

Trigonometric identities (Part I) ($\sin^2\theta + \cos^2\theta = 1$)**Recall and Answer --**

1. $(\sin 30^\circ + \cos 30^\circ) - (\sin 60^\circ + \cos 60^\circ) =$ _____
2. $\sin^2 30^\circ + \cos^2 30^\circ =$ _____
3. If $\sin A = 1/2$ and $\cos A = \sqrt{3}/2$, then $\sin^2 A + \cos^2 A =$ _____



-Did u notice something about the value of $\sin^2\theta + \cos^2\theta$?
- Today we will learn about Trigonometric Identities?
-absolutely correct!

- Means, an equation with one or more trigonometric ratios and satisfied by all possible values of angles.

**First Identity-**

$\sin^2\theta + \cos^2\theta = 1$ which can be rewritten as $\Rightarrow 1 - \sin^2\theta = \cos^2\theta$ (by moving $\sin^2\theta$ to the other side of '=')
Similarly, $1 - \cos^2\theta = \sin^2\theta$

Let's See some examples based on this identity-**Example1- Write all other trigonometric ratios in terms of $\sec A$**

Solution: We know that the identity $\sin^2 A + \cos^2 A = 1$

$$\begin{aligned}\sin^2 A &= 1 - \cos^2 A \\ \sin^2 A &= 1 - \frac{1}{\sec^2 A} \\ &= \frac{\sec^2 A - 1}{\sec^2 A} \\ \sin A &= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} \\ &= \frac{\sqrt{\sec^2 A - 1}}{\sec A}\end{aligned}$$

$$\operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

Example2- If $x = a \sin \theta$ and $y = a \cos \theta$ then find the value of $x^2 + y^2$.

Solution:- GIVEN: $x = a \sin \theta$ and $y = a \cos \theta$

$$\begin{aligned}x^2 + y^2 &= (a \sin \theta)^2 + (a \cos \theta)^2 \text{ On Putting the value of } x \text{ \& } y \\ &= a^2 \sin^2 \theta + a^2 \cos^2 \theta \\ &= a^2 (\sin^2 \theta + \cos^2 \theta) \\ &= a^2 (1) \text{ [using } \sin^2 \theta + \cos^2 \theta = 1] \\ \Rightarrow x^2 + y^2 &= a^2 \text{ (Hence proved)}\end{aligned}$$

Example 3- If $\sin \theta + \sin^2 \theta = 1$, Find the value of $\cos^2 \theta + \cos^4 \theta$.

Solution: Given $\sin \theta + \sin^2 \theta = 1$

$$\begin{aligned}\Rightarrow \sin \theta &= 1 - \sin^2 \theta \\ \text{(using identity } 1 - \sin^2 \theta &= \cos^2 \theta) \\ \Rightarrow \sin \theta &= \cos^2 \theta \\ \Rightarrow \sin^2 \theta &= \cos^4 \theta \text{ (squaring both sides)} \\ 1 - \cos^2 \theta &= \cos^4 \theta \\ \Rightarrow \cos^2 \theta + \cos^4 \theta &= 1 \\ \text{(using identity } 1 - \sin^2 \theta &= \cos^2 \theta)\end{aligned}$$

Try Yourself**1.Fill in the blanks-**

- (i) _____ + $\cos^2 \theta = 1$.
- (ii) $1 - \sin^2 \theta =$ _____.
- (iii) $1 -$ _____ = $\sin^2 \theta$

2. Show:

$$\sec^2 x + \operatorname{cosec}^2 x = \sec^2 x \times \operatorname{cosec}^2 x$$

3. If $\sec^2 \theta (1 - \sin \theta)(1 + \sin \theta) = k$, find the value of k .

4. Write all other trigonometric ratios in terms of $\sin x$.