Directorate of Education, G.N.C.T of Delhi

Subject: Mathematics Worksheet No.32 Dated: 16.09.2020

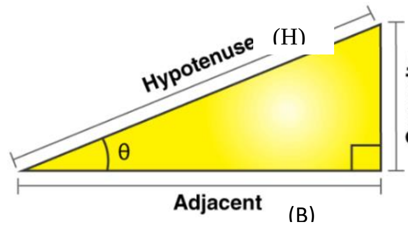
Class: X Student's Name:_________________ Teacher's Name:_________________

PROBLEMS BASED ON TRIGONOMETRIC RATIOS

Let's Recall

Six trigonometric ratios--

Trigonometric ratios	Abbreviations	Relationship with sides of a right triangle
Sine	Sin	Opposite side/ Hypotenuse (P/H)
Cosine	Cos	Adjacent side / Hypotenuse (B/H)
Tangent	Tan	Opposite side/Adjacent side (P/B)
Cosecant	Cosec	Hypotenuse / Opposite side (H/P)
Secant	Sec	Hypotenuse / Adjacent side (H/B)
Cotangent	cot	Adjacent side/Opposite side (B/P)



Opposite Side- side opposite to the mentioned angle.

Adjacent Side- Side adjacent to mentioned angle

Sin $\theta = \frac{1}{\text{Cosec } \theta}$ Cosec $\theta = \frac{1}{\text{Sin } \theta}$

Cos $\theta = \frac{1}{\text{Sec } \theta}$ Sec $\theta = \frac{1}{\text{Cos } \theta}$

Tan $\theta = \frac{1}{\text{Cot } \theta} = \frac{\text{Sin } \theta}{\text{Cos } \theta}$ Cot $\theta = \frac{1}{\text{Tan } \theta} = \frac{\text{Cos } \theta}{\text{Sin } \theta}$

Example1:- If 7tan Φ = 4, then find the value of $\frac{7\sin\Phi - 3\cos\Phi}{7\sin\Phi + 3\cos\Phi}$.

Solution:- 7tan Φ = 4

⇨ tan Φ=4/7

Now consider, $\frac{7\sin\Phi - 3\cos\Phi}{7\sin\Phi + 3\cos\Phi}$

Divide numerator and denominator by cosΦ,

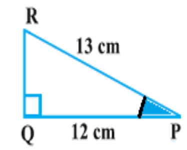
$\frac{7\sin\Phi/\cos\Phi - 3\cos\Phi/\cos\Phi}{7\sin\Phi/\cos\Phi + 3\cos\Phi/\cos\Phi}$

⇨ $\frac{7\tan\Phi - 3}{7\tan\Phi + 3}$ => $\frac{7*4/7 - 3}{7*4/7 + 3}$ => $\frac{4-3}{4+3}$

⇨ $\frac{7\sin\Phi - 3\cos\Phi}{7\sin\Phi + 3\cos\Phi} = \frac{1}{7}$

Example2:- In given fig. Find tan P – cot R,

Solution: In the given triangle PQR, the given triangle is right angled at Q and



PR = 13cm,

PQ = 12cm

Since the given triangle is right angled triangle,

to find the side QR, apply the Pythagoras theorem

In a right- angled triangle,

$PR^2 = QR^2 + PQ^2$

Substitute the values of PR and PQ

$13^2 = QR^2+12^2$

$169 = QR^2+144$

Therefore, $QR^2 = 169−144$

$QR^2 = 25$

$QR = \sqrt{25} = 5$

Therefore, the side QR = 5 cm

To find tan P – cot R:

tan (P) = Opposite side/Adjacent side = QR/PQ = 5/12

Cot (R) = Adjacent side/Opposite side = QR/PQ = 5/12

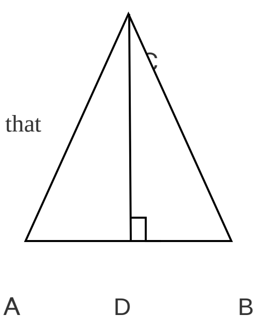
Therefore,

tan (P) – cot (R) = 5/12 – 5/12 = 0

Therefore, tan(P) – cot(R) = 0

Example3:. If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠ A = ∠ B.

Solution:



Let us assume the triangle **ABC** in which CD⊥AB

Give that the angles A and B are acute angles, such that

Cos (A) = Cos (B) ....(∗)

Cos (A)= AD/AC and Cos (B)= BD/BC

from (∗) AD/AC = BD/BC

Now, interchange the terms, we get

AD/BD = AC/BC

Let take a constant value

AD/BD = AC/BC = k …………(**)

Now consider the equation as

AD = k BD … (1)

AC = k BC … (2)

By applying Pythagoras theorem in ΔCAD and ΔCBD we get,

$CD^2 = BC^2 – BD^2$ … (3)

$CD^2 = AC^2 − AD^2$ ….(4)

From the equations (3) and (4) we get,

$AC^2−AD^2 = BC^2−BD^2$

Now substitute the equations (1) and (2) in (3) and (4)

$k^2(BC^2−BD^2)=(BC^2−BD^2)$ => $k^2=1$ =>k=1

Putting this value in equation (**), we obtain

AC = BC

∠A=∠B (Angles opposite to equal side are equal-isosceles triangle)

TRY YOURSELF -

1. If ∠ B and ∠ Q are acute angles such that sin B = sinQ, then prove that ∠ B = ∠ Q.

2. If cot θ = 7/8, evaluate :

(i) $\frac{(1 + \sin\theta)(1 – \sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

3. Consider Δ ACB, right-angled at C, in which AB = 29 units, BC = 21 units and ∠ ABC = θ Determine the values of (i) $\cos^2\theta + \sin^2\theta$, (ii) $\cos^2\theta – \sin^2\theta$.