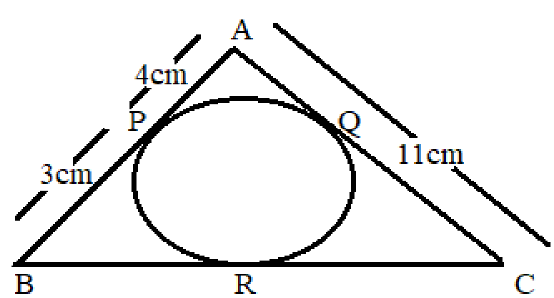
10. **In the given figure, find the value of (PR + OR)**



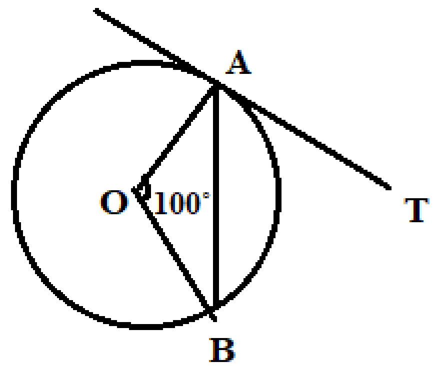
11. **In the given figure, find the value of ($\angle ACB + \angle CAO$).**



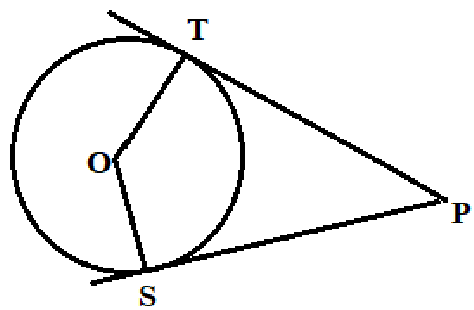
12. **In the given figure, find the length BC.**



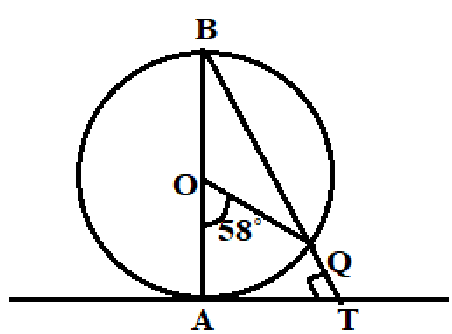
13. In the given figure, O is the Centre of circle, find the value of ∠OAB.



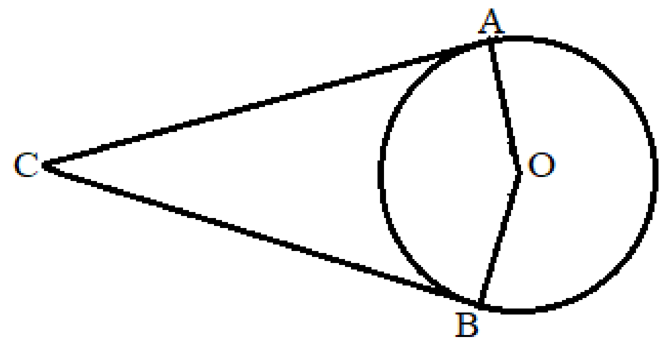
14. In the given figure, O is the Centre of circle with radius r. if OP = 2r, find the value of ∠OST.



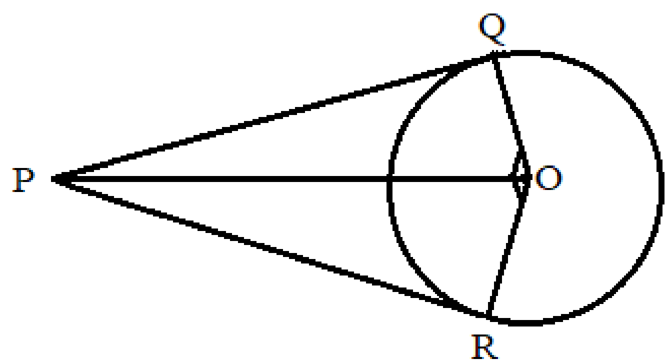
15. In the given figure, AB is the diameter of circle with Centre O and AT is tangent. Find the value of ∠ATQ.



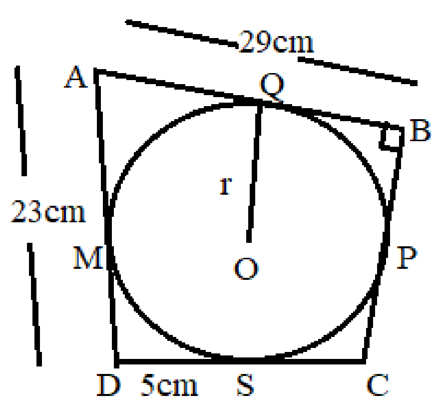
16. In the given figure, Find the value of $\angle AOB$, if $\angle ACB + \angle CBO = 120^{\circ}$.



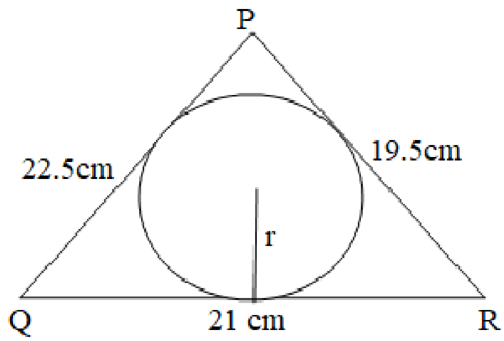
17. In the given figure, $\angle OPQ = 40^{\circ}$, find the value of $\angle ROQ$.



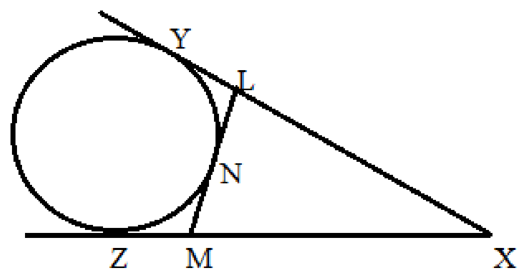
18. In the given figure, O is the Centre of circle with radius r, find the radius of circle.



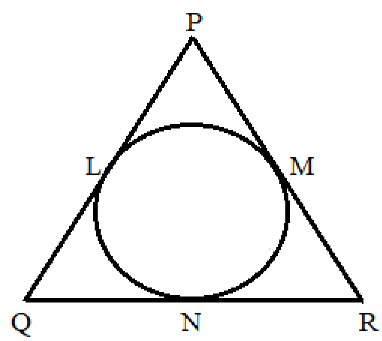
19. **In the given figure, find the radius of circle, if area of ΔPQR = 189 sq.cm.**



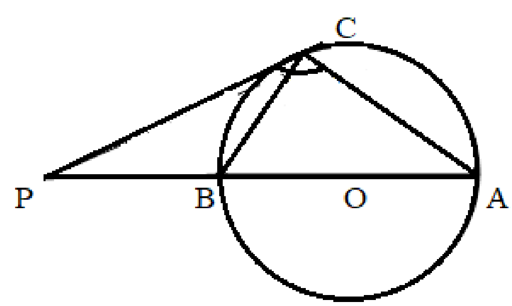
20. **Find the perimeter ofΔXLM, if XY = 18cm.**



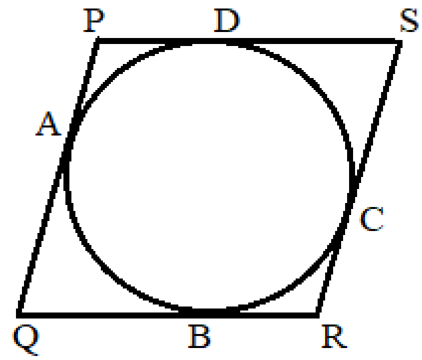
21. **Find the perimeter ofΔPQR, where PM = a cm, RN= b cm, QL = c cm.**



22. **The tangent at a point ‘C’ of a circle and a diameter AB when extended intersect at ‘P’. If ∠PCA = 110˚, find the value of ∠CBA.**

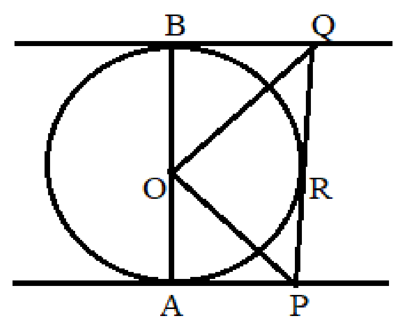


23. In the given figure, find the perimeter of Quadrilateral PQRS, if PA = 3cm, DS = 4cm, SR = 7cm, QB = 4cm.

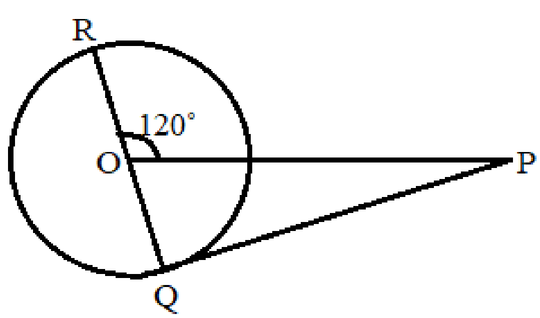


24. In the given figure as shown in question no 23, find the value of x such that PS = 7cm, SR= 12cm, QR= 15cm, QA = 7cm, AP = x cm.

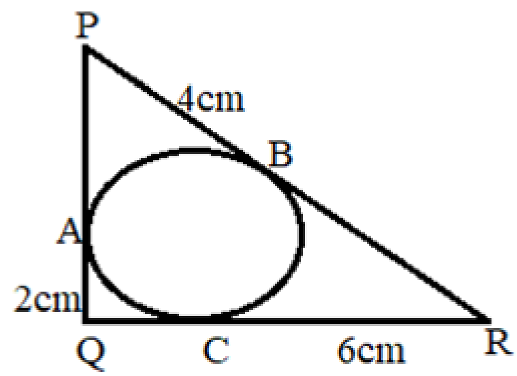
25. If BQ, QP and AP are tangents to the circle with Centre O then find the value of $\frac{4\angle QOP}{5}$.



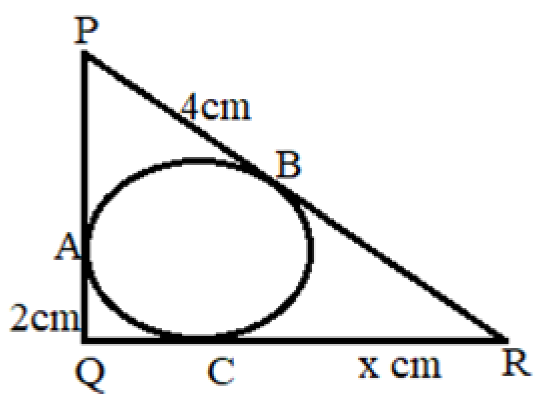
26. PQ is tangent drawn from an external point P to a circle with Centre O, QOR is the diameter of circle. If ∠POR =120˚. What is the measure of ∠OPQ?



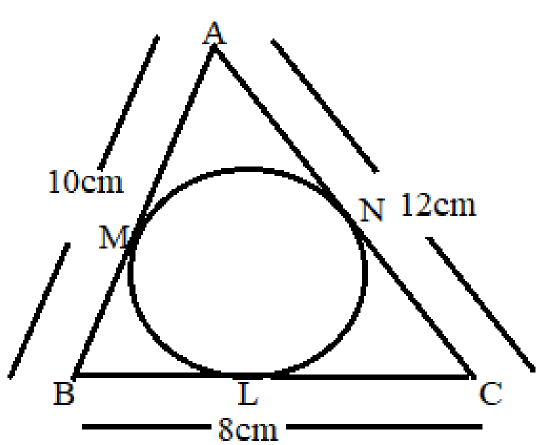
27. In the given figure, find the perimeter of ΔPQR.



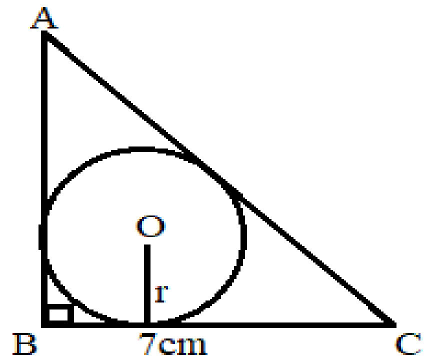
28. In the given figure, find x, if perimeter of ΔPQR is 52cm.



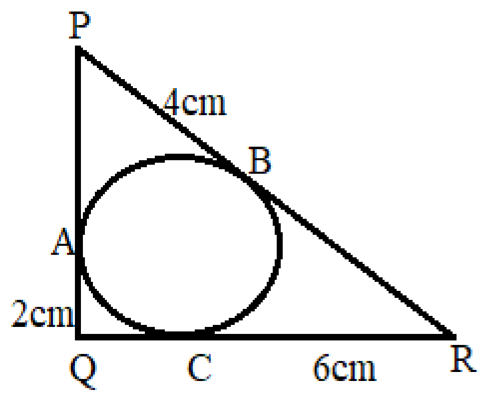
29. In the given figure, find BL.



30. In the given figure, find the value of r, if AC – AB = 1cm.

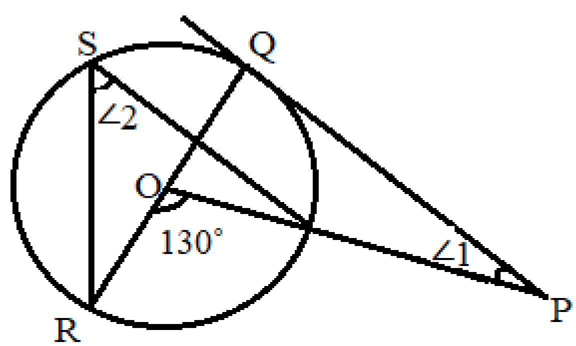


31. In the given figure, ΔPQR is a right angled triangle, right angled at Q, then find the value of (sinP + sinR − cosecQ).

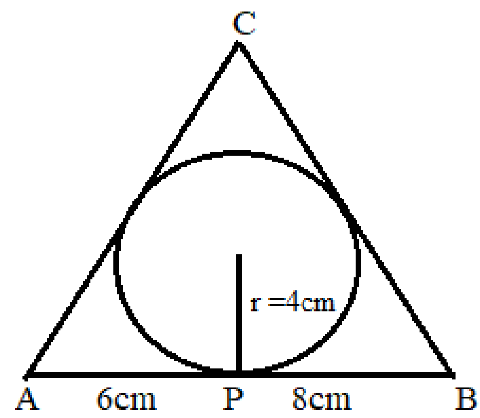


32. A point P is 26 cm away from the Centre of circle. Find the length of tangent drawn from P to the circle whose diameter is 20cm.

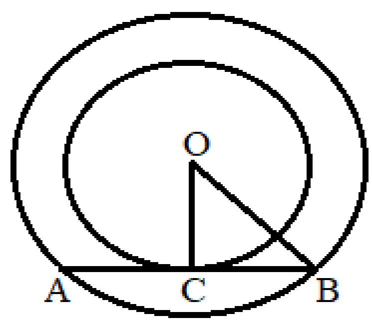
33. In the given figure, if ∠POR = 130°, PQ is a tangent from the external point P. Find ∠1 + ∠2.



34. In the given figure, find AC + BC, if area of ΔABC = 84sq.cm.

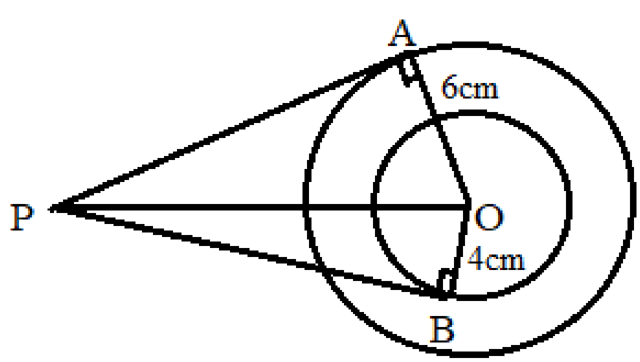


35. In the given figure, if AB =48cm and OC = 7cm, then what is the difference of radii of concentric circle.

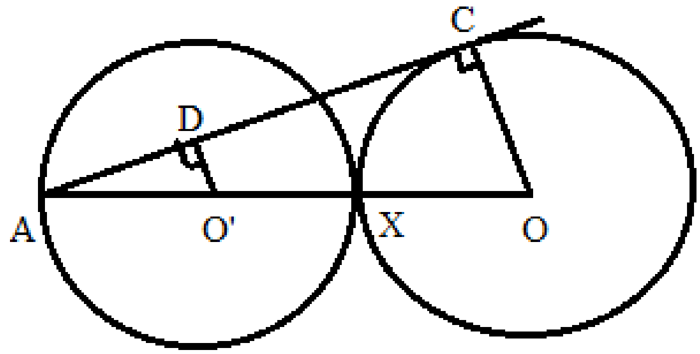


36. In the given figure as shown in question no 35, if the difference of radii of concentric circle is 4cm. Find the OC, where OB = 10cm.

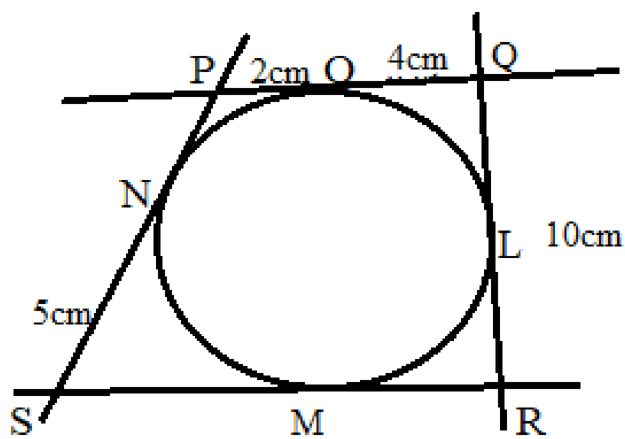
37. In the given figure, if PO – PA = 2cm, find BP.



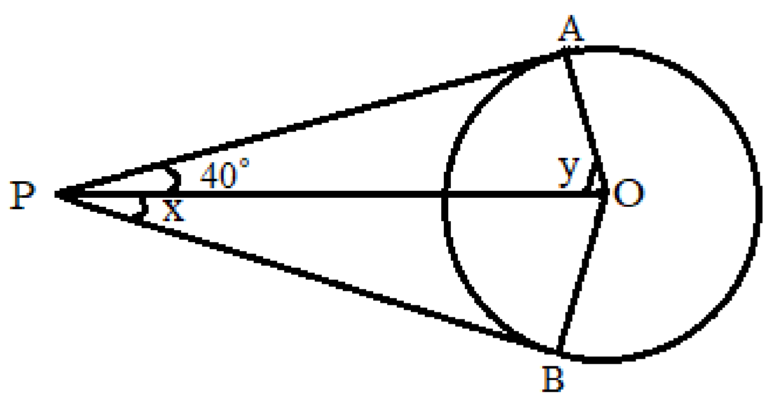
38. **In the given figure, two equal circles with Centre O and O′ touches each other at X, find the value of DO′: CO.**



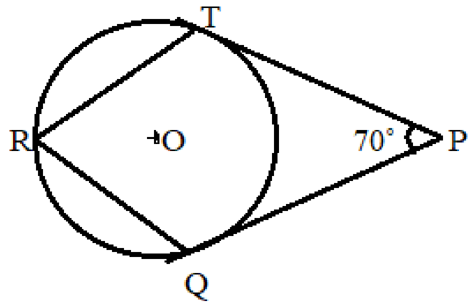
39. **In the given figure, find the value of (PQ + QR + RS -SP).**



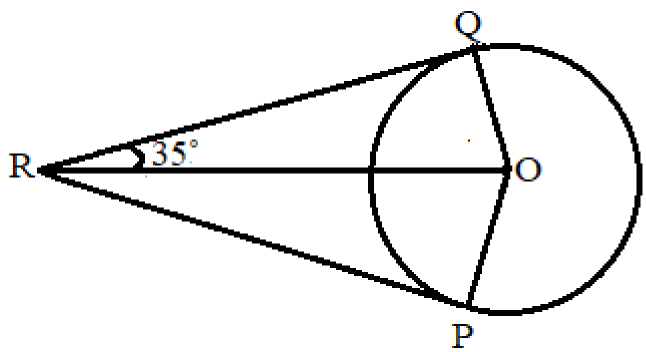
40. **In the given figure, find the value of (4y – 5x).**



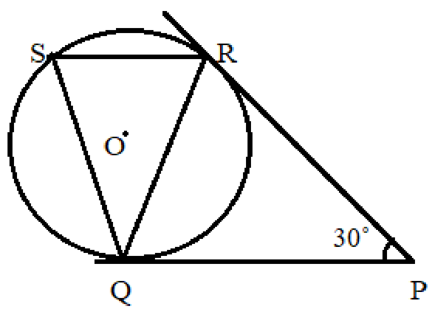
41. In the given figure, ∠TPQ = 70°, find ∠TRQ.



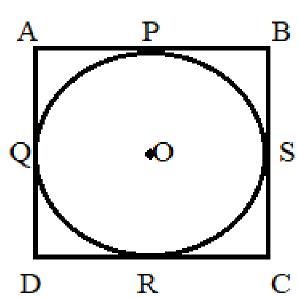
42. In the given figure, find the value of (∠QOP - ∠ROP)



43. In the given figure, SR∥QP, find the value of ∠RQS.

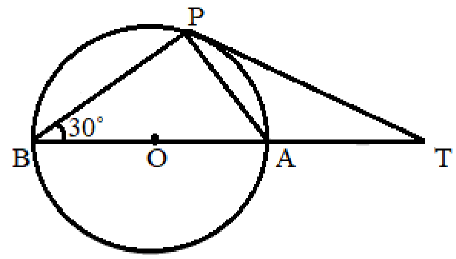


44. In the given figure, if AB = 15.5cm and CD = 16cm, find the perimeter of Quadrilateral ABCD.

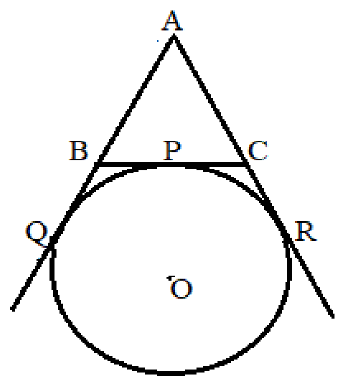


45. In the figure as shown in the question no 44, if AB = x cm, and CD = y cm, Find the value of (AD + BC).

46. In the given figure, O is the Centre of circle, if $\angle PBT = 30^{\circ}$, find the ratio BA: AT.

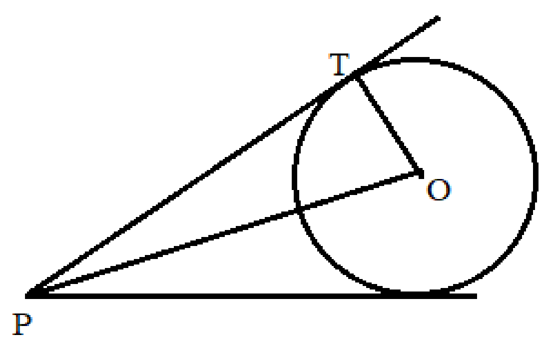


47. In the given figure, find AQ if AB = 2x cm, AC = 4y cm, BC = 6z cm.



48. In the figure as shown in question no 47, find perimeter of ΔABC where AQ = 6.5cm.

49. In the given figure, OT: TP = 3: 4 and OP = 10cm, then find the radius of the circle.



50. In the given figure, as shown in question no. 49, if PT = 8cm, PT + OT =14cm, find OP: OT.

ANSWERS

1	$\sqrt{52}$ cm or $2\sqrt{13}$ cm	26	30°
2	132 cm	27	24 cm
3	$\sqrt{160}$ cm or $4\sqrt{10}$ cm	28	20
4	5 cm	29	3 cm
5	25°	30	3 cm
6	2 cm	31	$\frac{2}{5}$
7	7 cm	32	24 cm
8	80°	33	105°
9	21 cm	34	28 cm
10	17 cm	35	18 cm
11	145°	36	6 cm
12	10 cm	37	$\sqrt{84}$ cm or $2\sqrt{21}$ cm
13	40°	38	1 : 3
14	30°	39	20 cm
15	61°	40	0
16	150°	41	55°
17	100°	42	55°
18	11 cm	43	30°
19	6 cm	44	63 cm
20	36 cm	45	(x + y) cm
21	2(a + b + c) cm	46	2 : 1
22	70°	47	(x + 2y + 3z) cm
23	28 cm	48	13 cm
24	3	49	6 cm
25	72°	50	5 : 3

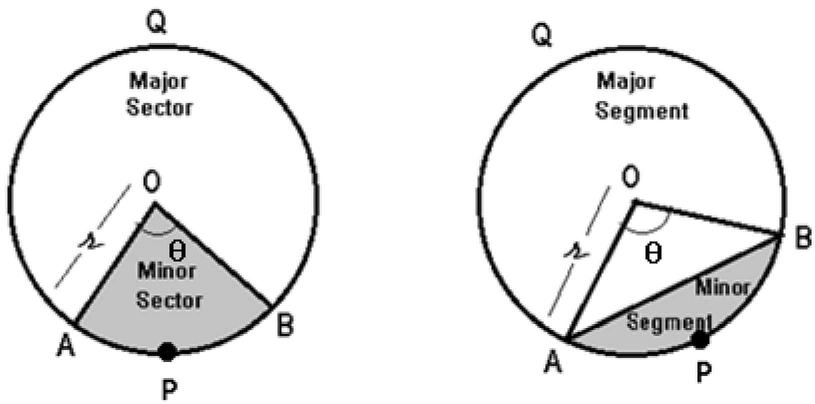
CHAPTER 10

AREAS RELATED TO CIRCLES

POINTS TO REMEMBER

PERIMETER AND AREA OF A CIRCLE

- **The area of a circle is the measurement of the region enclosed by its boundary. Area of the circle = πr^2**
- **The perimeter of a circle is the length of its boundary. Perimeter of a circle is also known as circumference of a circle. Perimeter of the circle = $2\pi r$**
- **Area of sector of circle**



The portion of circle enclosed between two radii and arc of a circle is called sector of a circle.

Area of sector OAPB = $\frac{\theta}{360^\circ}\pi r^2$

Length of an arc of sector OAPB = length of arc AB = $\frac{\theta}{360^\circ}2\pi r$

Perimeter of the sector = $\frac{\theta}{360^\circ}2\pi r + 2r$

- **Area of segment of circle**

  Any chord AB divides the circle into two parts. The bigger part is known as major segment and smaller one is called minor segment.

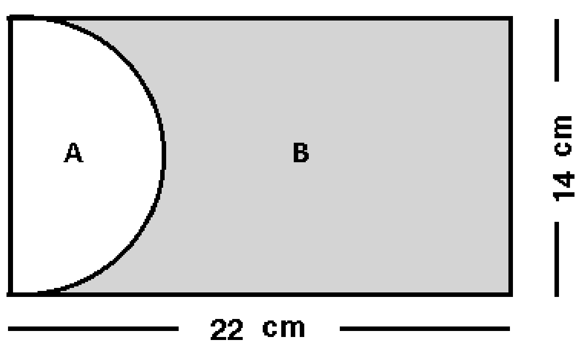
  Area of minor segment APB = Area of sector OAPB – Area of ΔOAB

  $$= \frac{\theta}{360^\circ}\pi r^2 - \frac{1}{2}r^2 \sin\theta$$

  Area of major segment = πr^2 − Area of minor segment

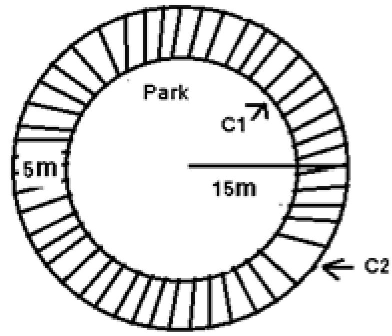
QUESTIONS

1. **Find the area of a segment (in terms of π) of a circle with central angle of 30° and a radius of 8 cm.**
2. **Find the area of a sector with an arc of length of 30 cm and a radius of 10 cm.**
3. **In a circle of radius 21 cm an arc subtends an angle of 30° at centre. Find the length of arc.**
4. **Find the area and perimeter of a semi-circle whose diameter is 'R'.**
5. **If the sum of the areas of two circles with diameters 40 cm and 42 cm is equal to the area of a circle with R, then find the value of R.**
6. **If the sum of circumference of two circles with radii 13 cm and 12 cm is equal to the circumference of a circle of radius R, then find the value of R^2.**
7. **If the perimeter of a circle is equal to that of a square, then find the ratio of their areas (in terms ofπ).**
8. **Find the area of the largest triangle that can be inscribed in a semi-circle of radius 85 m.**
9. **Find the area of the largest circle (in terms ofπ) that can be inscribed in a square of side 56 cm.**
10. **The side of a square is 14 cm. Find the area of circle circumscribed about this square.**
11. **In the given figure, find the area and perimeter of the region A.**

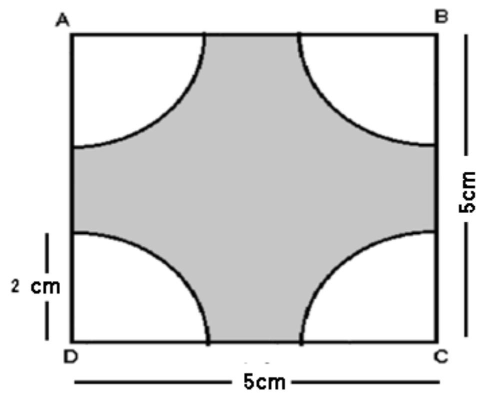


12. **In fig. given in Q.11, find the area of region B.**
13. **In fig. given in Q.11, find the ratio of the areas of region B and region A.**
14. **A path of width 5m is built around a circular park of radius 15m. Find the**

(i) sum of the perimeter of the circles C_1 and C_2. (ii) The area of path.



15. In fig. ABCD is a square of side 5cm. A quadrant of a circle of radius 2cm is drawn at each vertex of the square. Find the area (in terms of π) of the shaded region.



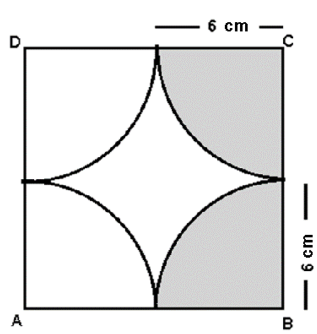
16. Find the perimeter (in terms of π) of the shaded region in Q.15.

17. In fig., ABCD is a square of side 12cm. A quadrant of a circle of radius 6cm is drawn at each vertex of the square. Find

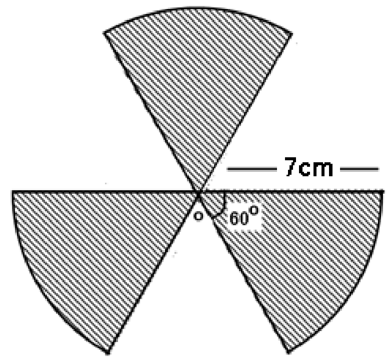
(i) Area (in terms of π) of the shaded region.

(ii) Perimeter (in terms of π) of the shaded region.

(iii) Area (in terms of π) of the unshaded region.

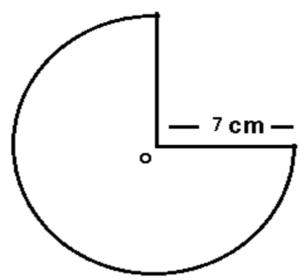


18. **In fig. find the total area of 3 equal sectors of the given circle with Centre O (use$\pi = \frac{22}{7}$).**



19. **Find the perimeter of the figure given in Q.18.**

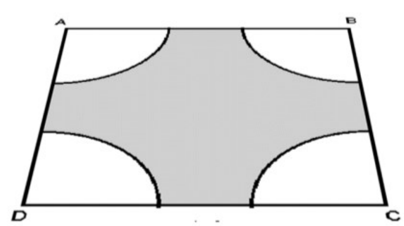
20. **Find the perimeter of the figure given below, if O is the Centre of the circle from which a quadrant is cut (use $\pi = \frac{22}{7}$).**



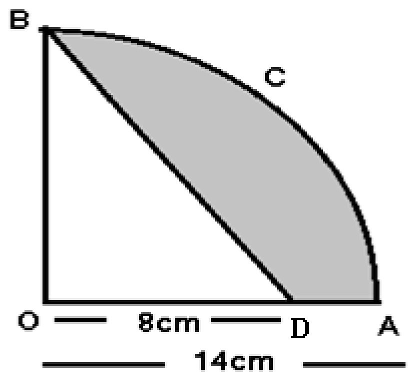
21. **A steel wire when bent in the form of a square encloses an area of 121 sq.cm. The same wire is bent in the form of a circle. Find the area of circle.**

22. **A bicycle wheel makes 500 revolutions in moving 1.1 km. Find the diameter of the wheel (in cm).**

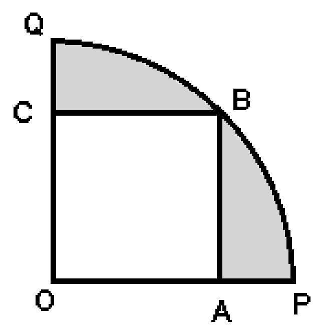
23. **In the given fig. ABCD is a trapezium in which AB|| DC, AB= 18cm, DC= 32cm and the distance between AB and DC is 14 cm. If arcs of length of equal radii 7cm have been drawn with centres A, B, C and D, then find the area of the shaded region.**



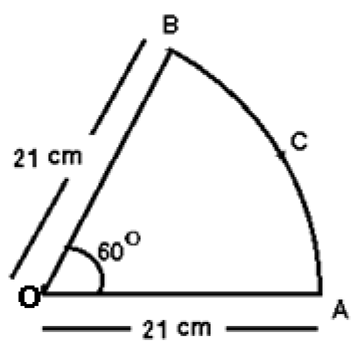
24. OACB is a quadrant of a circle with Centre O and radius 14cm. If OD= 8cm, then find the area of shaded region (use $\pi = \frac{22}{7}$).



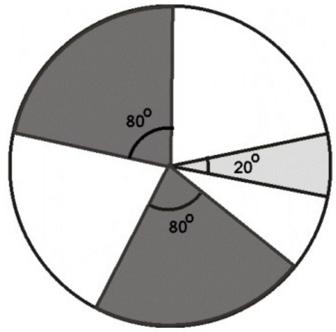
25. Square OABC is inscribed in a quadrant OPBQ of a circle. If OA=20cm, find the area of shaded region (use $\pi = \frac{22}{7}$).



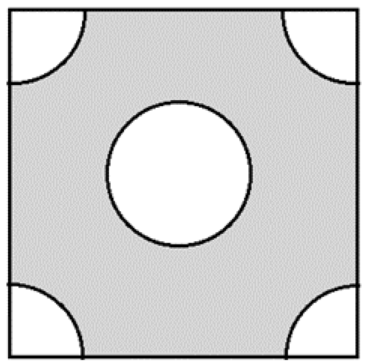
26. In the given figure, find the perimeter of OACB.



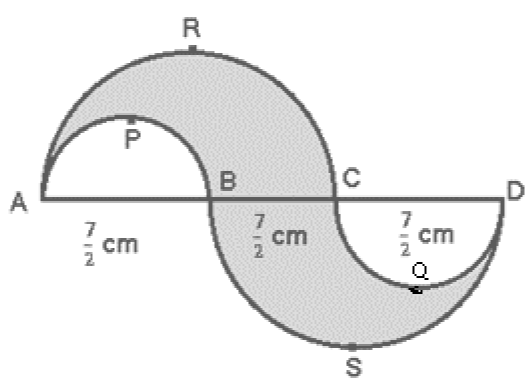
27. Find the perimeter of the protractor if its diameter is 14cm.
28. Find the diameter of a circle whose area is equal to the sum of the areas of two circles of diameters $2(a^2 - b^2)$cm and 4ab cm.
29. In the given fig., three sectors of a circle of diameter 7cm, making angles of 20°, 80° and 80° at the Centre are shaded. Find the area of shaded region.



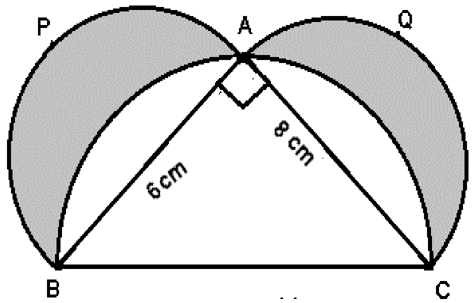
30. In the given fig. ABCD is a square of side 5cm. A quadrant of a circle of radius 1cm is drawn at each vertex of the square and a circle of diameter 2cm is also drawn in the Centre. Find the area of shaded region (use π = 3.14).



31. In the given fig. APB and CQD are semi circles of diameter $\frac{7}{2}$ cm each, while ARC and BSD are semi circles of diameter 7cm each. Find the
(i) Perimeter of shaded region
(ii) Area of shaded region.

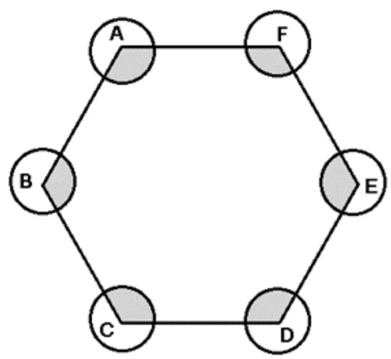


32. In the given fig. ΔABC is right angled at A. Semi circles are drawn on AB, AC and BC as diameters. If AB=6cm and AC= 8cm, then find the perimeter of the shaded region.

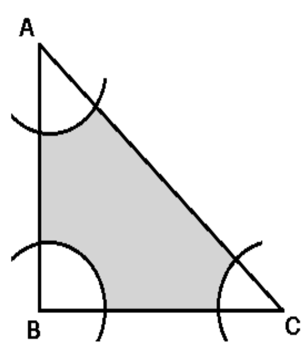


33. Find the area of shaded region in Q.32.

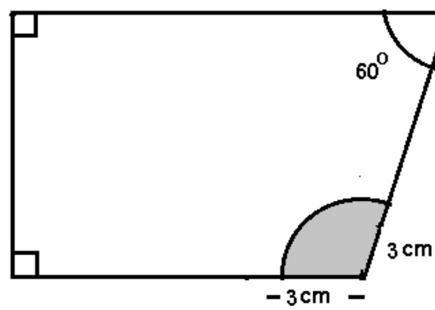
34. ABCDEF is a regular hexagon. With vertices A, B, C, D, E and F as the centers, circles of radius 7cm are drawn as shown in fig. , find the area of shaded portion.



35. With vertices A, B and C of a ΔABC as centres, arcs are drawn with radius 4cm each as shown in fig. if AB=10cm, BC= 24cm and CA= 26cm, then find the area of shaded region(in terms of π).

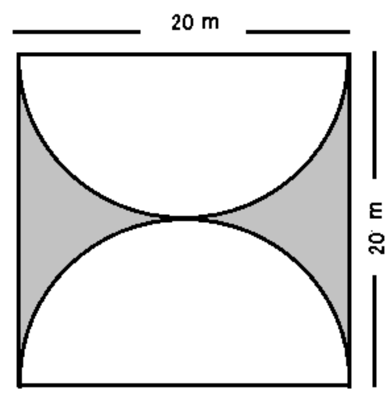


36. In fig. , find the perimeter (in terms ofπ) of the shaded region.



37. In fig. given in Q.36, find the area (in terms ofπ) of the shaded region.

38. In fig., find the area (in terms ofπ) of the shaded region.

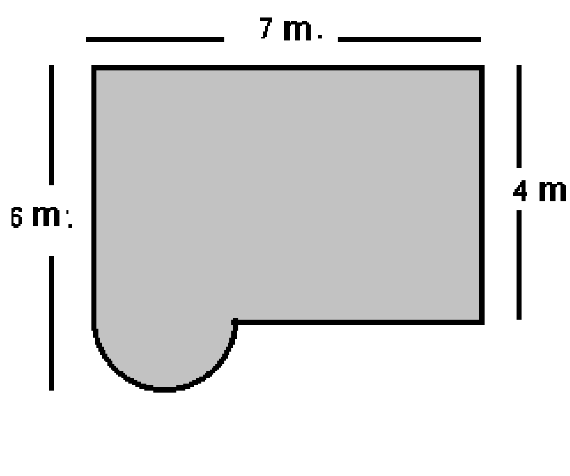


39. In fig.given in Q.38 find the perimeter (in terms ofπ) of the shaded region.

40. In fig., find the following (in terms ofπ):

(i) Area of shaded portion.

(ii) Perimeter of shaded portion



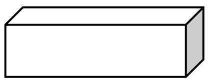
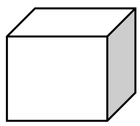
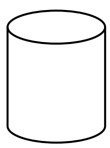
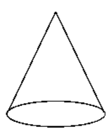
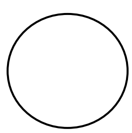
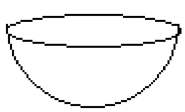
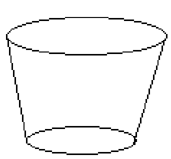
ANSWERS

1	$(\frac{16\pi}{3} - 16)$sq.cm	21	154 sq.cm
2	150 sq.cm	22	70 cm
3	11 cm	23	196 sq.cm
4	Area = $\frac{\pi R^2}{8}$sq. units, Perimeter = $R(\frac{\pi}{2} + 1)$units	24	98 sq.cm
5	29 cm	25	228 sq.cm
6	625 sq.cm	26	64cm
7	4:π	27	36cm
8	7225 sq.m	28	$2(a^2 + b^2)$cm
9	784 π sq.cm	29	19.25 sq.cm
10	308 sq.cm	30	18.72 sq.cm
11	Area =77 sq.cm. Perimeter = 36cm	31	(i) 33cm (ii) $28\frac{7}{8}$ sq.cm
12	231 sq.cm	32	$\frac{264}{7}$ cm
13	3:1	33	24 sq.cm
14	(i) 220m (ii)550 sq.m	34	308 sq.cm
15	(25 - 4 π)sq.cm	35	(120-8π)sq.cm
16	$(4\pi + 4)$cm	36	(2π+6)cm
17	(i) 18 π sq.cm (ii)$[6(\pi + 4)]$cm (iii)[144-18 π]sq.cm	37	3 π sq.cm
18	77sq.cm	38	(400-100π)sq.m
19	64cm	39	(40+20 π)m
20	47cm	40	(i) (28+2 π) sq.m (ii) (18+2π)m

CHAPTER – 11

SURFACE AREAS AND VOLUMES

POINTS TO REMEMBER

Name of the Solid	Curved Surface Area	Total Surface Area	Volume
Cuboid 	$2h(l + b)$	$2(lb + bh + hl)$	lbh
Cube 	$4(\text{edge})^2$	$6(\text{edge})^2$	$(\text{edge})^3$
Right Circular Cylinder 	$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$
Right Circular Cone 	πrl	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$
Sphere 	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Hemisphere 	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
Frustum of a Cone 	$\pi(r_1 + r_2)l$ Where $l = \sqrt{h^2 + (r_1 - r_2)^2}$ and $r_1 > r_2$	$\pi(r_1 + r_2)l + \pi {r_1}^2 + \pi {r_2}^2$	$\frac{1}{3}\pi h({r_1}^2 + {r_2}^2 + r_1 r_2)$

QUESTIONS

1. Three cubes each of volume 125cu.cm are joined end to end to form a cuboid. Find the total surface area of the resulting cuboid.
2. Find the curved surface of a right circular cone (in terms ofπ) of height 15cm and base diameter 16cm.
3. Find the ratio of volumes of a cylinder, a cone and a hemisphere having same base radius and same height.
4. A hemisphere and a cone have equal bases. If their heights are also equal, then find the ratio of their curved surface.
5. Three solid spheres of diameters 6cm, 8cm and 10cm are melted to form a single solid sphere. Find the diameter of the new sphere.
6. Volume of two spheres are in the ratio 125:216. Find the ratio of their surface areas.
7. A hollow pipe is 21cm long and its external diameter is 8cm. If the thickness of the pipe is 1cm, then find the volume of the pipe (in terms ofπ).
8. Find the volume (in terms ofπ) of the largest right circular cone that can be cut out of a cube whose edge is 12cm.
9. Find the percentage increase in the surface area of a cube when it's each edge is tripled.
10. If the total surface area of a solid hemisphere is 462 sq.cm, find its curved surface area.
11. If the volumes of two cones are in the ratio 1:4 and their diameters are in the ratio 4:5, find the ratio of their heights.
12. If each side of a cube is decreased by 25%, then find the ratio of volumes of original cube and the resulting new cube.
13. The dimensions of a metallic cuboid are 100cm$\times$ 80cm$\times$64cm. It is melted and recast into a cube. Find the total surface area of the cube so formed.
14. A cone of height 20cm and base radius 5cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the diameter of the sphere.

15. How many lead balls, each of radius 2cm can be made from a solid sphere of radius 16cm?
16. How many cubes of edge 5cm can be put in a cubical box of 1m edge?
17. The radii of two cylinders are in the ratio of 4:3 and their heights are in the ratio of 5:6, find the ratio of their volumes.
18. 66 cu.cm of silver is drawn into a wire 1mm in diameter. Calculate the length of the wire in meters.
19. Three solid cubes of sides 1cm, 6cm and 8cm are melted to form a new cube. Find the total surface area of the cube so formed.
20. The diagonal of a cube is $12\sqrt{3}$ cm. Find its volume.
21. An iron sphere of diameter of diameter 18cm is drawn into a wire of diameter 4mm. Find the length of the wire.
22. If the capacity of a cylindrical tank is 1848 cu.m and its base diameter is 14m, then find the depth of the tank.
23. The radii of the bases of a cylinder and a cone are in the ratio 3:4 and their heights are in the ratio 2:3, find the ratio of their volumes.
24. The radii of the base of two cylinders A and B are in the ratio 3:2 and their heights are in the ratio n: 1. If the volume of cylinder A is 3 times that of cylinder B, then find the value of n.
25. Two iron balls spherical in shape each of diameter 6cm are immersed in the water contained in a cylindrical vessel(half filled) of radius 6cm. Find the level of the water that will be raised in the vessel.
26. The lateral surface area of a cylinder is 1056 sq.cm and its height is 10cm. Find its radius.
27. Find the mass of a solid cone of silver metal having base diameter 14cm and vertical height 30cm given that density of silver is 10g/cu.cm.
28. Garvit was making a mathematical model, in which he placed 4 cubes each of edge 20cm one above the other. Find the surface area of resulting cuboid.
29. A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

30. **Find the weight of a hollow sphere of metal having internal and external diameters as 8cm and 10cm respectively if 1 cu.cm of metal weighs 21g.**

31. **The volume of cuboid is 36 times the volume of a cube. If the dimensions of the cuboid are 9cm, 18cm and 48cm, then find the total surface area of cube.**

32. **A solid spherical steel ball of radius 'r' was silver polished and then cut into 4 similar pieces.**

(i) Find the non-polished area of each piece

(ii) Find the ratio of the polished area to the non-polished area of each piece.

33. **The height of a circular cylinder is increased 6 times and base area is decreased by $\frac{1}{9}^{th}$ times. By what factor its lateral surface area is increased/decreased?**

34. **Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.**

35. **The radius and height of a cylinder are in the ratio 5:7 and its volume is 550 cubic cm. Find its curved surface area.**

36. **A spherical lead ball of radius 15cm is melted and small lead balls of diameter 10mm are made. Find the total possible number of small lead balls so formed.**

37. **The radius of a metallic cylinder is 3cm and its height is 5cm. It is melted and moulded into small cones each of height 1cm and base radius 1 mm. Find the number of cones so formed.**

38. **If a solid cone of volume 27π cu.cm is kept inside a hollow cylinder whose radius and height are equal to that of cone, then find the volume of water needed to fill the empty space.**

39. **A conical flask is full of water having base radius r and height h. This water is poured into an empty cylindrical flask of base radius 'mr'. Find the height of water in the cylindrical flask.**

40. **What is the height of a cylinder that has the same volume and radius as that of a sphere of diameter 12cm?**

41. **If the radius of a sphere is increased by 2cm, its surface area increased by 352 sq.cm, then find the radius of sphere before change.**

42. **If the height of frustum of a cone is 4cm and radii of two bases are 3cm and 6cm respectively, then find the curved surface area of frustum of cone (in terms of π).**

43. **If the volume of a cube is 3375 cubic metre then, find its total surface area.**

44. **If semi vertical angle of a cone of height 3cm is 60°, then find the diameter of cone.**

45. **If semi vertical angle of a cone of height 5cm is 30°, then find the slant height of the cone.**

46. **If semi vertical angle of a cone of diameter 12cm is 45°, then find the volume of cone (in terms of π).**

47. **The surface area of three adjacent faces of a cuboid are 36 sq.m, 27sq.m and 12 sq.m respectively. Find its volume.**

48. **A cone, hemisphere and cylinder stand on the same base and have equal height.**

(i) Find the ratio of their curved surface areas.

(ii) Find the ratio of their volumes.

49. **The radii of the base of a cylinder and a cone are in the ratio $\sqrt{3}:\sqrt{2}$ and their heights are in the ratio $\sqrt{2}:\sqrt{3}$. Find the ratio of their volumes.**

50. **A solid spherical ball of radius 3cm is melted and recast into three solid spherical balls. The radii of two of the balls are 1.5cm and 2cm. Find the radius of the third ball.**

ANSWERS

1	350sq.cm	26	16.8cm
2	136π sq.cm	27	15.4kg
3	3:1:2	28	7200 sq.cm
4	$\sqrt{2}:1$	29	2:1
5	12cm	30	5368g or 5.368kg
6	25:36	31	216sq.cm
7	147 π cu.cm	32	(i) πr^2 (ii) 1:1
8	144 π cu.cm	33	LSA is increased 2 times
9	800%	34	7:9
10	308 sq.cm	35	220 sq.cm
11	25:64	36	27000
12	64:27	37	13500
13	38400 sq.cm	38	54 π cu.cm
14	10cm	39	$\left(\frac{h}{3m^2}\right)$
15	512	40	8cm
16	8000	41	6cm
17	40:27	42	45 π sq.cm
18	84m	43	1350 sq.cm
19	486 sq.cm	44	$6\sqrt{3}$ cm
20	1728 cu.cm	45	$\frac{10\sqrt{3}}{3}$ cm
21	243m	46	72 π cu.cm
22	12m	47	108 cu.m
23	9:8	48	(i) $1:\sqrt{2}:\sqrt{2}$ (ii) 1:2:3
24	$n=\frac{4}{3}$	49	$3\sqrt{3}:\sqrt{2}$
25	2cm	50	2.5 cm [hint : $1.5^3 + 2^3 + 2.5^3 = 3^3$]

CHAPTER -12

STATISTICS AND PROBABILITY

POINTS TO REMEMBER

- **Mean :**

Mean of ungrouped data: $\bar{x} = \dfrac{\text{sum of all observations}}{\text{total number ofobservations}}$

Mean of grouped data:

(i) By direct method $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

(ii) By assumed mean method $\bar{x} = A + \dfrac{\sum f_i d_i}{\sum f_i}$ where A = Assumed mean

(iii)By step deviation method $\bar{x} = A + \dfrac{\sum f_i u_i}{\sum f_i} \times h$ $d_i = x_i - A$, $u_i = \dfrac{x_i - A}{h}$

- **Mode :**

Mode of ungrouped data = observation having maximum frequency

Mode of grouped data = $l + \dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

where l = lower limit of the modal class

f_1 = frequency of the modal class

f_0 = frequency of class preceding the modal class

f_2 = frequency of class succeeding the modal class

h = Size of the class interval

- **Median :**

Median of ungrouped data = $\left(\frac{n+1}{2}\right)^{th}$ observation (if n is odd),

$$= \frac{\left(\frac{n}{2}\right)^{th}\text{observation} + \left(\frac{n}{2}+1\right)^{th}\text{observation}}{2} \text{ (if n is even)}$$

Median of grouped data = $l + \dfrac{\frac{n}{2} - cf}{f} \times h$

where l = lower limit of median class

cf = Cumulative frequency of class preceding the median class

f = Frequency of median class

h = Class size

- **Empirical formula**

  Mode = 3 Median – 2 Mean
- **Probability of an event , P(E) =** $\frac{\text{number of favourable outcomes}}{\text{total no.of outcomes}}$
- **Probability of any event E lies from 0 to 1 i.e. 0** $\leq \mathrm{P(E)} \leq 1$
- **P(Impossible event) = 0**
- **P(Sure event) = 1**
- **For any event E, P(E)+P(not E) = 1 or P(E)+P($\bar{E}$) =1**

QUESTIONS

1. **Find the arithmetic mean of 1, 2, 3… (n-1).**
2. **Find the mode of a distribution whose mean is 8.32 and the median is 8.05.**
3. **Find the median of first ten prime numbers.**
4. **The point of intersection of the ogives (more than and less than type) is given by (20, 30). Find the ratio of median and total frequency.**
5. **Two distributions M and N with total number of observations 25 and 75, and mean 3 and 4 respectively are combined. What is the mean of the resulting distribution?**
6. **If the median of** $\frac{x}{7}, \frac{x}{6}, \frac{x}{5}, \frac{x}{3}, \frac{x}{2}, x \text{ and } \frac{x}{4}$ **is 9 (where x>0), find the value of x.**
7. **The mean of 3, 7, 5 and x is 5 and the mean of 12, 7, 6, x and y is 10. What is the value of y?**
8. **If the median of the data 5,8,x-3,x-5,15 and 25 written in ascending order is 15,then find the value of x.**
9. **If the mean of first n natural numbers is** $\frac{5n}{9}$**, then find n.**
10. **Find the difference between median and mean of the given data**

    17, 19,20,22,23 and 25.
11. **If** $\sum f_i$ **=15,** $\sum f_i x_i = 3p + 36$ **and mean of the distribution is 4, then find** p**.**
12. **The mean of 300 items was 50. Later on it was discovered that two items were misread as 67 and 26 instead of 76 and 62. Find the correct mean.**

13. If $x_i{}'s$ are the mid points of the class intervals of grouped data, $f_i{}'s$ are the corresponding frequencies and $\bar{x}$ is the mean, then find $\sum f_i(x_i - \bar{x})$.

14. In the following table :

Marks obtained	No. of students
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

Find the frequency of the class intervals (30-40) and (40-50)

15. Find P, if 18 is the mean of following distribution:

x_i	10	15	20	25
f_i	5	10	P	8

16. The following distribution represents the age of 35 females :

Age in years	<15	<30	<45	<60	<75	<90
No. of females	3	15	20	25	28	35

(i) How many females are there in the age group 45-90?

(ii) How many females are senior citizens? (if more than or equal to 60 years old are supposed to be senior citizens)

17. Two dice are thrown simultaneously. What is the probability of obtaining a total of at least 10?

18. Find the probability of choosing perfect square numbers between 2 and 100.

19. 17 cards numbered 1, 2, 3…, 17 are put in a box and mixed thoroughly. One person draws a card from the box. Find the probability that the number on the card is

(i) Prime

(ii) Odd

(iii) Divisible by 3

(iv) Divisible by 3 and 2 both.

20. Cards marked with the numbers 3 to 101 are placed in a box and mixed thoroughly. One card is drawn at random from this box. Find the probability of getting a perfect square or cube on the card drawn.

21. Two dice are thrown simultaneously. What is the probability that

(i) 5 will not come up on either of dice.

(ii) 5 will come up on at least one dice.

22. All the black face cards are removed from a pack of 52 playing cards. The remaining cards are well shuffled and then a card is drawn at random. Find the probability of getting

(i) A face card

(ii) A red card

(iii) A black card

23. Find the probability that a leap year selected at random will contain 53 Sundays and 53 Mondays.

24. From two digit numbers, a number is chosen at random. Find the probability that it is a multiple of 5 or 7.

25. A box contains cards numbered 3, 5, 7, 9… 35, 37. A card is drawn at random from the box. Find the probability that the number on the card is a prime number.

26. A bag contains some red, blue and orange balls. The probabilities of selecting a red ball and a blue ball from this bag are $\frac{1}{4}$ and $\frac{1}{3}$ respectively. If this bag contains 10 orange balls then find the total number of balls in the bag.

27. A jar contains 54 marbles out of which some are blue, some are red and some are yellow. The probability of selecting a blue marble at random is $\frac{1}{3}$ and probability of selecting a yellow marble is at random is $\frac{5}{9}$. How many red marbles are there in the jar?

28. Two dice are rolled once. Find the probability of getting perfect square as product of numbers on both dice.

29. **All kings, queens and aces are removed from a pack of 52 cards. The remaining are well shuffled and then a card is drawn from it. Find the probability that the card drawn is**
    (i) **A black face card**
    (ii) **A red card**
30. **What is the probability of winning a game if the probability of losing it is 0.092?**
31. **The mean of three numbers p, q and r is 9 and the mean of five numbers p, q, r, s and t is 14. Find the mean of s and t.**
32. **For the following frequency distribution :**

Class	**0-5**	**5-10**	**10-15**	**15-20**	**20-25**
Frequency	**10**	**15**	**12**	**20**	**9**

Find the sum of the lower limits of the median class and modal class.

33. **Find the mean of a grouped frequency distribution, where** $\sum f_i u_i = 27, \sum f_i = 30$ **and** $u_i = \frac{x_i - 25}{10}$.
34. **Out of 1000 small coloured bulbs** 9^3 **are of white color,** 5^3 **are of red color,** 2^6 **are of green color and rest are blue coloured. What is the probability that the bulb chosen is blue coloured.**
35. **A card is drawn from a well shuffled deck of 52 cards. Find the probability that the card drawn will be**
    (i) **A king or queen**
    (ii) **A red face card**
    (iii) **Neither a king nor a queen**
    (iv) **Either a red card or a black face card**
    (v) **Not a king**
36. **A bag contains slips with all natural numbers between 3 and 32, what is the probability that a slip chosen contains multiple of 4?**
37. **A bag contains 12 balls out of which x are white.**
    (i) **If one ball is drawn at random, what is the probability that it will be a white ball?**

(ii) If 6 more white balls are put in the bag, the probability of drawing a white ball will be double than that in [(i) case]. Find x.

38. A bag contains 6 red balls and some white balls. If the probability of drawing a white ball is double that of a red ball, find the number of white balls in the bag.

39. A coin is tossed successively three times. Find the probability of getting exactly one head or two heads.

40. Find the mode of the data if x=13

$$13, 15, 17, 10, x+2, 2x-11, x+3$$

41. The mean monthly salary of 10 members of a group is ₹ 1445, one more member whose monthly salary is ₹1500 has joined the group. Find the mean monthly salary of 11 members of the group.

42. The mean of 6 numbers is 18. If one number is excluded, the mean of remaining numbers is 16. Find the excluded number.

43. If the mean of five observations x, x+2, x+4, x+6 and x+8 is 11, find the mean of last three observations.

44. In a well shuffled pack of playing cards, find the probability of getting a card with even number.

45. There are three children in a family. Find the probability of having atmost one girl in the family.

46. A card is drawn from an ordinary pack of playing cards and a person bets that it is a heart or an ace. Find the probability of him not winning the bet.

47. Find the probability of getting a prime number if one card is selected at random from cards numbered 11, 12, 13… 49, 50.

48. Find the mean of the data given below :

$$x-5, 2x+5, x+3, 3x+7, 2x+1, 2x+9, 3x+8.$$

49. In a factory, the daily wages of 5 workers are ₹400, ₹500, ₹420, ₹350 and ₹300. If daily wages of each worker is increased by ₹50, find the mean wages.

50. If the mode of the given data is 48, then find the value of 2x+8

$$36, 42, 48, 53, 36, 48, x+2, 50, 42$$

ANSWERS

1	$\frac{n}{2}$	26	24
2	7.51	27	6
3	12	28	$\frac{2}{9}$
4	1:3	29	(i) $\frac{1}{20}$ (ii) $\frac{1}{2}$
5	3.75	30	0.908
6	36	31	21.5
7	y = 20	32	25
8	19	33	34
9	9	34	$\frac{41}{500}$
10	0	35	(i) $\frac{2}{13}$ (ii) $\frac{3}{26}$ (iii) $\frac{11}{13}$ (iv) $\frac{8}{13}$ (v) $\frac{12}{13}$
11	8	36	$\frac{1}{4}$
12	50.15	37	(i) $\frac{x}{12}$ (ii) x = 3
13	0	38	12
14	3,6	39	$\frac{3}{4}$
15	7	40	15
16	(i) 15 (ii) 10	41	₹1450
17	$\frac{1}{6}$	42	28
18	$\frac{8}{97}$	43	13
19	(i) $\frac{7}{17}$ (ii) $\frac{9}{17}$ (iii) $\frac{5}{17}$ (iv) $\frac{2}{17}$	44	$\frac{5}{13}$
20	$\frac{4}{33}$	45	$\frac{1}{2}$
21	(i) $\frac{25}{36}$ (ii) $\frac{11}{36}$	46	$\frac{9}{13}$
22	(i) $\frac{3}{23}$ (ii) $\frac{13}{23}$ (iii) $\frac{10}{23}$	47	$\frac{11}{40}$
23	$\frac{1}{7}$	48	2x + 4
24	$\frac{29}{90}$	49	₹444
25	$\frac{11}{18}$	50	100

मैंटल मैथ्स

MENTAL MATHS

शिक्षा निदेशालय दिल्ली सरकार

DIRECTORATE OF EDUCATION GOVT. OF DELHI

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