

MENTAL MATHS

QUESTION BANK
CLASS

9

DIRECTORATE OF EDUCATION GOVT. OF N.C.T. OF DELHI

MENTAL
MATHS
CLASS
IX

2024-25

DIRECTORATE OF EDUCATION

GOVT. OF NCT OF DELHI



सत्यमेव जयते

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MESSAGE

The eloquent words of Galileo Galilei resonate: 'The laws of nature are written by the hand of God in the language of mathematics.' In this profound observation, the great astronomer awakened humanity to the paramount importance of mathematics. Within our school education system, mathematics holds a pivotal role, with a dedicated focus on foundational numeracy and literacy.

This year marks a significant milestone, as the project extends its reach to Government-Aided schools and introduces Level IV for classes 11th and 12th as well.

In the competitive arena, where time is of the essence, a strong command over mathematics is indispensable. These skills are not only prized in competitive exams but also wield significant influence in the realms of entrepreneurship and innovation. Mental Maths, with its transformative impact, enhances students' number sense, fosters an understanding of relationships between quantities, and cultivates logical thinking for problem-solving.

The meticulously crafted Mental Maths Question Banks recognize the diverse abilities, needs, and interests of students. As the saying goes, 'Nothing great can be achieved without consistent and persistent hard work'. Heartfelt congratulations to the State Core Team members, District Coordinators and Subject Experts for their silent and steadfast dedication to bring forth these impactful publications.

(Ashok Kumar)



MESSAGE

Beyond mere numbers and equations, Mathematics serves as a foundational language, intricately woven into the fabric of everything from the technology we rely on to the scientific principles shaping our understanding of the cosmos.

Enter Mental Maths – a captivating art of calculation sans paper or tools, a dance of numbers performed within the confines of the mind. It's not just about crunching numbers; it's about empowerment. Mental Maths nurtures the comprehension of place value, fortifies basic operations, and establishes a robust foundation for grappling with more complex mathematical concepts in the future.

Engaging in Mental Maths includes exercising multiple cognitive processes – memory, attention, and critical thinking. Studies reveal that regular Mental Maths exercises contribute to maintaining cognitive reserve, postponing the onset of age-related memory loss, and fending off other cognitive declines. In essence, Mental Maths keeps our minds agile and adaptable, akin to the benefits of physical activity for our bodies. It becomes the catalyst for swift decision-making and adept situational adaptation.

A heartfelt commendation goes to the dedicated State Core Team members and subject experts who meticulously crafted the Mental Maths Question Banks. These resources, tailored for students in Government and Government-Aided Schools of the Directorate of Education are a testament to their sincere efforts and the wise guidance of the Project Director of Mental Maths. It brings me immense pleasure to present this Mental Maths Question Bank to students, encouraging them to weave the magic of Mental Maths into the tapestry of their daily lives.

(BHUPESH CHAUDHARY)

विकास कालिया
क्षेत्रीय शिक्षा निदेशक
उत्तर एवं मध्य क्षेत्र,
पुरस्कार एवं कल्याण शाखाएँ,
पत्राचार विद्यालय एवं
रा. मुक्त विद्यालयी शिक्षा शाखाएँ
परियोजना निदेशक: मेंटल मैथ्स



सत्यमेव जयते

VIKAS KALIA
Regional Director of Education
Central & North,
Awards & Welfare Branches,
Patrachar Vidyalaya &
NIOS (Branches)
Project Director: Mental Maths

MESSAGE

At the tender age of 16, RPraggnanandhaa, the prodigious talent in Indian chess, sent waves through the global chess community by outsmarting Chess Grandmaster Magnus Carlsen in a lightning-fast game at the Airthings Masters Rapid Chess Tournament. His secret weapon was the remarkable ability for mental calculations. This young genius effortlessly combines his exceptional talent with lightning-quick numerical intuition, fortifying his strategic thinking skills.

At the age of 20, Neelakanta Bhanu Prakash of Hyderabad secures his place as the fastest human calculator on the planet, clinching India's first gold in the Mental Calculation World Championship at the Mind Sports Olympiad in London. Holding an impressive tally of 4 world records and 50 Limca records for speed calculation, his journey is even more remarkable considering a childhood setback. A skull fracture at the age of 5 kept him away from school for a year, but he turned adversity into opportunity, delving into puzzle-solving and mathematics games to hone his cognitive skills.

Mental Mathematics isn't just about acing exams; it's a cognitive superpower that equips the brain to think strategically, break down challenges into manageable steps, and devise creative solutions. This skill transcends academic boundaries, proving invaluable when estimating shopping costs, calculating expenses, or planning a trip. Imagine confidently tallying a shopping bill without reaching for any gadgets.

Recognizing that each student has a unique learning style, Mental Maths Question Banks cater to diverse needs, offering a plethora of materials. Through collaborative efforts, students engage in exhilarating Mental Maths competitions, learning from one another and building self-confidence.

A heartfelt acknowledgment goes to the Mental Maths State Core Team, District and Zonal Coordinators, and HOSs for their unwavering dedication to bringing the Mental Maths superpower to students across all Government and government-aided schools of the Directorate of Education. Gratitude extends to the esteemed Secretary Education and the Director of Education for their guidance and constructive feedback, steering the Mental Maths Project toward continuous improvement.

(VIKAS KALIA)
PROJECT DIRECTOR (MMP)

ACKNOWLEDGEMENT

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LEVEL-3
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2	IX	AVIN JANGID	VINOD KUMAR	20170290248	SOSE KHICHRI PUR	1002401	KAMALJEET SINGH
3	IX	SIDDHANT KUSHWAHA	SANTOSH KUSHWAHA	20190034262	SOSE KALKA JI	1925438	AJAY VIR SINGH

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S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
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3	IX	PRINCE MISHRA	MITHILESH MISHRA	20190224308	SOSE SEC-23 ROHNI	1413345	DEEPAK

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STATE LEVEL MENTAL MATH QUIZ COMPETITION RESULT 2023-2024
LEVEL-3
REGION-SOUTH (1st POSITION)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
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REGION -CENTRAL (2nd POSITION)

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REGION- EAST (4th POSITION)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
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3	IX	KRISH KUMAR	VIRENDRA KUMAR SAW	20200197665	DBRA SOSE GAUTAM PURI	1105250	RAJ KUMAR

REGION -NORTH (5th POSITION)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
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CONSTITUTION OF INDIA

¹[PART IV A

FUNDAMENTAL DUTIES

Article 51A. Fundamental duties. — It shall be the duty of every citizen of India—

- a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;
- b) to cherish and follow the noble ideals which inspired our national struggle for freedom;
- c) to uphold and protect the sovereignty, unity and integrity of India;
- d) to defend the country and render national service when called upon to do so;
- e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;
- f) to value and preserve the rich heritage of our composite culture;
- g) to protect and improve the natural environment including forests, lakes, rivers and wildlife, and to have compassion for living creatures;
- h) to develop the scientific temper, humanism and the spirit of inquiry and reform;
- i) to safeguard public property and to abjure violence;
- j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;]

²[(k) who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.]

1. Ins. by the Constitution (Forty-second Amendment) Act, 1976, Sec. 11 (w.e.f. 3-1-1977).

2. Ins. by the Constitution (Eighty-sixth Amendment) Act, 2002, Sec. 4 (w.e.f. 1-4-2010).

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a ¹**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens:

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity;

and to promote among them all

FRATERNITY assuring the dignity of the individual and the ²[unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty- sixth day of November, 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

-
1. Subs. by the Constitution (Forty-second Amendment Act, 1976, Sec. 2, for "SOVEREIGN DEMOCRATIC REPUBLIC" (w.e.f. 3.1.1977)
 2. Subs. by the Constitution (Forty-second Amendment Act, 1976, Sec. 2, for "Unity of the Nation" (w.e.f. 3.1.1977)

SCHEDULE OF MENTAL MATHS QUIZ COMPETITIONS
FOR THE YEAR 2024-2025
DIRECTORATE OF EDUCATION
GOVT OF NCT OF DELHI

❖ Practice to students from Question Bank	:	01.04.2024 to 19.10.2024
❖ School Level Quiz Competitions	:	21.10.2024 to 30.10.2024
❖ Cluster Level Quiz Competition	:	14.11.2024 to 20.11.2024
❖ Zonal Level Quiz Competition	:	25.11.2024 to 30.11.2024
❖ District Level Quiz Competition	:	07.12.2024 to 13.12.2024
❖ Regional Level Quiz Competition	:	26.12.2024 to 31.12.2024
❖ State Level Quiz Competition	:	18.01.2025 to 31.01.2025

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CHAPTER 1

NUMBER SYSTEM

POINTS TO REMEMBER

- **Natural Numbers:** $N = \{1, 2, 3, 4, \dots\}$. They are also called as counting numbers.
- **Whole Numbers:** $W = \{0, 1, 2, 3, \dots\}$
- **Integers:** $Z/I = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- **Rational Numbers:**

$$Q = \left\{ x = \frac{p}{q} \text{ where } p, q \in Z, q \neq 0 \text{ and HCF}(p, q) = 1 \right\} \text{ e.g. } \frac{3}{4}, \frac{4}{5}, \frac{15}{17}, \frac{-101}{20}$$

Also defined as “terminating decimals or non-terminating repeating decimal”
e.g. 2.5, 3.128, 0.3, 40.157, 0.333... ($= 0.\overline{3}$) etc.

- **Irrational Numbers:**
 Q' = Non-terminating, non-repeating decimals are called irrational numbers,
e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi, 1.314738\dots, 2.410112\dots$ etc.
- **Real Numbers (R):** Rational numbers and irrational numbers taken together form real numbers.
- For any real number ‘a’, we have

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases} \text{ and } |a| \text{ is called absolute value of 'a'}$$

- If ‘x’ and ‘y’ are any two rational numbers, then $(x + y), (x - y), x \times y, \frac{x}{y} (y \neq 0)$ are also rational numbers.
- If ‘r’ is a non-zero rational number and ‘s’ is an irrational number, then $(r + s), (r - s), r \times s, \frac{s}{r} (r \neq 0), \frac{r}{s} (s \neq 0)$ are also irrational numbers.
- For positive real numbers ‘a’ and ‘b’, the following hold:
(i) $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

$$(ii) \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$(iii) (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$$

$$(iv) (a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$$

$$(v) (\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$$

- Let $a > 0$ be a real number and p and q be rational numbers then

$$(i) a^p \cdot a^q = a^{p+q}$$

$$(ii) (a^p)^q = a^{pq}$$

$$(iii) \frac{a^p}{a^q} = a^{p-q}$$

$$(iv) a^p b^p = (ab)^p$$

$$(v) a^0 = 1 (a \neq 0)$$

Questions:

1. Find the unit digit of:

$$(i) (13)^{41}$$

$$(ii) (24)^{31}$$

$$(iii) (79)^{78}$$

$$(iv) (46)^{100}$$

$$(v) (57)^{21}$$

2. Find two rational numbers between:

$$(i) -3 \text{ and } -2$$

$$(ii) \frac{1}{3} \text{ and } \frac{1}{2}$$

$$(iii) 2.\overline{14} \text{ and } 2.\overline{58}$$

3. If $\frac{2}{7} = 0.\overline{285714}$, then find the value of:

$$(i) \frac{1}{7}$$

(ii) $\frac{3}{7}$

(iii) $\frac{4}{7}$

(iv) $\frac{5}{7}$

(v) $\frac{6}{7}$

4. Classify as rational and irrational numbers from the following:

(i) $\sqrt{400}$

(ii) $\sqrt{1100}$

(iii) $(\sqrt{2} + \sqrt{3})^2$

(iv) $(2\sqrt{2} - \sqrt{7})(2\sqrt{2} + \sqrt{7})$

(v) $16^{\frac{1}{4}} + (625)^{\frac{1}{4}}$

(vi) 0.236

(vii) 2.371371...

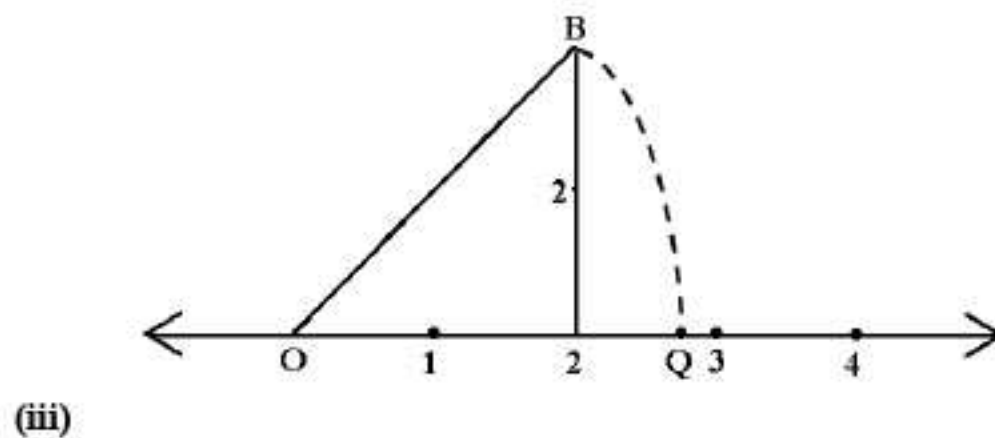
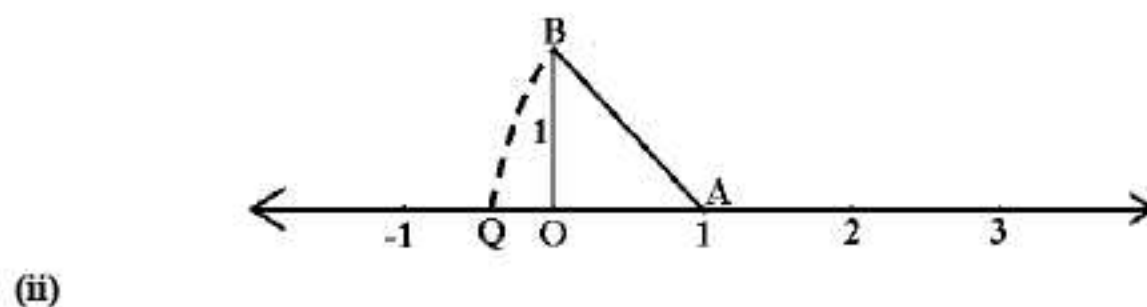
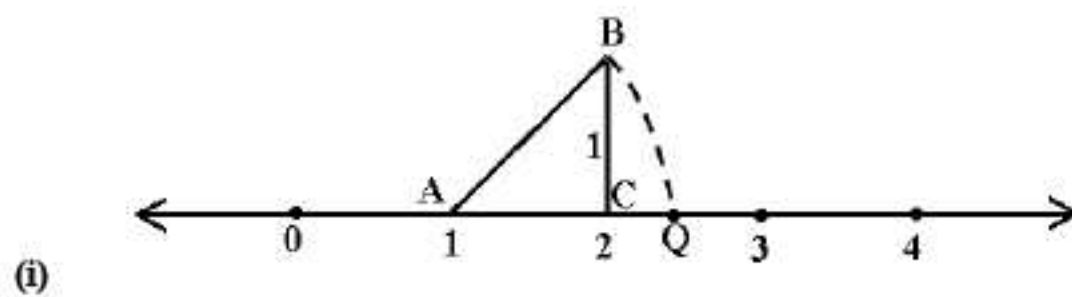
(viii) 1.1010010001...

(ix) $\frac{3}{\sqrt{5}}$

(x) $(2\sqrt{3} - \sqrt{5})(\sqrt{3} + \sqrt{5})$

5. Find one third of 3^{100} .

6. Find the measurement of OQ from the given number line:



7. Convert the following in the form of $\frac{p}{q}$:

- (i) 0.45
- (ii) 2.512
- (iii) $0.\overline{53}$
- (iv) $3.\overline{47}$
- (v) $0.\overline{23} + 0.\overline{22}$

8. Find the greatest number.

- (i) $\sqrt{0.04}, \sqrt{0.004}, \sqrt{4.0}, \sqrt{0.4}$
- (ii) $\sqrt{2}, \sqrt[4]{2}, \sqrt[3]{2}, \sqrt[5]{2}$
- (iii) $\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{3}}, \sqrt{\frac{1}{4}}, \sqrt{\frac{1}{5}}$
- (iv) $\sqrt[4]{81}, \sqrt[3]{81}, \sqrt{81}, \sqrt[5]{81}$
- (v) $2^{\frac{1}{5}}, 3^{\frac{1}{6}}, 4^{\frac{1}{5}}, 5^{\frac{1}{6}}$

9. Simplify:

- (i) $(3\sqrt{2} + 7\sqrt{3}) + (7\sqrt{2} - 5\sqrt{3})$
- (ii) $(\sqrt{13} - \sqrt{6})(\sqrt{13} + \sqrt{6})$
- (iii) $(\sqrt{3} - \sqrt{5})^2$
- (iv) $(\frac{2}{3}\sqrt{7} - \frac{1}{2}\sqrt{2} + 6\sqrt{11}) + (\frac{1}{3}\sqrt{7} + \frac{3}{2}\sqrt{2} - \sqrt{11})$
- (v) $\sqrt{27} - 2\sqrt{12} + 5\sqrt{3}$

10. Find the rationalizing factor of:

- (i) $\sqrt[3]{4}$ (ii) $\sqrt[5]{81}$ (iii) $2 - \sqrt{3}$ (iv) $\sqrt{3} + \sqrt{5}$ (v) $\sqrt{120} \times \sqrt{45}$

11. Rationalize the denominator:

- (i) $\frac{2}{\sqrt{37}-6}$ (ii) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ (iii) $\frac{\sqrt{6}+\sqrt{3}}{\sqrt{2}}$ (iv) $\frac{3}{\sqrt{48}-\sqrt{18}}$ (v) $\frac{\sqrt{5}}{\sqrt{45}-\sqrt{15}}$

12. Find the value of 'a + b':

- (i) $\frac{2}{\sqrt{3}-1} = a + b\sqrt{3}$
- (ii) $\frac{\sqrt{2}-1}{\sqrt{2}+1} = a + b\sqrt{2}$
- (iii) $\frac{\sqrt{5}+2}{\sqrt{5}-2} = a + b\sqrt{5}$

13. If $x + \frac{1}{x} = 2$, then find the value of:

(i) $x^3 + \frac{1}{x^3}$ (ii) $x^5 + \frac{1}{x^5}$

14. Evaluate: $\frac{21\sqrt{24}}{14\sqrt{54}}$

15. Find the value of $\frac{3^{40} + 3^{39} + 3^{38}}{3^{41} + 3^{40} + 3^{39}}$

16. Find the value of $(8^{-\frac{4}{3}} \div 2^2)^{\frac{1}{6}}$

17. Find the value of $\{(23 + 2^2)^{\frac{1}{3}} + (140 - 19)^{\frac{1}{6}}\}^2$

18. Simplify:

(i) $[(16)^{-\frac{1}{5}}]^{\frac{5}{6}}$

(ii) $(0.027)^{\frac{1}{3}}$

(iii) $[(625)^{-\frac{1}{6}}]^{-\frac{1}{4}}$

(iv) $(\sqrt[3]{64})^{-\frac{1}{6}}$

(v) $\sqrt{x^6 y^{-6}}$

(vi) $(256)^{0.16} (256)^{0.09}$

(vii) $-\sqrt{32} + 32\sqrt{2}$

(viii) $11\sqrt{5} + 2\sqrt{125}$

(ix) $\left(\frac{64}{125}\right)^{-\frac{1}{3}}$

(x) $[5(8^{\frac{1}{3}} + 27^{\frac{1}{3}})^3]^{\frac{1}{4}}$

19. Find the value of x :

(i) $27^x = \frac{9}{3^x}$

(ii) $4^x - 4^{x-1} = 24$

(iii) $2^5 \div 2^x = (\sqrt[5]{2})^{10}$

(iv) $5^{x-2} \cdot 3^{2x-3} = 135$

(v) $2^{x-7} \cdot 5^{x-4} = 1250$

$$(vi) \quad \left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{27}$$

20. Find: $\sqrt[4]{\frac{1}{16}} + (0.001)^{-\frac{1}{3}} - (27)^{\frac{1}{3}}$

21. If $x^{-3} = 64$, then find the value of $x^{\frac{3}{2}} + x^0$.

22. If $\sqrt{2} = 1.4142$, then find the value of $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$.

23. If $x = \frac{2}{3+\sqrt{7}}$, then find the value of

(i) $(x - 3)^4$ (ii) $(x + \sqrt{7})^3$

24. If $\sqrt{7 + a\sqrt{10}} = \sqrt{2} + \sqrt{5}$, then find the value of a .

25. If $\sqrt{15 - a\sqrt{14}} = \sqrt{8} + \sqrt{7}$, then find the value of a .

26. If $\sqrt[3]{5^n} = 25$, then find the value of $5^{\frac{n}{\sqrt{64}}}$.

27. If $x = 2 + \sqrt{3}$, then find the value of:

(i) $x + \frac{1}{x}$

(ii) $x - \frac{1}{x}$

(iii) $x^2 + \frac{1}{x^2}$

(iv) $x^2 - \frac{1}{x^2}$

28. If $x = 11 - 2\sqrt{30}$, then find the value of:

(i) $\sqrt{x} + \frac{1}{\sqrt{x}}$

(ii) $\sqrt{x} - \frac{1}{\sqrt{x}}$

(iii) $x - \frac{1}{x}$

29. If $x = (\sqrt{2} - 1)$, then find the value of $\left(x - \frac{1}{x}\right)^3$.

30. If $x - \frac{1}{x} = 2$, then find the value of $x^3 - \frac{1}{x^3}$.

31. Give an example of a pair of two irrational numbers whose:

(i) Difference is a rational number.

(ii) Difference is an irrational number.

- (iii) Sum is a rational number.
- (iv) Sum is an irrational number.
- (v) Product is a rational number.
- (vi) Product is an irrational number.
- (vii) Quotient is a rational number.
- (viii) Quotient is an irrational number.

32. Find the value of:

(i) $\sqrt{(2020)(2022) + 1}$ (ii) $\sqrt{(2010)(2030) + 100}$

33. Find the value of:

(i) $(2\sqrt{5} - 3\sqrt{3}) + (2\sqrt{3} - \sqrt{5}) + (\sqrt{3} - \sqrt{5})$

(ii) $\sqrt{5 + 2\sqrt{6}} - \sqrt{8 - 2\sqrt{15}}$

(iii) $\sqrt{12} + \sqrt{18} - (\sqrt{3} + \sqrt{2})$

(iv) $(\sqrt{72} - \sqrt{18}) \div \sqrt{12}$

(v) $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75}$

34. Find the value of:

(i) $\sqrt[3]{2} \times \sqrt[4]{2} \times \sqrt[11]{32}$

(ii) $\sqrt[6]{12} \div (\sqrt{3} \times \sqrt[3]{2})$

(iii) $\sqrt{2} \times \sqrt[3]{3} \times \sqrt[4]{4}$

35. Find the value of: $\frac{1}{\sqrt{4}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{2}+1}$

36. What can be the maximum number of digits in the repeating block of digits in the decimal expansion of $\frac{1}{17}$.

37. Find the value of $(x + y)$.

(i) $(2\sqrt{2} + 5\sqrt{3}) + (2\sqrt{3} - 3\sqrt{2}) = x\sqrt{2} + y\sqrt{3}$

(ii) $4\sqrt{3} - 3\sqrt{12} + 2\sqrt{75} = x\sqrt{2} + y\sqrt{3}$

(iii) $\sqrt{8} + \sqrt{32} - \sqrt{2} = x\sqrt{2} + y\sqrt{3}$

38. If $\frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} = a - b\sqrt{7}$, then find the value of $a + b$.

39. Solve the following:

(i) $(1 - \frac{1}{3})(1 - \frac{1}{4})(1 - \frac{1}{5}) \dots (1 - \frac{1}{n})$

(ii) $\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c$

(iii) $\left(\frac{2^a}{2^b}\right)^{a+b} \times \left(\frac{2^b}{2^c}\right)^{b+c} \times \left(\frac{2^c}{2^a}\right)^{c+a}$

(iv) $\sqrt{5 + \sqrt{13 + \sqrt{5 + \sqrt{16}}}}$

(v) $\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7}}}}}$

40. If $\sqrt{1 + \frac{27}{169}} = \left(1 + \frac{x}{13}\right)$, then find the value of x .

ANSWERS

Q.No.	ANSWER	Q.No.	ANSWER
1	(i) 3 (ii) 4 (iii) 1 (i) 6 (v) 7	15	$\frac{1}{3}$
2	(i) $-\frac{5}{2}, -\frac{11}{4}$ (ii) $\frac{5}{12}, \frac{11}{24}$ (iii) 2.3, 2.5 (or any suitable answer)	16	$\frac{1}{8}$
3	(i) 0.142857 (ii) 0.428571 (iii) 0.571428 (iv) 0.714285 (v) 0.857142	17	400
4	(i), (iv), (v), (vi), (vii) are rational numbers (ii), (iii), (viii), (ix), (x) are irrational numbers	18	(i) $\frac{1}{4}$ (ii) 0.3 (iii) $\sqrt{5}$ (iv) $\frac{1}{2}$ (v) $\frac{x^3}{y^3}$ (vi) 4 (vii) $28\sqrt{2}$ (viii) $21\sqrt{5}$ (ix) $\frac{5}{4}$ (x) 5
5	3^{99}	19	(i) $\frac{1}{2}$ (ii) $\frac{5}{2}$ (iii) 3 (iv) 3 (v) 8 (vi) 3
6	(i) $1 + \sqrt{2}$ (ii) $\sqrt{2} - 1$ (iii) $\sqrt{8}$	20	$1\frac{1}{2}$
7	(i) $\frac{9}{20}$ (ii) $\frac{314}{125}$ (iii) $\frac{53}{99}$ (iv) $\frac{344}{99}$ (v) $\frac{5}{11}$	21	$1\frac{1}{8}$
8	(i) $\sqrt{4.0}$ (ii) $\sqrt{2}$ (iii) $\sqrt{\frac{1}{2}}$ (iv) $\sqrt{81} = 9$ (v) $5^{\frac{1}{2}}$	22	0.4142
9	(i) $10\sqrt{2} + 2\sqrt{3}$ (ii) 7 (iii) $8 - 2\sqrt{15}$ (iv) $\sqrt{2} + \sqrt{7} + 5\sqrt{11}$ (v) $4\sqrt{3}$	23	(i) 49 (ii) 27
10	(i) $\sqrt[3]{2}$ (ii) $\sqrt[5]{3}$ (iii) $2 + \sqrt{3}$ (iv) $\sqrt{3} - \sqrt{5}$ (v) $\sqrt{6}$	24	2
11	(i) $2(\sqrt{37} + 6)$ (ii) $7 + 4\sqrt{3}$ (iii) $\frac{2\sqrt{3} + \sqrt{6}}{2}$ (iv) $\frac{4\sqrt{3} + 3\sqrt{2}}{10}$ (v) $\frac{3 + \sqrt{3}}{6}$	25	-4
12	(i) 2 (ii) 1 (iii) 13	26	10
13	(i) 2 (ii) 2	27	(i) 4 (ii) $2\sqrt{3}$ (iii) 14 (iv) $8\sqrt{3}$
14	1	28	(i) $2\sqrt{6}$ (ii) $2\sqrt{5}$ (iii) $4\sqrt{30}$
		29	-8
		30	14
		31	(i) $\sqrt{2}, \sqrt{2}$ (ii) $3\sqrt{5}, 2\sqrt{5}$ (iii) $\sqrt{2}, (-\sqrt{2})$ (iv) $\sqrt{2}, \sqrt{3}$ (v) $\sqrt{2}, \sqrt{8}$ (vi) $\sqrt{2}, \sqrt{5}$ (vii) $\sqrt{8}, \sqrt{2}$ (viii) $\sqrt{5}, \sqrt{3}$ or any suitable answer for (i) to (viii)

Q.No.	ANSWER	Q.No.	ANSWER
32	(i) 2021 (ii) 2020	37	(i) 6 (ii) 8 (iii) 5
33	(i) 0 (ii) $2\sqrt{3} + \sqrt{2} - \sqrt{5}$ (iii) $\sqrt{3} + 2\sqrt{2}$ (iv) $\sqrt{\frac{3}{2}}$ (v) $\sqrt{3}$	38	$\frac{2}{3}$
34	(i) 2 (ii) $\frac{1}{\sqrt[3]{3}}$ (iii) $2\sqrt[3]{3}$	39	(i) $\frac{2}{n}$ (ii) 1 (iii) 1 (iv) 3 (v) $7^{\frac{31}{31}}$
35	1	40	1
36	16		

CHAPTER – 2

POLYNOMIALS

POINTS TO REMEMBER:

- **Polynomial** : An algebraic expression $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ is called a polynomial in x variable.
 $a_0, a_1, a_2, \dots, a_n$ are respectively the coefficients of $x^0, x^1, x^2, \dots, x^n$ and $a_0, a_1x, a_2x^2, \dots, a_nx^n$ are called its terms. $a_0, a_1, a_2, \dots, a_n$ are real numbers, while exponent of x are whole numbers.
- **Degree of a polynomial** : The degree of a polynomial $p(x)$ is the index of the highest power of the variable in a non – zero polynomial and is denoted by $\deg p(x)$.
- **Zero polynomial** : The polynomial $p(x) = 0$ is called Zero polynomial and degree of the zero polynomial is not defined.
- **Constant polynomial** : A non – zero constant is called a constant polynomial.
e.g. 0.5, 11, 105 etc. Degree of a constant polynomial is always zero.
- **Linear polynomial** : A polynomial of degree 1 is called a linear polynomial. Its standard form is $ax + b$, where a and b are real constants and $a \neq 0$. e.g. $3x + 1, 5x$.
- **Quadratic polynomial** : A polynomial of degree 2 is called a quadratic polynomial. Its standard form is $ax^2 + bx + c$, where a, b and c are real constants and $a \neq 0$.
e.g. $4x^2 + 5x - 1$.
- **Cubic polynomial** : A polynomial of degree 3 is called a cubic polynomial. Its standard form is $ax^3 + bx^2 + cx + d$, where a, b, c and d are real constants and $a \neq 0$. e.g. $x^3 + 1$.
- **Bi-quadratic polynomial** : A polynomial of degree 4 is called a Bi-quadratic polynomial. Its standard form is $ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d and e are real constants and $a \neq 0$. e.g. $3x^4 + 2x^3 + 4x^2 + 8$.
- **Monomial** : A polynomial having only one non-zero term is called a Monomial
e.g. $7x, 11x^3, -\frac{2}{3}xy, 4xyz$ etc.

- **Binomial** : A polynomial having two non-zero terms is called a Binomial e.g.
 $x^2 + 1, 3p^3 - 5p, x + y, r^2 + 2s^2r$ etc.
- **Trinomial** : A polynomial having three non-zero terms is called a Trinomial.
e.g. $x^3 + x - 5, p + q + r, x^2 + 2y^2x - 3$ etc.
- **Zero of a polynomial** : A real number 'a' is called a zero of the polynomial p(x) if $p(a) = 0$.
- **Remainder Theorem** : If a polynomial p(x) of degree $n \geq 1$ is divided by
 - (i) $(x - a)$, then remainder = p (a)
 - (ii) $(x + a)$, then remainder = p (-a)
- **Factor Theorem** : If p(x) is a polynomial of degree $n \geq 1$ and 'a' is any real number then
 - (i) $(x - a)$ is a factor of polynomial p(x), if $p(a) = 0$
 - (ii) $(x + a)$ is a factor of polynomial p(x), if $p(-a) = 0$
- **Algebraic Expressions and Identities** :
 - (i) $(x + y)^2 = x^2 + 2xy + y^2$
 - (ii) $(x - y)^2 = x^2 - 2xy + y^2$
 - (iii) $x^2 - y^2 = (x + y)(x - y)$
 - (iv) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
 - (v) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
 - (vi) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
 - (vii) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
 - (viii) $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$
If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$
 - (ix) $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$
 $(x - y)^3 = x^3 - 3xy(x - y) - y^3$
 - (x) $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
 $(x + y)^3 = x^3 + 3xy(x + y) + y^3$
 - (xi) $(x + a)(x + b) = x^2 + (a + b)x + ab$

QUESTIONS:

1. Which of the following are polynomials:

(i) 0

(ii) $\frac{4}{5}x^2 - \frac{7}{3}x + \frac{1}{2}$

(iii) $3^0 + 5^0 + 7^0$

(iv) $(\sqrt{3}x + \sqrt{2}y)^3$

(v) $x^2 + x + 3\sqrt{2}$

(vi) $5x + 3\sqrt{x} + \sqrt{7}x$

(vii) $\frac{4x^1 + 12xy + 9y^1}{2x + 3y}$

(viii) $2 + \frac{3}{x} + \frac{4}{x^1}$

2. Express the following polynomials in standard form and also find their degree:

(i) $(x^2 + \sqrt{7})(x^2 - \sqrt{7})$

(ii) $(x + 3)(x + 5)$

(iii) $-\frac{5}{7}$

(iv) $(x + y)^2 - 2xy$

(v) $x^3 + x^5 + 3x + 2x^2 + x^6$

(vi) $(x + 1)^2 - (x - 1)^2$

(vii) $(y^2 + \sqrt{x^5})(y^2 - \sqrt{x^5})$

(viii) $3^2x^2 + 3^{10}x^5 + 3^{45}x + 3^{100} + 3^{31}x$

3. Classify the following polynomials as Constant, Linear, Quadratic, Cubic or Bi-quadratic :

(i) $p(x) = x^4 + 4x^2 + 16$

(ii) $q(y) = 5y^3 + 3y^2 + y + 2$

(iii) $r(z) = z^2 + 5z + 6$

(iv) $s(t) = \sqrt{5t} + 3$

(v) $u(x) = 5$

4. Give an example of:

- (i) a polynomial of degree 0.
- (ii) a binomial of degree 10.
- (iii) a monomial of degree 15.
- (iv) a trinomial of degree 5.
- (v) a polynomial of degree 100.
- (vi) a polynomial of degree 4 with one zero 5.
- (vii) a polynomial of degree 2 having factor $(x-1)$.

5. Find the value of the following polynomials as indicated:

- (i) $p(1)$ if $p(x) = x^3 - 6x^2 + 11x + 6$.
- (ii) $p(0)$ if $p(x) = 2x^3 - 13x^2 + 17x + 12$.
- (iii) $p(-1)$ if $p(x) = x^3 + 6x^2 + 11x + 6$.
- (iv) $p(2)$ if $p(x) = x^3 - 2x^2 + 3x + 1$.
- (v) $p(-2)$ if $p(x) = x^2 - 5x + 2$.

6. Find the coefficient of y in the following:

- (i) $p(y) = y^3 - \frac{2}{3}y^2 - \frac{\pi}{2}y + 3$.
- (ii) $q(y) = 2y^2 + b^2ky + 5y$.
- (iii) $f(y) = \sqrt{3}hx^2y^2 - \sqrt{5}x^2y + \sqrt{3}y$.
- (iv) $h(y) = y^3 + 2y^2 + 3$.
- (v) $r(y) = yx^2 + 2yx - 3x$.

7. Find the remainder when $p(x)$ is divided by $g(x)$ & hence check whether $g(x)$ is a factor of $p(x)$ or not:

- (i) $p(x) = x^4 - 3x^2 + 2x + 1, g(x) = x - 1$.
- (ii) $p(x) = 2x^4 - 6x^3 + 2x^2 - x + 2, g(x) = x + 1$.
- (iii) $p(x) = x^4 - 3x^2 + 4, g(x) = x - 2$.
- (iv) $p(x) = x^{101} + 101, g(x) = x + 1$.
- (v) $p(x) = x^3 - ax^2 + 6x - a, g(x) = x - a$.
- (vi) $p(x) = x^3 + x^2 + x + 1, g(x) = x + 1$.

8. Find the value of 'k', if g(x) is a factor of f(x) in the following:

(i) $f(x) = x^5 - kx^4 + k + x - 2, g(x) = x - k.$

(ii) $f(x) = x^3 + kx^2 - 17x + 15, g(x) = x - 3.$

(iii) $f(x) = 4x^2 + 12x + 5k - 1, g(x) = x - 1.$

(iv) $f(x) = x^3 + 3x^2 + kx - 50, g(x) = x - 5.$

(v) $f(x) = kx^2 - \sqrt{2}x + 1, g(x) = x - 1.$

9. Find the product of the following:

(i) $(x + \sqrt{2})(x + 3\sqrt{2})$

(ii) $(x + 2y)(x - 2y)(+4y^2)$

(iii) $(x^2 + 4 - 2x)(x^2 + 4 + 2x)$

(iv) $\left(\frac{2x}{5} - \frac{y}{3}\right)\left(\frac{y^1}{9} + \frac{4x^1}{25} + \frac{2xy}{15}\right)$

(v) $(0.5y^2 - 1.2z^2)(0.5y^2 + 1.2z^2)$

(vi) $\left(3a - \frac{1}{2}b\right)\left(\frac{1}{2}b + 3a\right)\left(\frac{1}{4}b^2 + 9a^2\right)$

10. Expand the following:

(i) $(a^2b - b^2a)^2$

(ii) $\left(\frac{x}{2} - \frac{y}{5}\right)^2$

(iii) $(x - 2y + 3z)^2$

(iv) $(3x + 2y)^3$

(v) $\left(y - \frac{1}{y}\right)^3$

(vi) $\left(\frac{1}{2}a - \frac{1}{3}b + 1\right)^2$

11. Factorize the following:

(i) $a^{12}x^4 - a^4x^{12}$

(ii) $x^2 + 2xz + z^2 - a^2 + 2ab - b^2$

(iii) $1 - 2xy - (x^2 + y^2)$

(iv) $x^2 + 5\sqrt{5}x + 30$

(v) $(x + y)^3 - 5(x + y)^3$

(vi) $2xy(x - y)^2 + x^2(x - y)^2 + y^2(x - y)^2$

(vii) $a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3$

- (viii) $x^2 - y^2 - x - y$
 (ix) $24a^6b + 3a^3b^4$
 (x) $a^5b^5 - a^2b^2$
 (xi) $(a + b)^2 + 2(a + b) + 1$
 (xii) $x^2 - y^2 + 2yz - z^2$
 (xiii) $x^3 - 0.216$
 (xiv) $4a^2 + 12ab + 9b^2 - 8a - 12b$
 (xv) $x^4 + 4x^2 + 3$

12. Find the remainder when $x^4 - y^4$ is divided by $x - y$.
 13. Find the coefficient of $5x^2 + 3x + 9$ in the expression $15x^4 + 9x^3 + 27x^2$.
 14. If $a = b = c$ and $(a + b + c)^2 - xa^2 = 0$, then find the value of x .
 15. Factorize: $(x^2 - y^2)^3 + (y^2 - z^2)^3 + (z^2 - x^2)^3$.
 16. If $x + y = 12$ and $xy = 35$, then find the value of $x^2 + y^2$.
 17. If $ab + bc + ca = 0$, then find the value of $\frac{1}{a^2 - bc} + \frac{1}{b^2 - ca} + \frac{1}{c^2 - ab}$.
 18. Simplify : $\frac{1.5 \times 1.5 \times 1.5 - 0.5 \times 0.5 \times 0.5}{1.5 \times 1.5 + 1.5 \times 0.5 + 0.5 \times 0.5}$
 19. Find the value of $\frac{(20202020)^2}{(20202019)^2 + (20202021)^2 - 2}$.
 20. Find the perimeter of the rectangle whose area is $14x^2 - 11x - 15$.
 21. Simplify: $(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$
 22. Find the value of $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$,
 if $a + b + c = 3x$.
 23. If a, b and c are all non-zeroes and $a + b + c = 0$, then find the value of
 $\frac{a^3}{bc} + \frac{b^3}{ca} + \frac{c^3}{ab}$.
 24. If $a^2 + b^2 + c^2 = ab + bc + ca$, then find the value of
 $\frac{(a-b)^2 + (b-c)^2 + (c-a)^2}{2}$.
 25. Find the square root of $a^2 + 4b^2 + 9c^2 + 6ac + 4ab + 12bc$.
 26. Find the cube root of $\frac{27}{x^3} - \frac{8}{x^6} - \frac{54}{x^4} + \frac{36}{x^5}$.

27. If $a^2 + b^2 + c^2 = 16$ and $ab + bc + ca = 10$, then find the value of $(a + b + c)$.

28. If $x = \frac{a-b}{a+b}$, $y = \frac{b-c}{b+c}$, $z = \frac{c-a}{c+a}$, then find the value of $\frac{(1+x)(1+y)(1+z)}{(1-x)(1-y)(1-z)}$.

29. If $(x + 1)$ and $(x - 1)$ are factors of $ax^3 + x^2 - 2x + b$, then find the product of a and b .

30. Simplify: $\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right)$.

31. If $x - \frac{1}{x} = 3$, then find the value of $x^3 - \frac{1}{x^3}$.

32. If $x + \frac{1}{x} = 3$, then find the value of $x^2 + \frac{1}{x^2}$.

33. If $x + \frac{1}{x} = 3$, then find the value of $x^3 + \frac{1}{x^3}$.

34. If $a + b + c = 0$, then find the value of $\frac{(b+c)^3}{3bc} + \frac{(c+a)^3}{3ac} + \frac{(a+b)^3}{3ab}$.

ANSWERS

Q.No	ANSWER	Q. No	ANSWER
1.	(i), (ii), (iii), (iv), (v), (vii)	6.	(iv) 0 (v) $x^2 + 2x$
2.	(i) $x^4 - 7$, degree 4 (ii) $x^2 + 8x + 15$, degree 2 (iii) $\frac{-5}{7}x^0$, degree 0 (iv) $x^2 + y^2$, degree 2 (v) $x^6 + x^4 + x^3 + 2x^2 + 3x$, degree 6 (vi) $4x$, degree 1 (vii) $y^4 - x^5$, degree 5 (viii) $3^{16}x^5 + 3^{45}x^3 + 3^2x^2 + 3^{31}x + 3^{100}$, degree 5	7.	(i) Remainder = 1, No (ii) Remainder = 13, No (iii) Remainder = 8, No (iv) Remainder = 100, No (v) Remainder = 5a, No (vi) Remainder = 0, Yes
3.	(i) Bi-quadratic polynomial (ii) Cubic polynomial (iii) Quadratic polynomial (iv) Linear polynomial (v) Constant polynomial	8.	(i) $k = 1$ (ii) $k = 1$ (iii) $k = -3$ (iv) $k = -30$ (v) $k = \sqrt{2} - 1$
4.	(i) $p(x) = -\frac{5}{9}$ (ii) $f(x) = x^{10} - 3x^2$ (iii) $g(y) = 3y^{15}$ (iv) $h(t) = -7t^5 - 2t + 1$ (v) $q(y) = 5y^{100} - y^{82} + 7y^{17} - 1$ (vi) $f(x) = x^4 - 625$ (vii) $p(x) = x^2 - 1$ or any other relevant answer .	9.	(i) $x^2 + 4\sqrt{2}x + 6$ (ii) $x^4 - 16y^4$ (iii) $x^4 + 4x^2 + 16$ (iv) $\frac{x^3}{125} - \frac{y^3}{27}$ (v) $0.25y^4 - 1.44z^4$ (vi) $81a^4 - \frac{b^4}{16}$
5.	(i) 12 (ii) 12 (iii) 0 (iv) 7 (v) 16	10.	(i) $a^2b^2(a^2 - 2ab + b^2)$ (ii) $\frac{x^1}{4} + \frac{y^1}{25} - \frac{xy}{5}$ (iii) $x^2 + 4y^2 + 9z^2 - 4xy - 12yz + 6xz$ (iv) $27x^3 + 8y^3 + 54x^2y + 36xy^2$ (v) $y^3 - \frac{1}{y^3} - 3y + \frac{3}{y}$ (vi) $\frac{1}{4}a^2 + \frac{1}{9}b^2 + 1 - \frac{ab}{3} - \frac{2b}{3} + \frac{1}{a}$
6.	(i) $-\frac{\pi}{2}$ (ii) b^2k+5 (iii) $\sqrt{3} - \sqrt{5}x^2$		

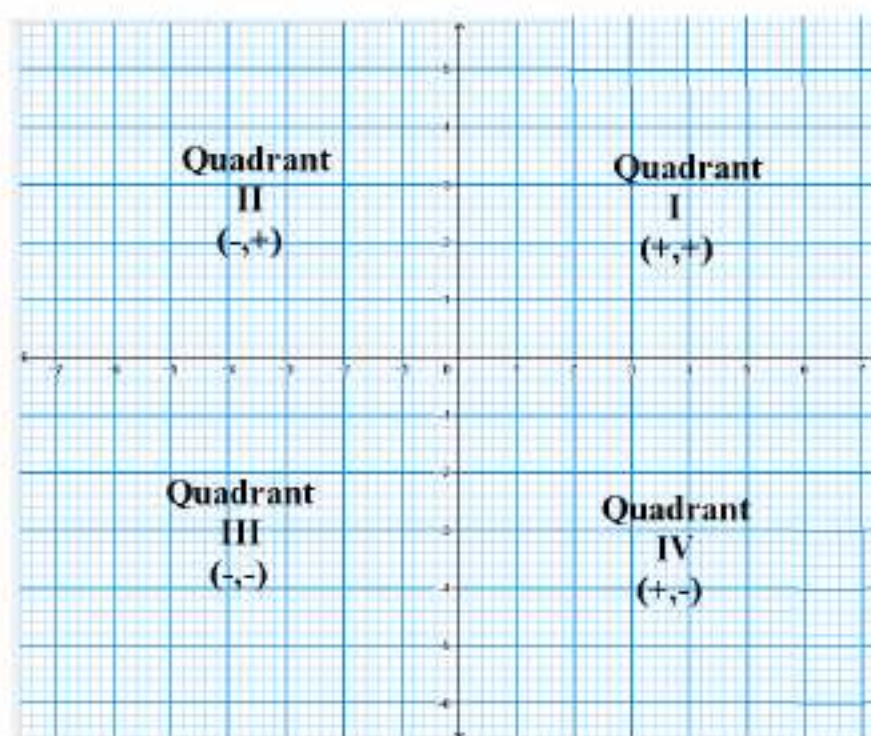
Q. NO.	ANSWER	Q. NO.	ANSWER
11.	(i) $a^4 x^4 (a^4 + x^4) (a^2 + x^2) (a+x)(a-x)$	13.	$3x^2$
	(ii) $(x+z+a-b)(x+z-a+b)$	14.	9
	(iii) $(1-x-y)(1+x+y)$	15.	$3(x+y)(y+z)(z+x)$ $(x-y)(y-z)(z-x)$
	(iv) $(x+3\sqrt{5})(x+2\sqrt{5})$	16.	74
	(v) $-4(x+y)(x+y)(x+y)$	17.	0
	(vi) $(x-y)(x-y)(x+y)(x+y)$	18.	1
	(vii) $(ax-b)(ax-b)(ax-b)$	19.	$\frac{1}{2}$
	(viii) $(x+y)(x-y-1)$	20.	$18x+4$
	(ix) $3a^3b(2a+b)(4a^2-2ab+b^2)$	21.	a^6-b^6
	(x) $a^2b^2(ab-1)(a^2b^2+ab+1)$	22.	0
	(xi) $(a+b+1)(a+b+1)$	23.	3
	(xii) $(x-y+z)(x+y-z)$	24.	0
	(xiii) $(x-0.6)(x^2+0.6x+0.36)$	25.	$a+2b+3c$
	(xiv) $(2a+3b)(2a+3b-4)$	26.	$\frac{3}{x} - \frac{2}{x^2}$
	(xv) $(x^2+1)(x^2+3)$	27.	± 6
		28.	1
		29.	-2
		30.	$x^8 - \frac{1}{x^8}$
		31.	36
		32.	7
		33.	18
12.	0	34.	1

CHAPTER 3

COORDINATE GEOMETRY

POINTS TO REMEMBER

- The coordinate axes $X'OX$ and $Y'OY$ are called **x-axis** and **y-axis** respectively. $X'OX$ is the horizontal line and $Y'OY$ is the vertical line perpendicular to $X'OX$.
- The coordinate axes divide the plane into four parts called **quadrants**.
- Point of intersection of x-axis and y-axis is called the **origin** and is denoted by $O(0, 0)$.
- A point is represented in a plane as $P(x, y)$.
- OX and OY are called **positive directions** of x-axis and y-axis respectively whereas OX' and OY' are called **negative directions** of x-axis and y-axis respectively.
- The x-coordinate of a point is also called the **abscissa**.
- The y-coordinate of a point is also called the **ordinate**.
- If a point is in I quadrant, then its coordinates are of the form **(+ve, +ve)**, in II quadrant it is of the form **(-ve, +ve)**, in III quadrant it is of the form **(-ve, -ve)** and in IV quadrant it is of the form **(+ve, -ve)**.



- $P(x, y)$ represents a point P whose coordinates are x and y .
 - (a) Abscissa of point P = Perpendicular distance of point P from y -axis
 - (b) Ordinate of point P = Perpendicular distance of point P from x -axis
 - (c) The abscissa of point P is positive if P lies on the right of the y -axis, zero if it is on the y -axis and negative if it lies on the left of y -axis.
 - (d) The ordinate of point P is positive if P lies above x -axis, zero if it is on the x -axis and negative if it lies below the x -axis.
- Coordinates of a point lying on
 - (a) x -axis is of the form $(x, 0)$
 - (b) y -axis are of the form $(0, y)$

QUESTIONS

1. Find the coordinates of the reflection of the following points about y -axis.

- (i) $(3, -5)$
- (ii) $(6, 0)$
- (iii) $(-4, -7)$
- (iv) $(-5, 3)$
- (v) $(0, -4)$

2. Find the coordinates of the reflection of the following points about x -axis.

- | | | |
|-----------------|----------------|----------------|
| (i) $(2, 4)$ | (ii) $(-4, 3)$ | (iii) $(0, 5)$ |
| (iv) $(-3, -6)$ | (v) $(-6, 0)$ | |

3. Find the quadrant in which the following points lie if:

- (i) Its coordinates are $(-5, 6)$.
- (ii) Its abscissa is -5 and ordinate is -6 .
- (iii) Its y -coordinate is 3 and x -coordinate is -4 .
- (iv) Its perpendicular distance from x -axis is 5 units and distance from y -axis is 6 units in negative direction.
- (v) Its abscissa is a negative integer and ordinate is a positive integer.

4. Find the abscissa and ordinate of the following points:

- (i) (3, 0)
- (ii) (-4, 3)
- (iii) (0, -5)
- (iv) (0.5, 2)
- (v) (1.4, -2.3)

5. Find the point whose:

- (i) Abscissa is 5 and ordinate is 10 less than its abscissa.
- (ii) Ordinate is double the abscissa and abscissa is -2.
- (iii) Ordinate is 3 and abscissa is additive inverse of ordinate.
- (iv) Abscissa is reciprocal of ordinate and ordinate is 4.
- (v) Sum of abscissa and ordinate is 0 and ordinate is 3.

6. Write the perpendicular distance of:

- (i) (0, -4) from y-axis.
- (ii) (2, 8) from x-axis.
- (iii) (-1, 3) from y-axis.
- (iv) (3, 5) from x-axis.
- (v) (-4, -6) from y-axis.

7. Find the axis to which following lines are parallel:

- (i) $y = 0$
- (ii) $3x - 6 = 0$
- (iii) $x = -2$
- (iv) $x = 0$
- (v) $2y + 8 = 0$

8. Find the coordinates of points whose:

- (i) Abscissa is $-\frac{3}{2}$ and ordinate is 5.

(ii) Ordinate is $\frac{5}{4}$ and lies on y-axis.

(iii) Ordinate is 5 and abscissa - 2.

9. Fill in the blanks:

(i) The point of intersection of the coordinate axes is called _____.

(ii) The measure of angle between the coordinate axes is _____.

(iii) The abscissa and ordinate of the origin are _____.

(iv) The ordinate of any point on x-axis is _____.

(v) The abscissa of any point on y-axis is _____.

10. Find the difference of:

(i) abscissa of P (2, 3) and abscissa of Q (-2, 0)

(ii) ordinate of A (-1, 5) and ordinate of B (2, 3)

(iii) abscissa of L (3, 5) and ordinate of M (-3, -4)

11. Find the ordinate of a point lying on x-axis.

12. Find the type of triangle formed by joining the following points in the Cartesian Plane:

(i) A (-4, 0), B (0, 4) and C (4, 0)

(ii) P (0, 0), Q (3, 3) and R (0, 3)

(iii) L (0, 6), M (0, -6) and N ($6\sqrt{3}$, 0)

13. Find the type of quadrilateral formed by joining the following points in the Cartesian plane:

(i) P (3, 0), Q (3, 3), R (-3, 3) and S (-3, 0)

(ii) A (1, 3), B (1, -1), C (7, -1) and D (7, 3)

14. Find the area of the geometrical figure formed by following points:

(i) A (1, 2), B (-4, 2), C (-4, -1) and D (1, -1)

(ii) A (2, 0), B (0, 8) and O (0, 0)

(iii) O (0, 0), P (1, 0) and Q (0, 2)

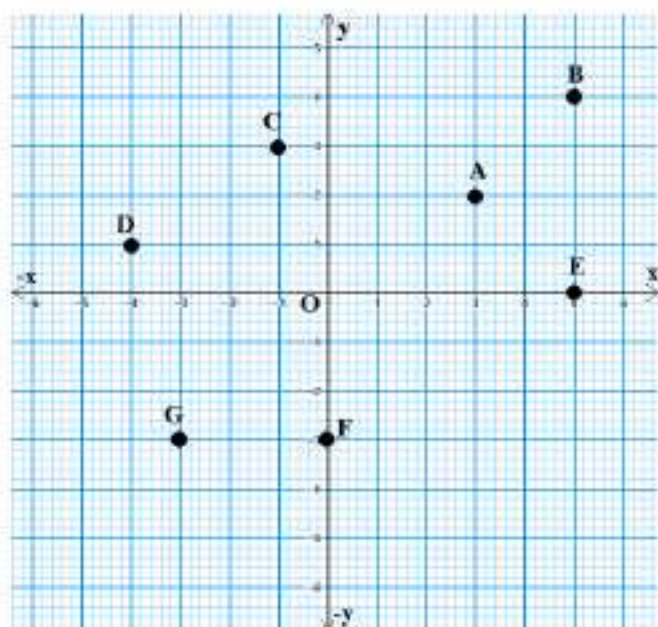
15. Find the missing coordinates of the following:

(i) If the points P (1, 0), Q (5, 0), R (5, 2) and S (x, y) form a rectangle.

(ii) If the points A (0, 0), B (2, 0), C (2, 2) and D (x, y) form a square.

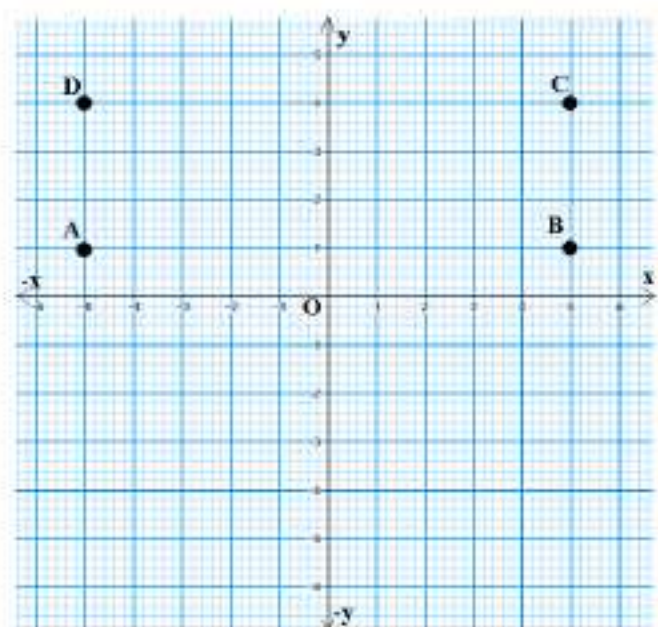
- (iii) If $O(0, 0)$ is the centre of the circle and points $A(0, 4)$, $B(-4, 0)$, $C(0, -4)$ and $D(x, 0)$ lie on the circle.
16. If the points $A(2, 0)$, $B(-6, 0)$ and $C(3, a - 3)$ are collinear, then find the value of a .
 17. The coordinates of two points are $A(5, 3)$ and $B(-2, 7)$, then find the mean of abscissa of A and ordinate of B .
 18. If the perpendicular distance of a point A from x -axis be 4 units along the negative direction of the y -axis, then find the ordinate of A .
 19. How many points lie on both the axes?
 20. PQR is an equilateral triangle with coordinates of P and R as $(0, -5)$ and $(0, 5)$ respectively. Find the length of the side of $\triangle PQR$.
 21. The points whose abscissa and ordinate have different signs, can lie in which quadrants if abscissa is positive?
 22. If sum of abscissa and ordinates of a point (other than origin) is zero, then where can the point lie if ordinate is positive?
 23. A point in III quadrant having perpendicular distance from x -axis and y -axis are 4 units and 3 units respectively. What is the coordinates of the point?
 24. A and B are points on a line which is parallel to x -axis and distance between A and B is 5 units. If coordinates of A is $(-3, 4)$, then what will be the abscissa of B if B is on right side of A ?
 25. B is the mid-point of AC . If coordinates of A and C are $(3, -5)$ and $(3, 6)$ respectively, then find the coordinates of B ?
 26. A point P lie in III quadrant. If ordinate of P is an integer greater than its abscissa and abscissa of P is -2 , then what will be the coordinate of P ?
 27. According to the graph, find the coordinates of

(i) A	(ii) B	(iii) C	(iv) D
(v) E	(vi) F	(vi) G	



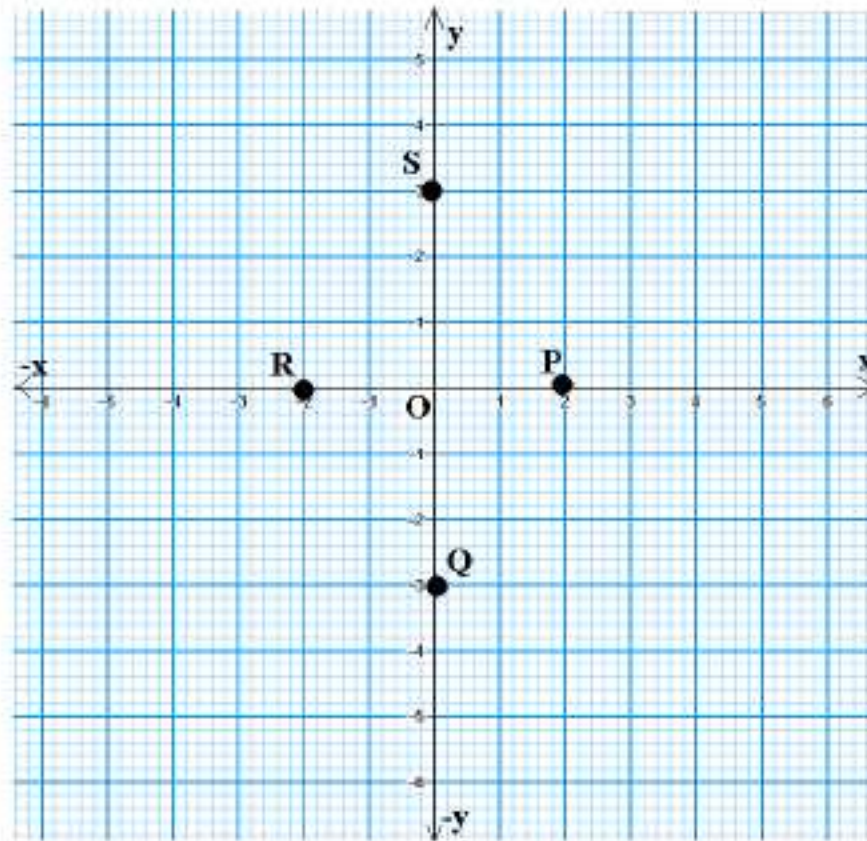
28. Join the points A, B, C, D and answer the following:

- Name the quadrilateral formed by A, B, C and D.
- Find area of quadrilateral ABCD.



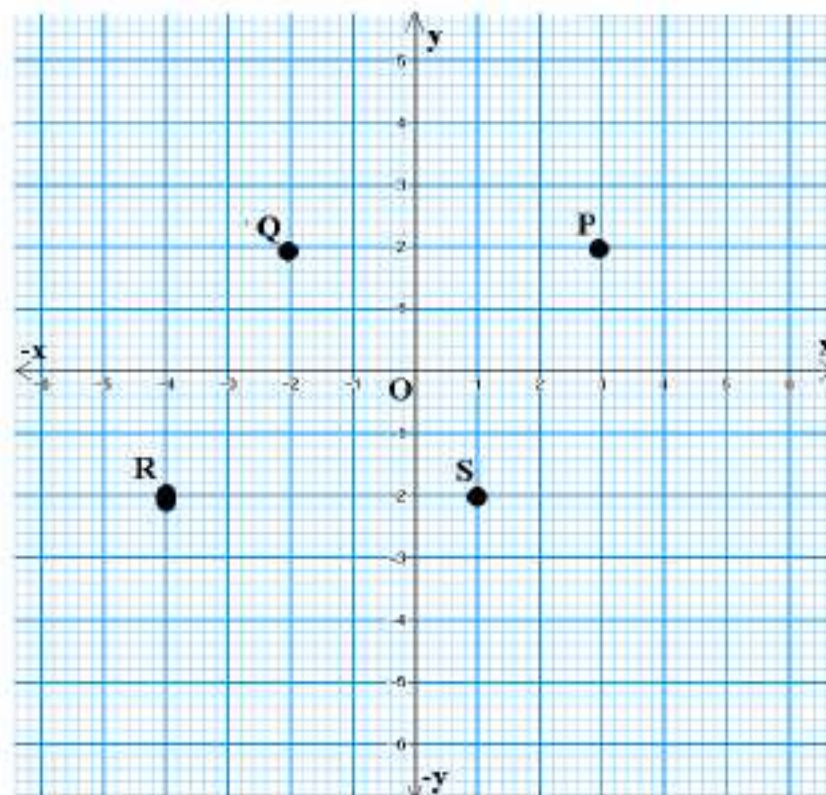
29. According to the given figure, answer the following:

- Find the coordinates of P, Q, R, S.
- Name the quadrilateral formed by joining these points.
- Find the sum of lengths of its diagonals.

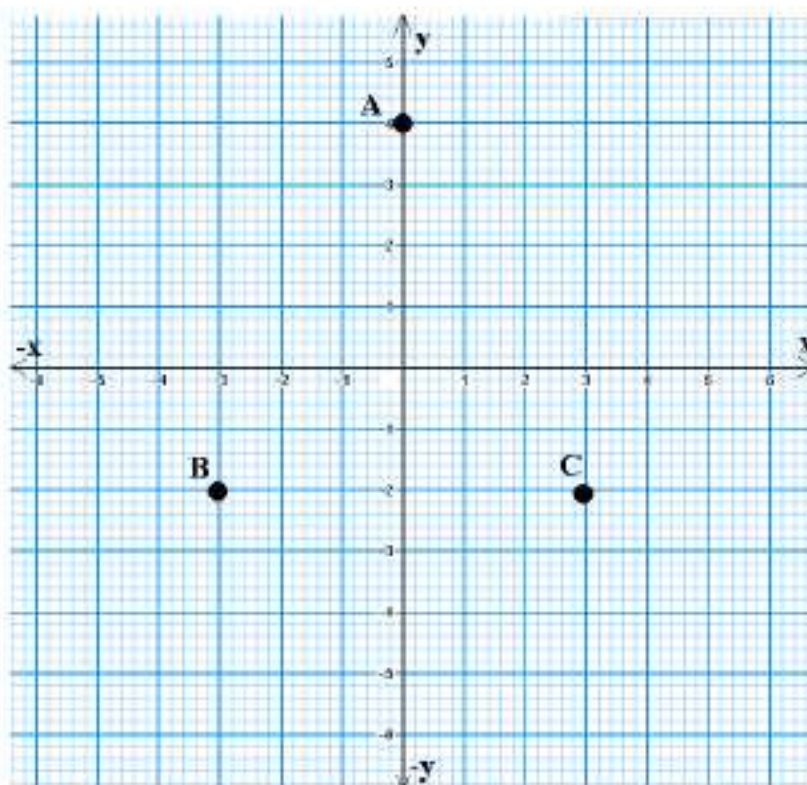


30.If PQRS is a parallelogram then find the co-ordinates of

- (i) P (ii) Q (iii) R (iv) S
 (v) Point of intersection of its diagonals.



31. Find the area of $\triangle ABC$.



32. A circle is drawn with centre O and radius 4 units. Find the coordinates of:

(i) A

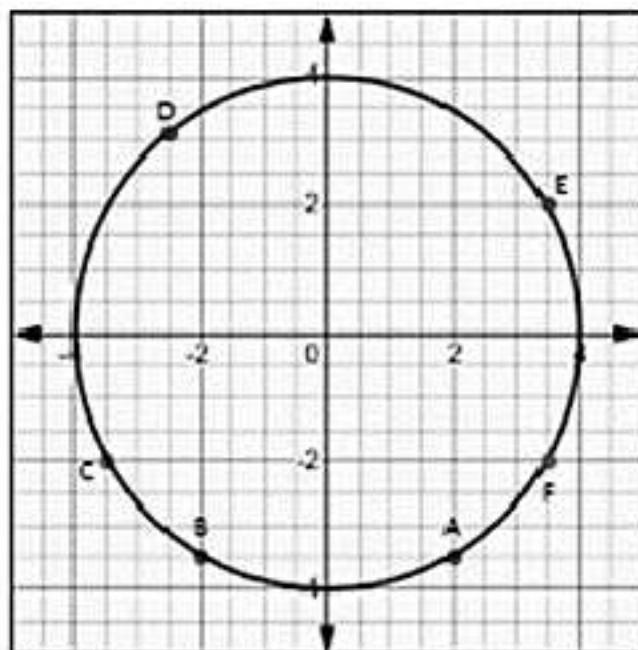
(ii) B

(iii) C

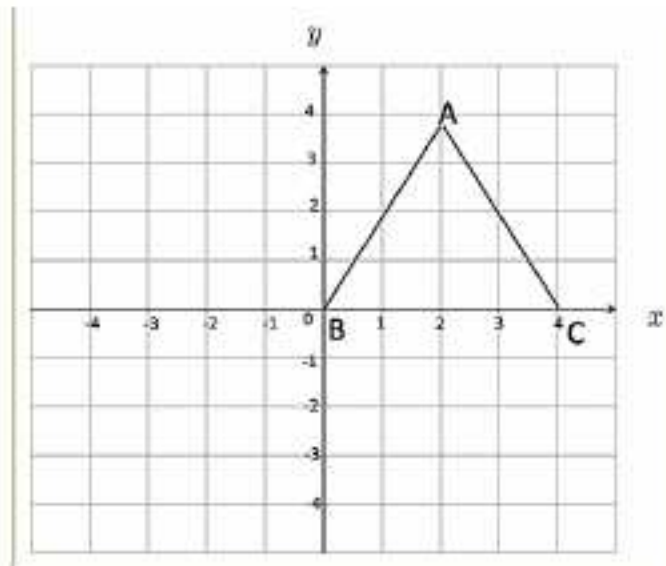
(iv) D

(v) E

(vi) F



33. $\triangle ABC$ is an equilateral triangle. Find the coordinates of A.



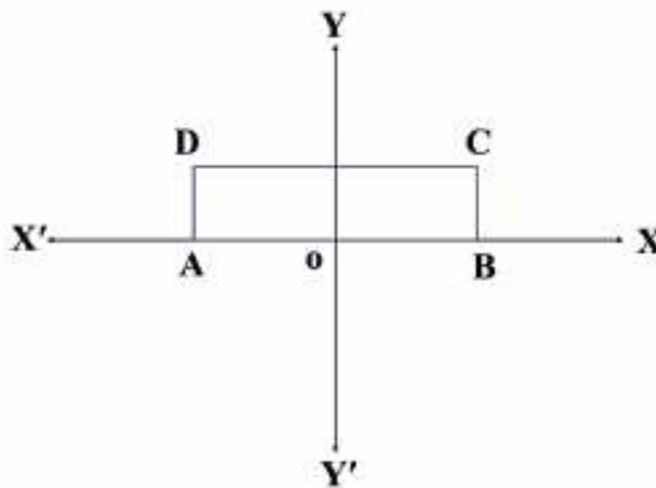
34. ABCD is a rectangle with length 6 units and breadth 2 units. O is the mid-point of AB. Find the coordinates of

(i) A

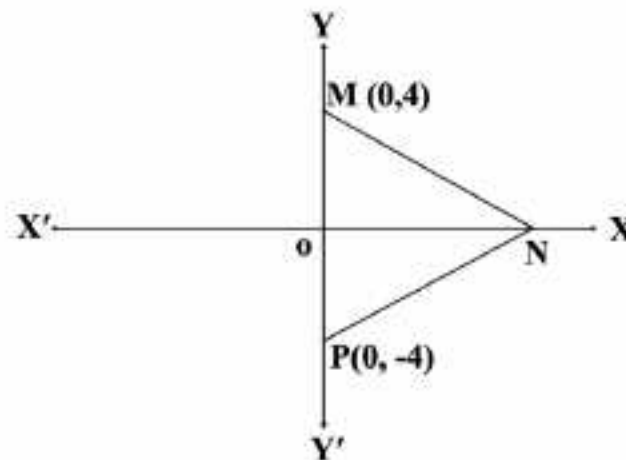
(ii) B

(iii) C

(iv) D



35. $\triangle MNP$ is an equilateral triangle. Find the coordinates of N.



ANSWERS

Q.No.		ANSWER	Q. NO.		ANSWER
1.	(i)	$(-3, -5)$	6.	(i)	0 (zero) unit
	(ii)	$(-6, 0)$		(ii)	8 units
	(iii)	$(4, -7)$		(iii)	1 unit
	(iv)	$(5, 3)$		(iv)	5 units
	(v)	$(0, -4)$		(v)	4 units
2.	(i)	$(2, -4)$	7.	(i)	It represents x – axis
	(ii)	$(-4, -3)$		(ii)	y – axis
	(iii)	$(0, -5)$		(iii)	y – axis
	(iv)	$(-3, 6)$		(iv)	It represents y – axis
	(v)	$(-6, 0)$		(v)	x – axis
3.	(i)	II	8.	(i)	$\left(-\frac{3}{2}, 5\right)$
	(ii)	III		(ii)	$\left(0, \frac{5}{4}\right)$
	(iii)	II		(iii)	$(-2, 5)$
	(iv)	II	9.	(i)	Origin
	(v)	II		(ii)	90°
4.	(i)	Abscissa = 3, Ordinate = 0		(iii)	0 and 0
	(ii)	Abscissa = -4, Ordinate = 3		(iv)	0
	(iii)	Abscissa = 0, Ordinate = -5		(v)	0
	(iv)	Abscissa = 0.5, Ordinate = 2	10.	(i)	4
	(v)	Abscissa = 1.4, Ordinate = -2.3		(ii)	2
5.	(i)	$(5, -5)$		(iii)	7
	(ii)	$(-2, -4)$	11.		0 (zero)
	(iii)	$(-3, 3)$	12.	(i)	Isosceles triangle
	(iv)	$\left(\frac{1}{4}, 4\right)$		(ii)	Isosceles right triangle
	(v)	$(-3, 3)$		(iii)	Equilateral triangle

Q.No.		ANSWER	Q. NO.	ANSWER
13.	(i)	Rectangle	24.	2
	(ii)	Rectangle	25.	(3, 0.5)
14.	(i)	15 sq units	26.	(-2, -1)
	(ii)	8 sq units	27.	A(3,2), B(5,4), C(-1,3), D(-4,1), E(5,0), F(0,-3), G(-3, -3)
	(iii)	1 sq unit	28.	(i) Rectangle (ii) 30 sq units
15.	(i)	S(1, 2)	29.	(i) P (2,0), Q(0,-3), R(-2,0), S(0,3) (ii)Rhombus (iii) 10 units.
	(ii)	D(0, 2)	30.	(i) P(3,2), (ii) Q(-2,2), (iii) R(-4,-2), (iv) S(1,-2), (v) (-1/2, 0)
	(iii)	x = 4	31.	18 sq units
16.		3	32.	(i) A(2,-3.5), (ii) B(-2, -3.5), (iii) C(-3.5,-2), (iv) D(-2.5,3), (v) E(3.5,2), (vi) F(3.5, -2)
17.		6	33.	(2, 2√3)
18.		-4	34.	(i) (-3, 0)
19.		Only one i.e. origin(0,0)		(ii) (3, 0)
20.		10 units		(iii) (3, 2)
21.		IV quadrant		(iv) (-3, 2)
22.		II quadrant	35.	(4√3, 0)
23.		(-3, -4)		

CHAPTER 4

LINEAR EQUATIONS IN TWO VARIABLES

POINTS TO REMEMBER

- An equation of the form $ax + by + c = 0$, where a , b and c are real numbers, such that a and b are not both zero, is called a linear equation in two variables.
- A linear equation in two variables has infinitely many solutions.
- The graph of every linear equation in two variables is a straight line.
- The graph of $x = a$ is a straight line parallel to the y -axis, where a is any real number. If $a = 0$ then it represents y -axis.
- The graph of $y = a$ is a straight line parallel to the x -axis, where a is any real number. If $a = 0$ then it represents x -axis.
- Every solution to the linear equation is a point on the graph of the linear equation.
- Every point on the graph of a linear equation is a solution of that linear equation.
- $x = p$ and $y = q$ is a solution of the linear equation $ax + by + c = 0$, if $ap + bq + c = 0$, where p and q are real numbers.

QUESTIONS

1. In how many quadrants will the graph of linear equation be drawn if its solution points always have 8 as the sum of its coordinates?
2. Find the equation of a straight line which is parallel to y -axis and is at a distance of 5 units on left of y -axis.
3. Find the coordinates of the point which is common on the graphs of linear equation $x = y$ and $x = -y$.
4. Find any one solution of equation of $2x + 3y = 5$.
5. Find any one solution of equation of $y = \frac{x}{2}$.

6. Find the coordinates of the point where the equation $2x + y = 7$ intersects at y-axis.
7. Compare $2x = -6y$ with $ax + by + c = 0$ and then find the value of $2b - c$.
8. Compare $x - \frac{y}{2} + 8 = 0$ with $ax + by + c = 0$ and then find the value of $2a + b - c$.
9. Compare $2x = 0$ with $ax + by + c = 0$ and find the value of $(a + b + c)$.
10. If $x = 1$ and $y = -1$ is a solution of equation $3x + py = 6$, then find the value of p .
11. If $x = 2$ and $y = 3$ is a solution of equation $\frac{3x+5}{3} - y = k$, then find value of k .
12. Find the number of linear equations in x and y that can have $x = -2$, $y = 4$ as one of its solutions.
13. If $a = 0$, then how many solutions does a linear equation in two variables $ax + by + c = 0$ have?
14. If $b = c = 0$, then how many solutions does a linear equation in two variables $ax + by + c = 0$ have?
15. Which of the following points lie on the graph of linear equation $x = 4y + 5$?
A(0,9); B(9,0); C(9,1); D(1,9); E(13,2); F(2,13); G(1,1); H(-1,-1); I(-1,1); J(1,-1)
16. Find the value of p so that linear equation $3px + y = 5$ has $x = \frac{2}{3}$, $y = 1$ as one of its solutions.
17. On the line $y = 2$, find the value of ordinate of a point whose abscissa is -7 .
18. Express y in terms of x : $x - 2y = 4$.
19. Express x in terms of y : $2x + y = 9$.
20. Find the values of c for which, the linear equation $4x - 9y + 1 = 0$ has $(c+1, 2c-1)$ as one of its solution.
21. Find a point on the graph of the linear equation $5x + 2y = 10$ whose ordinate is $2\frac{1}{2}$ times its abscissa.
22. If the point $(1, \frac{9}{4})$ lies on the graph of the equation $mx + 4y = 9$, then find the value of m .

23. Express the following statement as a linear equation in two variables:
 "Age of x exceeds age of y by 4 years".
24. Express the following statement as a linear equation in two variables:
 "After 4 years, the age of father (y years) will be two times the age of his son (x years)"
25. Find the solution of $\sqrt{x + y + 22} - 6 = -1$ when $y = -7$.
26. Find the standard form of linear equation $\frac{9}{x} - \frac{11}{y} = \frac{1}{x} + \frac{1}{y}$ if $x \neq 0$ and $y \neq 0$.
27. "Cost of an Air Conditioner (₹ x) is ₹5000 more than 3 times the cost of a cooler (₹y)". Express this statement as a linear equation in standard form.
28. Find the point where graph of linear equation having following solutions intersects x-axis.

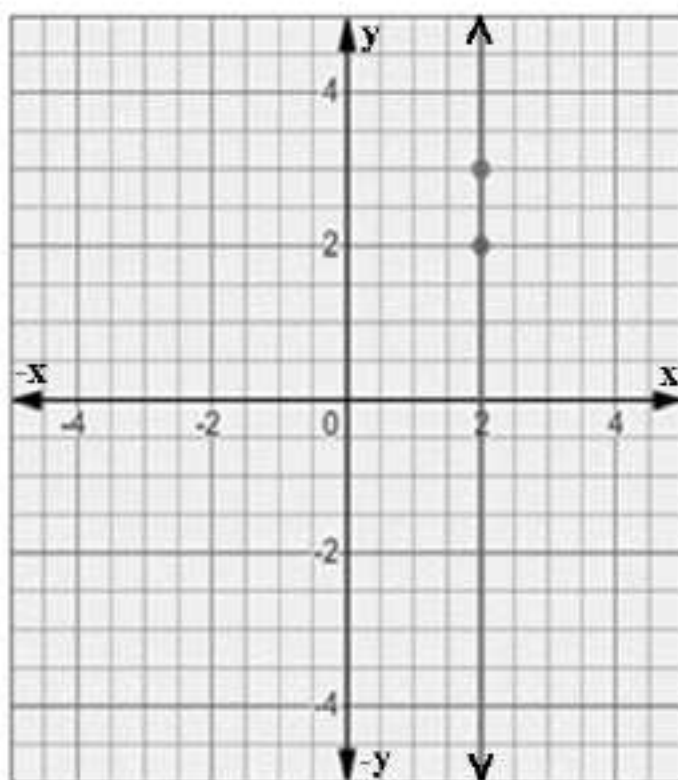
x	4	-3	1	2
y	2	9	5	4

29. Find the point where graph of linear equation having following solutions intersects y-axis.

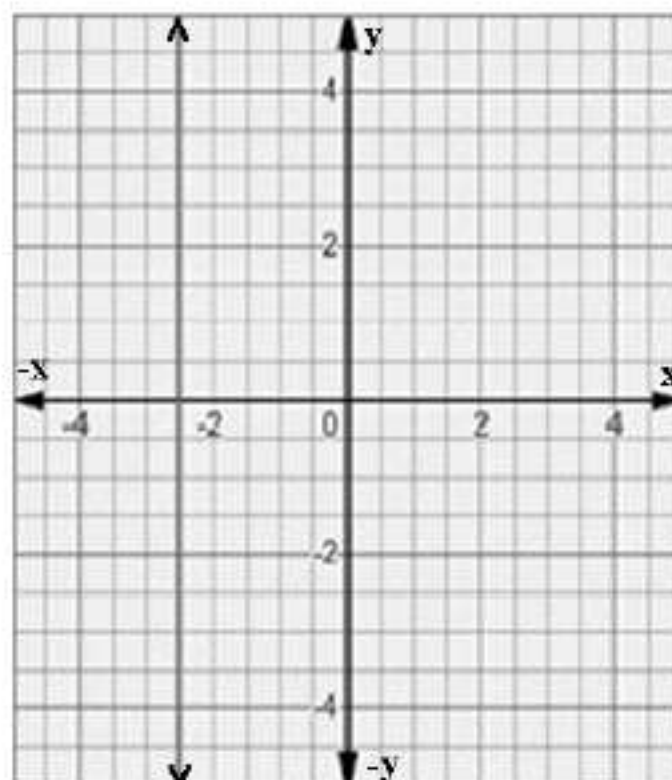
x	12	14	10	7
y	6	8	4	1

30. For the linear equation $3x + 5y - 11 = 0$, find the point where the equation intersects:
 (i) x-axis
 (ii) y-axis.
31. If the point $(2k - 3, k + 2)$ lies on the graph of the equation $2x + 3y + 15 = 0$, then find the value of k.
32. Find the common solution of the linear equations $x - 2y = 1$ and $2x + y = 7$.
33. Find the value of c for which the linear equation $2x + cy = 8y$ has equal values of x and y.
34. Let y varies directly as x. If $y = 36$ when $x = 12$, then represent this as a linear equation in two variables.

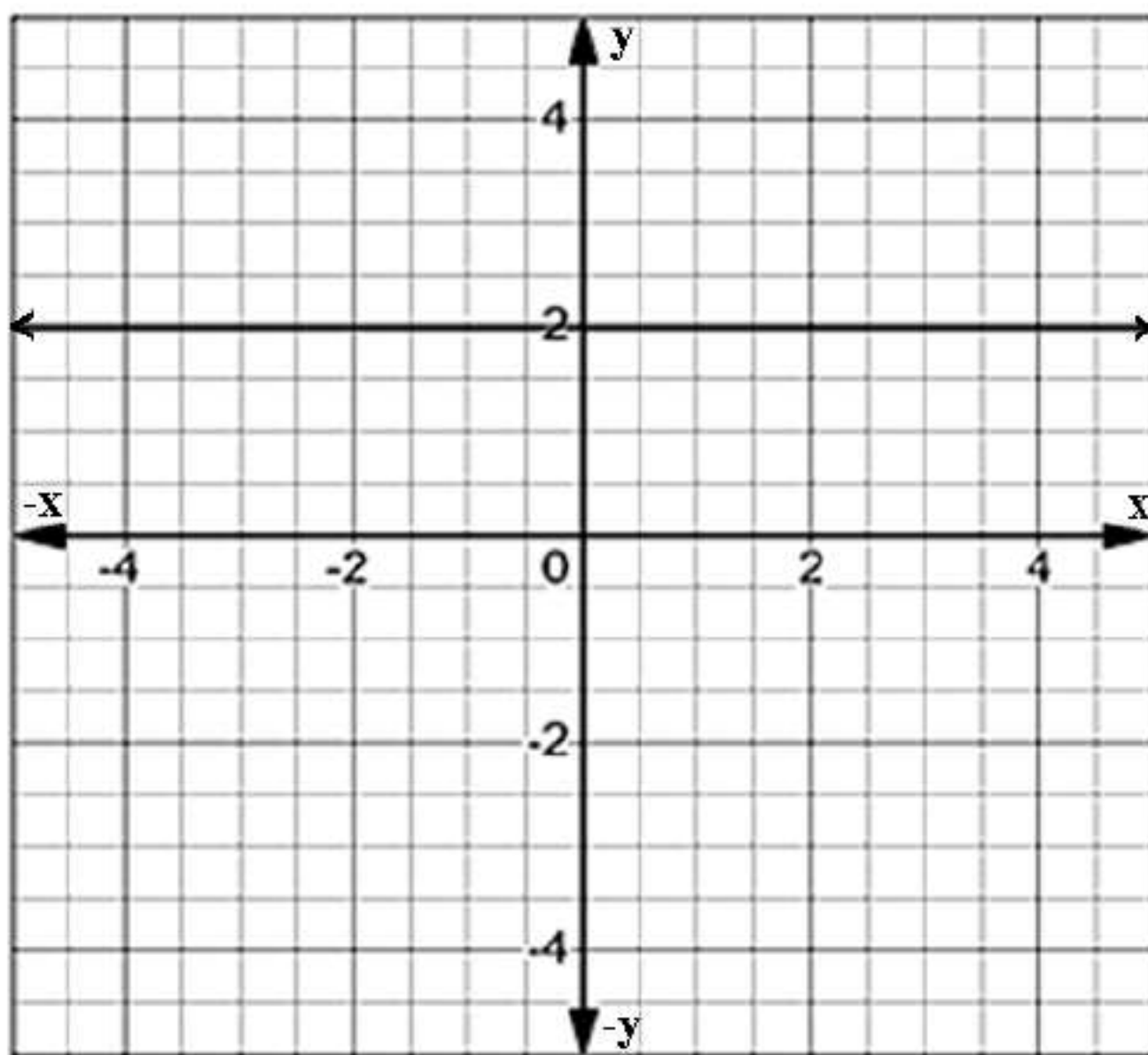
35. If the graph of a linear equation is inclined at an angle of 45° with x-axis at origin in first quadrant, then find the equation of the line.
36. Express the following as a linear equation in two variables:-
 "Parking charges of a motor cycle in a parking lot for first 2 hours is ₹40 and then ₹10 for subsequent hours. The parking charges paid for a motor cycle are ₹x and time of parking is y hours".
37. Find the solution of linear equation $\frac{12}{x} + \frac{15}{y} = \frac{20}{xy}$ which lies on x - axis.
38. Find area of a triangle formed by origin and the points where equation $2x + y = 4$ intersects x-axis and y-axis.
39. The amount of money (₹y) that a person gets after working x hours in a day is shown by the relation $y = 500 + 70x$. Find the amount of money that a person will get if he works 7 hours on first day and 6 hours on second day.
40. Observe the graph and find the corresponding equation for it.



I



II



III

ANSWERS

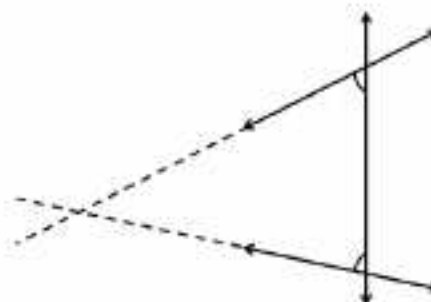
Q.NO.	ANSWER	Q.NO.	ANSWER
1.	3	21.	$(1, \frac{5}{2})$
2.	$x + 5 = 0$	22.	0
3.	(0, 0)	23.	$x - y - 4 = 0$
4.	(1, 1) or any other solution	24.	$2x - y + 4 = 0$
5.	(2, 1) or any other solution	25.	(10, -7)
6.	(0, 7)	26.	$3x - 2y = 0$
7.	12	27.	$x - 3y - 5000 = 0$
8.	$-6\frac{1}{2}$	28.	(6,0) [Linear Equation is $x + y = 6$]
9.	2	29.	(0,-6) [Linear Equation is $x - y = 6$]
10.	-3	30.	(i) $(\frac{11}{3}, 0)$ (ii) $(0, \frac{11}{5})$
11.	$\frac{2}{3}$	31.	$-\frac{15}{7}$
12.	Infinite	32.	(3, 1)
13.	Infinite [solutions are $(x, -\frac{x}{2})$ where x is any real number]	33.	6
14.	Infinite [solutions are (0,y) where y is any real number]	34.	$3x - y = 0$
15.	C(9,1), E(13,2), J(1,-1)	35.	$x - y = 0$
16.	2	36.	$x - 10y - 20 = 0$
17.	2	37.	$(\frac{4}{3}, 0)$
18.	$y = \frac{x-4}{2}$	38.	4 sq units
19.	$x = \frac{9-y}{2}$	39.	₹1910
20.	1	40.	(i) $x - 2 = 0$ (ii) $x + 2.5 = 0$ (iii) $y - 2 = 0$

CHAPTER – 5

INTRODUCTION TO EUCLID GEOMETRY

POINTS TO REMEMBER

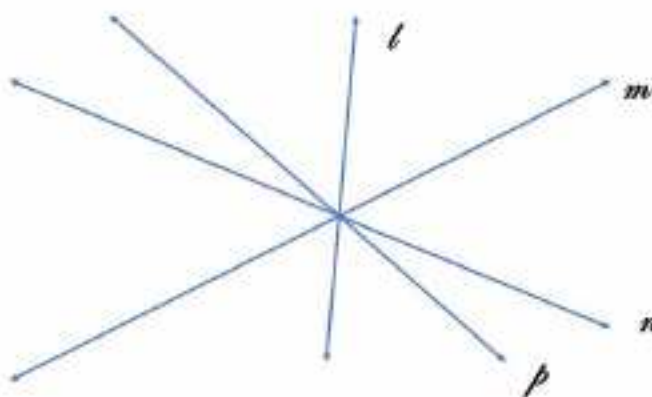
- Axioms and postulates are the assumptions which are obvious universal truths. They are not proved.
- Theorems are statements which are proved using definitions, axioms, previously proved statements and deductive reasoning.
- Some of Euclid's axioms are:
 - (i) Things which are equal to the same thing are equal to one another.
 - (ii) If equals are added to equals, the wholes are equal.
 - (iii) If equals are subtracted from equals, the remainders are equal.
 - (iv) Things which coincide with one another are equal to one another.
 - (v) The whole is greater than the part.
 - (vi) Things which are double of the same things are equal to one another.
 - (vii) Things which are halves of the same things are equal to one another.
- Euclid's postulates are following:
 - Postulate 1: A straight line may be drawn from any one point to any other point.
 - Postulate 2: A terminated line can be produced indefinitely.
 - Postulate 3: A circle can be drawn with any centre and any radius.
 - Postulate 4: All right angles are equal to one another.
 - Postulate 5: If a straight line falling on two other straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on the side on which the sum of angles is less than two right angles.



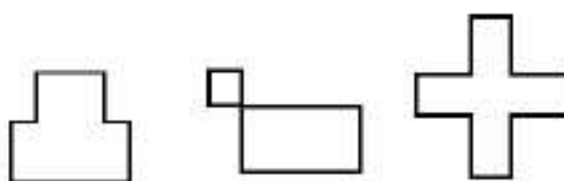
- Points that lie on the same straight line are called 'collinear points'.
(In the given figure points A, B, C and D are collinear points.)



- Lines in a plane or space are said to be concurrent, if they intersect at a single point. (In the figure given below, lines l, m, n and p are concurrent.)



- A rectilinear figure is a plane figure or shape which are formed by straight lines, all of whose sides meet at right angles. The interior angle at each vertex of a rectilinear figure is 90° or 270° . examples of Rectilinear figures are:



QUESTIONS

Answer each of the following questions using appropriate Euclid's axiom/postulate:

1. Fill in the blanks:

- A Line separates a plane into _____ parts.
- Rectilinear figure is formed by _____.
- A surface is that, which has length, _____ but no _____.
- A point has _____ number of dimension.
- A surface has _____ number of dimensions.

(vi) The number of planes passing through three non-collinear points is _____.

(vii) A solid has _____ number of dimensions.

2. Give an example of a geometrical line.
3. Find the number of lines that can pass through a given point.
4. Does a line segment have any length?
5. Give the Euclid's axiom stating relation between whole and the part.
6. Find the least number of distinct points required to determine a unique line.
7. Find the number of lines that can be determined by two distinct points in a plane.
8. Find the number of end points of a ray.
9. Find the maximum numbers of points in which two distinct lines intersect.
10. Find the maximum number of points that can lie on a line.
11. Find the maximum numbers of points in which two distinct planes intersect.
12. Find the number of line segments that can be drawn passing through two distinct points P and Q.
13. Two salesmen make equal sales of chocolates in the month of March. In June, each salesman doubles his sale to that of the month of March. Compare their sales in June.
14. If $a > b$ and $b > c$, then find the relation between a and c.
15. Which of the following statements are true?
 - (i) Two intersecting lines cannot be both parallel to the same line.
 - (ii) A line segment has no definite length.
 - (iii) A ray has no end point.
 - (iv) A line has a definite length.
 - (v) A line \overleftrightarrow{AB} is same as the line \overleftrightarrow{BA} .
 - (vi) A ray \overrightarrow{AB} is same as the ray \overrightarrow{BA} .
 - (vii) Two distinct points always determine a unique line.
 - (viii) Three lines are concurrent if they have a common point.
 - (ix) Two lines may intersect at two points.
 - (x) Two lines 'l' and 'm' are parallel, only if they have no point in common.

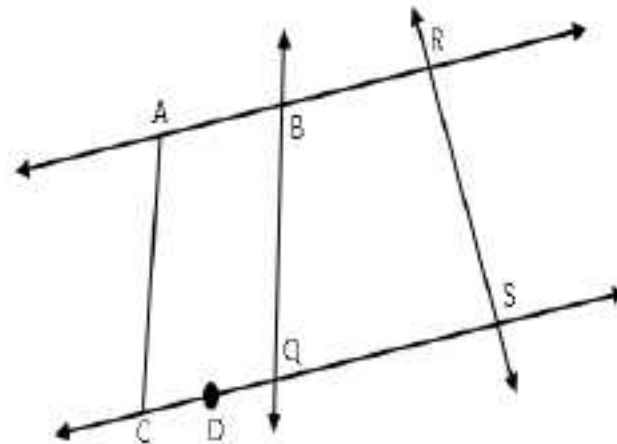
16. If $a + b = 10$, then find the value of $a + b + x$.

17. If B lies between A and C and $AC = 8\text{cm}$, $BC = 3\text{cm}$, then find the value of $2BA + 3BC - AC$.

18. A point P lies in between M and N. If C is the mid point of MP, then find $MN - NP - MC$.

19. In the given figure name the following

- (i) 3 line segments
- (ii) 4 collinear points
- (iii) A pair of non-intersecting line segments.

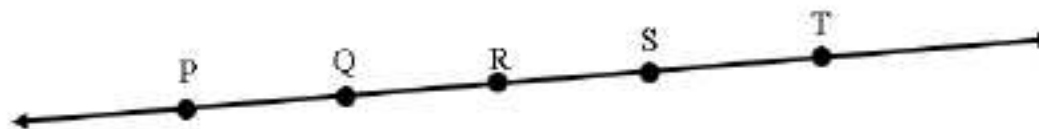


20. Given 4 distinct points in a plane, find the number of lines that can be drawn using them, when

- (i) all the 4 points are collinear.
- (ii) no three of the four points are collinear.

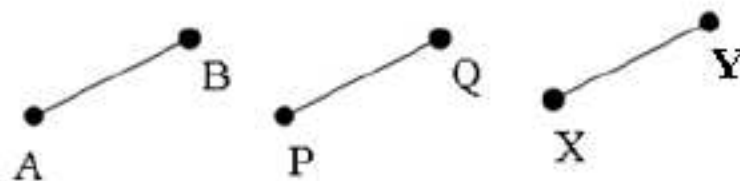
21. According to the given figure, state which statement is true and which is false.

- (i) $PQ + QR = PR$
- (ii) $PR + PS = PS$
- (iii) Line segments PQ and PS are coincident.
- (iv) Points R, S and T lie on the line PQ.



22. Which of the following statements are True and which are false?

- (i) If two circles are equal, then their radii are equal.
- (ii) In figure, if $AB = PQ$ and $PQ = XY$, then $AB = XY$.



(iii) The edges of a surface are curves.

(iv) A solid has three dimensions.

(v) If a quantity B is a part of another quantity A, then A can be written as sum of B and some third quantity C.

23. If $x + y = 10$ and $x = z$, then find the value of $y + z$.

24. Name the line segments determined by three collinear points X, Y and Z.

25. Fill in the blanks:

(i) If two squares are congruent, then their _____ are equal.

(ii) A pyramid is a solid figure with base as a _____ and side faces as _____.

(iii) Two distinct _____ in a plane cannot have more than one point in common.

(iv) Given a line and a point, not on the line, there is one and only one perpendicular line which passes through the given point and is _____ to the line.

(v) If line AB, AC, AD, AR are parallel to a line l, then the points A, B, C, D and R are _____.

26. Multiple Choice Questions: Choose the correct Option.

(i) "Lines are parallel if they do not intersect" is stated in the form of

- (a) An Axiom (b) A Definition (c) A Postulate (d) A Proof

(ii) Which of the following needs a Proof?

- (a) Theorem (b) Axiom (c) Definition (d) Postulate

(iii) Boundaries of solids are

- (a) Lines (b) Surfaces (c) Points (d) Curves

(iv) Boundaries of surfaces are

(a) Surfaces

(b) Lines

(c) Points

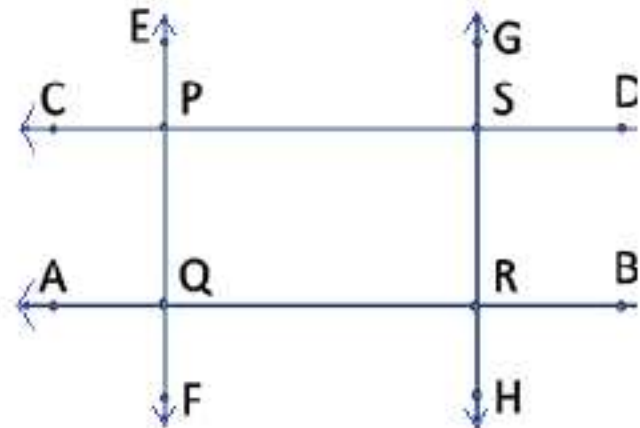
(d) Curves or Straight
Line

27. Which shape can be the base of a solid pyramid ?

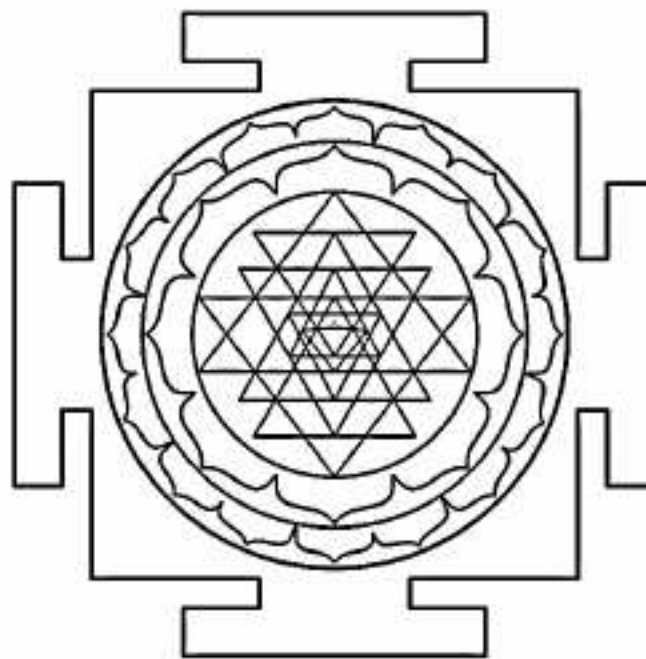
28. What is the shape of side faces of a pyramid?

29. From the given figure name the following:

- a) A line passing through P and Q.
- b) One rectilinear figure.



30. How many interwoven isosceles triangles are there in 'Sriyantra' (as depicted in Atharva Veda)?



ANSWERS

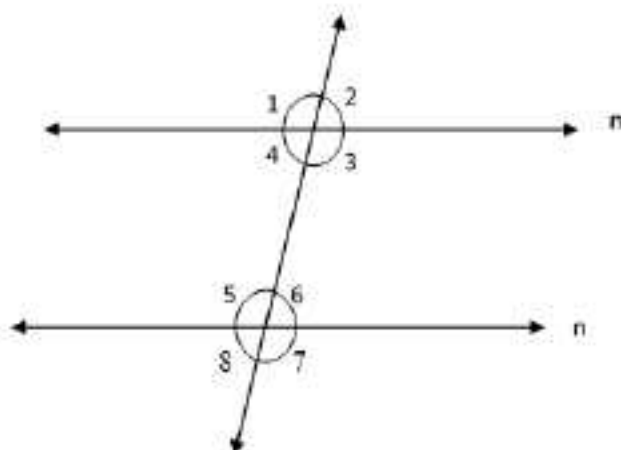
Q.NO.	ANSWER	Q.NO.	ANSWER
1.	(i) 2 (ii) Straight Lines (iii) breadth, thickness (iv) 0 (v) 2 (vi) 1 (vii) 3	16.	$10 + x$
2.	Meeting place of two walls or any other Suitable example	17.	11cm
3.	Infinitely many	18.	CP
4.	Yes (breadth less)	19.	(i) AC, BQ, RS or any other three line segments (ii) C, D, Q, S (iii) AC, BQ or any other pair
5.	Whole is greater than the part	20.	(i) one (ii) six
6.	2	21.	(i) True (ii) False (iii) False (iv) True
7.	1 (Unique)	22.	(i) True (ii) True (iii) False (iv) True (v) True
8.	1	23.	10
9.	1	24.	XY, YZ, ZX
10.	Infinite	25.	(i) sides (ii) polygon, triangles (iii) lines (iv) perpendicular (v) collinear
11.	Infinite number of points	26.	(i) (c) a Postulate (ii) (a) Theorem (iii) (b) Surfaces (iv) (d) curves or straight lines
12.	1	27.	Any Pyramid
13.	Equal	28.	Triangle
14.	$a > c$	29.	(i) \overline{EF} (ii) PQRS
15.	(i) True (ii) False (iii) False (iv) False (v) True (vi) False (vii) True (viii) True (ix) False (x) True	30.	9

CHAPTER – 6

LINES AND ANGLES

POINTS TO REMEMBER

- If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° & vice-versa. This property is known as the Linear Pair Axiom.
- If two lines intersect each other, then vertically opposite angles are equal.
- If a transversal intersects two parallel lines, then (refer figure)



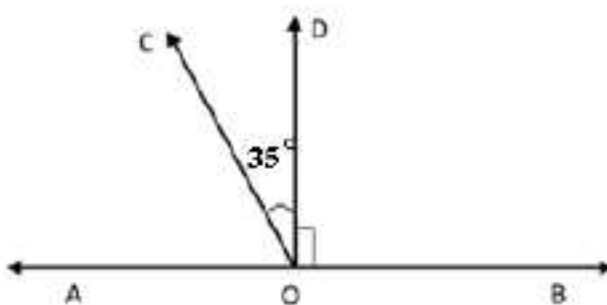
- (a) Each pair of corresponding angles are equal e.g. $\angle 1 = \angle 5$, $\angle 4 = \angle 8$, $\angle 2 = \angle 6$ and $\angle 3 = \angle 7$.
- (b) Each pair of alternate interior angles are equal. e.g. $\angle 4 = \angle 6$, $\angle 3 = \angle 5$
- (c) Each pair of interior angles on the same side of the transversal are supplementary. e.g. $\angle 4 + \angle 5 = 180^\circ$, $\angle 3 + \angle 6 = 180^\circ$.
- If a transversal intersects two lines such that, either any one pair of corresponding angles is equal, any one pair of alternate interior angles is equal or any one pair of interior angles on the same side of the transversal is supplementary, then the lines are parallel.
- Two intersecting lines cannot be parallel to the same line.
- Lines which are parallel to a given line are parallel to each other.

- The sum of three interior angles of a triangle is 180° .
- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of two interior opposite angles.

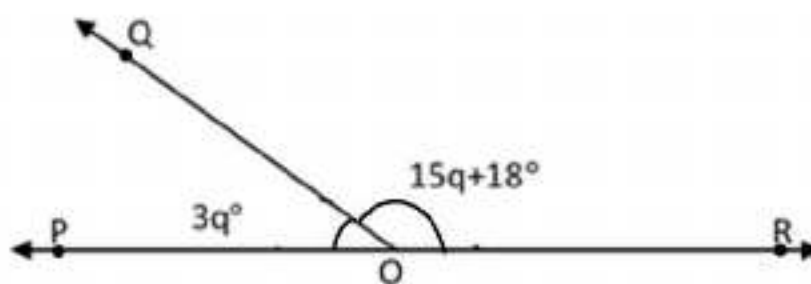
QUESTIONS:

1. How many points are common in two parallel lines?
2. Find the measurement of an angle which equals one-fifth of its supplement.
3. Two angles whose measures are x and y form a linear pair. If $3x - 4y = 50^\circ$, then find the value of $3x$.
4. If two parallel lines are intersected by a transversal, then name the shape formed by the bisectors of its interior angles.
5. Name the type of triangle formed, if its one angle is equal to the sum of other two angles.
6. In $\triangle ABC$ if $\angle A - \angle B = 30^\circ$ and $\angle B - \angle C = 15^\circ$, then find the measure of $\angle B$.
7. In $\triangle ABC$ if $\angle A = \angle B = 70^\circ$ and the bisectors of $\angle B$ and $\angle A$ meet each other at O , then find $\angle BOA$.
8. If x , y and z are the interior angles of a triangle and $\frac{x}{2} = \frac{y}{4} = \frac{z}{3}$, then find the greatest angle of the triangle.
9. If a wheel has six spokes equally spaced, then find the measure of the angle between two adjacent spokes.
10. Find the measure of an angle, if four times its complement is 10° less than twice its supplement.
11. How many lines can be drawn parallel to line l from a point p not lying on l ?
12. An angle is 26° more than its complement. Find its measures.
13. Two supplementary angles are in the ratio $3:2$. Find the measurement of the smaller angle.
14. The ratio of the complement and supplement of an angle is $2 : 5$. Find the angle.

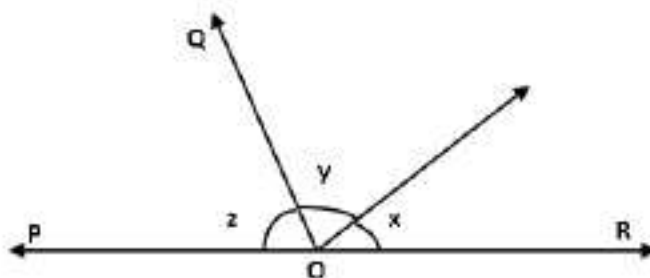
15. The three angles of a triangle are $\frac{x}{2} + 10^\circ$, $\frac{x}{3} + 30^\circ$ and $x - 25^\circ$. Find the value of x .
16. If side BC of $\triangle ABC$ is produced to D such that $\angle ACD = 120^\circ$ and $\angle B = \frac{1}{2}\angle A$, then find $\angle A$.
17. The exterior angle of a triangle is 108° . If the interior opposite angles of the triangle are in the ratio 4:5, then find the sum of largest interior angle and smallest interior angle.
18. The exterior angle of a triangle is 93° . If one of the interior opposite angles is 19° more than the other, then find the smallest angle of the triangle.
19. If $\frac{x}{2} - 19^\circ$ and $\frac{x}{3} + 14^\circ$ are complement to each other, then find the value of x .
20. In $\triangle ABC$, $\angle A = 3\angle B = 6\angle C$, then find the measure of $\angle A + \angle B - \angle C$.
21. In the given figure if $OD \perp AB$ and $\angle DOC = 35^\circ$, then find $\angle BOA - \angle BOC$.



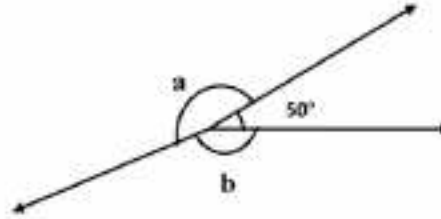
22. Find the value of q in the given figure:



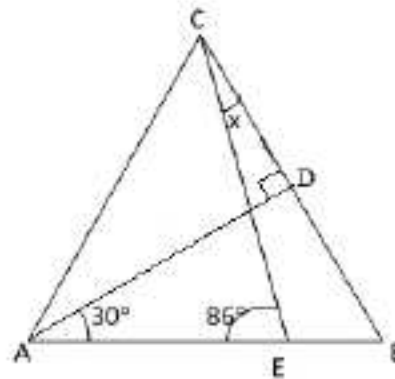
23. In the given figure, find the value of $\frac{1}{2}z$ if $x = 58^\circ$ and $y = 42^\circ$.



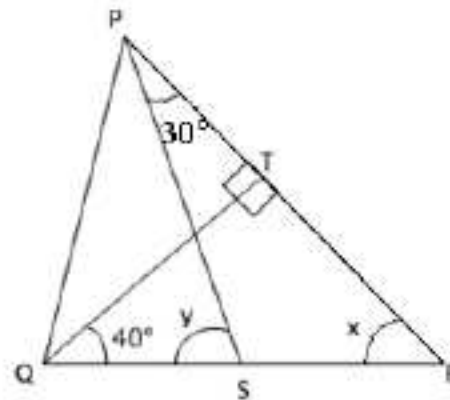
24. Find the value of $a + b$ in the given figure.



25. If $\angle ADB$ is a right angle, then find the value of x .



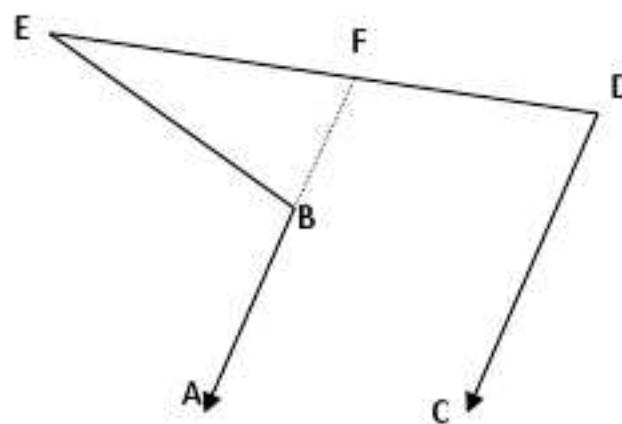
26. Find the value of $x + y$.



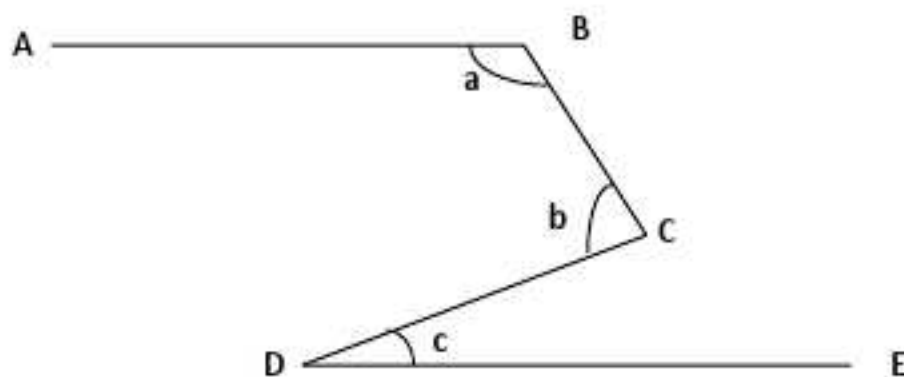
27. Angles of a triangle are in the ratio 2: 4: 3. Find the smallest angle of the triangle.

28. If $(5y + 62)^\circ$ and $(22 + y)^\circ$ are supplementary angles, then find y .

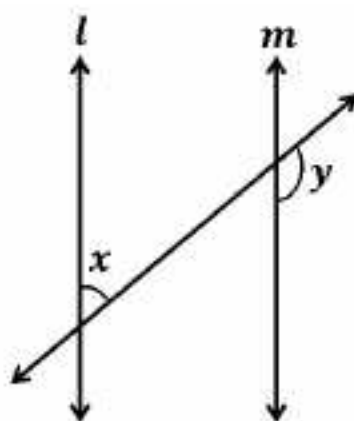
29. In the given figure, if $AB \parallel CD$, $\angle ABE = 130^\circ$ and $\angle BED = 20^\circ$, then find $\angle EDC$.



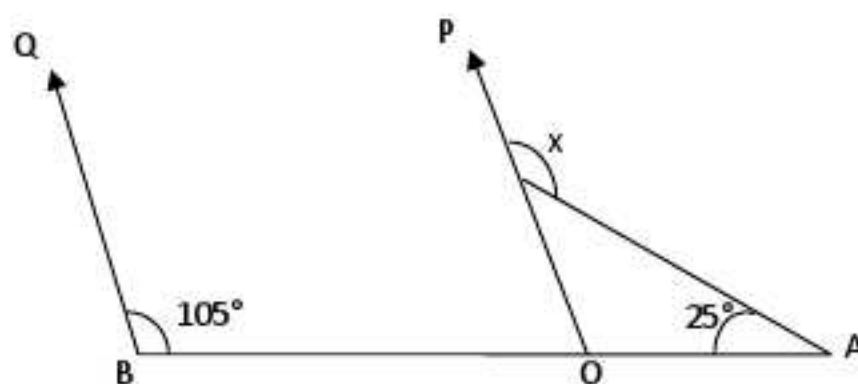
30. In the given figure, $AB \parallel DE$. Find $a + b - c$.



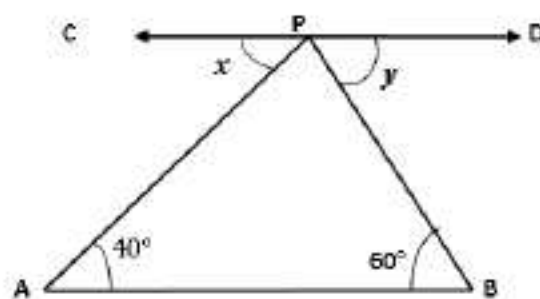
31. In the given figure, find the value of x if $y = 130^\circ$ and $l \parallel m$.



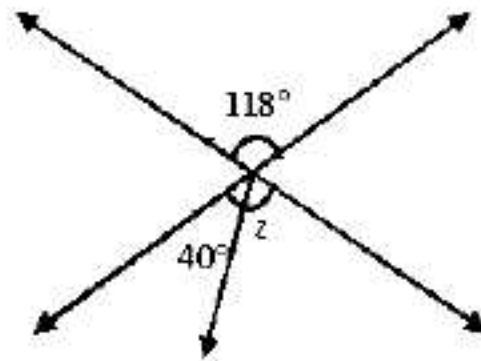
32. Find the value of x , if $BQ \parallel OP$.



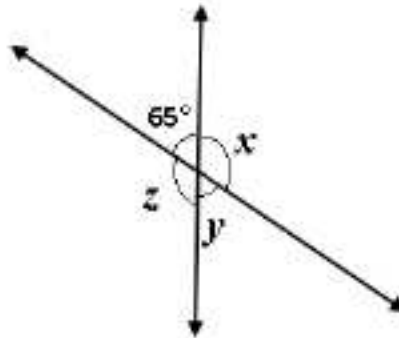
33. If AB and CD are parallel, then find the value of $x + y$.



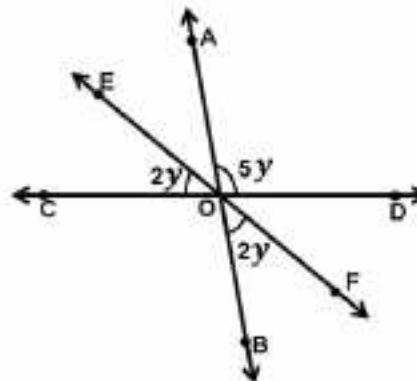
34. Find the value of angle z .



35. Find the value of $x + z - y$.

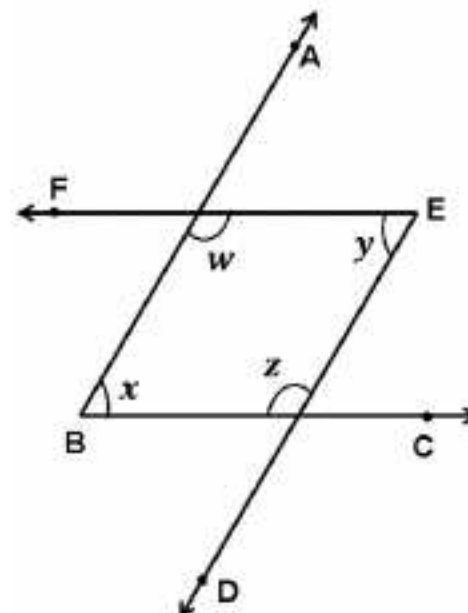


36. Find the value of y if AB , CD and EF are three lines concurrent at O .

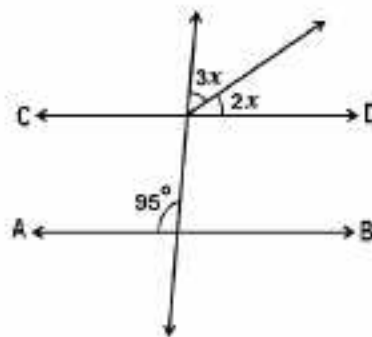


37. If arms of one angle $\angle ABC$ are parallel to the arms of another angle $\angle DEF$, then find the relation between:

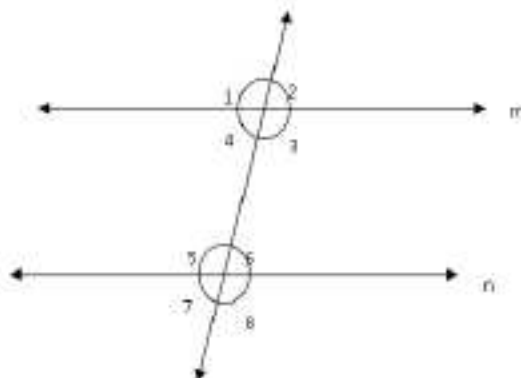
- (i) x and y
- (ii) z and y



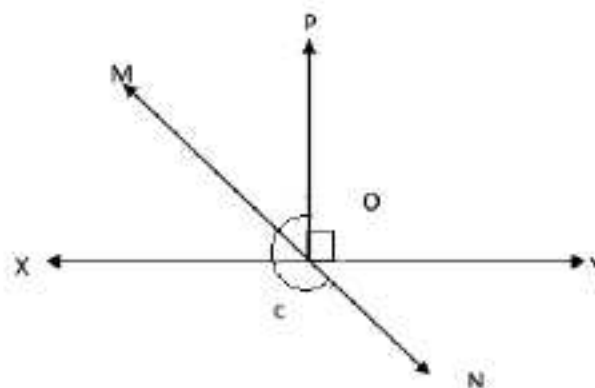
38. If $AB \parallel CD$, then find the value of x .



39. If $m \parallel n$ and $\angle 1 = 118^\circ$, then find $\angle 7$.

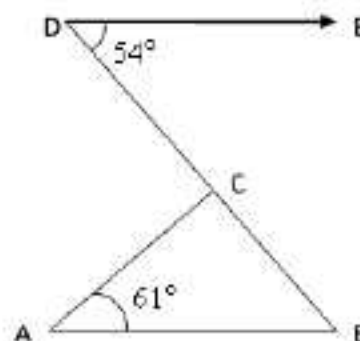


40. In given figure, lines XY and MN intersect at O , if $\angle POY = 90^\circ$, $\angle POM : \angle MOX = 2 : 3$. Find c .

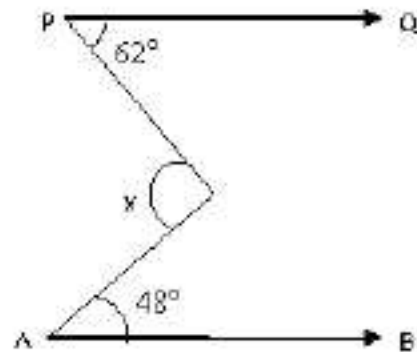


41. The complement of an angle is equal to one third of its supplement, find the angle.

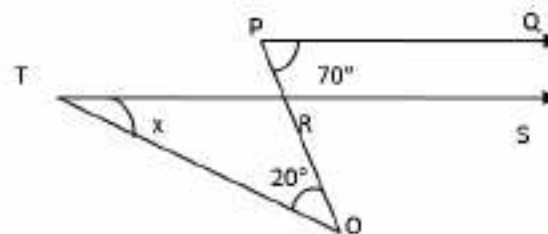
42. If $AB \parallel DE$, then find the value of $\angle ACB$.



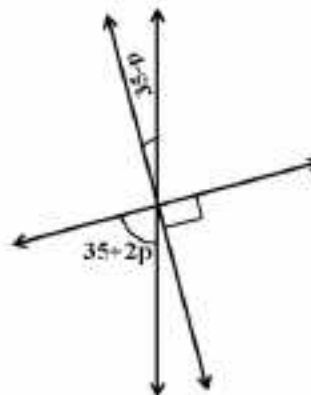
43. If $AB \parallel PQ$, then find the angle x .



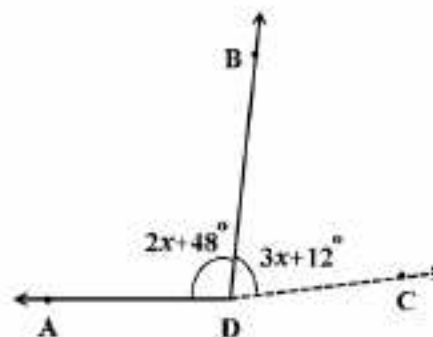
44. If $PQ \parallel TS$, then find the value of x .



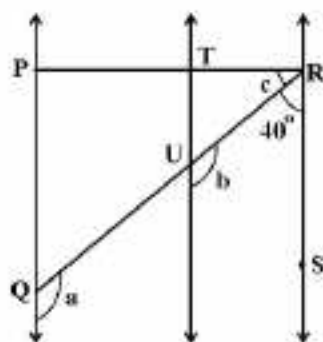
45. In the given figure, find the complement of p .



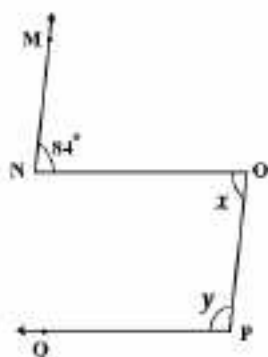
46. In the given figure, find the value of x so that ADC becomes a straight line.



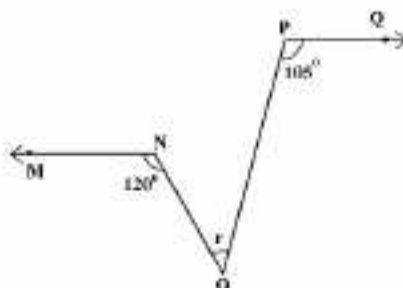
47. In the given figure, $PQ \parallel TU \parallel RS$. Also $\angle QRS = 40^\circ$, then find the value of $a - c$.



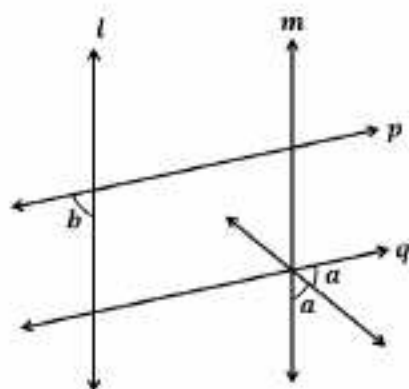
48. In the given figure, find the value of $2x - y$, if $NO \parallel QP$ and $MN \parallel OP$.



49. In the given figure, if $MN \parallel PQ$, then find the value of r .



50. In the given figure, $l \parallel m$ and $p \parallel q$. Find the value of a if $b = 78^\circ$.



ANSWERS

Question	Answer	Question	Answer
1	Zero	26	130°
2	30°	27	40°
3	330°	28	$y = 16$
4	Rectangle	29	110°
5	Right Angled Triangle	30	180°
6	55°	31	50°
7	110°	32	130°
8	80°	33	100°
9	60°	34	78°
10	5°	35	165°
11	One	36	20°
12	58°	37	(i) Equal (ii) Supplementary
13	72°	38	17°
14	30°	39	62°
15	90°	40	126°
16	80°	41	45°
17	120°	42	65°
18	37°	43	110°
19	114°	44	50°
20	140°	45	70°
21	55°	46	24°
22	$q = 9$	47	90°
23	40°	48	72°
24	310°	49	45°
25	26°	50	51°

CHAPTER – 7

TRIANGLES

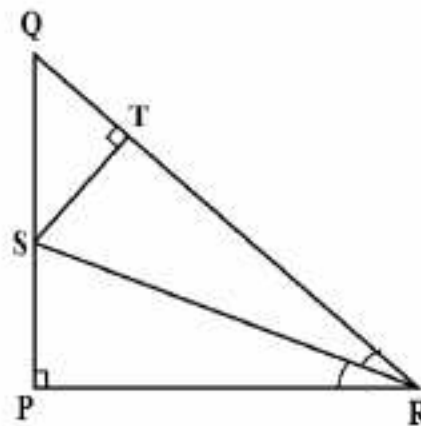
POINTS TO REMEMBER

- Two figures are congruent, if they are of same shape and same size.
- If two triangles ABC and XYZ are congruent under the correspondence $A \leftrightarrow X$, $B \leftrightarrow Y$ and $C \leftrightarrow Z$, then symbolically $\triangle ABC \cong \triangle XYZ$.
- SAS Congruence Rule: If two sides and the included angle of one triangle are equal to corresponding two sides and the included angle of the other triangle, then the two triangles are congruent.
- ASA Congruence Rule: If two angles and the included side of one triangle are equal to corresponding two angles and the included side of the other triangle, then the two triangles are congruent.
- AAS Congruence Rule: If two angles and one side of one triangle are equal to corresponding two angles and the corresponding side of the other triangle, then the two triangles are congruent.
- RHS congruence Rule: If in two right triangles, hypotenuse and one side of a triangle are equal to the hypotenuse and corresponding one side of other triangle, then the two triangles are congruent.
- SSS Congruence rule: If three sides of one triangle are equal to the corresponding three sides of another triangle, then the two triangles are congruent.
- Angles opposite to equal sides of a triangle are equal.
- Sides opposite to equal angles of a triangle are equal.
- Each angle of an equilateral triangle is 60° .
- Of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.
- In a triangle, angle opposite to the longer side is greater.

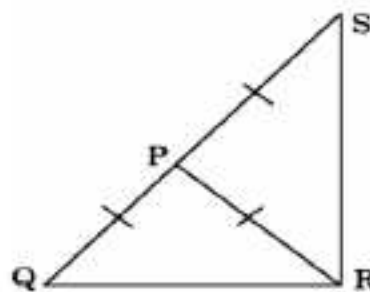
- In a triangle, side opposite to the greater angle is longer.
- Sum of any two sides of a triangle is greater than the third side.
- Difference between any two sides of a triangle is less than its third side.
- A Right-angled triangle was earlier known as **ORTHOGONAL TRIANGLE** or **RECTANGLED TRIANGLE**, indicating a triangle which has one right angle or a triangle whose two sides are perpendicular to each other.

QUESTIONS:

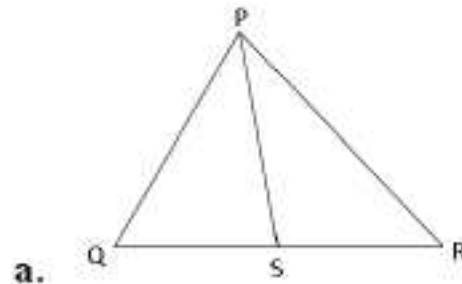
1. In $\triangle PQR$, right angled at P , if $\angle R \leq 40^\circ$, then find the minimum measure of $\angle Q$.
2. In $\triangle ABC$, if $\angle A = 40^\circ$ and $\angle B = 60^\circ$, then find ratio of the shortest and the longest side of triangle.
3. In $\triangle XYZ$, right angled at Y , find the sum (in terms of sides) of two smaller sides.
4. In the given figure, $\triangle PQR$ and $\triangle TQS$ are right angled triangles. If $PQ = PR$ and bisector of $\angle R$ intersects PQ at S , then find the side of $\triangle PQR$ which is equal to $PR + PS$.



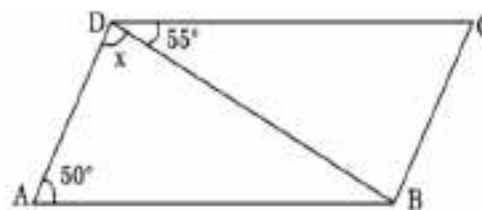
5. In $\triangle PQR$, if $\angle P - \angle Q = 20^\circ$ and $\angle Q - \angle R = 5^\circ$, then find the measure of $\angle Q$.
6. In the given figure, if $PQ = PR = PS$, then find the value of $(3\angle QRS - 2\angle SQR - \angle QSR)$.



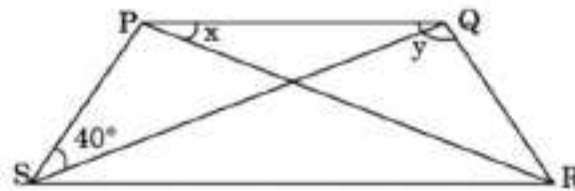
7. If the length of three sides of a triangle are 5 cm, 12 cm and 13 cm respectively, then find the length (in cm) of the altitude drawn on the hypotenuse of the given triangle.
8. In the given figure, if $PQ = PS = SR$ and $\angle QPS = 40^\circ$, then find the value of $\angle QPR$.



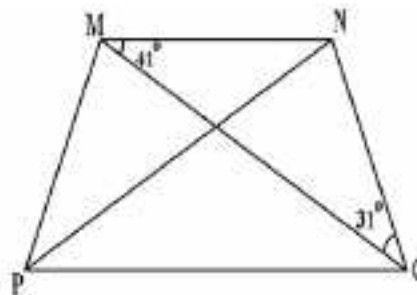
9. In the given figure, if $BC = AD$ and $AB = DC$, then find the value of x .



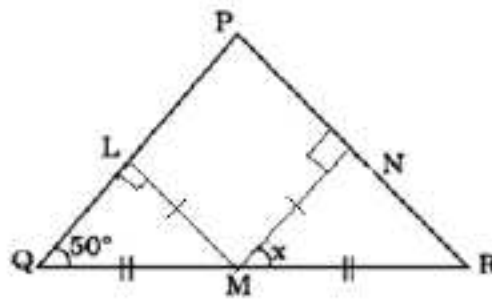
10. In the given figure, if $PQ \parallel SR$ and $PQ = QR$, then find the value of $2x + y$.



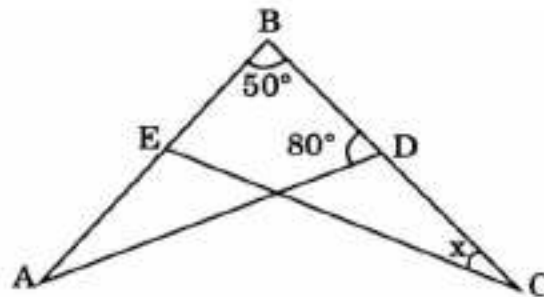
11. In a triangle PQR, internal angular bisectors of $\angle Q$ and $\angle R$ intersect at a point O. If $\angle P = 110^\circ$, then find the value of $\angle QOR$.
12. In a $\triangle XYZ$, XA is the angle bisector onto YZ. If the semi-perimeter of the triangle is 11 cm and $XY = 10$ cm, $YZ = 6$ cm, then find the ratio of $YA:AZ$.
13. In the given figure, if $MP = NO$ and $MO = NP$, then find the value of $\angle MPN + \angle PMN$.



14. 14. In the given figure, if $LM = MN$, $QM = MR$, $ML \perp PQ$, $MN \perp PR$ and $\angle Q = 50^\circ$, then find x .

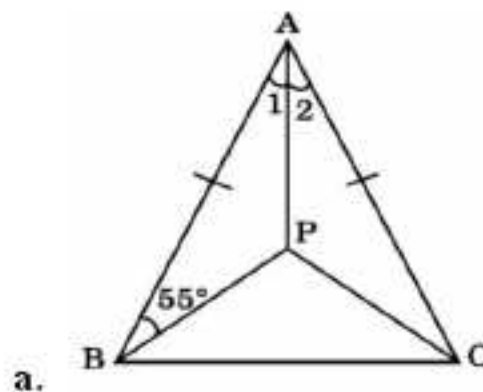


15. In the given figure, if $AB = BC$ and $\angle A = \angle C$, then find x .

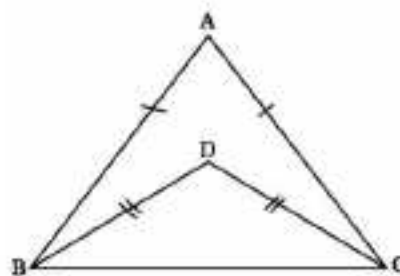


16. In $\triangle XYZ$, $\angle X - \angle Y = 16^\circ$ and $\angle Y - \angle Z = 28^\circ$. Find the value of $\angle Y$.

17. In $\triangle ABC$, $AB = AC$, $\angle 1 = \angle 2$ and $\angle A = 40^\circ$. Find $\angle PBC$.

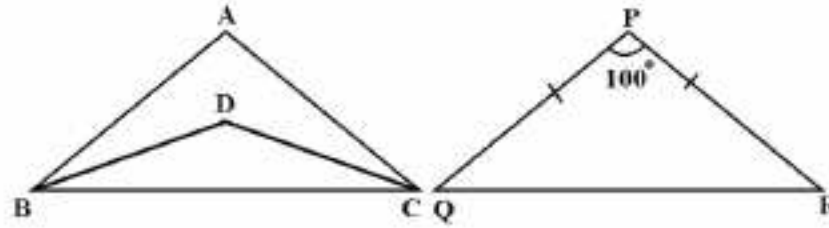


18. In the given figure, find $\angle ABD$: $\angle ACD$.

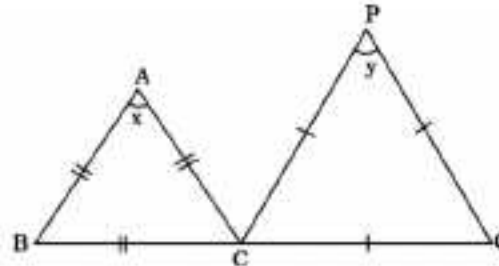


19. In $\triangle ABC$ right angled at C, if $AC = 12$ cm, $BC = 5$ cm, then find AB .

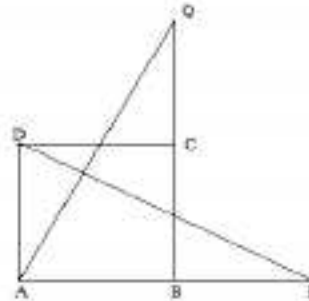
20. In the given figure, $\triangle ABC \cong \triangle PQR$. If DB and DC are the angle bisectors of $\angle B$ and $\angle C$ respectively, then find the value of reflex $\angle BDC$.



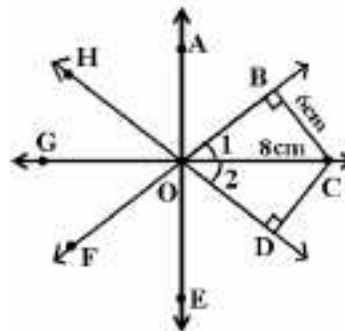
21. In the given figure, find the value of $(x + y)$.



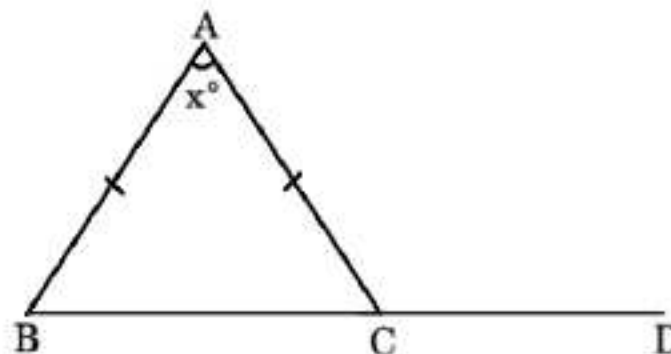
22. In the given figure, ABCD is a square. Sides AB and BC are produced to points P and Q such that $BP = CQ$. If $DP = 7$ cm, then find AQ.



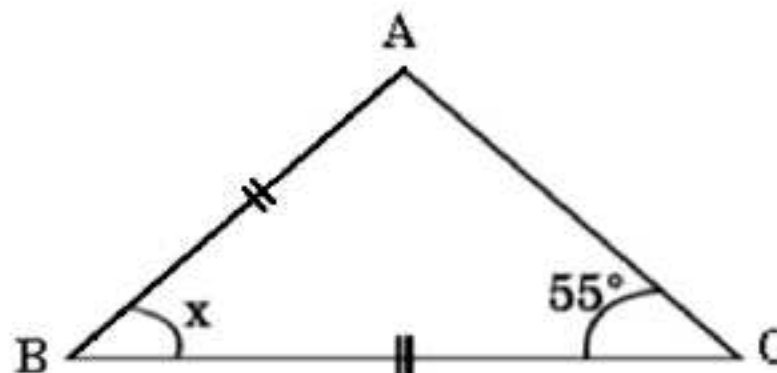
23. In the given figure, if $BC = 6$ cm, $OC = 8$ cm and $\angle 1 = \angle 2$, then find the length of OD.



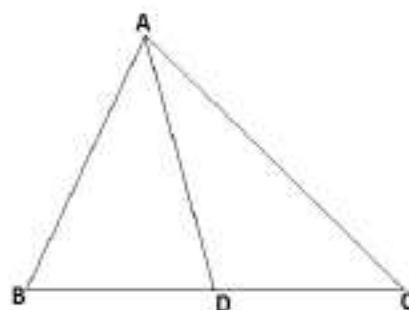
24. In the given figure, if $AB = AC$ and $\angle ACD = 110^\circ$, then find x .



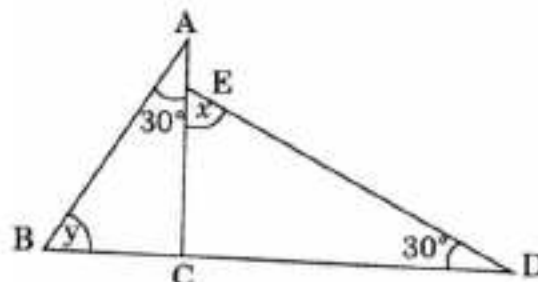
25. Find the value of x in the given triangle.



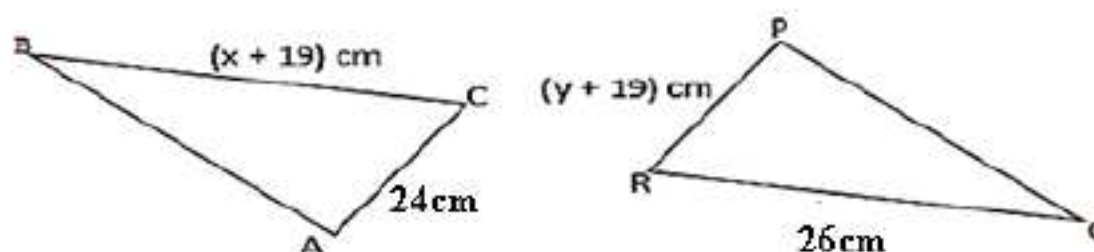
26. In $\triangle ABC$, if $\angle B = 65^\circ$, $\angle C = 45^\circ$ and the bisectors of $\angle BAC$ meets BC at D , then find $\angle DAC : \angle C$.



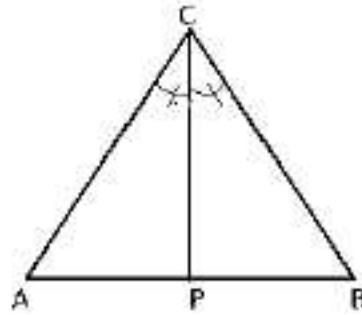
27. In the given figure, if $\angle A = \angle D = 30^\circ$, then find $(x + y)$.



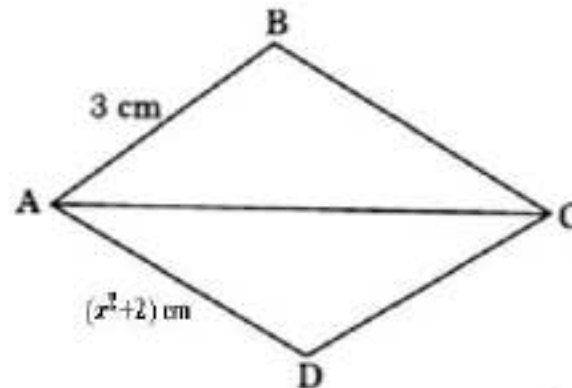
28. If $\triangle ABC \cong \triangle PQR$, then find the value of $(x - y)^2 + (x + y)^2$.



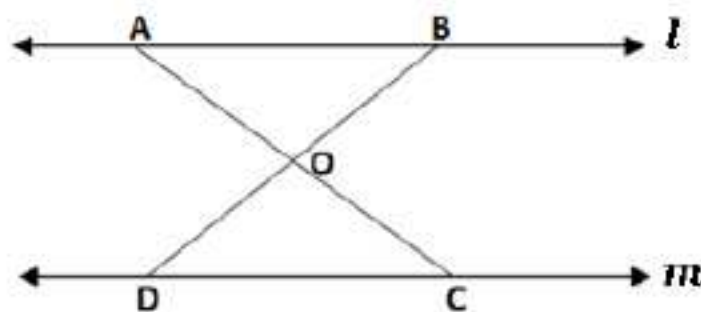
29. In the given figure, if CP is the bisector of $\angle C$ and $CA = CB$, then $\triangle CAP$ and $\triangle CBP$ are congruent by which congruency criteria?



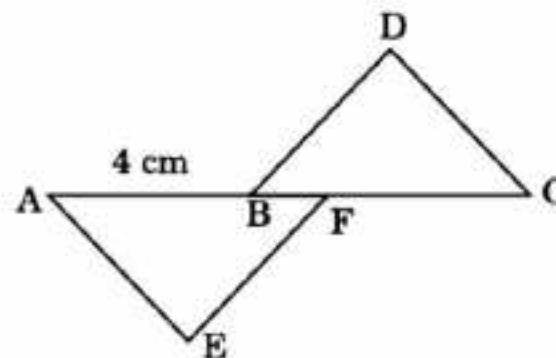
30. In the given figure, AC bisects $\angle A$ and $\angle C$. If $AB = 3$ cm, then find x .



31. In the given figure, $l \parallel m$ and $AB = DC$. If $OB = 3.5$ cm, $OC = 3.5$ cm and $AB = 6$ cm, then find $AC + BD$.

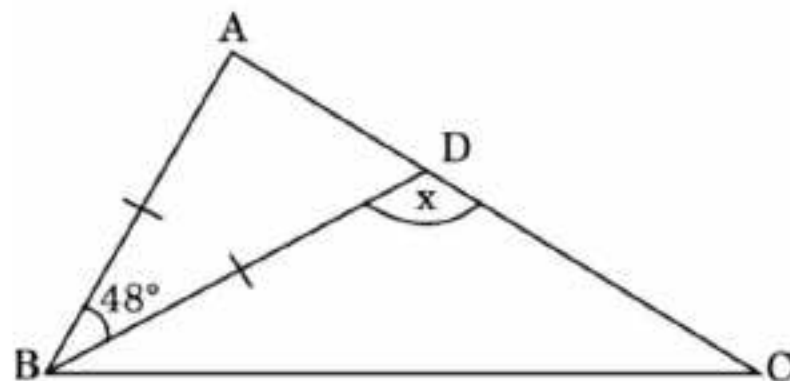


32. In the given figure, $\triangle DBC \cong \triangle EFA$. If $AB = 4$ cm, then find FC .

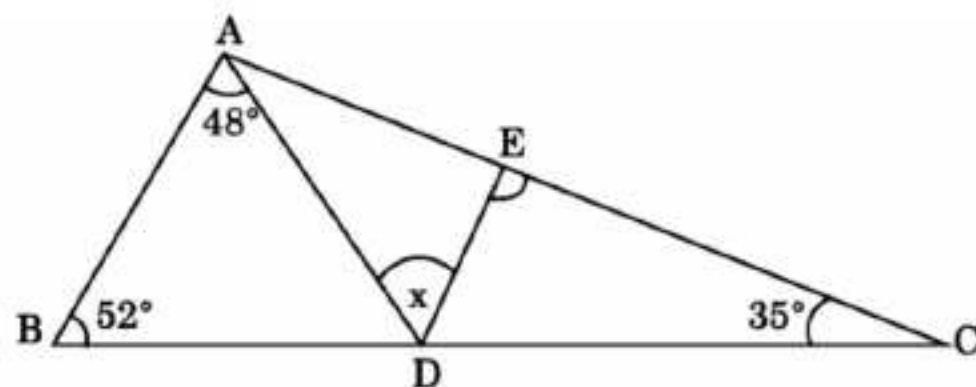


33. In $\triangle XYZ$, $\angle Y = 32^\circ$, $\angle Z = 68^\circ$ and bisector of $\angle YXZ$ meets YZ at P .
Arrange XP , YP and ZP in descending order.

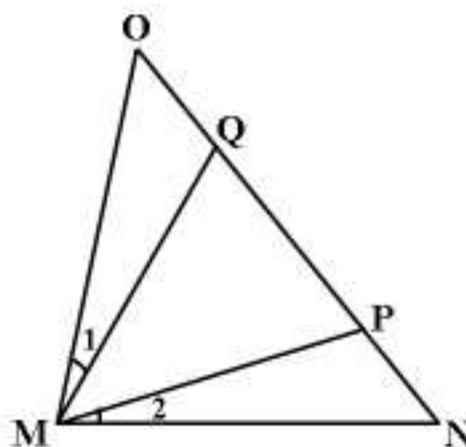
34. In the given figure, $AB = DB$. If $\angle ABD = 48^\circ$, then find x .



35. In the given figure, if $\angle DEC = 90^\circ$, then find x .



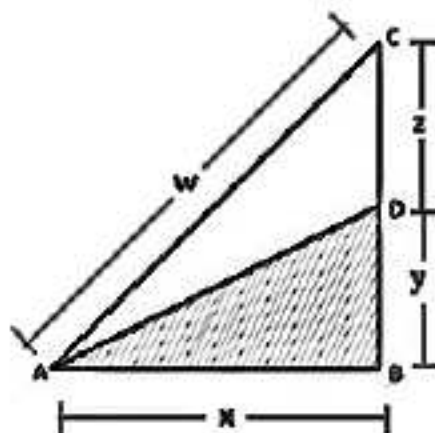
36. In the given figure, if $MQ = MP$, $\angle 1 = \angle 2$ and $\angle NMO = 78^\circ$, then find the value of $\angle O$.



37. Find the area of the largest triangle that can be fitted into a rectangle of length ' l ' units and width ' w ' units.

38. In $\triangle ABC$, $\angle A = 40^\circ$, $\angle B$ is 25% more than $\angle A$. Name the longest side of $\triangle ABC$.

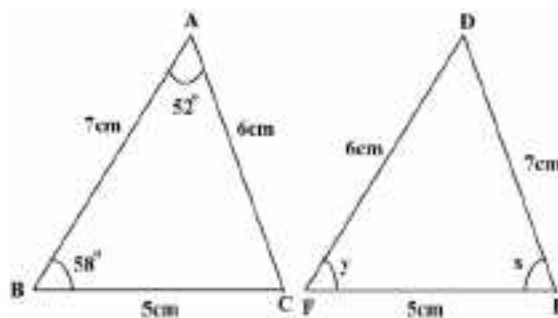
39. If the shaded area is one half of the area of $\triangle ABC$ and $\angle ABC$ is a right angle, then find the length of line segment AD in terms of y and w only.



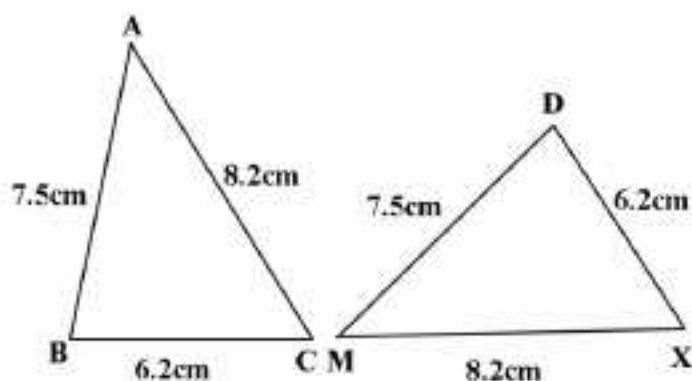
40. The length of three sides of a triangle are in the ratio of 5:12:13. The difference between largest side and the smallest side of this triangle is 1.6 cm. Find the area of the triangle.

41. If all three altitudes of a triangle are equal, then which type of triangle is it?

42. In the given figure, $\triangle ABC$ and $\triangle DEF$ are congruent triangles. Find the value of $2x + 3y$.

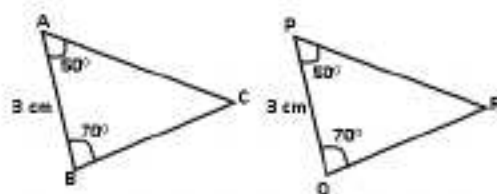


43. Name the triangle congruent to $\triangle CAB$.

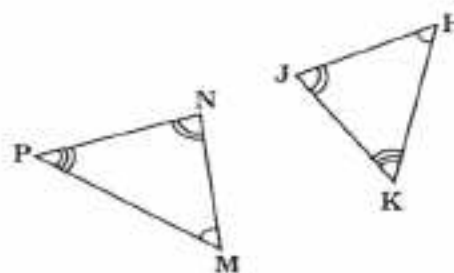


44. Which congruence criteria is used for the congruency of the following triangles?

i



ii

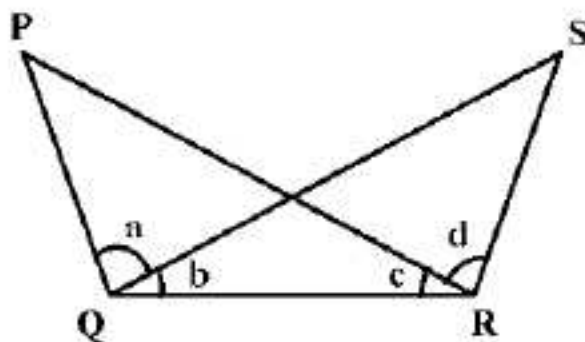


45. There are two congruent circles with centres C_1 and C_2 . If radius of circle with center C_1 is 5cm, then find the circumference of circle with center C_2 (in terms of π).

46. The perimeter of an equilateral triangle $\triangle XYZ$ is $72\sqrt{3}$ units. Find the height of $\triangle XYZ$.

47. In $\triangle PQR$, if $PQ = QR$ and $\angle Q = 55^\circ$, then find $2\angle P - \angle R$.

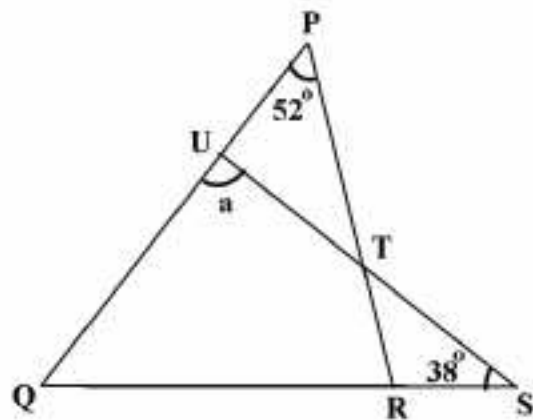
48. In the given figure, if $a = d$ and $b = c$, then fill in the blanks:-



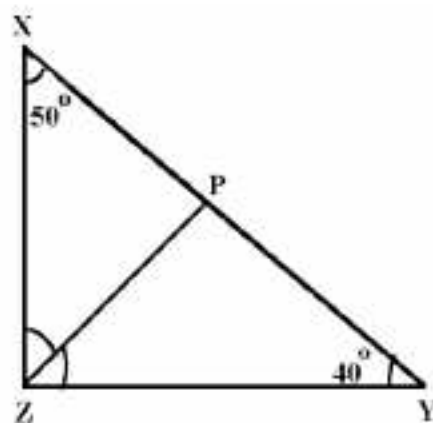
(i) $PR =$ _____

(ii) $\angle S =$ _____

49. In the given figure, if $RT = RS$, then find the value of a .



50. In $\triangle XYZ$, $\angle X = 50^\circ$, $\angle Y = 40^\circ$ and bisector of $\angle XZY$ meet XY at P . Arrange XP , YP and ZP in ascending order.



ANSWERS

1.	50°	25.	70°
2.	1:2	26.	7:9
3.	XY+YZ	27.	120°
4.	QR [Hint: $\triangle PSR \cong \triangle TSR$ So PR=TR, PS=TS. Also, TS=TQ]	28.	148
		29.	SAS
		30.	1 cm
5.	55°	31.	14 cm
6.	135°	32.	4cm
7.	6.5 cm	33.	YP > XP > ZP
8.	75°	34.	114°
9.	75°	35.	45°
10.	180°	36.	51°
11.	145°	37.	$\frac{lw}{2}$ sq units
12.	5:3	38.	AB
13.	139°	39.	$\sqrt{w^2 - 3y^2}$
14.	40°	40.	1.2 sq cm
15.	50°	41.	Equilateral
16.	64°	42.	326°
17.	15°	43.	$\triangle XMD$
18.	1 : 1	44.	(i)ASA (ii) Triangles may not be congruent
19.	13 cm	45.	10 π cm
20.	220°	46.	36 units
21.	120°	47.	$62\frac{1}{2}^\circ$
22.	7 cm	48.	(i) QS (ii) $\angle P$
23.	$2\sqrt{7}$ cm	49.	90°
24.	40°	50.	XP<ZP<YP

CHAPTER – 8

QUADRILATERALS

POINTS TO REMEMBER

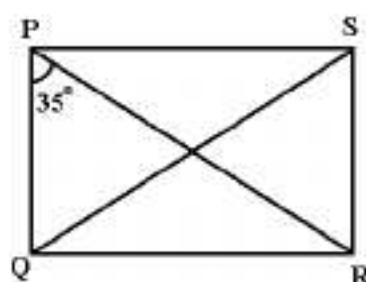
- Sum of the angles of a quadrilateral is 360° .
- A quadrilateral is called a parallelogram, if both pair of its opposite sides are parallel.
- A diagonal of a parallelogram divides it into two congruent triangles.
- In a parallelogram,
 - (i) both pair of opposite angles are equal.
 - (ii) both pair of opposite sides are equal.
 - (iii) diagonals bisect each other.
- A quadrilateral is a parallelogram, if any one of the following statement is true
 - (i) Both pair of opposite angles are equal.
 - (ii) Both pair of opposite sides are equal.
 - (ii) Diagonals bisect each other.
 - (iv) A pair of opposite sides are equal and parallel.
- A parallelogram becomes a rectangle, if one of its interior angles is a right angle.
- A parallelogram becomes a rhombus, if one pair of its adjacent sides are equal.
- A parallelogram becomes a square, if one of its interior angles is a right angle and one pair of its adjacent sides are equal.
- If diagonals of a quadrilateral bisect each other and are equal, then it becomes a rectangle and vice versa.
- If diagonals of a quadrilateral bisect each other at right angles, then it becomes a rhombus and vice versa.
- If diagonals of a quadrilateral bisect each other at right angles and are equal, then it becomes a square and vice versa.

- The line-segment joining the midpoints of any two sides of a triangle is parallel to the third side and is half of it.
- A line through the midpoint of a side of a triangle parallel to another side bisects the third side.
- The quadrilateral formed by joining the midpoints of the sides of a quadrilateral in order, is a parallelogram.
- Two congruent figures have equal areas but the converse may not be true.
- Parallelograms on the same base (or equal bases) and between the same parallels are equal in area.
- Area of a parallelogram is the product of its base and the corresponding altitude.
- Parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels.
- If a parallelogram and a triangle are on the same base (or equal bases) and between the same parallels, then the area of triangle is half the area of the parallelogram.
- Triangles on the same base (or equal bases) and between the same parallels are equal in area.
- Area of a triangle is half the product of its base and the corresponding altitude.
- Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.
- A median of a triangle divides it into two triangles of equal areas.

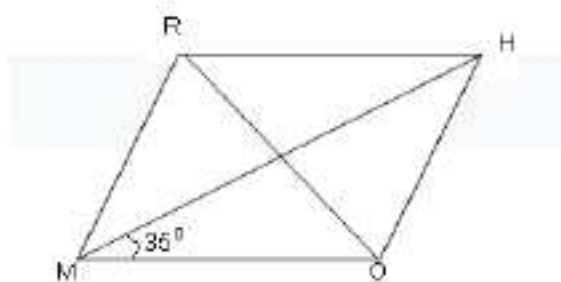
QUESTIONS:

1. The angles of a quadrilateral are $2x$, $3x$, $4x$ and 90° in order. Find the value of x .
2. The angles of a quadrilateral are in the ratio $2:5:9:20$ in order. Find the value of smallest angle.
3. If angles of a parallelogram are p , q , r and s in order. Find the value of $p + q - r - s$.
4. If angles of a trapezium are x , $2x$, $2y$ and y in order, then find the value of $x + y$.

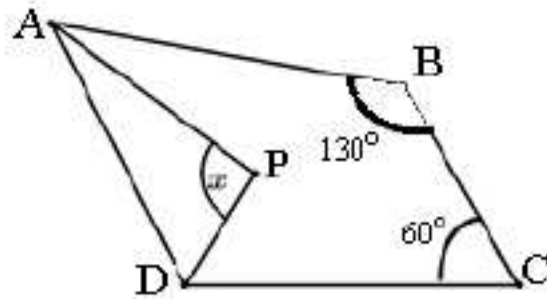
5. The angles of a quadrilateral are $3x^\circ$, $(x + 35)^\circ$, $(x - 25)^\circ$ and $2x^\circ$. Find the largest angle.
6. The angles of a quadrilateral are in the ratio 12:17:19:24. Find the value of largest angle.
7. In a quadrilateral, if two angles are complimentary and other two angles are in the ratio of 2:7, then find the measure of larger of these two angles.
8. In a quadrilateral, if one angle is of measure 100° and other three angles are in the ratio 1: 5:7, then find the measure of smaller of these three angles.
9. The angles of a quadrilateral are x° , $(x - 10)^\circ$, $(x + 52)^\circ$ and $3x^\circ$. Find the smallest angle.
10. The perimeter of a parallelogram is 30 cm. If the shorter side measures 6 cm, then find the measure of the longer side.
11. Diagonals of a quadrilateral ABCD bisect each other. If $\angle A = 55^\circ$, then find $\angle B$.
12. ABCD is a parallelogram. If its both the diagonals are equal, then find the measure of $\angle ABC$.
13. PQRS is a rectangle with $\angle QPR = 35^\circ$. Find the sum of $\angle SQR$ and $\angle SPR$.



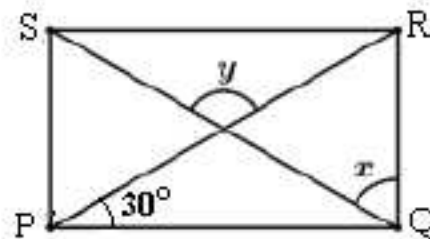
14. In the given rhombus RHOM, find the value of $\angle ROH$.



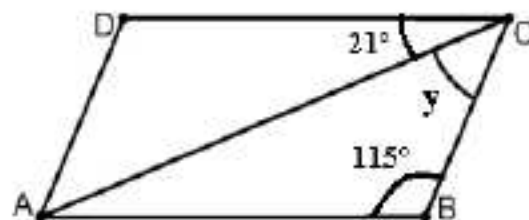
15. In the given figure, ABCD is a quadrilateral and AP and DP are the bisectors of $\angle A$ and $\angle D$. Find the value of x .



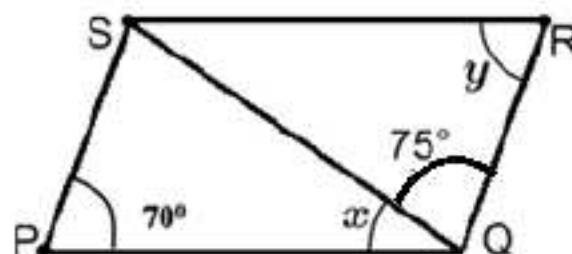
16. In the given figure, PQRS is a rectangle. If $\angle RPQ = 30^\circ$, then find the value of $(x + y)$.



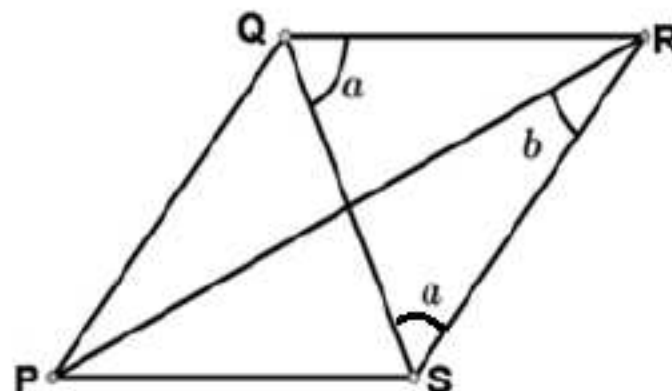
17. In the given figure, ABCD is a parallelogram. Find y .



18. Find the value of $\left(\frac{x+y}{5}\right)$ in the given parallelogram PQRS.

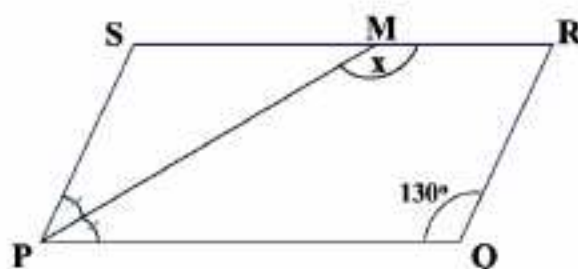


19. PQRS is a rhombus with $\angle QPS = 50^\circ$. Find the value of $a + b$ from the figure.

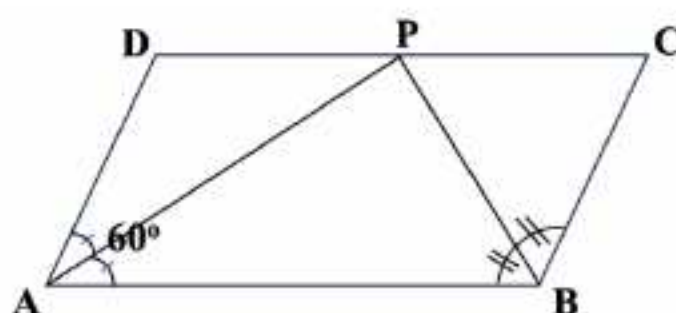


20. In a parallelogram ABCD, if $\angle A = (2x + 15)^\circ$, $\angle B = (3x - 25)^\circ$, then find the value of x .

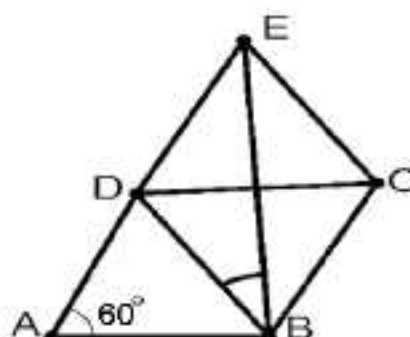
21. PQRS is a parallelogram, PM bisects $\angle P$. Find x .



22. ABCD is a parallelogram, $\angle DAB = 60^\circ$, $AD = 6\text{cm}$, AP and BP are bisectors of $\angle A$ and $\angle B$ respectively. Find perimeter of ABCD.

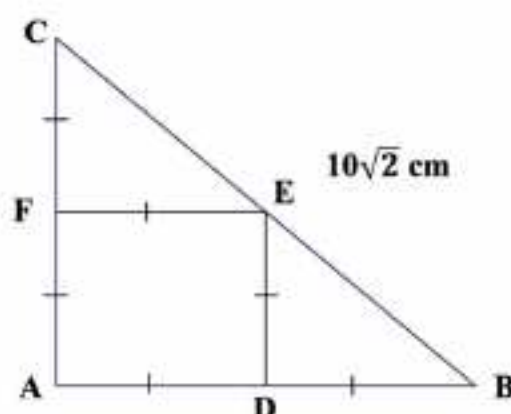


23. Two opposite angles of a parallelogram are $(3x - 2)^\circ$ and $(63 - 2x)^\circ$. Find the measure of smaller angle of the parallelogram.
24. If the ratio of numerical values of altitude and area of a parallelogram is 2:15, then find the length of its corresponding base.
25. In the given figure, ABCD is a rhombus with $\angle A = 60^\circ$. If $\triangle DEC$ is an equilateral triangle, then find $\angle DBE$.

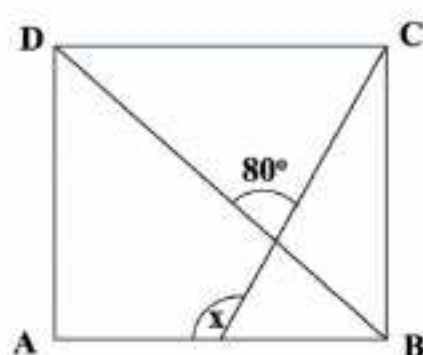


26. A diagonal of a rectangle is inclined at 25° on one side of the rectangle. Find the acute angle between the diagonals.

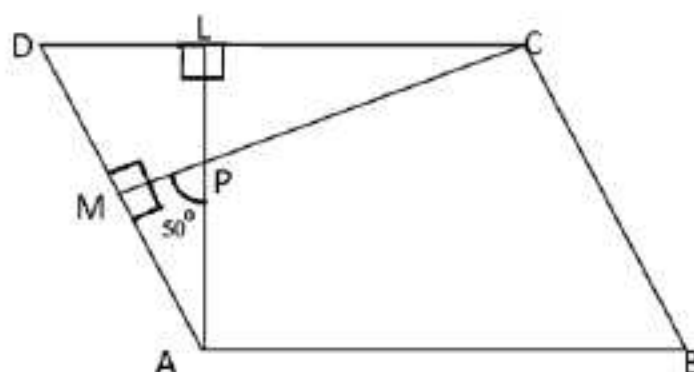
27. $\triangle ABC$, $\triangle CEF$ and $\triangle BDE$ are isosceles right angled triangles. ADEF is a square. Find the area of the square if $BC = 10\sqrt{2}$ cm.



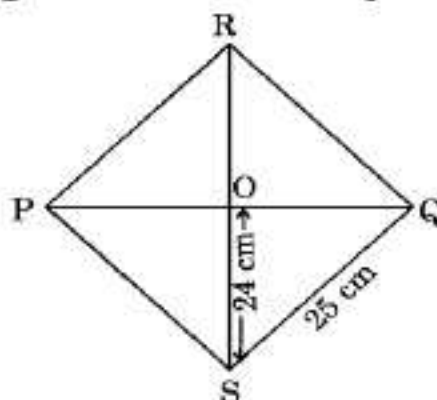
28. ABCD is a square. Find x .



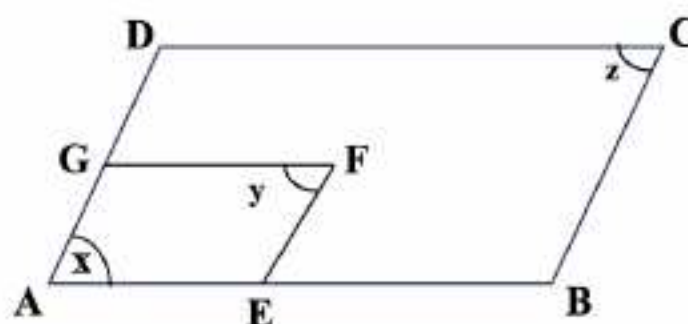
29. D, E and F are the midpoints of sides BC, CA and AB of $\triangle ABC$. If the perimeter of $\triangle ABC$ is 12.8 cm, then find the perimeter of $\triangle DEF$.
30. If the ratio of numerical values of base and area of a parallelogram is 1:9, then find the length of its corresponding altitude.
31. If the difference of two adjacent angles of a parallelogram is 30° , then find the larger angle.
32. If one angle of a parallelogram is 36° less than twice its adjacent angle, then find the smaller angle.
33. In the given figure, ABCD is a parallelogram. If $CM \perp AD$, $AL \perp DC$ and $\angle MPA = 50^\circ$, then find the larger angle of the parallelogram.



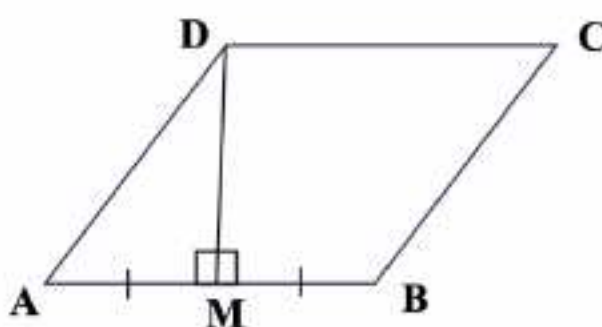
34. Find the sum of lengths of diagonals in rhombus PQRS.



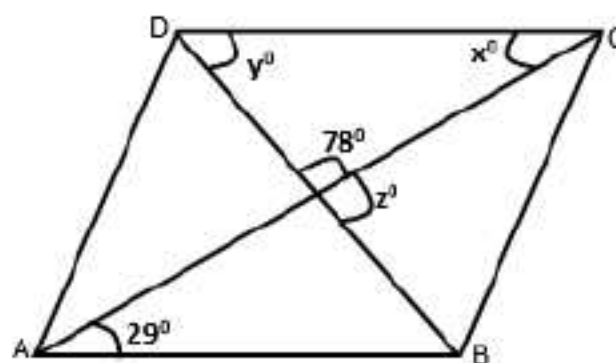
35. ABCD and AEFG are parallelograms, $\angle AGF = 3\angle GAE$. Find $x + y + z$.



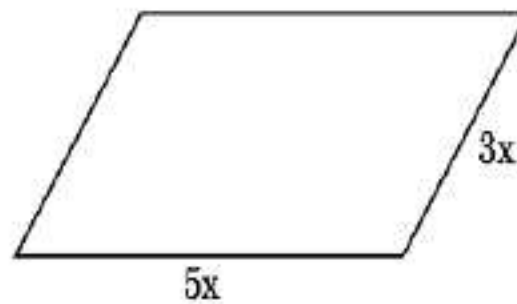
36. ABCD is a rhombus in which altitude DM bisects AB. Find the larger angle of the rhombus.



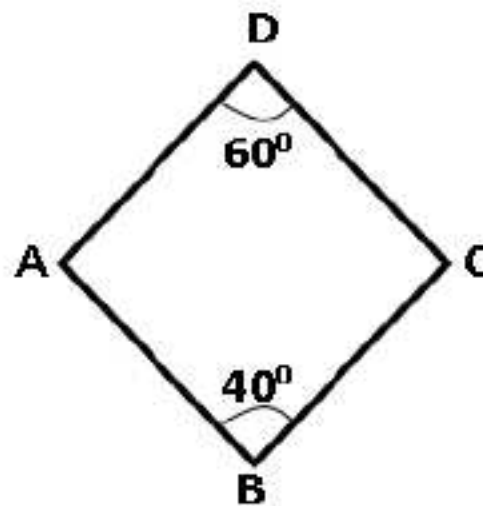
37. In the given figure, ABCD is a parallelogram, find the value of $x + y$.



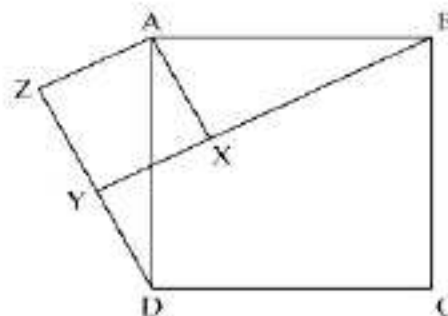
38. Perimeter of a parallelogram is 128m. Find the value of x .



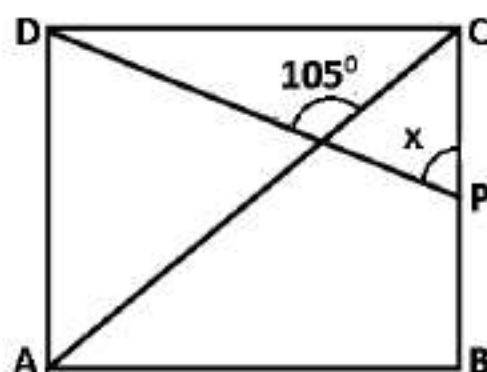
39. In the given figure, if $AD = DC$ and $AB = BC$, then find $2\angle A - \angle C$.



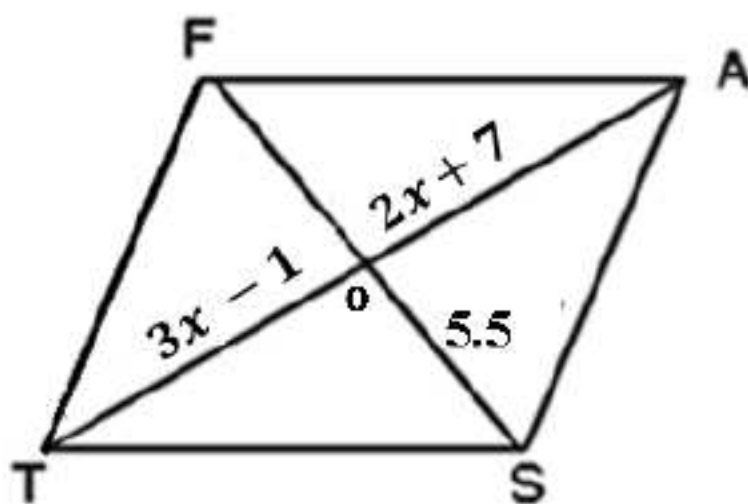
40. The length and breadth of a rectangle are in the ratio $4 : 3$. If the diagonal measures 25 cm, then find the perimeter of the rectangle.
41. Area of rectangle ABCD and parallelogram ABEF are equal in area. If base $AB = 8$ cm and height $BC = 3$ cm. Find the perimeter of parallelogram ABEF, if C is the midpoint of EF.
42. In the given figure, X is a point in the interior of square ABCD. XYZ is also a square. If $DY = 3$ cm and $AZ = 2$ cm, then find BY.



43. In the given figure, if ABCD is a square, then find the value of x .



44. The two parallel lines l and m intersect with the transversal line t' . What type of quadrilateral will be formed by bisectors of internal angles?
45. Find the length of the longest diagonal in the parallelogram FAST.



ANSWERS

Q. No.	Answers	Q. No.	Answers
1	30°	24	7.5 units
2	20°	25	30°
3	0	26	50°
4	120°	27	25 sq cm
5	150°	28	125°
6	120°	29	6.4cm
7	210°	30	9 units
8	20°	31	105°
9	43°	32	72°
10	9cm	33	130°
11	125°	34	62cm
12	90°	35	135°
13	110°	36	120°
14	55°	37	102
15	95°	38	8m
16	180°	39	130°
17	44°	40	70cm
18	21°	41	26cm
19	90°	42	7cm
20	38°	43	60°
21	155°	44	Rectangle
22	36cm	45	46 units
23	37°		

CHAPTER – 9

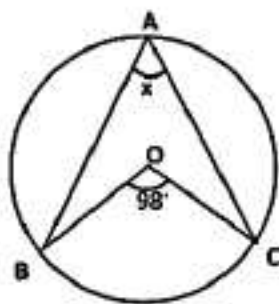
CIRCLES

POINTS TO REMEMBER

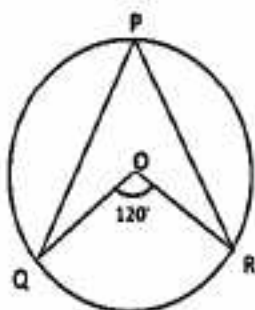
- A circle is a collection of all the points in the plane, which are equidistant from a fixed point in the plane.
- Equal chords of a circle (or of congruent circles) subtend equal angles at the centre.
- If the angles subtended by two chords of a circle (or of congruent circles) at the centre (corresponding centre) are equal, then the chords are equal.
- The perpendicular from the centre of the circle to a chord bisects the chord.
- The line drawn through the centre of the circle to bisect a chord is perpendicular to the chord.
- There is one and only one circle passing through three non-collinear points.
- Equal chords of a circle (or of congruent circles) are equidistant from the centre (or corresponding centres).
- Chords equidistant from the centre (or corresponding centres) of a circle (or of congruent circles) are equal in length.
- If two arcs of a circle are congruent, then their corresponding chords are equal and conversely, if two chords of a circle are equal, then their corresponding arcs (minor/major) are congruent.
- Congruent arcs of the circle subtend equal angles at the centre.
- The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.
- Angle in a semicircle is a right angle.
- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, then the four points lie on the circle.
- The sum of the either pair of opposite angles of a cyclic quadrilateral is 180° .
- If the sum of the pair of opposite angles of a quadrilateral is 180° , then the quadrilateral is cyclic.

QUESTIONS:

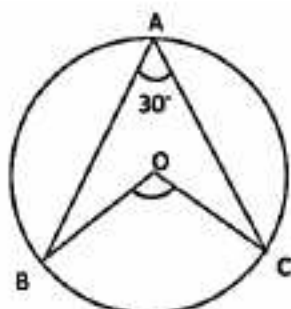
1. In the given figure, O is the centre of the circle. Find the value of x .



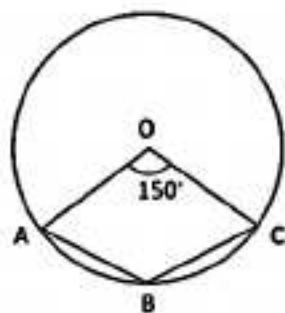
2. In the given figure, O is the centre of the circle. Find the complement of $\angle QPR$.



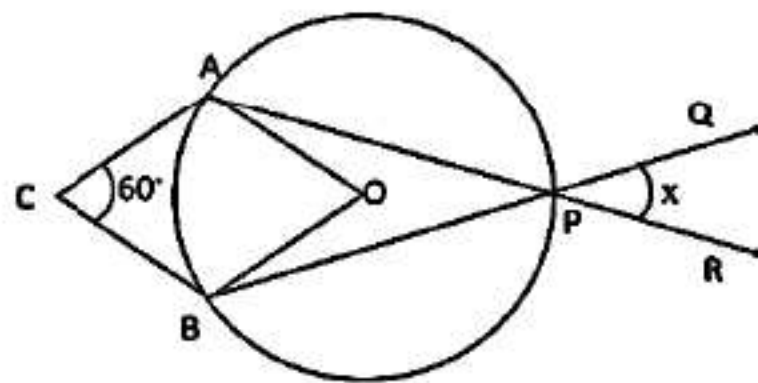
3. In the given figure, O is the centre of the circle. Find the supplement of $\angle BOC$.



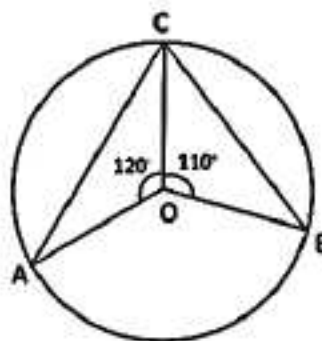
4. ABCD is a cyclic quadrilateral. If $\angle B = 105^\circ$, then find the measure of $\angle D$.
5. In the given figure, O is the centre of circle and $\angle AOC = 150^\circ$. Find $\angle ABC$.



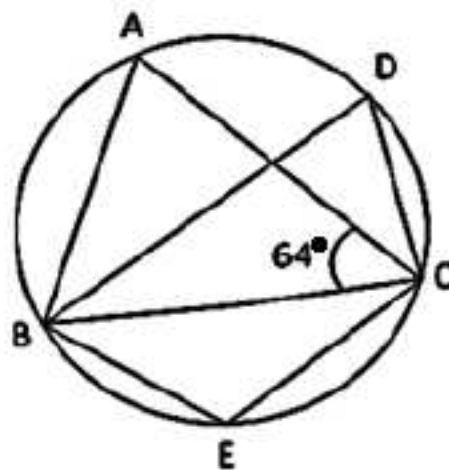
6. In the given figure O is the centre of circle and AOBC is a rhombus. $\angle ACB = 60^\circ$. Find the value of x .



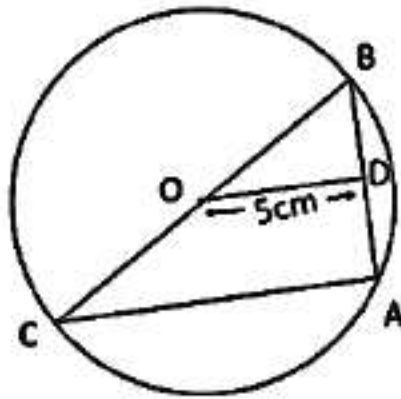
7. In the given figure, O is the centre of the circle. Find the value of $\angle ACB$.



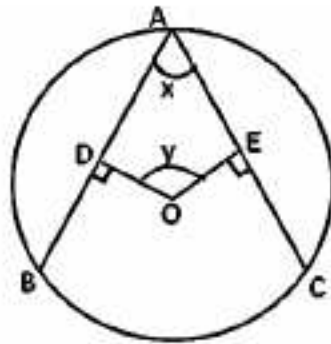
8. Any cyclic parallelogram is called a
9. The radius of a circle is 26 cm and length of the perpendicular from the centre to the chord AB is 10cm. Find the length of AB.
10. In the given figure, $AB = AC$ and $\angle ACB = 64^\circ$. Find the value of $\angle BEC$.



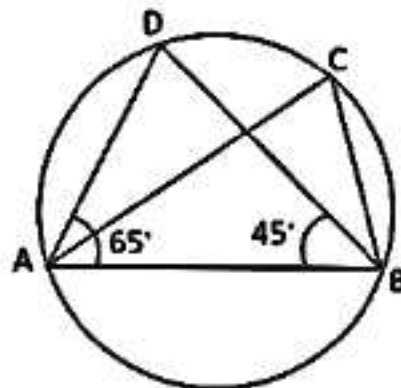
11. In the given figure, $OD \perp AB$. If O is the centre of the circle and $OD = 5\text{cm}$, then find AC.



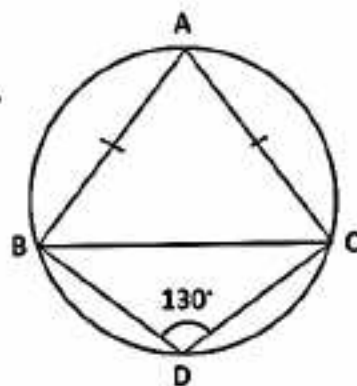
12. AB and AC are two equal chords of a circle with centre O. $OD \perp AB$ and $OE \perp AC$. Find the value of $(x + y)$.



13. In the given figure, if $\angle DAB = 65^\circ$, $\angle ABD = 45^\circ$, then find the value of $\angle ACB + \angle ABD$.

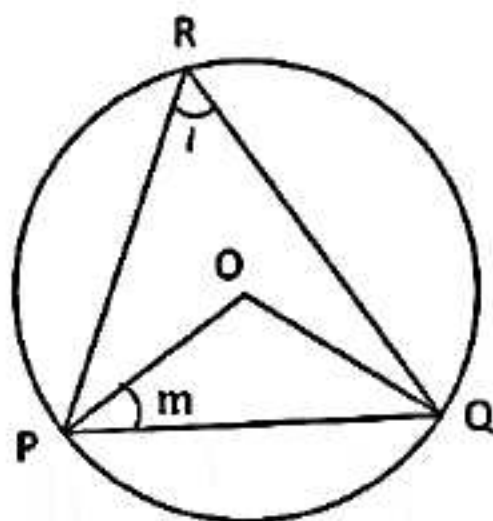


14. In the given figure, $\triangle ABC$ is an isosceles triangle and ABDC is a cyclic quadrilateral, $\angle BDC = 130^\circ$. Find the value of $\angle ABC$.

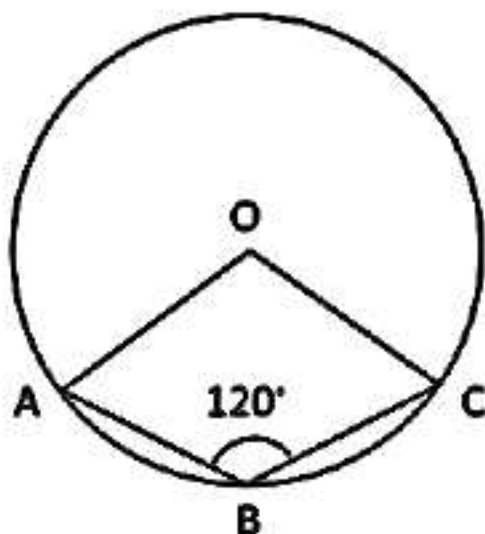


15. In the given figure, O is the centre of the circle. PQ is a chord of the circle and R is any point on the circle. If $\angle PRQ = l$ and $\angle OPQ = m$, then find

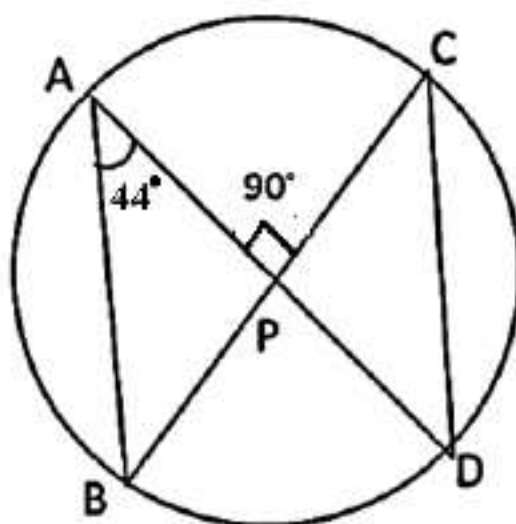
$$l + m.$$



16. If the length of an arc of a circle is proportional to the angle subtended by it at the centre, then find the ratio of length of arc(ABC): circumference.



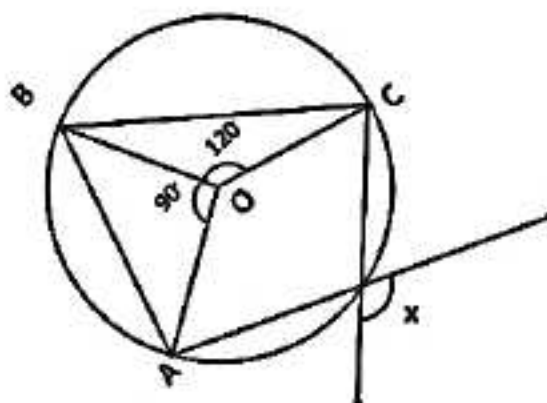
17. Chord AD and BC intersect each other at right angles at point P. If $\angle DAB = 44^\circ$, then find the value of $\angle ADC$.



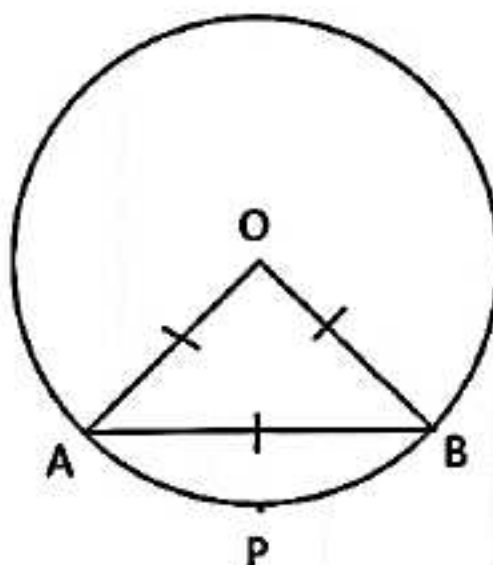
18. Find the radius of the circumcircle of an equilateral triangle of side a units.

19. If A , B and C are three points on a circle with centre O such that

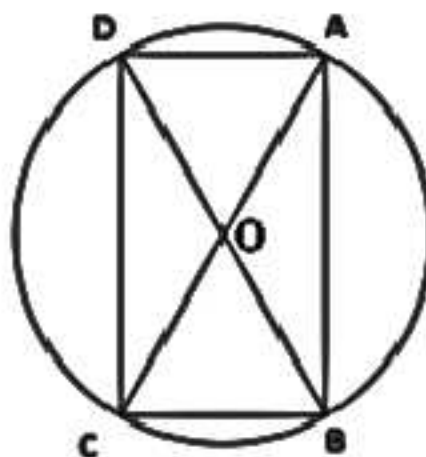
$$\angle AOB = 90^\circ \text{ and } \angle BOC = 120^\circ, \text{ then find } x.$$



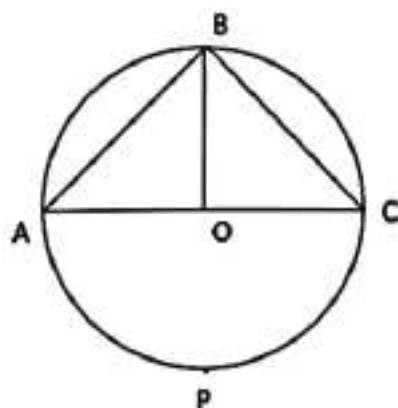
20. The length of the chord of a circle is equal to its radius, find the angle subtended by this chord at the midpoint P on the minor arc of the circle.



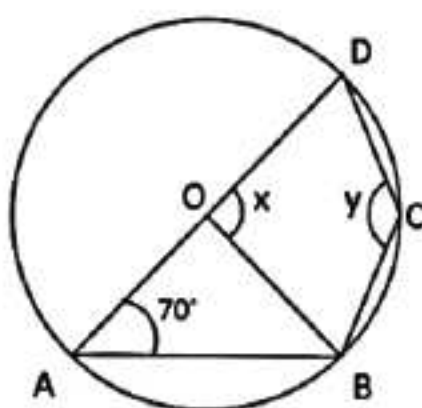
21. O is the centre of circle with $AC = 30\text{cm}$ and $DA = 10\sqrt{5}\text{cm}$, find the length of DC .



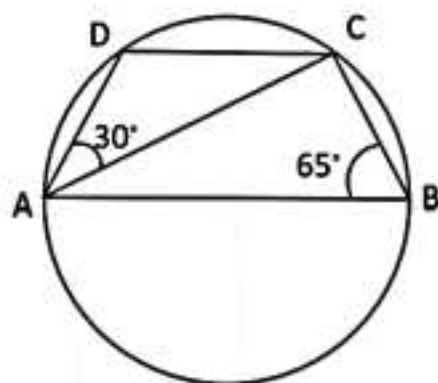
22. In the given figure, if O is the circumcentre of $\triangle ABC$, then find the value of $\angle OBC + \angle BAC$.



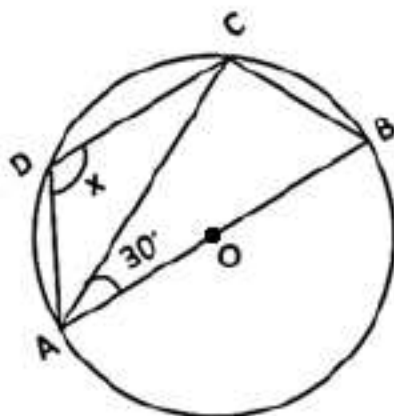
23. In the given figure, find the value of $x + y$.



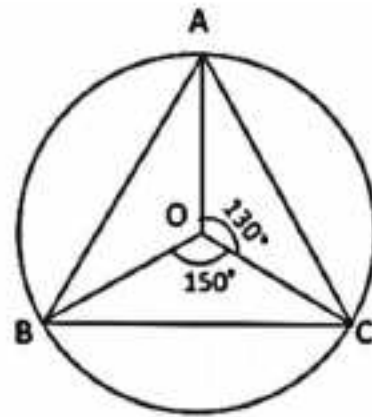
24. In the given figure $AB \parallel CD$, $\angle ABC = 65^\circ$ and $\angle DAC = 30^\circ$. Find the measure of $\angle CAB$.



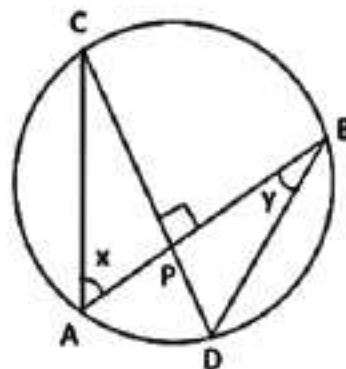
25. Find x , if O is the centre of circle.



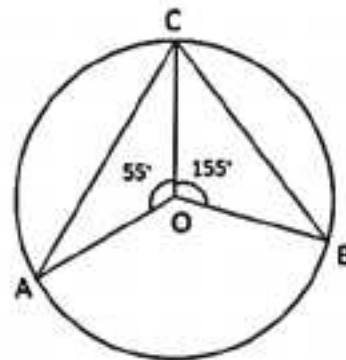
26. $\triangle ABC$ is a triangle inscribed in a circle with centre O. If $\angle AOC = 130^\circ$ and $\angle BOC = 150^\circ$, then find $\angle ACB$.



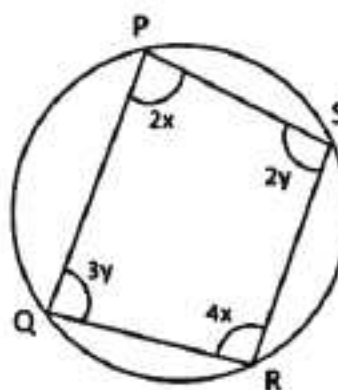
27. In the given figure, if chords AB and CD of the circle intersect each other at right angle, then find the value of $x + y$.



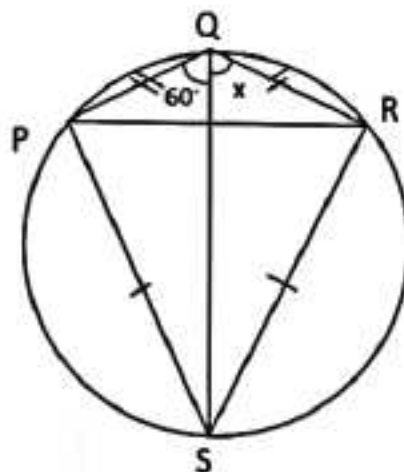
28. In the given figure, angles subtended by chords AC and BC at the centre O of the circle are 55° and 155° respectively. Find $\angle ACB$.



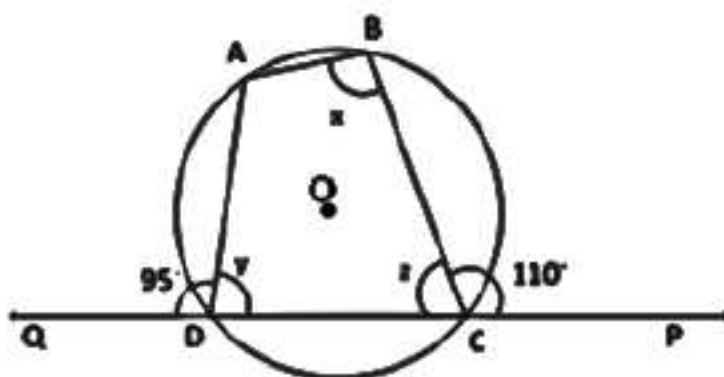
29. In the given figure, if PQRS is a cyclic quadrilateral. Find the ratio of x and y .



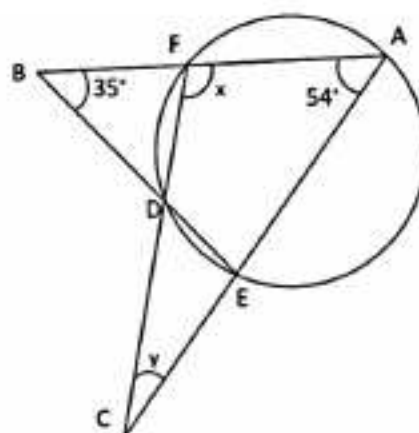
30. In the given figure, PQRS is a cyclic quadrilateral in which $PS = RS$, $\angle SQR = x$ and $\angle PQS = 60^\circ$. Find the value of x .



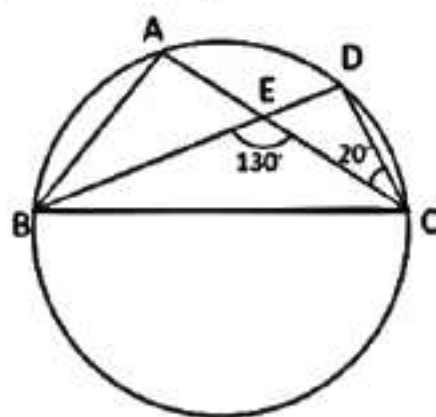
31. In the given figure, O is the centre of the circle. Find the value of $x + y + z$.



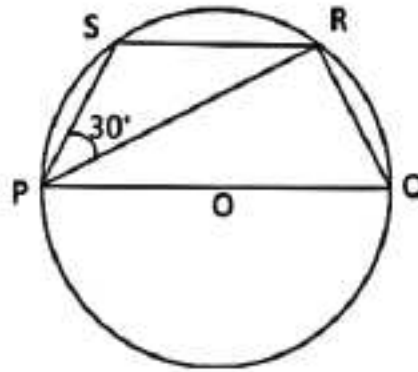
32. In the given figure, AEDF is a cyclic quadrilateral. Find the value of $x + y$.



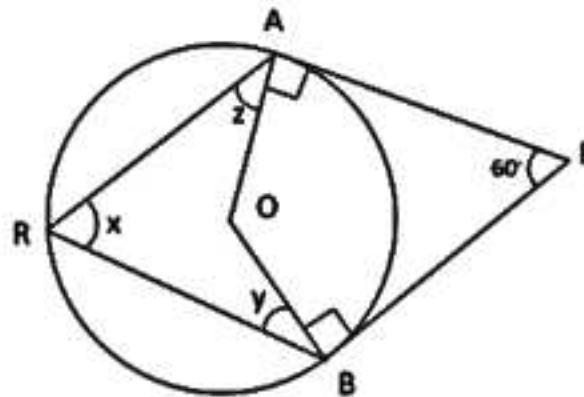
33. A, B, C and D are four point on a circle AC and BD intersect at a point E such that $\angle BEC = 130^\circ$, $\angle ECD = 20^\circ$, then find $\angle BAC$.



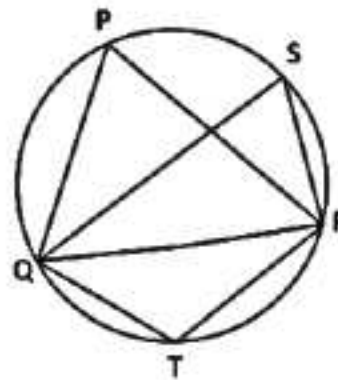
34. In the given figure, O is the centre of the circle. Find the measure of $\angle SPQ$ if $PS = RQ$.



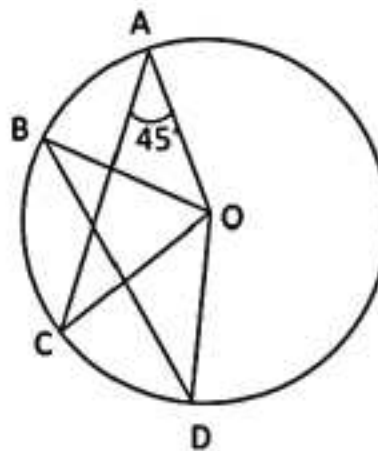
35. Find the value of $y + z$ from the given figure.



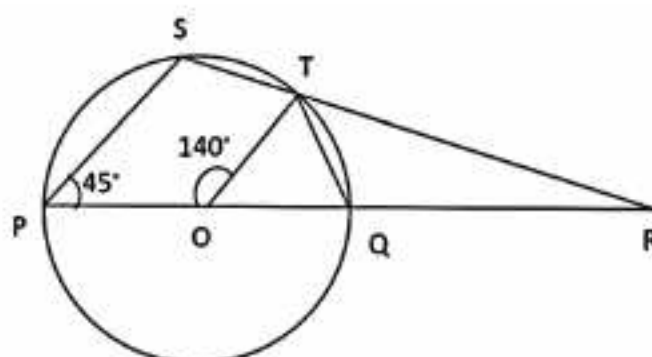
36. In the given figure, if $\triangle PQR$ is an equilateral triangle and $\triangle TQR$ is an isosceles triangle, then find the value of $\angle RQT$.



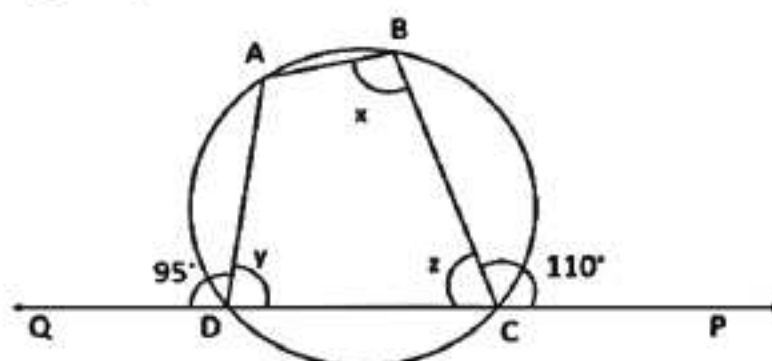
37. In the given figure, if $AC = BD$ and $\angle OAC = 45^\circ$, then find $\angle B$.



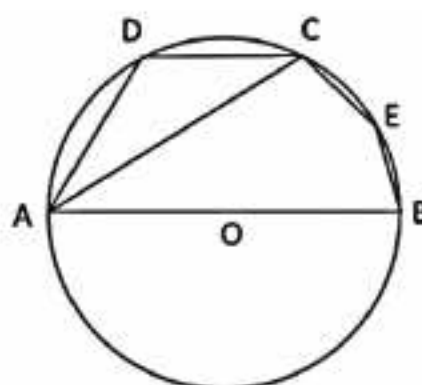
38. In the given figure, find $(\angle RTQ + \angle RQT)$.



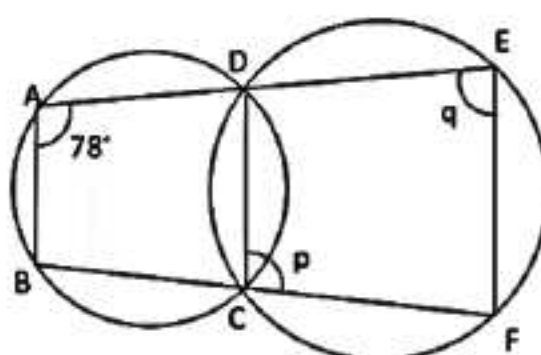
39. In the given figure, ABCD is a cyclic quadrilateral. Side CD is produced to both sides so that $\angle BCP = 110^\circ$ and $\angle ADQ = 95^\circ$. Find the value of $(x + y - z)$.



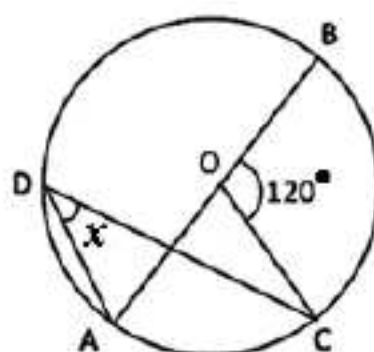
40. In the given figure, AB is the diameter of a circle. If $\angle ADC = 120^\circ$, then find $\angle CAB$.



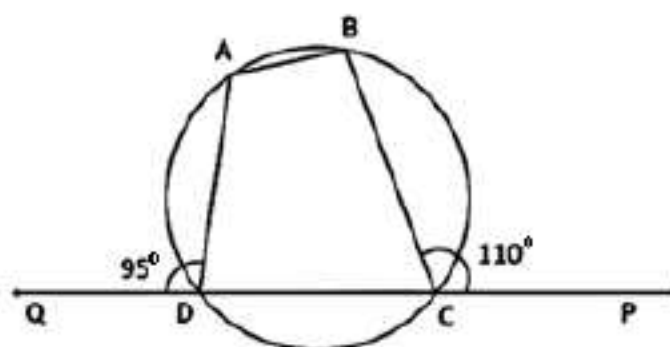
41. In the given figure, if $\angle BAD = 78^\circ$, $\angle DCF = p^\circ$ and $\angle DEF = q^\circ$, then find the value of q .



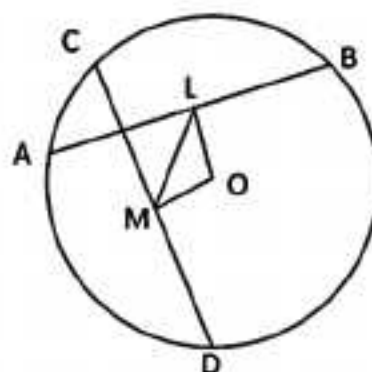
42. In the given figure, O is the centre of the circle. If $\angle BOC = 120^\circ$, then find the value of x .



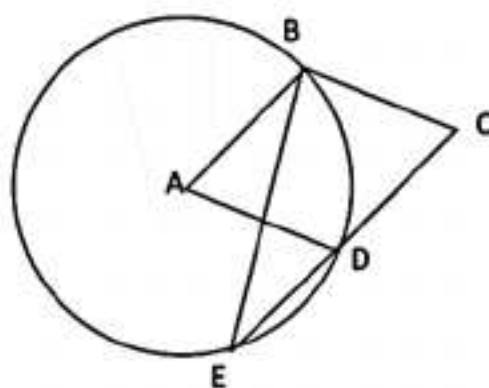
43. In figure, ABCD is a cyclic quadrilateral. Side CD is produced to both sides so that $\angle BCP = 110^\circ$ and $\angle ADQ = 95^\circ$. Find the value of $(\angle A + \angle B)$.



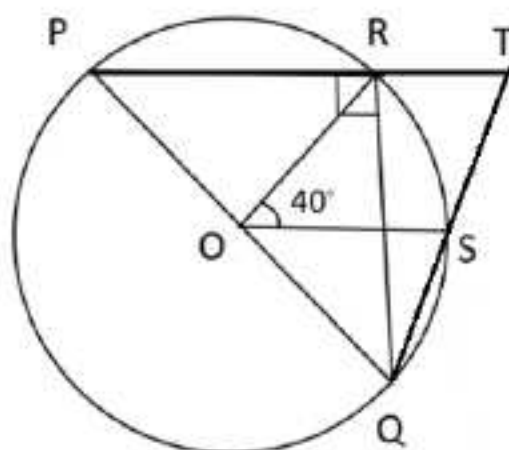
44. The measure of AB and CD are equal and $\angle LOM = 160^\circ$, OL and OM are perpendiculars on the chord from the centre O. Find the measure of $\angle OLM$.



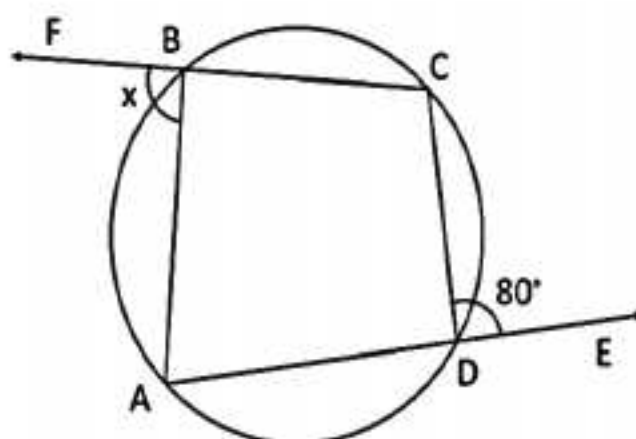
45. A is the centre of circle. ABCD is a parallelogram and CDE is a straight line, find the ratio of $\angle DEB : \angle BCD$.



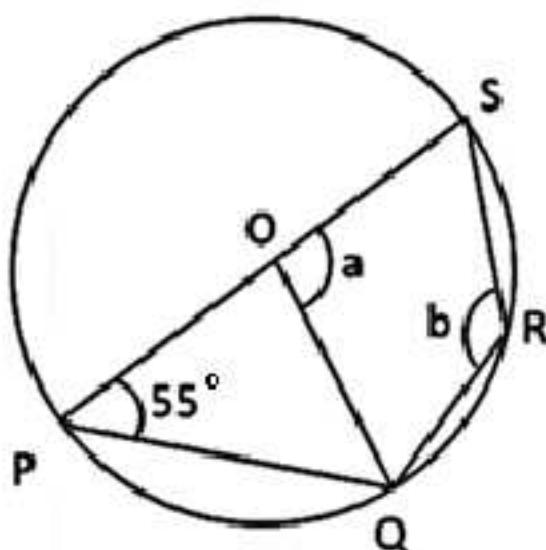
46. In the given figure, O is the centre and PQ is diameter. If $\angle ROS = 40^\circ$, then find the measure of $\angle RTS$.



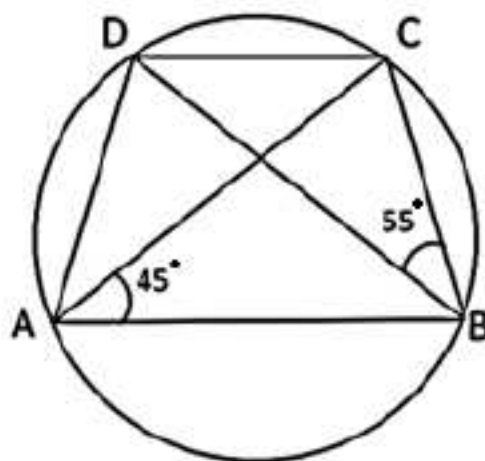
47. In the given figure, ABCD is a cyclic quadrilateral. Find the value of x .



48. In the given figure, O is the centre of the circle and $\angle SPQ = 55^\circ$. Find the values of $a + b$.

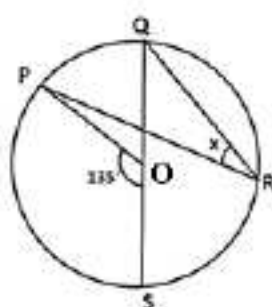


49. In the given figure, ABCD is the cyclic quadrilateral in which AC and BD are its diagonal. If $\angle DBC = 55^\circ$ and $\angle BAC = 45^\circ$, then find $\angle BCD$.

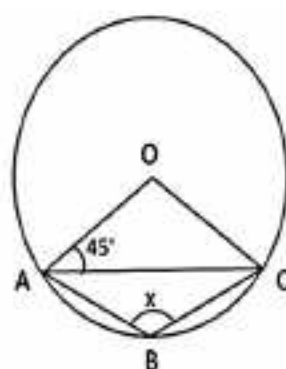


50. If O is the centre of circle, then find the value of x in each of the following figures.

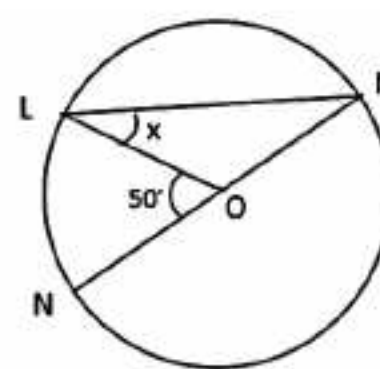
(a)



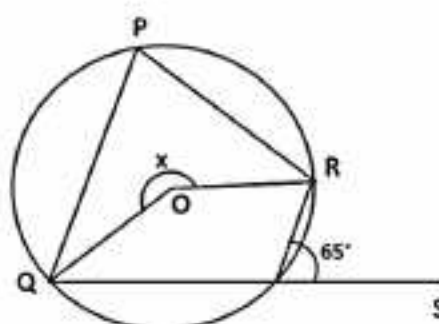
(b)



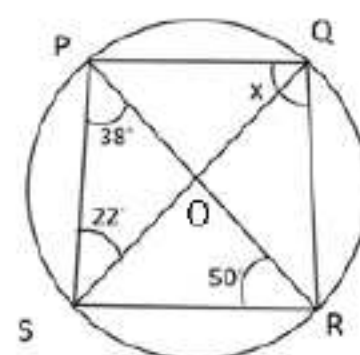
(c)



(d)



(e)



ANSWERS

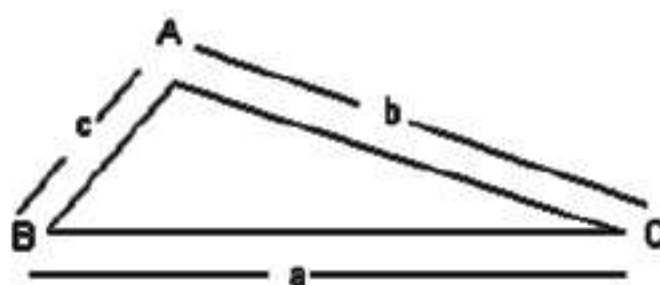
Q.NO.	ANSWERS	Q.NO.	ANSWERS
1.	49°	28.	75°
2.	30°	29.	5:6
3.	120°	30.	60°
4.	75°	31.	250°
5.	105°	32.	126°
6.	30°	33.	110°
7.	65°	34.	60°
8.	Rectangle	35.	60°
9.	48cm	36.	30°
10.	128°	37.	45°
11.	10cm	38.	155°
12.	180°	39.	110°
13.	115°	40.	30°
14.	65°	41.	102°
15.	90°	42.	30°
16.	1:3	43.	205°
17.	46°	44.	10°
18.	$\frac{a}{\sqrt{3}}$ units	45.	1:2
19.	105°	46.	70°
20.	150°	47.	100°
21.	20cm	48.	235°
22.	90°	49.	80°
23.	250°	50	(a) 22.5°
24.	35°		(b) 135°
25.	120°		(c) 25°
26.	40°		(d) 230°
27.	90°		(e) 88°

CHAPTER – 10

HERON'S FORMULA

POINTS TO REMEMBER

- Heron's Formula:



Consider a $\triangle ABC$ in which $BC = a$ units, $CA = b$ units and $AB = c$ units

Then semi-perimeter $(s) = \frac{1}{2}(a + b + c)$

By Heron's Formula, the area of the $\triangle ABC$ is given by

$$\text{ar}(\triangle ABC) = \sqrt{s(s-a)(s-b)(s-c)} \text{ sq units}$$

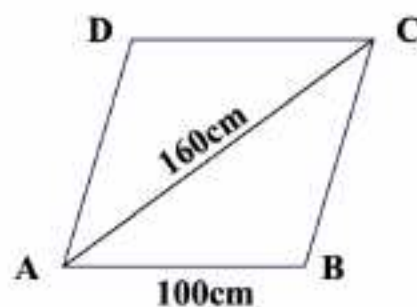
- Application of Heron's Formula:

The area of a Quadrilateral whose sides and one diagonal are given, can be calculated by dividing the quadrilateral into two triangles and using Heron's Formula.

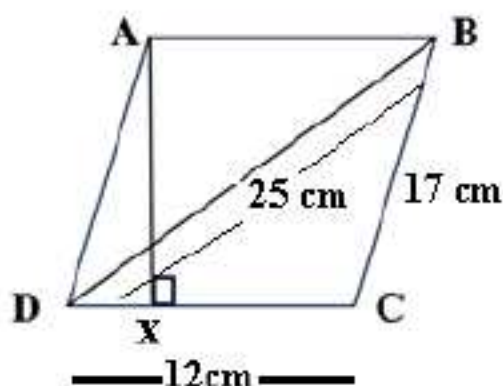
QUESTIONS:

1. The area of an isosceles right triangle is 8 sq cm. Find the length of its hypotenuse.
2. If a side of an equilateral triangle is 8 cm, then find its area.
3. The perimeter of a triangle is 540 m and its sides are in the ratio 25:17:12. Find its area.
4. Find area of the triangle whose two sides are 8 cm and 11 cm and perimeter is 32 cm.
5. The height of an equilateral triangle is 6 cm. Find area of the triangle.

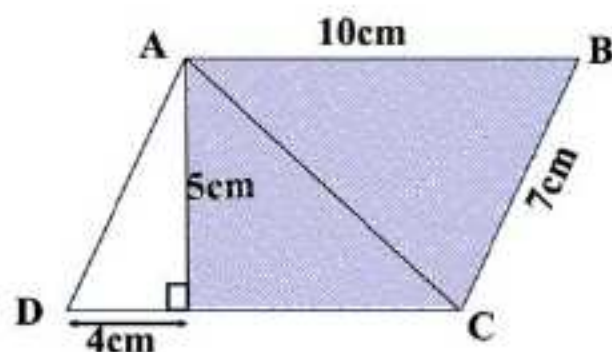
6. The length of the median of an equilateral triangle is $\sqrt{3}$ cm. Find area of the triangle.
7. Find the length of sides of an equilateral triangle, if length of its median is $2\sqrt{3}$ cm.
8. If $s - a = 10\text{cm}$, $s - b = 6\text{cm}$ and $s - c = 5\text{cm}$, then find s .
9. If $s - a = 7\text{cm}$, $s - b = 5\text{cm}$ and $s - c = 3\text{cm}$, find a .
10. The perimeter of an isosceles triangle is 16 cm. The ratio of one of the equal sides to its base is 3 : 2. Find the area of the triangle.
11. Find the area of square having length of diagonal $5\sqrt{2}$ cm.
12. In $\triangle ABC$, $a = 3b = 6c$ and $s = a$, then find the value of a .
13. The base and hypotenuse of a right triangle are 5cm and 13cm respectively. Find the length of altitude from the vertex containing right angle to the hypotenuse.
14. Find area of the rhombus ABCD.



15. In the given parallelogram, find the length of the altitude AX from vertex A on the side DC.



16. Find area of the shaded region.



17. Find the area of an equilateral triangle of side $2a$ cm.

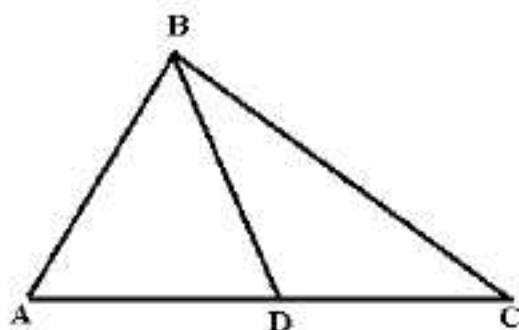
18. A square and an equilateral triangle have same perimeter. If the length of the diagonal of the square is $24\sqrt{2}$ cm, then find the area of the equilateral triangle.

19. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm and the parallelogram stands on the base 28 cm, then find the height of the parallelogram.

20. The area of a parallelogram ABCD in which $AB = 12$ cm, $BC = 9$ cm and diagonal $AC = 15$ cm is k sq cm. Find the value of $(k - 100)$.

21. The area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm, is $k\sqrt{30}$ sq cm. Find the value of k .

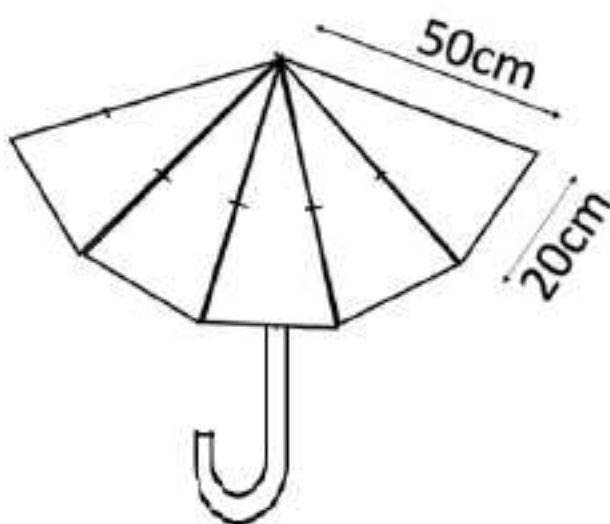
22. In given figure $AD:DC = 3:2$. If the area of $\triangle ABC = 40$ sq cm, then find the area of the triangle $\triangle BDC$.



23. The length of sides of a triangle are in the ratio 3:4:5 and its perimeter is 144 cm. Find the height corresponding to the longest side.

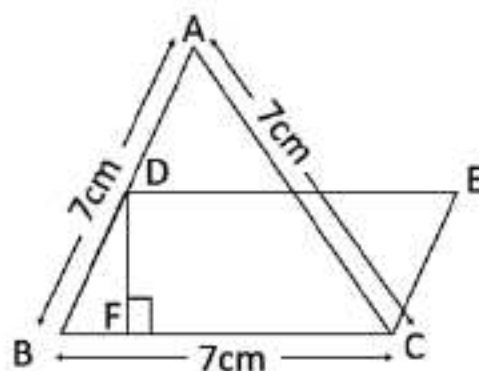
24. If the sides of a triangle are 50 cm, 78 cm and 112 cm, then find the smallest altitude.

25. The sides of a triangle are $AB=17$ cm, $BC=25$ cm, and $CA=28$ cm. Find the length of the altitude drawn from B.
26. The length of sides of a right angled triangle are 5cm, 12cm and 13 cm. Find the length of the shortest altitude.
27. Find the area of the umbrella.



28. A field is in the shape of a trapezium whose parallel sides are 77cm and 60cm. The non-parallel sides are 25cm and 26cm. Find the area of trapezium.
29. A rhombus shaped sheet with perimeter 80cm and one diagonal 24cm, is painted on both sides at the rate of ₹10 per sq cm. Find the total cost of painting.
30. If each side of the triangle is doubled, then find the percentage increase in its area.
31. Find the percentage increase in area of triangle, if its each side is tripled.
32. The difference between the sides containing a right angle in a right angled triangle is 14cm. The area of a triangle is 120 sq. cm. Find the semi-perimeter of the triangle.
33. $\triangle ABC$ is an equilateral triangle of side $4\sqrt{3}$ cm. P, Q and R are the mid-points of AB, CA and BC respectively. Find the area of $\triangle PQR$.
34. Find the area of the quadrilateral ABCD in which $AD = 24$ cm, $\angle BAD = 90^\circ$, $\triangle BCD$ forms an equilateral triangle of side 26cm.

35. In the given figure, $\triangle ABC$ has sides $AB = 7\text{cm}$, $AC = 7\text{cm}$ and $BC = 7\text{cm}$. On the base BC a parallelogram $DBCE$ of same area as that of $\triangle ABC$ is constructed. Find the height DF of the parallelogram.



36. Find the cost of fencing a triangular park having area $20\sqrt{2}$ sq m and two of its sides are 11m and 6m, if the rate of fencing is ₹10/m.
37. Perimeter of the rhombus is 100 m and one of its diagonal is 40m. Find the area of rhombus.
38. How many times will the area increase, if each side of a triangle is doubled?
39. If a square and rhombus have same perimeter and area of the square is S and area of the rhombus is R then, which of the following is true?
- a) $S > R$ b) $R = S$ c) $R > S$ d) data insufficient.
40. If a square and an equilateral triangle have same perimeter and their areas are A_1 and A_2 respectively, then which of the following is true?
- a) $A_1 = A_2$ b) $A_1 > A_2$ c) $A_1 < A_2$ d) $A_2 = \frac{2}{3}A_1$

ANSWERS

Q.NO.	ANSWERS	Q.NO.	ANSWERS
1.	$4\sqrt{2}$ cm	21.	8
2.	$16\sqrt{3}$ sq cm	22.	16 sq cm
3.	9000 sq m	23.	28.8 cm
4.	$8\sqrt{30}$ sq cm	24.	30 cm
5.	$12\sqrt{3}$ sq cm	25.	15 cm
6.	$\sqrt{3}$ sq cm	26.	$4\frac{8}{13}$ cm
7.	4 cm	27.	$2000\sqrt{6}$ sq cm
8.	21 cm	28.	1644 sq cm
9.	8 cm	29.	₹ 7680
10.	$8\sqrt{2}$ sq cm	30.	300%
11.	25 sq cm	31.	800 %
12.	$\frac{3}{4}$	32.	30 cm
13.	$4\frac{8}{13}$ cm	33.	$3\sqrt{3}$ sq cm
14.	9600 sq m	34.	$(120 + 169\sqrt{3})$ sq cm
15.	15 cm	35.	$\frac{7\sqrt{3}}{4}$ cm
16.	40 sq cm	36.	₹320
17.	$\sqrt{3}a^2$ sq units	37.	600 sq m
18.	$256\sqrt{3}$ sq cm	38.	4 times
19.	12 cm	39.	a) $S > R$
20.	8	40.	b) $A_1 > A_2$

Chapter -11

Surface Area and Volumes

Points to remember

1. Units of measurement of Length, Area and Volume :

LENGTH
Myriametre = 10^4 m
Kilometre = 10^3 m
Hectometre = 10^2 m
Decametre = 10 m
Decimetre = 10^{-1} m
Centimetre = 10^{-2} m
Millimetre = 10^{-3} m

AREA
1 sq cm = 100 sq mm
1 sq dm = 100 sq cm
1 sq m = 100 sq dm
1 sq hm = 1 Hectare = 100 sq dam = 10000 sq m
1 sq km = 100 sq hm = 100 hectare

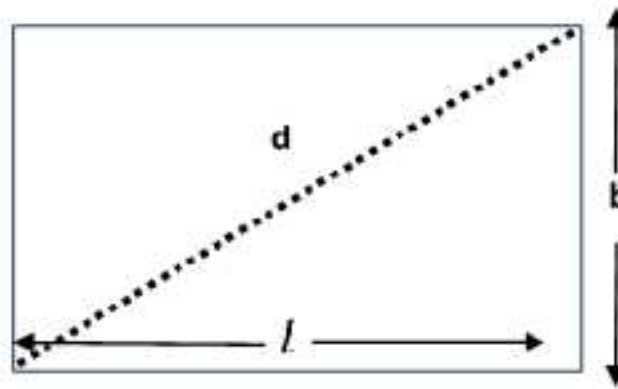
VOLUME
1 cu cm = 1 ml = 1000 cu mm
1 litre = 1000 ml = 1000 cu cm
1 cum = 10^6 cu cm = 1000 l = 1 kl
1 cu dm = 1000 cu cm
1 cu m = 1000 cu dm
1 cu km = 10^9 cum

2. Plane figure :

(i) Area of rectangle = $l \times b$

Perimeter of rectangle = $2(l + b)$

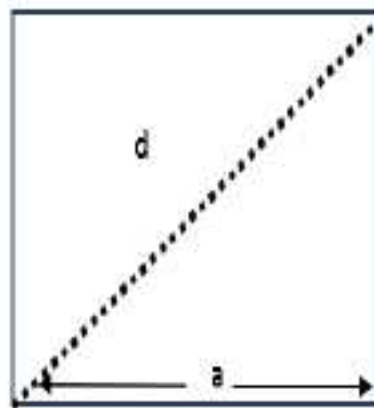
Length of diagonal = $d = \sqrt{l^2 + b^2}$



(ii) Area of square = $(side)^2 = a^2$

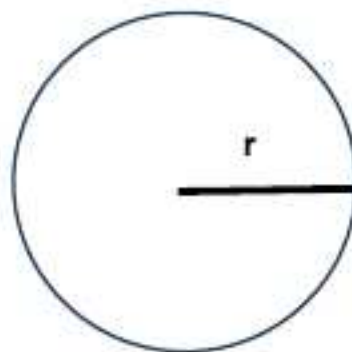
Perimeter of square = $4a$

Length of diagonal = $d = a\sqrt{2}$



(iii) Area of circle = πr^2

Circumference = $2\pi r$



5. Cuboid :

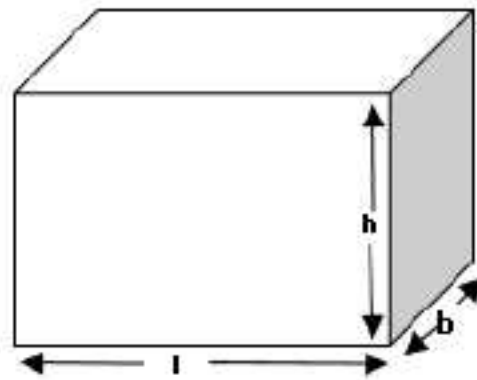
$$\text{Volume of cuboid} = l \times b \times h$$

$$\text{Total Surface area of Cuboid} = 2(lb + bh + hl)$$

$$\text{Lateral Surface area of Cuboid} = 2h(l + b)$$

$$\text{Diagonal of cuboid} = \sqrt{l^2 + b^2 + h^2}$$

$$\text{Area of four walls of room} = 2h(l + b) = (\text{perimeter of base}) \times \text{height}$$



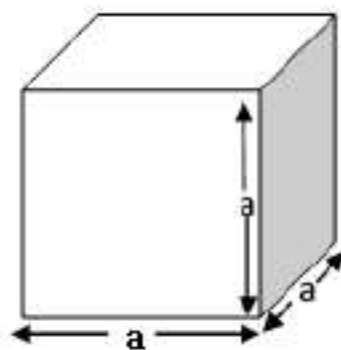
6. Cube :

$$\text{Volume of cube} = a^3$$

$$\text{Total Surface area of Cube} = 6a^2$$

$$\text{Lateral Surface area of Cube} = 4a^2$$

$$\text{Diagonal of cube} = a\sqrt{3}$$



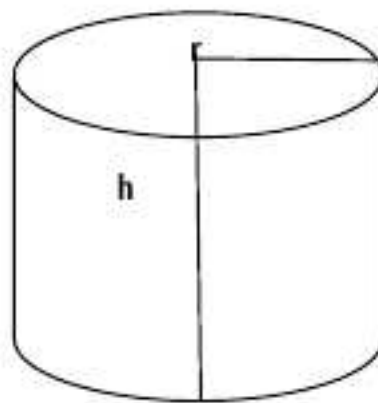
7. Right Circular Cylinder :

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\text{Total Surface area of Cylinder} = 2\pi r(h + r)$$

$$\text{Area of each base} = \text{area of circle} = \pi r^2$$

$$\text{Curved Surface area of Cylinder} = 2\pi r h = (\text{perimeter of base}) \times \text{height}$$



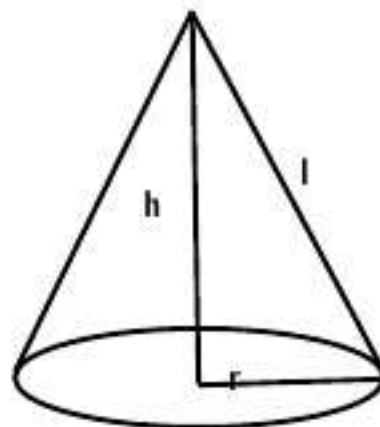
8. Right Circular Cone :

$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Total Surface area of Cone} = \pi r(l + r)$$

$$\text{Area of base} = \text{area of circle} = \pi r^2$$

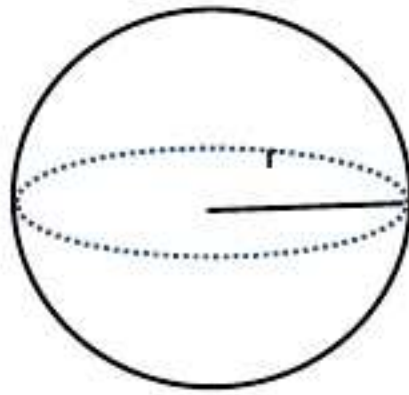
$$\text{Curved Surface area of Cone} = \pi r l$$



9. Sphere :

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$\text{Total surface area of sphere} = 4 \pi r^2$$

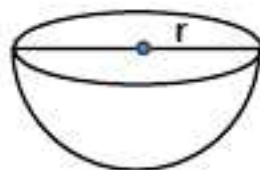


10. Hemi-Sphere :

$$\text{Volume of Hemi-sphere} = \frac{2}{3} \pi r^3$$

$$\text{Total surface area of Hemi-sphere} = 3 \pi r^2$$

$$\text{Curved surface area of Hemi-sphere} = 2 \pi r^2$$



QUESTIONS

1. Find the total surface area of a hemisphere of radius 30 cm.
(in terms of π).
2. A shopkeeper has one Bondi Laddoo of radius 4 cm, with the same material, how many Laddoos each of the radius 2cm can he make ?
3. A cubical pit of side 5 cm is dug in a field, find the surface area of the pit.
4. The curved surface area of cone is 60π sq cm. If slant height of the cone be 8 cm, then find the radius of the base.
5. The total surface area of a cube is 96 sq cm. Find its volume.
6. Find the length of longest rod that can be placed in a cube of side 12 cm.
7. A conical pit of radius 7 cm and height 24 cm is dug in a field. Find the curved surface area of the conical pit.
8. If volume of the two cubes are in the ratio 27:8, then find the ratio of their edge.
9. If we assume the earth to be a sphere of radius 7000 km and if $\frac{3}{4}$ of earth's surface is covered by water, then find the area of land in sq. km.
10. If the volume of cube is 2.197 cu cm, then find the length of its edge.
11. A conical tent is made for students, whose radius of base is 7 cm and height is 24 cm. Find the length of cloth 5m wide required to make conical tent.
12. Find the length of longest rod that can be kept in the room of dimension $5\text{m} \times 4\text{m} \times 3\text{m}$.
13. The length of the diagonal of a cube is $8\sqrt{3}$ cm, find its volume.
14. Four walls and ceiling of a room with dimension $8\text{m} \times 5\text{m} \times 10\text{m}$ are to be painted. Find the area to be painted.
15. Find the difference between the total surface area and lateral surface area of a cube whose edge measures 5.5 cm.
16. How many cubes whose edge measures 3 cm can be formed by melting a cubical block of metal of edge 15 cm?
17. Two cubes have edge 10 m. Their edges have been joined and form a cuboid. Find the surface area of cuboid thus formed.
18. How many solid spheres each of radius $\frac{2}{3}$ cm can be made from a solid

sphere of radius 2 cm?

19. If the volume and surface area of a sphere is numerically same, then find its radius.
20. The surface area of a sphere is 484π sq cm. Find its radius.
21. Four equal cubes have side 5 cm each. They are joined together edge to edge. Find the difference in total surface area of cubes and cuboid thus formed.
22. A boy casted a cone of 4cm height and 27cm radius into a solid sphere. Find the radius of the sphere
23. On a particular day, the rainfall recorded on a terrace 6m long and 5m broad is 15cm. Find the quantity of water collected on the terrace
24. The total surface area of a rectangular block is 2200 sq cm. Find its volume if the dimensions are in ratio 1: 2: 3.
25. Volume of a solid sphere is 36π cu cm. Find its radius.
26. Copper sphere of radius 3 cm is beaten and drawn into wire of diameter 0.6 cm. Find the length of the wire.
27. Curved surface area of a cone is thrice the curved surface area of the other cone. Slant height of second cone is thrice the slant height of first. Find ratio of their radii.
28. The diameters of two cones are equal. If their slants heights are in the ratio 5: 4, then find the ratio of their curved surface areas.
29. The circumference of the base of a 12m high wooden solid cone is 22 m. find the slant height of the cone.
30. If the radius of the base of the right circular cone is '3r' units and its height is equal to the radius of the base, then find its volume. (In the terms of r and π)
31. A cone and a hemisphere are standing on equal bases and have same height. Find the ratio of their volumes.
32. If the curved surface area of a cone and sphere are equal and the radius of their bases is also same then find the ratio of radius of sphere and the slant height of cone.
33. Find the percentage increase /decrease in the volume of a sphere if its radius is doubled.

34. The volumes of two sphere are in the ratio 64:27. Find the ratio of their surface areas.
35. Find the volume of greatest sphere that can be cut off from the cubical log of wood of side 21 cm.
36. If each edge of a cube is increased by 50%, then find the percentage increase in its surface area.
37. Find the maximum volume of a cone that can be carved out of a solid hemi sphere of radius 'r' units.
38. The radii of two cones are in the ratio 2:1 and their volumes are equal. Find the ratio of their heights.
39. Two cones have their heights in the ratio 1:3 and radii in the ratio 3:1. Find the ratio of their volumes.
40. Volume and surface area of a solid hemisphere are numerically equal. Find the diameter of the hemisphere.
41. If a cone and a sphere have equal radii and equal volumes. Find the ratio of the diameter of the sphere to the height of the cone
42. If the heights of two right circular cones are in the ratio 1:2 and the circumference of their bases are in the ratio 3:4, then find the ratio of their volumes.
43. Metallic sphere of radii 6cm, 8cm and 10cm respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.
44. A hemispherical bowl of internal diameter 18 cm contains a liquid. This liquid is to be filled in conical bottles of radius 3 cm and height 6 cm. How many bottles are required to empty the bowl?
45. The radius of the wire is decreased to its one third. Then how many times the length increases, if the volume remains same?
46. If the radius of the base of the right circular cylinder is halved, keeping the same height, then find the ratio of the volume of the new cylinder to the volume of the original cylinder.
47. If the sum of length, breadth and depth of a cuboid is 20 cm and its diagonal is $5\sqrt{3}$ cm, then find its surface area.
48. If the radius of sphere is doubled, then find the ratio of the volume of the first sphere to that of the second.

49. If each edge of the cuboid of surface area ' S ' is doubled, then find the surface area of the new cuboid.
50. If the slant height and the radius of the base of a right circular cone are ' L ' and ' R ' respectively, then find the ratio of the areas of the lateral surface and the base.

ANSWERS

1	2700π sq cm	26	400 cm or 4 m
2	8	27	9:1
3	125 sq cm	28	5:4
4	7.5 cm	29	12.5 m
5	64 cu cm	30	$9\pi r^3$ cu units
6	$12\sqrt{3}$ cm	31	1:2
7	550 sq cm	32	1:4
8	3:2	33	700%
9	154000000 sq km	34	16:9
10	1.3 cm	35	4851
11	110 m	36	125%
12	$5\sqrt{2}$ m	37	$\frac{\pi}{3} r^3$ cu units
13	512 cu cm	38	1:4
14	300 sq m	39	3:1
15	60.5 sq cm	40	6 units
16	125	41	1:2
17	1000 sq cm	42	9:32
18	27	43	12 cm
19	3 unit	44	27
20	11 cm	45	9 times
21	150 sq cm	46	1:4
22	9 cm	47	275 sq cm
23	4.5 cu m	48	1:8
24	6000 cu cm	49	4 S
25	3 cm	50	L:R

CHAPTER –12

STATISTICS

POINTS TO REMEMBER

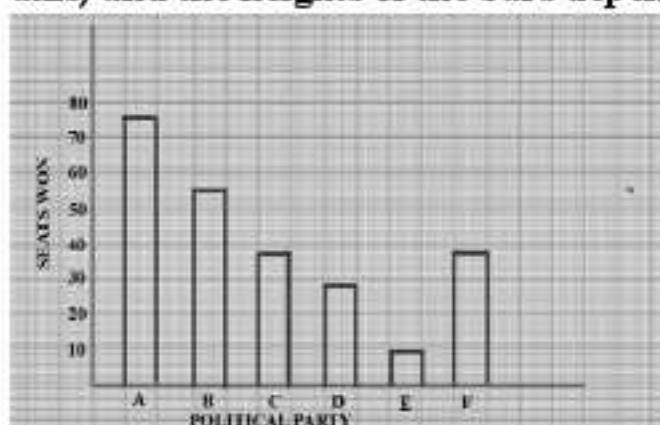
We shall study the following graphical representations in this section.

(A) Bar graphs

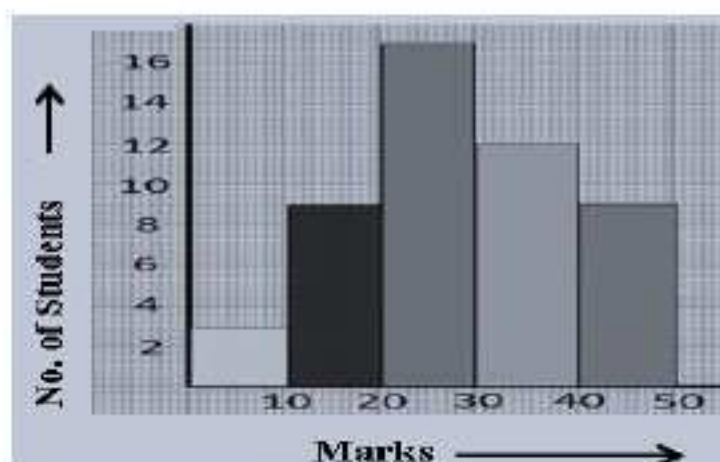
(B) Histograms

(C) Frequency polygons

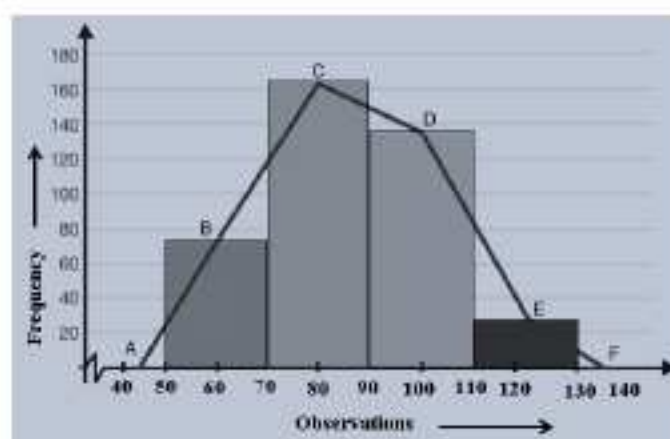
(A) Bar graphs :- A bar graph is a pictorial representation of data in which usually bars of uniform width are drawn with equal spacing between them on one axis (say x-axis), depicting the variable. The values of the variable are shown on the other axis (say y-axis) and the heights of the bars depend on the values of the variable.



(B) Histogram :- This is a form of representation like the bar graph, but it is used for continuous class intervals.



(C) Frequency Polygon: A frequency polygon is a line graph of class frequency plotted against class mid-point. It can be obtained by joining the midpoints of the tops of the rectangles in the histogram.



Range = Highest value – Lowest value

$$\text{Class mark} = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Measures of Central Tendency for ungrouped data

Mean: Mean is calculated by adding all the values of the observations and dividing it by the total number of observations, this is denoted by \bar{X} also Known as average.

$$\text{Mean} = \frac{\text{Sum of all the observations}}{\text{Total number of observation}}$$

If $x_1, x_2, x_3, x_4, x_5, \dots, x_n$ are n observations, then

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + \dots + x_n}{n}$$

Symbolic form

$$\bar{X} = \frac{\sum_{i=1}^n x_i}{n}$$

For Grouped data

$$\bar{X} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$$

Median: Median is that observation which divides given data (arranged in ascending or descending order) into exactly two equal parts.

Let n be the number of observations in the given data

Case 1: when n is odd

Median = value of $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation

Case 2: when n is even

Median = value of $\frac{1}{2} \left[\left(\frac{n}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ observation} \right]$

Mode: Mode is that value of the observation which occurs most frequently (Whose frequency is maximum).

- Trial – Trial is an action which results in one or more outcomes.

Example:- (i) Tossing a coin

(ii) Rolling a die

- Probability of an event E is given by.

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

- The probability of an event always lies from 0 to 1.

$$0 \leq P(E) \leq 1$$

- If $P(E) = 1$, then E is a sure event.
- If $P(E) = 0$, then E is an impossible event.
- $P(E) + P(\text{Not } E) = 1$

Or

$$P(E) + P(\bar{E}) = 1$$

QUESTIONS:

1. Find the class mark of the interval 12 – 17.
2. Find the sum of upper class limit of the interval 10 – 20 and the lower class limit of the interval 30 – 40.
3. If 5, 15, 25, 35, 45, 55 are the class marks of the given class intervals, then find the class limits corresponding to class mark 35.
4. Following table shows the marks obtained by the students.

Marks	Number of students
1 - 4	6
4 - 6	30
6 - 8	44
8 - 12	16
12 - 20	4

Find the adjusted frequency of the class interval 8-12

5. Following table shows the marks obtained by the students.

Marks	Number of students
0 - 9	5
10 - 19	12
20 - 29	30
30 - 39	18
40 - 49	15

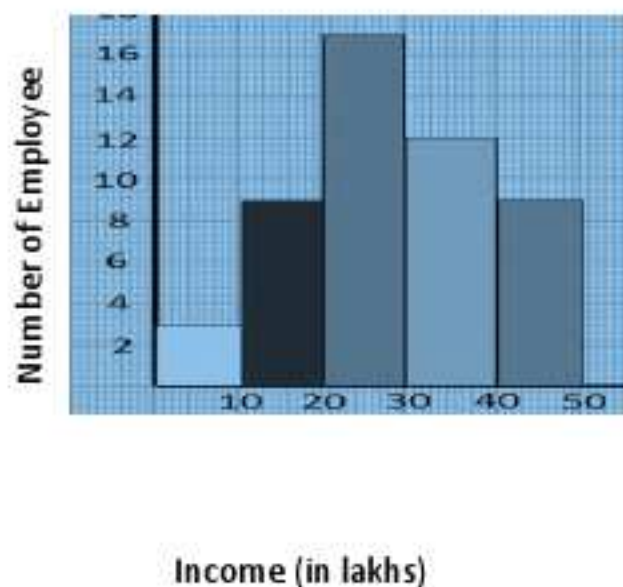
Find the true class limit of the class interval 20-29.

6. Following table shows the marks obtained by students

Marks	Number of students
0 - 10	5
10 - 20	12
20 - 30	30
30 - 40	18
40 - 50	15

Find the number of students who obtained more than 30 marks.

7. In the above table, find the number of students who obtained less than 30 marks.
8. Histogram shows the yearly income (in lakhs) of persons in a locality.



Find the number of people who earns more than 30 lakhs

9. From the above graph, find the income interval in which maximum number of people belongs.
10. From the above graph, find the number of people belonging to higher income interval.
11. Find the mean of all the factors of 8.
12. Find the median of prime numbers lying between 20 and 50.
13. Find the mean of first 10 whole numbers.
14. Find the mean of all odd numbers between 90 and 100.
15. Find the median of all prime numbers from 1 to 11.
16. Find the mean of the squares of first six natural numbers.
17. Find the mean of the cubes of first five natural numbers.
18. If the mean of data $x, x + 1, x + 3, x + 6$ is 7, then find the value of x .
19. Find the mean of first twelve odd natural numbers.
20. The mean of eight numbers is 25. If each number is multiplied by 4, then find the new mean.
21. Find the mean of all the factors of 24.
22. For what value of x , the mode for the following data is 17?

15, 16, 17, 14, 17, 16, 13, $15, \frac{(x-1)}{2}, 17, 16, 15$.

23. In a class the average score of girls in an examination is 80 and that of boys is 70. The average score of the whole class is 74. Find the ratio of girls and boys in the class.
24. Find the range of the data: 25.7, 16.3, 2.8, 21.7, 24.3, 22.7, 24.9.
25. Find the median of 23, 30, 37, 27, 47, 46, 24, 31.
26. The mean of five numbers is 18. If one number is excluded, their mean is 16. Find the excluded number.
27. Find the median of all prime numbers less than 25.
28. If a data is such that its maximum value is 29 and range is 20, then find its minimum value.
29. Find the difference between mean and median of first five multiples of 5.
30. If the median of x , $x + 2$, $x + 4$, $x + 6$, $x + 8$ is 24, then find $2x - 3$.
31. If the mean of 5 observations is 30 and each observation is divided by 6, then find the new mean.
32. Find the mode of following data:
15, 14, 19, 20, 14, 15, 16, 14, 15, 18, 14, 19, 15, 17, 15.
33. If the mean of the data 6, 8, 10, 3, 7 and m is 7, then find the value of $\frac{m}{2}$.
34. A cricketer has a mean score of 60 runs in eight innings. Find out how many runs are to be scored by him in ninth innings to raise his mean score to 68?
35. The mean of 20 numbers is 18. If 3 is added to each of the first ten numbers, then find the mean of new set of twenty numbers.
36. The mean of five numbers is 30. If one number is excluded their mean becomes 28. Find the excluded number.
37. The mean of 100 observations is 50. If one of the observation which was 50 is replaced by 150, then find the value of resulting mean.
38. Out of total of 20 observations arranged in ascending order, 10th and 11th observations are 41 and 43, find the median.
39. Find the median of the following data: 41, 43, 127, 99, 61, 92, 71, 58, 57. If 58 is replaced by 85, then find the new median.
40. 25, 35, 45, 55, 65, 75 are the class marks of a given frequency table. Find the class size.

41. A survey of families of two children is being conducted.

No. of families	No. of boys
25	0
50	1
75	2

A family is chosen at random. Find the probability that it has one girl child.

42. Find the probability of getting 53 Sundays in a non-leap year.
43. Find the probability of getting 53 Sundays in a leap year.
44. A letter is chosen from the word INDIA. Find the probability of getting the letter S.
45. A letter is chosen from the word 'MATHEMATICS'. Find the probability that it is a consonant.
46. A die is rolled once. Find the probability of getting
- a. A multiple of 2.
 - b. A prime number.
 - c. An odd number.
 - d. A number greater than 4.
 - e. Neither 4 nor 5.
47. From a pack of 52 playing cards, a card is drawn at random. Find the probability of getting
- a. A black king.
 - b. A king or a queen.
 - c. A red ace.
 - d. A '2' of club.
 - e. A '10' of red color.

48. A pair of coins is tossed once. Find the probability of getting

- a. Two heads.
- b. One tail.
- c. At least one head.
- d. At most one tail.

49. In a bag there are 5 white, 6 black and 3 green balls. If a ball is drawn at random, then find the probability that it is not green.

50. If $P(E) = 0.023$, then find $P(\text{not } E)$.

ANSWERS

1.	14.5	26.	26
2.	50	27.	11
3.	30 – 40	28.	9
4.	8	29.	0
5.	19.5-29.5	30.	37
6.	33 students	31.	5
7.	47 students	32.	15
8.	21 persons	33.	4
9.	20 – 30 lakhs	34.	132
10.	9 persons	35.	19.5
11.	$\frac{15}{4}$ or 3.75	36.	38
12.	37	37.	51
13.	4.5	38.	42
14.	95	39.	71
15.	5	40.	10
16.	15.16	41.	$\frac{1}{3}$
17.	45	42.	$\frac{1}{7}$
18.	1	43.	$\frac{2}{7}$
19.	12	44.	0
20.	100	45.	$\frac{7}{11}$
21.	7.5	46.	a. $\frac{1}{2}$ b. $\frac{1}{2}$ c. $\frac{1}{2}$ d. $\frac{1}{3}$ e. $\frac{2}{3}$
22.	35	47.	a. $\frac{1}{26}$ b. $\frac{2}{13}$ c. $\frac{1}{26}$ d. $\frac{1}{52}$ e. $\frac{1}{26}$
23.	2:3	48.	a. $\frac{1}{4}$ b. $\frac{1}{2}$ c. $\frac{3}{4}$ d. $\frac{3}{4}$
24.	22.9	49.	$\frac{11}{14}$
25.	30.5	50.	0.977



पढ़े चलो बढ़े चलो