Directorate of Education

Govt. of NCT of Delhi

Practice Test Material

2015-2016

Subject : Mathematics
Class : XII

Under the guidance of :

Addl. DE (School/Exam)

Prepared by :

1.	**Sanjay Lakra**	**Lecturer**	**RPVV, A-6, Paschim Vihar**
2.	**Suman Arora**	**Lecturer**	**RPVV, A-6, Paschim Vihar**
3.	**Sahansar Pal Singh**	**Lecturer**	**RPVV, Shalimar Bagh**
4.	**Sangeeta Malhotra**	**Lecturer**	**RPVV, Yamuna Vihar**
5.	**Pradeep Kumar Jain**	**Lecturer**	**RPVV, Raj Niwas Marg**
6.	**Rajesh Kumar Yadav**	**Lecturer**	**RPVV, Dwarka**
7.	**Om Prakash Yadav**	**Lecturer**	**RPVV, Dwarka**
8.	**Ashok Kumar**	**Lecturer**	**RPVV, Suraj Mal Vihar**
9.	**Deepak Sharma**	**Lecturer**	**RPVV, Suraj Mal Vihar**
10.	**Shashi Bala**	**Lecturer**	**RPVV, B-Block Yamuna Vihar**
11.	**R.K. Malik**	**Lecturer**	**RPVV, B-Block Yamuna Vihar**

PRACTICE TEST-1

CLASS: XII

SUBJECT: MATHS

RELATIONS AND FUNCTIONS (CHAPTER-1)

Time : 1 hr. **M.M. 33**

Very short answer questions

1. A = {1,2,3,4,5,6,7,8} and if R = {(x,y): y is one half of x; x, yεA} is a relation on A, then ratio R as a set of ordered pairs. 1

2. If f(x) = $\frac{4x+3}{6x-4}$; $x \neq \frac{2}{3}$, show that fof(x) = x for all $x \neq \frac{2}{3}$. What is the inverse of f? 1

3. What is the range of the function $f(x) = \frac{|x-1|}{x-1}$? 1

4. If * is defined on the set R of real numbers by a*b = $\frac{3ab}{7}$, find the identify element in R for the binary operation *. 1

OR

State the reason for the relation R in the set {1,2,3,4} given by R = {(1,2) (2,1)} not to be transitive. 1

Short answer questions

5. Find gof and fog if f: R $\to$ R and g : R $\to$ R are given by f(x) = |x| and g(x) = |5x–2|. 4

6. Consider f : $R^+ \to [-5,\infty)$ given by f(x) = $9x^2$ + 6x – 5. Show that f is inversible with $f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$. 4

7. Consider the binary operations * : R*R$\to$R and O : ROR$\to$R defined as a*b=|a–b| and aob = a for all a, bεR. Show that * is commutative, but not associative, O is associative but not commutative. Further show that * is distributive over 0. 4

OR

Show that the function f : A $\to$B defined as f(x) = $\frac{3x+4}{5x-7}$ where $A = R - \left\{\frac{7}{5}\right\}$, $B = R - \left\{\frac{3}{5}\right\}$ is one-one and onto. Hence find f^{-1}. 4

Long answer questions

8. Show that the relation R on the set A = {1,2,3,4,5} given by R = {(a,b): |a–b| is even} is an equivalence relation. Show that all the elements of {1,3,5} are related to each other and all the elements of {2,4} are related to each other, but no element of {1,3,5} is related to any element of {2,4}. 6

9. Let N denote the set of all natural numbers and R be the relation on N×N defined by (a,b) R (c,d) $\Leftrightarrow$ad(b+c)=bc(a+d). Check whether R is an equivalence relation on N×N. 6

PRACTICE TEST-2

CLASS: XII

SUBJECT: MATHS

INVERSE TRIGONOMETRIC FUNCTIONS (CHAPTER-2)

Time : 1 hr. **M.M. 33**

Very short answer questions

1. Evaluate $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$.

2. Evaluate $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

3. Write the value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\cos^{-1}\left(\frac{-1}{2}\right)$

4. Find the value of $\cos\ \tan^{-1}\sqrt{x}+\cot^{-1}\sqrt{x}$; $x\geq 0$.

Short answer questions

5. If $\tan^{-1}x^{2}+\cot^{-1}x^{2}=\frac{5\pi^{2}}{8}$ then find x.

6. Solve for x, $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$

7. If $\sin(\cot^{-1}(x+1))=\cos(\tan^{-1}x)$, then find x.

8. Prove that $\cos\left[\tan^{-1}\ \sin\ \cot^{-1}x\ \right]=\sqrt{\frac{x^{2}+1}{x^{2}+2}}$

9. Prove that $2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

10. Prove that $\tan^{-1}\left(\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right)=\tan^{-1}\left(\frac{a}{b}\right)-x$ if $\frac{a}{b}\tan x>-1$.

PRACTICE TEST-3

CLASS: XII

SUBJECT: MATHS

INVERSE TRIGONOMETRIC FUNCTIONS (SET-2)

Time : 1 hr. **M.M. 33**

Very short answer questions

1. Write the principal value of $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

2. Evaluate $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)$.

3. Write the value of $\tan\left[\sin^{-1}\alpha+\cos^{-1}\alpha\right]$

4. Write the value of $\tan^{-1}(-1)-\cot^{-1}(-1)+\sin^{-1}(1)$

Short answer questions

5. Prove $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

6. Solve $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{2\pi}{3}, x>0$

7. Prove that $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

8. Solve for x : $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x), x\neq\frac{\pi}{2}$

9. Find the value of $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$ where |x|<|, y > 0, xy < 1.

10. Prove that $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}=\frac{\pi}{4}$

OR

Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, 0<x<1$

PRACTICE TEST-4

CLASS: XII

SUBJECT: MATHS

MATRICES (CHAPTER-3)

Time : 1 hr. **M.M. 33**

Very short answer questions

1. If the matrix $\begin{bmatrix} 0 & 6-5x \\ x^2 & x+3 \end{bmatrix}$ is symmetric, then find the value(s) of x.

2. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, find A^2. Hence find A^6.

3. For what value of K, $\begin{bmatrix} K & 2 \\ 3 & 4 \end{bmatrix}$ is a singular matrix?

4. Find the number of all possible matrices of order 2×2 with each entry 1 or 2 or 3.

Short answer questions

5. Express $\begin{bmatrix} 6 & 1 & -5 \\ -2 & -5 & 4 \\ -3 & 3 & 1 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

6. Given matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find f(A) if $f(x) = 2x^2 - 3x + 5$

7. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, where $i = \sqrt{-1}$, then prove by the principle of mathematical induction that $A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}$

OR

A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Products	Market	
	I	II
x	10000	6000
y	2000	20000
z	18000	8000

If unit sale prices of x, y and z are Rs. 2.50, Re. 1.00 and Re. 1.00 respectively. Find the total revenue of both the markets and revenue of each market.

Long answer questions

8. Using elementary transformation, find the inverse of $A = \begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 3 \\ 3 & -2 & 7 \end{bmatrix}$.

9. Prove that the product of the matrices $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is a null matrix. When θ and ϕ differ by odd multiple of $\frac{\pi}{2}$.

PRACTICE TEST-5

CLASS: XII

SUBJECT: MATHS

DETERMINANTS (CHAPTER-4)

Time : 1 hr. **M.M. 33**

Very short answer questions

1. If A is square matrix of order 2 and |A| = 6 then find |adj A|.
2. If A is square matrix of order 3 and |A| = 5 then find |A adj A|.
3. If A is 3×3 matrix then find |A.A'| provided |A| = 4.
4. If A is square matrix of order 3 and |A| = 6, find |B| if B = 5 A^2.

Short answer questions

5. If $\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ find matrix A.

6. If $\begin{vmatrix}p & b & c\\a & q & c\\a & b & r\end{vmatrix}=0$, find value of $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}$

OR

Using properties of determinants prove that

$$\begin{vmatrix}b+c & c+a & a+b\\c+a & a+b & b+c\\a+b & b+c & c+a\end{vmatrix}=2(3abc-a^3-b^3-c^3)$$

7. If a, b, c are all +ve and are pth, qth and rth terms of G.P. prove that $\begin{vmatrix}\log a & p & 1\\\log b & q & 1\\\log c & r & 1\end{vmatrix}=0$

Long answer questions

8. If $\begin{bmatrix}1 & 1 & 2\\1 & -2 & -1\\1 & 3 & 1\end{bmatrix}$ find A^{-1} hence solve $\begin{aligned}x+y+z&=3\\x-2y+3z&=2\\2x-y+z&=2\end{aligned}$

9. A school decided to award some of its students for honesty, some for punctuality and some other for obedience. The sum of all the awardees is 12. Three times the sum of awardees for punctuality and obedience added to two times the number of awardees for honesty is 33. The sum of number of awardees for honesty and obedience is twice the number of awardee for punctuality. Using matrix method, find the number of awardees of each category, which value do you prefer to be rewarded most and why?

PRACTICE TEST-6

CLASS: XII

SUBJECT: MATHS

DETERMINANTS (2B) (CHAPTER-4)

Time : 1 hr. **M.M. 33**

Very short answer questions

1. Find the value of $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$; where $i=\sqrt{-1}$.

2. If area of triangle with vertices (K,0) (1,1) and (0,3) is 5 units. Find value of K.

3. If A is square matrix of order 3. Such that |adj A| = 81, find |A|.

4. If |A| = 14 then find det (adj (adj A)) where A is square matrix of order 3.

Short answer questions

5. If $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}=0$ prove that $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$ (Provided a, b, c $\neq 1$)

6. If $A=\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ prove that $A^2 - 4A - 5I = 0$. Hence find A^{-1}.

7. Using properties prove that

$$\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}=(a-b)(b-c)(c-a)(a^2+b^2+c^2)$$

Long answer questions

8. Using properties of determinants. Prove that $\begin{vmatrix} y^2+z^2 & z^2 & y^2 \\ z^2 & z^2+x^2 & x^2 \\ y^2 & x^2 & y^2+x^2 \end{vmatrix}$ is a perfect square for $x\neq 0, y\neq 0$ & $z\neq 0$.

Long answer questions

9. Find the product of matrices

$$A=\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \text{ and } B=\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$x + y + 2z = 1$$

and hence solve $3x + 2y + z = 7$

$$2x + y + 3z = 2$$

OR

Using matrix method, solve

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2 \quad z \neq 0, y \neq 0, z \neq 0$$

PRACTICE TEST-7

CLASS: XII

SUBJECT: MATHS

CONTINUITY AND DIFFERENTIABILITY (CHAPTER-5)

Time : 1 hr. **M.M. 33**

Very short answer questions

1. Check the continuity of the function f(x) = [x] at x = 1.5.

2. Write the value of x for which f(x) = |x–5| is not differentiable in R.

3. If $y=\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2$, find $\frac{dy}{dx}$ at $x=\frac{\pi}{6}$.

4. Differentiate Cot(5x°) w.r.t. x.

Short answer questions

5. Show that $f(x)=\begin{cases}\frac{\log(1+ax)-\log(1-bx)}{x} & \text{if } x\neq 0\\ a+b+x^2 & \text{if } x=0\end{cases}$, is continuous at x = 0.

6. Show that the function f(x) defined by f(x) $f(x)=\begin{cases}3x-2 & \text{if } 0<x\leq 1\\ 2x^2-x & \text{if } 1<x\leq 2\\ 5x-4 & \text{if } x>2\end{cases}$ is not differentiate at x = 2.

7. If x = a sec θ, y = b tan θ, then prove that $\frac{d^2y}{dx^2}=\frac{-b^4}{a^2y^3}$.

Long answer questions

8. If x = sin t, y = sin pt, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$

OR

Given that $\cos\frac{x}{2}.\cos\frac{x}{4}.\cos\frac{x}{8}.....=\frac{\sin x}{x}$ then prove that $\frac{1}{2^2}\sec^2\frac{x}{2}+\frac{1}{2^4}\sec^2\frac{x}{4}+.....=\operatorname{cosec}^2x-\frac{1}{x^2}$

9. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a(x^3-y^3)$ then prove that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

PRACTICE TEST-8

CLASS: XII

SUBJECT: MATHS

CONTINUITY AND DIFFERENTIABILITY-1 (CHAPTER-5)

Time : 1 hr. **M.M. 33**

Very short answer questions

1. If $f(x)=|\cos x|$, find $f'\left(\frac{\pi}{4}\right)$

2. Find the value of C in Rolle's theorem for the function $f(x)=x^3-3x$ in the partial $(0,\sqrt{3})$.

3. Differentiate $\log_x 4$ w.r.t. 'x'

4. If $f(x)=\begin{cases}\frac{\sin}{x} & , \quad x\neq 0\\ K & , \quad x=0\end{cases}$ is continuous at x = 0. Write the value of K.

Short answer questions

5. Find the value of 'a' and 'b' if $f(x)=\begin{cases}\frac{1-\sin^3 x}{3\cos^2 x} & \text{if} \quad x<\frac{\pi}{2}\\ a & \text{if} \quad x=\frac{\pi}{2}\\ \frac{b(1-\sin x)}{(\pi-2x)^2} & \text{if} \quad x>\frac{\pi}{2}\end{cases}$

   Is continuous at $x=\frac{\pi}{2}$.

6. Differentiate $\tan^{-1}\frac{\sqrt{1-x^2}}{x}$ w.r.t. $\cos^{-1}(2x.\sqrt{1-x^2})$ when $x\varepsilon\left[\frac{1}{\sqrt{2}},1\right]$.

7. If $x=a(\cos\theta+\theta\sin\theta), y=a(\sin\theta-\theta\cos\theta)$ then prove that $\frac{d^2y}{dx^2}=\frac{\sec^3\theta}{a\theta}$.

8. Show that the function f(x) given by $f(x)=\begin{cases}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & \text{if} \quad x\neq 0\\ 0 & \text{if} \quad x=0\end{cases}$

   Is discontinuous at x = 0.

Long answer questions

9. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{(x-c)} + 1$ then prove that $\frac{dy}{dx} = \frac{y}{x}\left[\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right]$.

OR

Differentiate w.r.t. 'x' $\cos^{-1} x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2}$

PRACTICE TEST-9

CLASS: XII

SUBJECT: MATHS

APPLICATION OF DERIVATIVES (CHAPTER-6)

Time : 1 hr. **M.M. 33**

Very short answer questions

1. Find the point on the curve $y = x^3 + 1$ where the tangent is parallel to x-axis.

2. Find the slope of normal to the curve $y = \cos^2 x$ at $x = \frac{\pi}{4}$.

3. Find the value (s) of 'a' such that the function $f(x) = ax^2 + 5$ has local maxima at $x = 0$.

4. Find the absolute minimum value of $|x|$ when $x \varepsilon R$.

Short answer questions

5. The government is running a campaign "MAKE INDIA POLIO FREE". To spread awareness in a particular society it is displaying an air balloon with the above tag line printed on it. The balloon is in the form of a right circular cone surmounted by a hemisphere, having a diameter equal to the height of the cone. It is being inflated by a pump. How fast is its volume changing with respect to its total height 'h', when h is 3 cm? Do you think that we should work seriously towards Polio eradication? Write the values that have been highlighted here.

6. Find the intervals for which the following function is increasing OR decreasing in $0 \leq x \leq 2\pi$ $f(x) = \sqrt{3}\sin x - \cos x$

7. Using differentiation find the approximate value of $\sqrt{49.5}$.

OR

Find the point on the curve $y = (x-1)^2$ where the tangent is parallel to the line joining the points (4,–1) and (5,0).

Long answer questions

8. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius 'a' show that the area of triangle is maximum when $\theta = \frac{\pi}{6}$

9. A wire of length 20m is to be cut into two pieces. One of the piece will be bent into shape of a square and the other into a shape of an equilateral triangle. What should be the length of the two pieces so that the sum of the areas of square and triangle is minimum.

PRACTICE TEST-10

CLASS: XII

SUBJECT: MATHS

APPLICATION OF DERIVATIVES (CHAPTER-6)

Time : 1 hr. **M.M. 33**

Short answer questions

1. In the first five months, the performance of a student in x months is governed by the relation $f(x) = 2x^3 - 9x^2 + 12x + 1$. Find the month in which the performance of the student is increasing or decreasing. What life skills should the student develop to improve his performance?

2. If the tangent to the curve $y = x^3 + ax + b$ at (2,–6) is perpendicular to the line $x – y – 9 = 0$, find a and b.

3. Find the condition for the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $xy = c^2$ to intersect orthogonally.

4. The volume of metal of a hollow sphere is constant. If the inner radius is increasing at the rate of 1cm/sec, find the rate of increase of the outer radius, when the radii are 3 cm and 6 cm respectively.

5. Oil is leaking out of a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of 2 cm^2/s in the surface area through a tiny hole at the vertex in the bottom. When slant height of the oil is 4 cm, find the rate of decrease of the slant height of oil. Does leaking of oil leads to wastage? Can we afford the wastage of natural resources?

Long answer questions

6. The total area of a rectangular poster is $\frac{3}{2} m^2$. The combined width of margins at the bottom and the top is 12cm and that at the sides is 8cm. What must be the dimensions of the poster in order that the area of printed matter may be maximum. This poster is to be presented in a debate competition on the topic “SAVE GIRL CHILD”. How can we save girl child?

7. Find the area of greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

PRACTICE TEST-11

CLASS: XII

SUBJECT: MATHS

INTEGRAL AND ITS APPLICATIONS

Time : 1 hr. **M.M. 33**

Very short answer questions

1. Evaluate $\int \sqrt{16-(5-x)^2}dx$

2. Evaluate $\int e^{-2\log\sin x}dx$

3. Evaluate $\int_0^{\pi/2}(\cos^{-1}\sqrt{x}+\sin^{-1}\sqrt{x})\ dx$

4. Evaluate $\int e^x\left(\frac{x-1}{x^2}\right)dx$

Short answer questions

5. Find the area bounded by the curve $x^2 = y$, the tangent to the curve at the point (2,4) and the x-axis.

6. Evaluate $\int_{-1}^{1/2}|x^3-x|\ dx$

7. Evaluate $\int_0^1 \log\left(\frac{1}{x}-1\right)\ dx$

8. Evaluate $\int \frac{\cos^6 x+\sin^6 x}{\sin^2 x.\cos^2 x}dx$

Long answer questions

9. Evaluate $\int \frac{x^2}{(x\sin x+\cos x)^2}dx$

OR

Find the ratio of areas of the regions in which the curves $y^2 = x$ and $x^2 = y$ divide the area of the square bounded by x = 0, x = 1, y = 0, y = 1.

PRACTICE TEST-11

CLASS: XII

SUBJECT: MATHS

INTEGRAL AND ITS APPLICATIONS-1

Time : 1 hr. **M.M. 33**

Very short answer questions

1. Evaluate $\int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

2. Evaluate $\int_{-\pi/6}^{\pi/6} (\tan x + x^{1003} - 1) dx$

3. Evaluate $\int \sec^2 x^\circ \, dx$ (x° denotes x degree)

4. If $\int_1^a (2x + 1) dx = 4$

   Find the value of a.

Short answer questions

5. Evaluate $\int (1 + x^2 + x^4 + x^6 + ... \infty) dx, |x| < 1$

   OR

   Evaluate as the limit of sum $\int_0^2 (e^x + x^2 - 5) dx$

6. Evaluate $\int \frac{e^x(4e^x + 3)}{\sqrt{2e^{2x} + 2e^x - 3}} dx$

7. Using integration, show that area of a circle is π times the square of its radius.

Long answer questions

8. Find the area bounded by the curve y = x|x|, x-axis and the ordinates x=1 and x=–1.

9. Evaluate $\int \frac{\tan\theta + \tan^3\theta}{\tan^3\theta + 1} d\theta$

PRACTICE TEST-12

CLASS: XII

SUBJECT: MATHS

DIFFERENTIAL EQUATIONS

Time : 1 hr. **M.M. 33**

Very short answer questions

1. Write the order of the differential equations

$$\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^2y}{dx^2}\right)^2 = \frac{d^3y}{dx^3}$$

2. Write the degree of the differential equation

$$\left(\frac{dy}{dx}\right)^2 - x\frac{d^2y}{dx^2} = \log x$$

3. From the differential equation of the system of concentric circles given by $x^2 + y^2 = a^2$

4. Write the integrating factor of the differential equation:

$$\frac{dy}{dx} + \left(\frac{1}{x^2}\right)y = \frac{1}{x}$$

Short answer questions

5. Solve the differential equation:

$$\frac{dy}{dx} + y\cot x = \sin 2x$$

6. Find the particular solution of the differential equation:

$$\log\left(\frac{dy}{dx}\right) = 3x + 4y \text{ given that } y = 0, x = 0.$$

7. Verify that $y = (x^3 - x)$ log cx is a solution of the differential equation

$$(x^3 - x)\frac{dy}{dx} - (3x^2 - 1)y = x^5 - 2x^3 + x$$

OR

Solve the differential equation :

$$(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$$

Long answer questions

8. Solve the differential equations:

$$(2x^2 + 3y^2 + 1)x\,dx + (4x^2 + 6y^2 + 3)y\,dy = 0$$

9. Find the general solution of :

$$(1 + \tan y)(dx - dy) + 2xdy = 0$$

where $-\frac{\pi}{4} < y < 3\frac{\pi}{4}$

PRACTICE TEST-12

CLASS: XII

SUBJECT: MATHS

DIFFERENTIAL EQUATIONS-1

Time : 1 hr. **M.M. 33**

Very short answer questions

1. Write the order and degree of the differential equation

$$\frac{dy}{dx}+\tan\left(\frac{dy}{dx}\right)=0$$

2. Write the general solution of the differential equation :

$$\frac{dy}{dx}=5^{x+y}$$

3. Write the order of the differential equation of family of curves

$$Ay=Bx^2$$

4. Write the integrating factor of the differential equation:

$$\frac{dy}{dx}+y\tan x=\sec x$$

Short answer questions

5. Form the differential equation of the family of curves given by :

$$(y-b)^2=4(x-a)$$

6. Find the particular solution of the differential equation:

$$\frac{dy}{dx}-3y\cot x=\sin 2x;\ y=2\ \text{ when}\ x=\frac{\pi}{2}$$

OR

Solve the differential equation:

$$\left(x\sin\frac{y}{x}\right)dy=\left(y\sin\frac{y}{x}-x\right)dx$$

7. Find the equation of a curve passing through the point (–2,3) given that the slope of the tangent to the curve at any point (x,y) is $\frac{2x}{y^2}$.

Long answer questions

8. Form the differential equation which is satisfied by :

$\sqrt{1-x^4}+\sqrt{1-y^4}=a(x^2-y^2)$ where a is an arbitrary constant.

9. The equation of electromotive force for an electric circuit containing resistance and self inductance is $E=R_i+L\frac{di}{dt}$ where E is the electromotive force given to the circuit, R is the resistance and L is the coefficient of induction. Find the current i at time t when

(a) E = 0 (b) E = a non-zero constant.

PRACTICE TEST-13

CLASS: XII

SUBJECT: MATHS

VECTORS

Time : 1 hr. **M.M. 33**

Very short answer questions

1. Write the vector along (–) ve y-direction whose magnitude is equal to the magnitude of the vector $\sqrt{3}\hat{i}-2\hat{j}+3\hat{k}$.

2. What is the projection of $\vec{b}$ on $\vec{a}$, if $\vec{a}=3\hat{i}-4\hat{j}$ and $\vec{b}=5\hat{k}-2\hat{j}$.

3. Position vectors of the vertices of a triangle are $2\hat{i}-\hat{j}, 4\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$. Write the position vector of centroid of the triangle.

4. What is the magnitude of the vector

   $(\hat{i}\times\hat{j}+\hat{j}\times\hat{k}+2\hat{k}\times3\hat{j})$

Short answer questions

5. Find the value of 'λ' for which the points with position vector

   $-\hat{j}-\hat{k}, 4\hat{i}+5\hat{j}+\lambda\hat{k}, 3\hat{i}+9\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar.

6. Find vector(s) of magnitude 9 units which are perpendicular to both the vector $4\hat{i}-\hat{j}+3\hat{k}$ and $-2\hat{i}+\hat{j}-2\hat{k}$.

7. If $\vec{a}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-4\hat{k}$ then express $\vec{b}$ in the form $\vec{b}=\vec{c}-\vec{d}$, where $\vec{c}$ is parallel to $\vec{a}$ and $\vec{d}$ is perpendicular to $\vec{a}$.

Long answer questions

8. $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors of magnitude 3,4, and 5 respectively. If each one is perpendicular to the sum of the other two vectors, find the magnitude of the vector $(\vec{a}+\vec{b}+\vec{c})$.

9. If $\vec{p},\vec{q},\vec{r}$ are three mutually perpendicular vectors of equal magnitude, prove that the vector $(\vec{p}+\vec{q}+\vec{r})$ is equally inclined to the vectors $\vec{p},\vec{q}$ and $\vec{r}$. Also, find the angle.

   OR

   If $\vec{\alpha},\vec{\beta},\vec{\gamma}$ are three coplanar vectors, prove that $\vec{\alpha}+\vec{\beta}$, $\vec{\beta}+\vec{\gamma}$, $\vec{\gamma}+\vec{\alpha}$ are also coplanar.

PRACTICE TEST-14

CLASS: XII

SUBJECT: MATHS

THREE-DIMENSIONAL GEOMETRY

Time : 1 hr. **M.M. 33**

Very short answer questions

1. Write the vector equation of the line

$$\frac{2x-1}{3}=5z=3-y$$

2. What is the distance of the plane 3x+4y+z=3 from the point (–2,1,–1)?

3. What is the angle between the line $\vec{r}=(\hat{i}+\hat{j}+\hat{k})+\lambda(3\hat{i}-\hat{j}-\hat{k})$ and the plane $\vec{r}.(2\hat{i}-\hat{j}-2\hat{k})=3$?

4. Write the equation of the plane parallel to the plane x+2y–z=3 and passing through the point (–1, –2, 3).

Short answer questions

5. Find coordinates of the point where the line through the points (3,–4,–5) and (2,–3,1) crosses the plane 2x+y+z=7.

6. Find the image of the point $(\hat{i}+3\hat{j}+4\hat{k})$ in the $\vec{r}.(2\hat{i}-\hat{j}+\hat{k})+3=0$.

7. Find the distance of the point (1,–2,3) from the plane x–y+z=5 measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$.

Long answer questions

8. If the points (1,1,p) and (–3,0,1) are equidistant from the plane 3x+4y+13=12z, find the value(s) of ‘p’.

9. Find the equation of the plane which is perpendicular to the plane 5x+3y+6z+8=0 and contains the line of intersection of the planes $\vec{r}.(i+2\hat{j}+3\hat{k})=4$ and $2x+y-z+5=0$.

PRACTICE TEST-15

CLASS: XII

SUBJECT: MATHS

LINEAR PROGRAMMING PROBLEM (L.P.P.)

Time : 1 hr. **M.M. 33**

Very short answer questions

Choose the correct answer in each of the following :

1. Objective function of a L.P.P. is
   a) a quadratic function
   b) a linear function to be optimized
   c) a constraint
   d) none of these
2. The optimal value of the objective function is attained at the points
   a) given by the intersection of in equations with x-axis only
   b) given by corner points of the feasible region.
   c) given by the intersection of in equations with the axes only
   d) none of these

Short answer questions

3. Solve the following L.P.P. graphically

   Minimize z = x – 2y

   Subject to constraints :

   $x - 3y \leq 0$

   $2x + 3y \leq 6$

   $3x + y \geq 0$

   $x, y \geq 0$

4. Solve the following L.P.P. graphically

   Maximize = z = x + 3y

   Subject to constraints

   $2x + 3y \geq 6$

   $x - y \geq 0$

   $x \geq 0,\ y \geq 0$

Long answer questions

5. Mohan wants to invest at most Rs. 12000 in Kisan Vikas Patras and National Saving Bonds. According to given condition, he has to invest at least Rs. 2000 in Kisan Vikas Patras and at least Rs. 4000 in National Saving Bonds. If the rate of interest on Kisan Vikas Patras is 8% per annum and the rate of interest on National Saving Bonds is 10% per annum, how much money should be

invest to earn maximum yearly income? Also find his maximum yearly income. Formulate the above L.P.P. mathematically and then solve it graphically. Do you think that your investment in Kisan Vikas Patras and National Saving Bonds helps in the well-being of the Nation? Do you think that a person should start saving at an early age for his retirement?

6. A toy company manufactures two types of gift items A and B. Market tests and the available resources have indicated that the combined production level should not exceed 1200 gifts items per week and the demand for gift items of type B is at most half of that for gift items of type A. Further, the production level of gift items of type A can exceed three times the production of gift items of other type by at most 600 units. If the company makes profit of Rs. 12 and Rs. 16 per item respectively on gift items A and B, how many of each should be produced weekly in order to maximize the profit? Formulate the above L.P.P. mathematically and then solve it graphically. What is the importance of exchanging gifts with friends and relatives?

7. If a 19 years old girl drives her law at 40 km/hr, she has to spend Rs. 2 /km on Petrol. If she drives it at a faster speed of 70 km/hour. The petrol cost increases to Rs. 5/km. She has Rs. 100 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express it as L.P.P. and then solve it. Is the girl eligible for a driving licence? What is the benefit of driving at economic speed?

PRACTICE TEST-16

CLASS: XII

SUBJECT: MATHS

PROBABILITY

Time : 1 hr. **M.M. 33**

Very short answer questions

1. Let A and B be two events such that P(A)=0.6, P(B)=0.2 and P(A/B)=0.5. Find the value of P(A'/B').

2. If events A and B are independent events such that $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$. Find the value of $P(A\cup B)$

3. The mean of binomial distribution is 12 and the standard deviation is 2. Find the binomial distribution.

4. What is the probability of 53 Sundays and 53 Mondays in a leap year?

Short answer questions

5. A card is drawn from a pack of 52 cards. Find the probability that the card drawn is a queen or a spade or a black car.

6. The probability of simultaneous occurrence of atleast one of two events A and B is p. If the probability that exactly one of A, B occurs is q, then prove that P(A') + P(B') = 2–2p+q.

7. A discrete random variable X has the following probability distribution:

X:	1	2	3	4	5	6	7
P(X):	C	2C	2C	3C	C^2	$2C^2$	$7C^2+C$

Find the value of C. Also find the mean of this distribution.

OR

Past records show that 80% of the operations performed by a certain doctor were successful. If he performs 4 operations in a day. What is the probability that at least 3 operations will be successful? Name two life skills to be successful in life.

Long answer questions:

8. For three persons A, B and C, the chances of being selected as a manager of a firm are in the ratio 4:1:2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8 and 0.5. If the change take place, find the probability that it is due to appointment of B. To prove yourself be a good manager, what qualities you should possess?

9. By examining chest X-ray, the probability that T.B. is detected when a person is actually suffering is 0.99. The probability that the doctor diagnoses incorrectly that a person has T.B. on the basis of X-ray is 0.001. In a certain city, 1 in 1000 persons suffer from T.B. A person is selected at random and is diagnosed to have T.B. What is the chance that he actually has T.B.? What precautions should one take to reduce chances of catching a disease?

PRACTICE TEST-18

CLASS: XII

SUBJECT: MATHS

MATRIX AND DETERMINE

Time : 1 hr. **M.M. 33**

1. Construct a matrix $A = [a_{ij}]_{2\times 2}$ where $a_{ij} = e^{3ix \tan jx}$.

2. Find x if $\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

3. A is a non-singular matrix of order 3 and |A| = –4, find |adj A|.

4. For what value of x, the matrix $\begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$ is a singular matrix.

5. Prove that :

$$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2b^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$$

6. Three shopkeepers A, B and C go to store to buy stationary A purchase 12 dozen note books, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen note books 6 dozen pens and 7dozen pencils. C purchases 11 dozen note books, 5 dozen pens, 8 dozen pencils. A notebooks cost Rs. 4, a pen Rs. 6 and pencil Rs. 2. Use matrix multiplication to calculate each individual bills.

7. Evaluate :

$$\begin{vmatrix} \sqrt{23}+\sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{46} & 5 & \sqrt{10} \\ 3+\sqrt{115} & \sqrt{15} & 5 \end{vmatrix}$$

8. If $D = \begin{vmatrix} \underline{|n} & \underline{|n+1} & \underline{|n+2} \\ \underline{|n+1} & \underline{|n+2} & \underline{|n+3} \\ \underline{|n+2} & \underline{|n+3} & \underline{|n+4} \end{vmatrix}$

Prove that $\frac{D}{(\underline{|n})^3} - 4$ is divisible by n?

9. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} = B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

be two square matrices. Find AB and hence solve the system of linear equations.

$x-y = 3 \qquad 2x + 3y + 4z = 17 \qquad y + 2z = 7$

10. In a survey of 20 richest people of three residential societies A, B, and C it is found that in society A, 5 believes in honesty, 10 in hard work, 5 in unfair means while in B, 5 in honesty, 8 in hard work, 7 in unfair means and in C, 6 in honesty, 8 in hard work, 6 in unfair means. If the per day income of 20 richest people of the society A, B, C are Rs. 32,500; Rs. 30,500; Rs. 31,000 respectively, then find the per day income of each type of people by matrix method:

    i. Which type of people has per day more income?

    ii. What do you think which type of the persons good for the country?

PRACTICE TEST-19

CLASS: XII

SUBJECT: MATHS

INVERSE TRIGONOMETRIC FUNCTION

Time : 1 hr. **M.M. 33**

1. Evaluate : $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right]$

2. Write the value of $\operatorname{cosec}^{-1}\operatorname{cosec}\left(\frac{3\pi}{4}\right)$

3. Evaluate $\tan^{-1}2+\tan^{-1}3$

4. Find the value of $\tan^{-1}\tan\frac{5\pi}{6}+\cos^{-1}\cos\frac{13\pi}{6}$

5. Find the value of : $\sin\left(2\cot^{-1}\left(\frac{-5}{12}\right)\right)$

6. Prove that : $2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

7. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$

   Prove that $x^2+y^2+z^2+2xyz=1$

8. Solve the equation : $\cot^{-1}x-\cot^{-1}(x+z)=\frac{\pi}{12}$

9. Prove that : $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

   $-\frac{1}{2}\le x\le 1$

10. Find the greatest and least value of : $(\sin^{-1}x)^2+(\cos^{-1}x)^2;\quad |x|\le 1$