

Problems based on trigonometric ratios of 30° , 45° and 60°

Recall and Answer

Example 1: Evaluate

$$\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$$

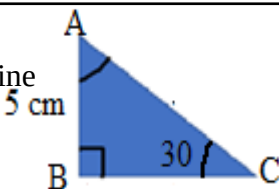
Solution: $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$\angle A$	30°	45°	60°
$\sin A$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos A$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	?
$\tan A$	$\frac{1}{\sqrt{3}}$	?	?

Example 2: In $\triangle ABC$, $\angle B = 90^\circ$,

$AB = 5\text{cm}$ and $\angle ACB = 30^\circ$. Determine the length of the sides BC and AC .



Solution: $\frac{AB}{BC} = \tan C$

$$\Rightarrow \frac{5}{BC} = \tan 30^\circ \Rightarrow \frac{1}{\sqrt{3}} \text{ (By trigonometric ratio)}$$

$$\Rightarrow BC = 5\sqrt{3} \text{ cm,} \quad \text{Again } \sin C = \frac{AB}{AC}$$

$$\Rightarrow \sin 30^\circ = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{5}{AC} \Rightarrow AC = 10 \text{ cm}$$

Example 3: If $\tan(A+B) = \sqrt{3}$ and

$$\tan(A-B) = \frac{1}{\sqrt{3}}; 0^\circ < A+B \leq 90^\circ,$$

$A > B$, find A and B .

Solution: Since $\tan(A+B) = \sqrt{3} = \tan 60^\circ$

$$\Rightarrow A+B = 60^\circ \text{-----(1)}$$

$$\text{Also } \tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow A-B = 30^\circ \text{-----(2)}$$

Solving (1) and (2) we get

$$A = 45^\circ \text{ and } B = 15^\circ.$$

Example 4: Evaluate

$$\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$\text{Solution: } \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ} = \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{3}{2}} = \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+3\sqrt{3}}{2\sqrt{3}}} = \frac{(3\sqrt{3}-4)(3\sqrt{3}-4)}{(3\sqrt{3}+4)(3\sqrt{3}-4)} = \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

Example 5: Choose the correct option and justify your choice $2 \tan 30^\circ / (1 - \tan^2 30^\circ) =$

(a) $\cos 60^\circ$ (b) $\sin 60^\circ$ (c) $\tan 60^\circ$ (d) $\sin 30^\circ$

Solution: (c) $\tan 60^\circ$

$$\text{Justification: } 2 \tan 30^\circ / (1 - \tan^2 30^\circ) = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3} = \tan 60^\circ$$

Try Yourself

Q 1. If $\sin(A-B) = \frac{1}{2}$ and $\cos(A+B) = \frac{1}{2}$,
 $0^\circ < A+B \leq 90^\circ$, find A and B .

Q 2. Evaluate

$$\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$$

Q 3. Evaluate

$$2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

Q 4. Choose the

correct option and

justify your choice

$$2 \tan 30^\circ / 1 + \tan^2 30^\circ =$$

(a) $\sin 60^\circ$

(b) $\cos 60^\circ$

(c) $\tan 60^\circ$

(d) $\sin 30^\circ$