Directorate of Education, G.N.C.T of Delhi

Subject: Mathematics **Worksheet No.36** **Dated: 25 September, 2020**

Class: X **Student's Name**:________________ **Teacher's Name:**________________

Trigonometric Identities (Part ll) 1+ $\tan^2\theta$= $\sec^2\theta$ and 1+$\cot^2\theta$ = $\text{cosec}^2\theta$

Recall and answer:

1. $\sin^2\theta + \cos^2\theta$ = _____
2. $1 - \sin^2\theta$ = _____
3. $1 - \cos^2\theta$ = _____

Do we have any other identities also for solving equations?

Yes, Let's learn together.

We can get second and third identities by using **first identity ($\sin^2\theta + \cos^2\theta = 1$)** as follows:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \quad (\div\cos^2\theta)$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \quad (\div\sin^2\theta)$$

$$1 + \cot^2\theta = \text{cosec}^2\theta$$

Our Second Identity is

$\tan^2\theta + 1 = \sec^2\theta$

We can write this identity in following ways also:

$\sec^2\theta - \tan^2\theta = 1$ or

$\sec\theta = \sqrt{(1+\tan^2\theta)}$ or

$\tan^2\theta = \sec^2\theta - 1$ or

$\tan^2\theta = \sqrt{(\sec^2\theta - 1)}$

Note: *When we move numbers and trigonometric ratios from one side to other side, sign changes.*

Our Third identity is:

$1+\cot^2\theta = \text{cosec}^2\theta$

We can write this in following ways for proving equations :

$\text{cosec}^2\theta - \cot^2\theta = 1$ or

$\text{cosec}\,\theta = \sqrt{(1+\cot^2\theta)}$ or

$\cot^2\theta = \text{cosec}^2\theta - 1$ or

$\cot^2\theta = \sqrt{(\text{cosec}^2\theta - 1)}$

Example1: Prove

$$\frac{(1+\cos A)}{(1-\cos A)} = \frac{\tan^2 A}{(\sec A - 1)^2}$$

Solution:

$$\text{LHS} = \frac{(1+\cos A)}{(1-\cos A)} = \frac{(1+1/\sec A)}{(1-1/\sec A}$$

$$= \frac{(\sec A+1)}{(\sec A-1)}$$

$$= \frac{(\sec A+1)}{(\sec A-1)} \text{ X } \frac{(\sec A-1)}{(\sec A-1)}$$

$$= \frac{(\sec^2 A-1)}{(\sec A-1)^2}$$

(using $\sec^2 A - 1 = \tan^2 A$)

$$= \frac{\tan^2 A}{(\sec A - 1)^2}$$

= RHS Hence proved

Example 2: Prove

$$\frac{1+\tan^2 A}{(1+\cot^2 A)} = \tan^2 A$$

Solution: $\text{LHS} = \dfrac{1+\tan^2 A}{(1+\cot^2 A)}$

Using $1+\tan^2 A = \sec^2 A$ and $1+\cot^2 A = \text{Cosec}^2 A$, we get

$$= \frac{\sec^2 A}{\text{cosec}^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A} \quad \{\sec A = \frac{1}{(\cos A)} \text{ and } \text{cosec } A = \frac{1}{(\sin A)}\}$$

$$= \left(\frac{\sin A}{\cos A}\right)^2 = \tan^2 A$$

= RHS Hence proved

Example 3: Prove

$$\text{cosec } A + \cot A = \frac{1}{(\text{cosec } A - \cot A)}$$

Solution: LHS = cosec A + cot A

$$= (\text{cosec } A + \cot A) \text{ X } \frac{\text{cosec } A - \cot A}{\text{cosec } A - \cot A}$$

$$= \frac{\text{cosec}^2 A - \cot^2 A}{\text{cosec } A - \cot A}$$

(Using $\text{cosec}^2 A - \cot^2 A = 1$)

$$= \frac{1}{(\text{cosec } A - \cot A)}$$

= RHS Hence proved

Try Yourself :

Q1. Fill in the blanks:

a) $\sec^2\theta$ - ________ = 1

b) 1+ $\tan^2\theta$ = ________

c) $\text{cosec}^2\theta$ - _______ = 1

d) $\cot^2\theta$ = ___________

Q2. Prove $\sec\theta\sqrt{(1-\sin^2\theta)} = 1$

Q3. Evaluate $\cos^2\theta \tan^2\theta + \tan^2\theta \sin^2\theta$ in terms of $\tan\theta$.

Q4. If x = $p\sec\theta + q\tan\theta$ and y= $p\tan\theta + q\sec\theta$ then prove that $x^2 - y^2 = p^2 - q^2$.