CLASS XII

MATHEMATICS

	Units	Weightage (Marks)
(i)	Relations and Functions	10
(ii)	Algebra (Matrices and Determinants)	13
(iii)	Calculus	44
(iv)	Vector and Three dimensional Geometry.	17
(v)	Linear Programming	06
(vi)	Probability	10
		Total : 100

Design

	Type of Questions	Weightage of each question	Number of questions	Total Marks
(i)	Very short answer (VSA)	01	10	10
(ii)	Short Answer (SA)	04	12	48
(iii)	Long Answer (LA)	06	07	42

Internal Choice

There will be internal choice in 4 questions of short answer type and in 2 questions of Long answer type.

NOTE

Questions requiring Higher Order thinking skills (HOTS) have been added in every chapter.

CHAPTER 1

RELATIONS AND FUNCTIONS

POINTS TO REMEMBER

1. **Empty relation** is the relation R in X given by R = $\phi \subset X \times X$.
2. **Universal relation** is the relation R in X given by R = $X \times X$.
3. **Reflexive relation** R in X is a relation with $(a, a) \in R, \ \forall \ a \in X$.
4. **Symmetric relation** R in X is a relation satisfying $(a, b) \in R \Rightarrow (b, a) \in R$.
5. **Transitive relation** R in X is a relation satisfying

   $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.
6. **Equivalence relation** R in X is a relation which is reflexive, symmetric and transitive.
7. A function $f : X \to Y$ is *one-one* (*or injective*) if

   $f(x_1) = f(x_2) \Rightarrow x_1 = x_2, \ \forall \ x_1, x_2 \in X$
8. A function $f : X \to Y$ is onto (or surjective) if given any $y \in Y$, $\exists \ x \in X$ such that $f(x) = y$.
9. A function $f : X \to Y$ is called *bijective* if it is one-one and onto.
10. For $f : A \to B$ and $g : B \to C$, the functions gof : $A \to C$ is given by $(gof)(x) = g[f(x)] \ \forall \ x \in A$.
11. A function $f : X \to Y$ is invertible if $\exists \ g : Y \to X$ such that go $f = I_x$ and fog $= I_y$.
12. A function $f : X \to Y$ is invertible if and only if f is one-one and onto.
13. A binary operation $*$ on a set A is a function $* : A \times A \to A$.
14. An operation $*$ on A is *commutative* if $a * b = b * a, \ \forall \ a, b \in A$.
15. An operation $*$ on A is *associative* if $(a * b) * c = a * (b * c) \ \forall \ a, b, c \in A$.
16. An element $e \in A$, is the identity element for $* : A \times A \to A$ if

    $a * e = a = e * a, \ \forall \ a, \in A$.
17. An element $a \in A$ is *invertible for* $* : A \times A \to A$ if there exists $b \in A$ such that $a * b = e = b * a$, where e is the identity for $*$. The element b is called inverse of a and is denoted by a^{-1}.

VERY SHORT ANSWER TYPE QUESTIONS

1. If A is the set of students of a school then write, which of following relations are. (Universal, Empty or neither of the two).

   $R_1 = \{(a, b) : a, b$ are ages of students and $|a - b| \geq 0\}$

   $R_2 = \{(a, b) : a, b$ are weights of students, and $|a - b| < 0\}$

   $R_3 = \{(a, b) : a, b$ are students studying in same class$\}$

   $R_4 = \{(a, b) : a, b$ are age of students and $a > b\}$

2. Is the relation R in the set $A = \{1, 2, 3, 4, 5\}$ defined as $R = \{(a, b) : b = a + 1\}$ reflexive?

3. If R, be a relation in set N given by

   $R = \{(a, b) : a = b - 3, b > 5\}$

   Does elements $(5, 7) \in R$?

4. If $f : \{1, 3\} \rightarrow \{1, 2, 5\}$ and $g : \{1, 2, 5\} \rightarrow \{1, 2, 3, 4\}$ be given by

   $f = \{(1, 2,), (3, 5)\}$, $g = \{(1, 3), (2, 3), (5, 1)\}$

   Write down gof.

5. Let $g, f : R \rightarrow R$ be defined by

   $g(x) = \frac{x + 2}{3}$, $f(x) = 3x - 2$. Write fog.

6. If $f : R \rightarrow R$ defined by

   $$f(x) = \frac{2x - 1}{5}$$

   be an invertible function, write $f^{-1}(x)$.

7. If $f(x) = \frac{x}{x + 1}$ $\forall$ $x \neq -1$, Write *fo f*(x).

8. Let * is a Binary operation defined on R, then if

   (i) $a * b = a + b + ab$, write 3 * 2

   (ii) $a * b = \frac{(a + b)^2}{3}$, Write (2 * 3) * 4.

   (iii) $a * b = 4a - 9b^2$, Write (1 * 2) * 3.

9. What is the number of bijective function from a set A to B, when A and B have same number of elements.

10. If $f, g : R \to R$ be defined by

$$f(x) = \frac{3x - 7}{8}, \quad g(x) = \frac{8x + 7}{3}, \text{ then}$$

What is fog (7).

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

11. Determine whether each of the following relations are

(i) Reflexive (ii) symmetric (iii) Transitive (iv) Equivalence relation..

(a) $R_1 = \{(a, b) : a \geq b, a, b \in R\}$

(b) $R_2 = \{(a, b) : (a - b) \text{ is a multiple of } 3, a, b \in R\}$

(c) $R_3 = \{(a, b) : (a - b) \text{ is an even integer}, a, b \in R\}$

(d) $R_4 = \{(a, b) : 3a - b = 0, a, b \in R\}$

(e) $R_5 = \{(a, b) : a \leq b^3, a, b \in R\}$

(f) $R_6 = \{(a, b) : a = b + 2, a, b \in R\}$

12. Check the following functions for one-one and onto.

(a) $f : R \to R, f(x) = \dfrac{3x - 2}{5}$

(b) $f : R \to R, f(x) = x^2 + 4$

(c) $f : R \to R, f(x) = |x - 3|$

(d) $f : A \to R, f(x) = \dfrac{x}{x + 1}$, where $A = R - \{-1\}$.

13. Let $f : R \to R$ be a function defined by $f(x) = \dfrac{2x - 3}{7}$. Show that f is invertible and hence find f^{-1}.

14. Let $f : R - \left\{\dfrac{-4}{3}\right\} \to R - \left\{\dfrac{4}{3}\right\}$ be a function given by $f(x) = \dfrac{4x}{3x + 4}$.

Show that f is invertible with $f^{-1}(x) = \dfrac{4x}{4 - 3x}$.

15. Show that function $f : A \to B$ defined as $f(x) = \frac{3x + 4}{5x - 7}$ where $A = R - \left\{\frac{7}{5}\right\}$, $B = R - \left\{\frac{3}{5}\right\}$ is invertible and hence find f^{-1}.

16. Let * be a binary operation on Q. Such that $a * b = a + b - ab$.

   (i) Prove that * is Commutative and associative.

   (ii) Find identify element of * in Q (if exists).

17. If * is a binary operation defined on $R - \{0\}$ defined by $a * b = \frac{2a}{b^2}$, then check * for cummutativity and associativity.

18. If $A = N \times N$ and binary operation * is defined on A as $(a, b) * (c, d) = (ac, bd)$.

   (i) Check * for commutativity and associativity.

   (ii) Find the identity element for * in A (If exists).

19. Show that the relation R defined by $(a, b) R(c, d) \Leftrightarrow a + d = b + c$ on the set $N \times N$ is an equivalence relation.

20. Let * be a binary operation on set Q defined by $a * b = \frac{ab}{4}$, *show that*

   (i) 4 is the identity element of * on Q.

   (ii) Every non zero element of Q is invertible with

   $$a^{-1} = \frac{16}{a}, \quad a \in Q - \{0\}.$$

H.O.T.S.

VERY SHORT ANSWER TYPE QUESTIONS (1 Mark)

21. Let $f : A \to B$ defined as $f(x) = (a - x^4)^{\frac{1}{4}}$ where $A, B \subset R$, what is fof (x).

22. *A* relation R in the set R of real numbers is defined as

   $R = \{(a, b) : a \leq b^2\}$. Is R reflexive?

SHORT ANSWER TYPE QUESTIONS (4 Marks)

23. Consider $f : R_+ \to [-5, \infty]$ as $f(x) = 9x^2 + 6x - 5$. Show that f is invertible and Hence find f^{-1}.

ANSWERS

1. R_1 – Universal relation.

   R_2 – Empty relation

   R_3 – Neither empty nor universal.

   R_4 – Neither empty nor universal.

2. Not reflexive.

3. (5, 7) $\notin$ R

4. gof = {(1, 3), (3, 1)}.

5. fog = I_R *i.e.* fog (x) = x.

6. $f^{-1}(x) = \frac{5x + 1}{2}$

7. $fof(x) = \frac{x}{2x + 1}$

8. (i) 11 (ii) $\frac{1369}{27}$ (iii) –209

9. n !

10. 7.

11. (a) Reflexive and Transitive but not symmetric

    (b) Equivalence relation

    (c) Equivalence relation

    (d) Neither reflexive nor symmetric nor transitive

    (e) Neither reflexive nor symmetric nor transitive

    (f) Neither reflexive nor symmetrix nor transitive.

12. (a) Bijective function (*i.e.,* one-one and onto).

    (b) Neither one-one nor onto.

    (c) Neither one-one nor onto.

    (d) one-one but not onto.

13. $f^{-1}(x) = \frac{7x + 3}{2}$.

15. $f^{-1}(x) = \dfrac{7y + 4}{5y - 3}$.

16. (ii) 0 is the identity element of * in Q.

17. (i) * is not commutative.

(ii) * is not associative.

18. (i) * is commutative as well as associative

(ii) (iii) is identity element.

ANSWERS OF HOTS

21. x

22. It is not reflexive.

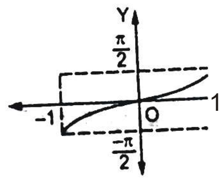
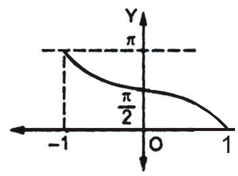
23. $f^{-1}(x) = \dfrac{\sqrt{x + 6} - 1}{3}$.

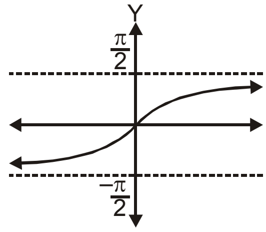
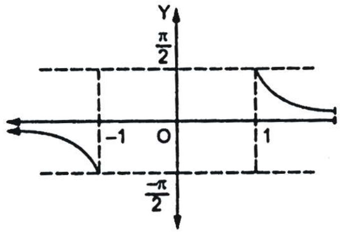
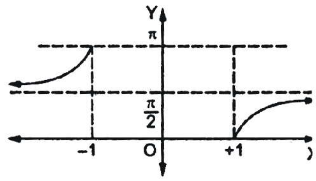
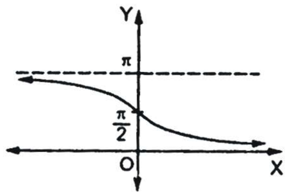
CHAPTER 2

INVERSE TRIGONOMETRIC FUNCTIONS

POINTS TO REMEMBER

1. $\sin^{-1} x$, $\cos^{-1} x$, ... etc., are angles.
2. $\sin^{-1} (\sin x) = x$ and $\sin (\sin^{-1} y) = y$, $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$, $y \in [-1, 1]$.
3. $\text{cosec}^{-1} = \sin\left(\frac{1}{x}\right)$; $\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$; $\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$
4. $\sin^{-1} (-x) = -\sin^{-1} x$; $\tan^{-1}(-x) = -\tan^{-1} x$; $\text{cosec}^{-1} (-x) = -\text{cosec}^{-1} x$.
5. $\cos^{-1} (-x) = \pi - \cos^{-1} x$; $\sec^{-1} (-x) = \pi - \sec^{-1} x$; $\cot^{-1} (-x) = \pi - \cot^{-1} x$.
6. $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ or $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$.
7. $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ or $\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$.
8. $\sec^{-1} x + \text{cosec}^{-1} x = \frac{\pi}{2}$ or $\text{cosec}^{-1} x = \frac{\pi}{2} - \sec^{-1} x$.
9. $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$; $xy < 1$.
10. $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$; $xy > -1$.
11. Infact every formula in trigonometry can be written in the language of inverse trigonometric functions.

Function	Domain	Range/Principal Value Branch	Graph
Inverse sine function $\sin^{-1} x = y \Leftrightarrow x = \sin y$	[–1, +1]	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	
Inverse cosine function $\cos^{-1} x = y \Leftrightarrow x = \cos y$	[–1, +1]	$[0, \pi]$	

Inverse tangent function ⇔ $\tan^{-1} x = y$	R	$\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$	
Inverse cosecant function $y = \text{cosec}^{-1} x$ iff $x = \text{cosec } y$	$]-\infty, -1] \cup [+1, +\infty[$	$\left[-\frac{\pi}{2}, 0\right[\cup \left]0, \frac{\pi}{2}\right]$	
Inverse secant function $y = \sec^{-1} x$ ⇒ $x = \sec y$	$]-\infty, -1] \cup [+1, +\infty[$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$	
Inverse cotangent function $y = \cot^{-1} x$ ⇒ $x = \cot y$	R	$]0, \pi[$	

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the principal value of

(i) $\sin^{-1}\left(-\sqrt{3}/2\right)$ (ii) $\sin^{-1}\left(\sqrt{3}/2\right)$.

(iii) $\cos^{-1}\left(-\sqrt{3}/2\right)$ (iv) $\cos^{-1}\left(\sqrt{3}/2\right)$.

(v) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ (vi) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

(vii) $\text{cosec}^{-1}(-2)$. (viii) $\text{cosec}^{-1}(2)$

(ix) $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ (x) $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

(xi) $\sec^{-1}(-2)$. (xii) $\sec^{-1}(2)$.

(xiii) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}\left(-1/\sqrt{3}\right)$

2. What is value of the following functions (using principal value).

(i) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$. (ii) $\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

(iii) $\tan^{-1}(1) - \cot^{-1}(-1)$. (iv) $\cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

(v) $\tan^{-1}\left(\sqrt{3}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$. (vi) $\operatorname{cosec}^{-1}\left(\sqrt{2}\right) + \sec^{-1}\left(\sqrt{2}\right)$.

(vii) $\tan^{-1}(1) + \cot^{-1}(1) + \sin^{-1}(1)$. (viii) $\cot^{-1}\left(\sqrt{3}\right) - \sin^{-1}\left(-\frac{1}{2}\right)$.

(ix) $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$. (x) $\cos^{-1}\left(\cos\frac{7\pi}{5}\right)$.

(xi) $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$. (xii) $\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{3\pi}{4}\right)$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

3. Show that $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right) = \frac{\pi}{4} + \frac{x}{2}$.

4. Prove $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) - \cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right) = \frac{\pi}{4}$ $x \in (0, \pi/2)$.

5. Prove $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) = \sin^{-1}\frac{x}{a} = \cos^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)$.

6. Prove $\cot^{-1}\left[2\tan\left(\cos^{-1}\frac{8}{17}\right)\right] + \tan^{-1}\left[2\tan\left(\sin^{-1}\frac{8}{17}\right)\right] = \tan^{-1}\left(\frac{300}{161}\right)$.

7. Prove $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$.

8. Solve $\cot^{-1}2x + \cot^{-1}3x = \frac{\pi}{4}$.

9. Solve $\tan^{-1} 2x - \tan^{-1} 3x = \frac{\pi}{4}$.

10. Prove $\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{2}$.

11. Prove $\tan^{-1}\frac{m}{n} - \tan^{-1}\frac{m-n}{m+n} = \frac{\pi}{4}$.

12. Prove $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right] = \frac{x+y}{1-xy}$.

13. Solve for x

$$\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \frac{1}{2}\tan^{-1}\frac{2x}{1-x^2} = \frac{2\pi}{3}.$$

H.O.T.S.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

14. $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right) = \tan^{-1}\frac{a}{b} - x$ if $\frac{a}{b}\tan x + 1 > 0$.

15. Prove $\cot\left\{\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)\right\} + \cos^{-1}\left(1-2x^2\right) + \cos^{-1}\left(2x^2-1\right) = \pi$.

16. Prove $\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \tan^{-1}\left(\frac{c-a}{1+ac}\right) = 0$. If $a, b, c > 0$

17. Find the value of $\cot^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

18. Find value of x for

$$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$$

19. Express the following in simplest form

$$\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right].$$

20. If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$ then prove that $a + b + c = abc$.

VERY SHORT ANSWER TYPE QUESTIONS

21. What is value of x if

$$\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = \frac{\pi}{2}.$$

22. If $f(x) = \cos^{-1}(\log x)$ then what is value of $f(1) + f(e)$

23. What is range of $y = \sin^{-1}[x]$, where [ ] is greatest integer function.

24. What is value of $\cot\left(\sec^{-1} x + \sin^{-1}\frac{1}{x}\right)$.

ANSWERS

1. (i) $-\frac{\pi}{3}$ (ii) $\frac{\pi}{3}$ (iii) $\frac{5\pi}{6}$ (iv) $\frac{\pi}{6}$

   (v) $\frac{-\pi}{6}$ (vi) $\frac{\pi}{6}$ (vii) $\frac{-\pi}{6}$ (viii) $\frac{\pi}{6}$

   (ix) $\frac{-\pi}{3}$ (x) $\frac{\pi}{3}$ (xi) $\frac{2\pi}{3}$ (xii) $\frac{\pi}{3}$

   (xiii) $\frac{\pi}{2}$.

2. (i) 0 (ii) $\frac{-\pi}{3}$ (iii) $\frac{\pi}{2}$ (iv) $\frac{\pi}{2}$

   (v) $\frac{2\pi}{3}$ (vi) $\frac{\pi}{2}$ (vii) π (viii) $\frac{\pi}{3}$

   (ix) $\frac{\pi}{5}$ (x) $\frac{3\pi}{5}$ (xi) $\frac{-\pi}{6}$ (xii) $\frac{\pi}{4}$.

8. $x = 1$.

9. $x = \frac{1}{6}$.

13. $\sqrt{3}$.

17. $\frac{-\pi}{4}$.

18. $x = 0$ or $\frac{\pi}{4}$.

19. $\sin^{-1} x - \sin^{-1} \sqrt{x}$.

21. $x = 1$.

22. $\frac{\pi}{2}$.

23. $\left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$.

24. 0.

CHAPTER 3 and 4

MATRICES AND DETERMINANTS

POINTS TO REMEMBER

Matrix : A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements of the matrix.

Order of Matrix : A matrix having 'm' rows and 'n' columns is called the matrix of order *mxn.*

Zero Matrix : A matrix having all the elements zero is called zero matrix or null matrix.

Diagonal Matrix : A square matrix is called a diagonal matrix if all its non diagonal elements are zero.

Scalar Matrix : A diagonal matrix in which all diagonal elements are equal is called a scalar matrix.

Identity Matrix : A scalar matrix in which each diagonal element is I, is called an identity matrix or a unit matrix.

$\therefore \quad A = [a_{ij}]_{n \times n}$

$a_{ij} = 0$ when $i \neq j$

$= 1$ when $i = j$ is a identity matrix.

Transpose of a Matrix : If $A = [a_{ij}]_{m \times n}$ be an $m \times n$ matrix then the matrix obtained by interchanging the rows and columns of A is called the transpose of the matrix. If $A = [a_{ij}]_{m \times n}$. Then transpose $A = A' = [a_{ij}]_{n \times m}$. Transpose of A is denoted by A' or A^T.

Symmetric Matrix : A square matrix $A = [a_{ij}]$ is said by symmetric if $A' = A$.. or $a_{ij} = a_{ji} \ \forall \ i \ \& \ j$.

Skew Symmetric Matrix : A square matrix $A = [a_{ij}]$ is said to be a skew symmetric matrix if $A' = -A$. or $a_{ij} = -a_{ji} \ \forall \ i \ \& \ j$.

Inverse of a Matrix : Inverse of a square matrix.

$$A^{-1} = \frac{Adj\ A}{|A|}, \text{ provided } |A| \neq 0.$$

where (Adj A) is the adjoint matrix which is the transpose of the cofactor matrix.

Singular Matrix : A square matrix is called singular if $|A| = 0$, otherwise it will be called a nonsingular matrix.

Determinant : To every square matrix $A = [a_{ij}]$ of order $n \times n$, we can associate a number (real or complex) called determinant of A. It is denoted by det A or $|A|$.

If A is a nonsingular matrix then its inverse exists and A is called invertible matrix.

$(AB)^1 = B^1A^1 \qquad$ Adj (AB) = (Adj B) (Adj A)

$(AB)^{-1} = B^{-1}A^{-1}$

$(A')^{-1} = (A^{-1})'$ Adj A' = (Adj A)$'$

If A is any non singular matrix of order n, then $|\text{adj } A| = |A|^{n-1}$

If A be any given square matrix of order n. Then A (adj A) = (adj. A) . $A = |A|I$.

Where I is the identity matrix of order n.

$|A\ B| = |A||B|$ where A and B are square matrices of same order.

Area of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) = $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

The points (x_1, y_1), (x_2, y_2), (x_3, x_3) are collinear $\Leftrightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$.

VERY SHORT ANSWER TYPE QUESTIONS (1 Mark)

1. What is the matrix of order 2 × 2 whose general element a_{ij} is given by $a_{ij} = \begin{cases} i - j & \text{if } i \geq j \\ i + j & \text{if } i < j \end{cases}$

2. If the matrix P is the order 2 × 3 and the matrix Q is of order 3 × m, then what is the order of the matrix PQ?

3. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ *find* A^2.

4. If $A = [1 \quad 3 \quad 2]$ and $B = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$, find AB.

5. What is the element a_{23} in the matrix $A = \lambda[a_{ij}]_{3 \times 3}$

   where $\lambda \in R$ and $a_{ij} = \begin{cases} |2i - j| & \text{if } i > j \\ 2i + j + 3 & \text{if } i \geq j \end{cases}$.

6. Let P and Q be two different matrices of order $3 \times n$ and $n \times p$ then what is the order of the matrix $4Q - P$, if it is defined.

7. Let A be a 5 × 7 type matrix, then what is the number of elements in the second column.

8. Write the matrix X if $3X - \begin{bmatrix} 8 & -2 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ 0 & 0 \end{bmatrix}$.

9. Give an example of two non zero 3 × 3 matrices A and B such that $AB = 0$.

10. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} = P + Q$ where P is symmetric and Q is skew-symmetric matrix, then find the matrix P.

11. If $A = \begin{bmatrix} \cos 20^\circ & \sin 20^\circ \\ \sin 70^\circ & \cos 70^\circ \end{bmatrix}$, what is $|A|$?

12. Find the value of the determinants $\begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$.

13. Find the value of xy if $\begin{vmatrix} 3x^3 & 8 \\ -4 & 4y^3 \end{vmatrix} = -4$.

14. Write the cofactor of the element 5 in the determinant $\begin{vmatrix} 2 & -3 & 6 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

15. Write the minor of the element b in the determinant $\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix}$.

16. If $\begin{vmatrix} 3x & 1 \\ 5 & -x \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ 5 & 2 \end{vmatrix}$, find the values (s) of x.

17. If $A = [a_{ij}]$ is 3×3 matrix and A_{ij} is denote the co-factors of the corresponding elements a_{ij}'s, then what is the value of $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$?

18. If A is a square matrix of order 3 and $|A| = -2$, find the value of $|-3A|$.

19. For what value(s) of λ, the points $(\lambda, 0)$, $(2, 0)$ and $(4, 0)$ are colinear?

20. If $0 < x < \frac{\pi}{2}$ and the matrix $\begin{bmatrix} 2 \sin x & 3 \\ 1 & 2 \sin x \end{bmatrix}$ is singular, find the value of x.

21. For what value of λ, the matrix $\begin{bmatrix} -3 & 5 \\ \lambda & \lambda + 1 \end{bmatrix}$ has no inverse?

22. If $A = \begin{bmatrix} 5 & -3 \\ 6 & 8 \end{bmatrix}$, find adj (adj A)

23. It $A = 2B$, where A and B are square matrices of order 3×3 and $|B| = 5$. What is $|A|$?

24. If the matrix $A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$, find AA'.

25. If $B = \begin{bmatrix} -2 & -1 \\ 3 & 0 \end{bmatrix}$, *and* $C = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$. Find $2B - 3C$.

26. Let A be a non singular matrix of order 3 × 3 such that $|A| = 5$. What is $|\text{adj } A|$?

27. Find a 2 × 2 matrix B such that.

$$\begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix} B = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}.$$

28. If $\begin{bmatrix} 2x + 1 & 3y \\ 0 & y^2 + 5y \end{bmatrix} = \begin{bmatrix} x + 3 & y + 2 \\ 0 & 6 \end{bmatrix}$, find x and y.

29. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & 3 \end{bmatrix}$. For what value of x, A will be a scalar matrix.

30. Find Δ if $\Delta = \begin{bmatrix} a - b & b - c & c - a \\ b - c & c - a & a - b \\ c - a & a - b & b - c \end{bmatrix}$.

31. Determine the value of x for which the matrix $A = \begin{bmatrix} -2 & 4 \\ 6 & 3x \end{bmatrix}$ is singular?

32. If $A = \begin{bmatrix} 5 & -2 \\ 3 & -2 \end{bmatrix}$, write the matrix $A(\text{adj } A)$.

33. Find the value of $\begin{bmatrix} P & 0 & 0 \\ a & q & 0 \\ b & c & r \end{bmatrix}$.

34. If A is a 2 × 2 matrix and $A\left(\text{adj } A\right) = \begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix}$, what is $|A|$.

35. If $A = \begin{vmatrix} x + y & y + z & z + x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$. Write the value of det A.

36. If $A = \begin{bmatrix} 4 & x + 2 \\ 2x - 3 & x + 1 \end{bmatrix}$ is symmetric matrix, then find x.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

37. Find x, y, z and w if $\begin{bmatrix} x - y & 2x + z \\ 2x - y & 3x + w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$.

38. Find A and B if $2A + 38 = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 6 & 2 \end{bmatrix}$.

39. Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 4 \\ -2 & 1 & 0 \\ 3 & 2 & 6 \end{bmatrix}$, *verify* that $(AB)C = A(BC)$.

40. Find the matrix X so that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

41. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$, verify that $(AB)' = B'A'$.

42. Express the matrix $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = P + Q$ where P is a symmetric and Q is a skew symmetric matrix.

43. Find the inverse of the following matrix by using elementary transformations $\begin{bmatrix} 7 & 6 \\ 2 & 2 \end{bmatrix}$.

44. Find the value of x such that $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$.

45. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 - xA + yI = 0$.

46. Find A (adj A) without finding (adj A) if $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$.

47. Given that $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$. Compute A^{-1} and show that $9I - A = 2A^{-1}$.

48. Given that matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$. Show that $A^2 - 4A + 7I = 0$. Hence find A^{-1}.

49. Show that $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 = 0$. Hence find A^{-1}.

50. Prove that the product of two matrices.

$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is zero when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

51. Show that :

$$\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix} = 0.$$

52. Using the properties of determinant, prove the following questions 52 to 56.

$$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2.$$

53. $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$

54. $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$

55. $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c).$

56. Show that :

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy).$$

57. (i) If the points (a, b) (a', b') and $(a - a', b - b')$ are collinear. Show that $ab' = a'b$.

(ii) If $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$ verity that $|AB| = |A||B|$.

58. Given $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$. Find the product AB and also find $(AB)^{-1}$.

59. Solve the following equations for x.

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0.$$

60. Verify that $(AB)^{-1} = B^{-1}A^{-1}$ for the matrices $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$.

61. Using matrix method to solve the following system of equations : $5x - 7y = 2$, $7x - 5y = 3$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

62. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$. Use this result to find A^5.

63. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verify that $A \cdot (\text{adj } A) = (\text{adj } A)\, A = |A|\, I_3$.

64. Find the matrix X for which

$$\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}.$$

65. Using elementary transformations, find the inverse of the matrices.

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}.$$

66. By using properties of determinants prove that

$$\begin{bmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{bmatrix} = \left(1+a^2+b^2\right)^3.$$

67. Solve the system of linear equations by using matrix in equations.

$$\begin{aligned} 2x - y + 4z &= 1 \\ 3x - z &= 2 \\ x - y - 2z &= 3 \end{aligned}$$

68. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, hence solve the system of linear equations :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

69. The sum of three numbers is 2. If we subtract the second number from twice the first number, we get 3. By adding double the second number and the third number we get 0. Represent it algebraically and find the numbers using matrix method.

70. Compute the inverse of the matrix.

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 5 \end{bmatrix} \text{ and verify that } A^{-1} A = I_3.$$

71. If the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix}$, then compute $(AB)^{-1}$.

72. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations.

$$x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1.$$

73. Solve the following system of equations using matrix method.

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2.$$

74. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Show that $A^3 - 6A^2 + 5A + 11I = 0$ and hence find A^{-1}.

H.O.T.S.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

75. How many matrices of order 2 × 3 are possible with each entry as 0 or 1.

76. If $x \in R$, $0 \le x \le \frac{\pi}{2}$, and $\begin{vmatrix} 2\sin x & -1 \\ 1 & \sin x \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ -4 & \sin x \end{vmatrix}$

Then find the value of x

77. If A is a square matrix of order 3 such that $|adj\ A| = 125$, find $|A|$.

78. If $A = \begin{vmatrix} 0 & 0 \\ -3 & 0 \end{vmatrix}$, find A^{20}.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

79. If $A = \begin{vmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{vmatrix}$ and I is the identity matrix of order 2, show that

$I + A = (I - A) \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$.

80. Using properties of determinants, show that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2).$$

81. If x, y, z are the 10^{th}, 13^{th} and 15^{th} terms of a G.P. find the value of $\Delta = \begin{vmatrix} \log x & 10 & 1 \\ \log y & 13 & 1 \\ \log z & 15 & 1 \end{vmatrix}$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

82. Show that $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz\,(x+y+z)^3$.

83. If $A = \begin{vmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{vmatrix}$, find A^{-1} and hence solve the system of equations

$3x + 4y + 7z = 14, \quad 2x - y + 3z = 4, \quad x + 2y - 3z = 0.$

ANSWERS

1. $\begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$.

2. $2xm$.

3. $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

4. [18]

5. 10λ.

6. 3×3.

7. 5.

8. $x = \begin{bmatrix} 5 & 1 \\ 2 & 0 \end{bmatrix}$.

9. $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

10. $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$.

11. 0.

12. $a^2 + b^2 + c^2 + d^2$.

13. $(-3)^{1/3}$.

14. 28.

15. $id - fg$.

16. $\pm\sqrt{\frac{2}{3}}$. [**Hint. :** $- 3x^2 - 5 = - 7$].

17. 0.

18. 54. [**Hint. :** order 3 $\Rightarrow |-3A| = (-3)^3\ |A|$].

19. x = any real number.

20. $\frac{\pi}{3}$.

21. $\lambda = -\frac{3}{8}$.

22. $\begin{bmatrix} 5 & -3 \\ 6 & 8 \end{bmatrix}$.

23. 40.

24. I_2.

25. $\begin{bmatrix} -13 & 2 \\ 9 & -6 \end{bmatrix}$.

26. 25.

27. $\begin{bmatrix} 6 & 5 \\ -5 & -6 \end{bmatrix}$.

28. $x = 2, y = 1$.

29. 3.

30. 0; [**Hint. :** $[R_1 \rightarrow R_1 + R_2 + R_3]$]

31. $x = -4$.

32. $\begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$.

33. *pqr.*

34. 12.

35. 0.

36. 5.

37. $x = 1$, $y = 2$, $z = 3$, $w = 4$.

38. $A = \begin{bmatrix} \frac{11}{7} & -\frac{9}{7} & \frac{9}{7} \\ \frac{1}{7} & \frac{18}{7} & \frac{4}{7} \end{bmatrix}$, $B = \begin{bmatrix} -\frac{5}{7} & -\frac{2}{7} & \frac{1}{7} \\ \frac{4}{7} & -\frac{12}{7} & -\frac{5}{7} \end{bmatrix}$.

40. $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

43. $\begin{bmatrix} 1 & -3 \\ -1 & \frac{7}{2} \end{bmatrix}$.

44. $x = -2$ or $x = -14$.

45. $x = 9$, $y = 14$.

46. $-14I_3$

47. $A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

48. $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$.

49. $A^{-1} = \frac{1}{17}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}$.

58. $AB = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$, $(AB)^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -2 \\ 2 & -1 \end{bmatrix}$.

61. $x = \frac{11}{24}$, $y = \frac{1}{24}$.

62. $A^5 = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$. **[Hint. :** $A^2 - 4A + 7I = 0$, $A^2 = 4A - 7I$, $A^3 = 4\,(4A - 7I - 7A)$)

64. $X = \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$. **[Hint. :** if $A \times B = P$, $X = A^{-1}\,P\,B^{-1}$]

65. $\frac{1}{5}\begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix}$.

67. $x = \frac{10}{19}$, $y = \frac{-31}{19}$, $z = \frac{-8}{19}$.

68. $A^{-1} = \frac{-1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$.

$x = 3$, $y = -2$, $z = 1$.

69. $x = 1, y = -2, z = 2.$ [**Hint. :** Suppose three numbers as *x, y, z*]

70. $A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}.$

71. $(AB)^{-1} = \frac{1}{19}\begin{bmatrix} 16 & 12 & 1 \\ 21 & 11 & -7 \\ 10 & -2 & 3 \end{bmatrix}.$ [**Hint. :** $(AB)^{-1} = B^{-1}A^{-1}$]

72. $x = 3, y = -2, z = -1.$

73. $x = 2, y = 3, z = 5.$ [**Hint. :** Let $\frac{1}{x} = u, \frac{1}{y} = v, \frac{1}{z} = w.$]

74. $A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}.$

75. 64.

76. $\frac{\pi}{6}, \frac{\pi}{2}.$

77. $+5\sqrt{5}$

78. 0

81. 0.

83. $x = 1, y = 1, z = 1.$

CHAPTER 5

CONTINUITY AND DIFFERENTIATION

POINTS TO REMEMBER

- **Continuity of a Function :** A function $f(x)$ is said to be continuous at $x = c$ if $\lim_{x \to c} f(x) = f(c)$

  i.e., $\underset{x \to c^-}{Lt} f(x) = \underset{x \to c^+}{Lt} f(x) = f(c)$

  $f(x)$ is continuous in $]a, b[$ if it is continuous at $x = c \;\forall\; c \in \left]a, b\right[$.

- $f(x)$ is continuous in $[a, b]$ iff it is continuous in (a, b) and $\lim_{x \to a^+} f(x) = f(a)$

  $\lim_{x \to b^-} f(x) = f(b)$

- $f(x)$ and $g(x)$ are continuous functions at $x = c$ and d is constant then,

  $f + g$, $f - g$, df, $f \cdot g$, $f + d$, $|f|$ are all continuous at $x = c$.

- $\dfrac{f(x)}{g(x)}$ is continuous at $x = c$ provided $g(c) \neq 0$.

- Every polynomial function is continuous on R.

- Every trigonometric function, Exponential function and logarithmic function are continuous in their respective domain.

- $f(x)$ is derivable at $x = c$ iff

  $\lim_{x \to c} \dfrac{f(x) - f(c)}{x - c}$ exists and value of this limit is called the derivative at $x = c$ and is denoted by $f'(c)$.

- $\dfrac{d}{dx}(u.v) = u\dfrac{dv}{dx} + v.\dfrac{du}{dx}$

- $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v \cdot \dfrac{dv}{dx} - u\dfrac{dv}{dx}}{v^2}$

- If $y = f(u)$, $x = g(u)$ then

  $\dfrac{dy}{dx} = \dfrac{f'(u)}{g'(u)}$.

- If y is a function of t and t is a function of x then, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$.
- If $x = \phi_1(t)$, $y = \phi_2(t)$ then $\frac{dy}{dx} = \frac{\phi_2'(t)}{\phi_2'(t)} = g(t)$ say then $\frac{d^2y}{dx^2} = g'(t) \cdot \left(\frac{dt}{dx}\right)$.
- **Rolle's theorem :** If $f(x)$ is continuous in $[a, b]$ and derivable in (a, b) and $f(a) = f(b)$ then there exists atleast one real no $c \in (a, b)$ *s.t.* $f'(c) = 0$.
- **L.M.V.T. :** If $f(x)$ is continuous in $[a, b]$ and derivable in (a, b) then these exists atleast one point $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write for what value of x, $f(x) = |3x + 1|$ is not derivable.
2. Write the set of points of discontinuity of the function, $g(x) = ||x - 1||$
3. What is derivative of $f(x) = |x - 1|$ at $x = 1$.
4. What are the points of discontinuity of the function $f(x) = \frac{x^2 + x + 1}{x^3 - x}$.
5. Write all the points of discontinuity of the function $f(x) = [x]$ in $[-1, 3]$, where $[x]$ denotes the greatest integer function.
6. At what point $f(x) = \operatorname{sgn}(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is discontinuous.
7. Is the function $e^{-x} \sin x$ is continuous on R?
8. If $f(x) = \begin{cases} \lambda x + 1, & x < 1 \\ 3 & x = 1 \\ 3x^2 & x > 1 \end{cases}$ then for which value of λ, $f(x)$ is continuous on R.
9. Write the value of k, for which $f(x) = \begin{cases} \frac{\sin 3x}{2x}, & x \neq 0 \\ \frac{k}{2}, & x = 0 \end{cases}$ is continuous $\forall\ x \in R$.
10. What is the derivative of x^6 *w.r.t.* x^2.
11. Given that $g(0) = 7$ and $f(x) = x\, g(x)$. Also $f'(x)$ and $g'(x)$ exist, then write value of $f'(0)$.
12. Write the derivatives of the following functions.

    (i) $\log_2 (2x - 1)$ (ii) $e^{3 \log x}$

(iii) $2^{\sqrt{x}}$ (iv) $\tan^{-1} x^2 + \cot^{-1} x^2$

(v) $\sin^{-1}\left(x \sqrt{x}\right) \quad 0 \le x \le 1.$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

13. Discuss the continuity of following functions at the indicated points.

(i) $f(x) = \begin{cases} \dfrac{x - |x|}{x}, & x \ne 0 \\ 2, & x = 0 \end{cases}$ at $x = 0.$

(ii) $f(x) = \begin{cases} \dfrac{\tan 3x}{5x}, & x \ne 0 \\ 5/3 & x = 0 \end{cases}$ at $x = 0.$

(iii) $f(x) = \begin{cases} x^2 \sin(1/x) & x \ne 0 \\ 0 & x = 0 \end{cases}$ at $x = 0.$

(iv) $f(x) = |x| + |x - 1|$ at $x = 1.$

(v) $f(x) = \begin{cases} \dfrac{\sin x}{|x|}, & x \ne 0 \\ 1 & x = 0 \end{cases}$ at $x = 0.$

(vi) $f(x) = \begin{cases} \dfrac{\sin 2x}{x}, & x < 0 \\ x + 2 & x \ge 0 \end{cases}$ at $x = 0.$

(vii) $f(x) = \begin{cases} x - [x], & x \ne 0' \\ 0 & x = 0' \end{cases}$ at $x = 1.$

14. For what value of K, $f(x) = \begin{bmatrix} 3 - 2x & 0 < x < 2 \\ 4x^2 - 3kx & 2 \le x < 5 \end{bmatrix}$ is continuous in it's domain.

15. For what values of a and b

$$f(x) = \begin{bmatrix} \dfrac{x + 2}{|x + 2|} + a & \text{if } x < -2 \\ a + b & \text{if } x = -2 \\ \dfrac{x + 2}{|x + 2|} + 2b & \text{if } x > -2 \end{bmatrix}$$ is continuous at $x = 2$.

16. Prove that $f(x) = |x + 3|$ is continuous at $x = -3$ but not derivable at $x = -3$.

17. If $f(x) = \begin{cases} x^p \sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0 & x = 0 \end{cases}$ is derivable at $x = 0$ the find the value of p.

18. If $y = (\log x)^x + x^{\log x}$ then find $\dfrac{dy}{dx} = ?$

19. If $y = \dfrac{1}{2}\left[\tan^{-1}\left(\dfrac{2x}{1-x^2}\right) + 2\tan^{-1}\left(\dfrac{1}{x}\right)\right]$, $x > 0$, find $\dfrac{dy}{dx}$.

20. If $y = \sin\left[2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right]$ then $\dfrac{dy}{dx} = ?$

21. If $3^x + 3^y = 3^{x+y}$ then Prove that $\dfrac{dy}{dx} = -3^{y-x}$.

22. If $y = \tan^{-1} x$ then show that $\left(1+x^2\right)\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} = 0$.

23. If $x\sqrt{1-y^2} + y\sqrt{1-x^2} = a$ then show that $\dfrac{dy}{dx} = -\sqrt{\dfrac{1-y^2}{1-x^2}}$.

24. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ then show that $\dfrac{dy}{dx} = +\sqrt{\dfrac{1-y^2}{1-x^2}}$.

25. If $(x+y)^{r+s} = x^r + y^s$ then prove that $\dfrac{dy}{dx} = \dfrac{y}{x}$.

26. If $y = \tan^{-1}\left(\dfrac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right)$ where $\dfrac{\pi}{2} < x < \pi$ find $\dfrac{dy}{dx}$.

27. Find the derivative of $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ with respect to $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$.

28. Find derivative of $\log(\sin x)$ w.r.t. $\log x$.

29. If $x = \sin\left(\dfrac{1}{a}\log y\right)$ then show that $(1 - x^2)\, y'' - xy' - a^2y = 0$.

30. If $x^y + y^x + x^x = a^b$ then find $\dfrac{dy}{dx} = ?$

31. If $x = a\cos^3\theta$, $y = a\sin^3\theta$ then find $\dfrac{d^2y}{dx^2}$.

32. If $x = ae^{\theta}(\sin\theta - \cos\theta)$

$y = ae^{\theta}(\sin\theta + \cos\theta)$ then show that $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{4}$ is 1.

33. If $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ then find $\dfrac{dy}{dx}$.

34. If $y = \dfrac{e^{2x}\left(\sin^{-1} x\right)^3}{x^4\left(\tan x\right)}$, find $\dfrac{dy}{dx}$.

H.O.T.S.

35. If $y = \sin^{-1}\left[\dfrac{12x + 5\sqrt{1-x^2}}{13}\right]$, $\dfrac{dy}{dx} = ?$

36. If $y^x = x^y$, find $\dfrac{dy}{dx}$.

37. If $\sin y = x \sin(a + y)$ then show that $\dfrac{dy}{dx} = \dfrac{\sin^2(a+y)}{\sin a}$ $\{x \neq n\pi, n \in z\}$

38. If $y = \sin^{-1} x$, find $\dfrac{d^2y}{dx^2}$ in terms of y.

39. If $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ then show that $\dfrac{d^2y}{dx^2} = \dfrac{-b^4}{a^2y^3}$.

40. If $y^3 - 3ax^2 + x^3 = 0$ then Prove that $\dfrac{d^2y}{dx^2} = \dfrac{-2a^2x^2}{y^5}$.

41. If $ax^2 + 2bxy + by^2 = 1$ the prove that $\dfrac{d^2y}{dx^2} = \dfrac{h^2 - ab}{(hx + by)^3}$.

42. Write the points of discontinuity of $f(x) = [x]$ in [3, 9].

43. Write the critical points of $f(x) = \dfrac{x^2}{2} - \log x$.

44. Evaluate $\lim_{x \to 3}\left(x - [x]\right)$, where [.] denotes the greatest integer function.

45. If for a function $f(x)$,

$f'(x) = (x - 2)^2 (x + 1)$ then write the interval in which $f(x)$ is increasing or decreasing.

46. If $f(x) = \begin{cases} 3ax + 7, & x < 1 \\ 2bx - 5 & x = 1 \\ 10x & x > 1 \end{cases}$ is continuous for all values of x, then find the value of a and b.

ANSWERS

1. $x = -1/3$
2. ϕ
3. Derivative does not exist.
4. –1, 0, 1.

5. 0, 1, 2, 3

6. $x = 0$

7. Yes

8. $\lambda = 2$

9. $k = 3$

10. $3x^4$

11. $f'(0) = 7$

12. (i) $\dfrac{2}{2x-1}\log_2 e$ (ii) $3x^2$ (iii) $\dfrac{2^{\sqrt{x}}}{2\sqrt{x}}\log_e 2$ (iv) 0

(v) $\dfrac{3\sqrt{x}}{2\sqrt{1-x^3}}$

13. (i) Discontinuous (ii) Discontinuous (iii) Continuous (iv) continuous

(v) Discontinuous (vi) Continuous (vii) Discontinuous

14. $K = \dfrac{17}{6}$.

15. $a = 0,\ b = -1$

17. $p > 1$.

18. $\left(\log_e x\right)^{x-1}\left[1+\log_e x \log_e\left(\log_e x\right)\right] + x^{\log_e x - 1}\left(2\log_e x\right)$

19. 0

20. $\dfrac{-x}{\sqrt{1-x^2}}$

26. –1/2

27. 1

28. $x \cot x$.

30. $\dfrac{dy}{dx} = \dfrac{-\left[y \,.\, x^{y-1} + y^x \log y + x^x\left(1+\log x\right)\right]}{x^y \log x + xy^{x-1}}$

31. $\dfrac{d^2y}{dx^2} = \dfrac{1}{3a}\sec^4\theta\ \text{cosec}\ \theta$

33. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}} - \dfrac{1}{2\sqrt{x-x^2}}$

34. $y\left[2 + \dfrac{3}{\sqrt{1-x^2}\,\sin^{-1}x} - \dfrac{4}{x} - 2\ \text{cosec}\ 2x\right]$

35. $\dfrac{-1}{\sqrt{1-x^2}}$

36. $\dfrac{y\left(y - x\log y\right)}{x\left(x - y\log x\right)}$.

38. $\sec^2 y \tan y$

42. 4, 5, 6, 7, 8, 9.

43. –1, 0, 1

44. Limit Does not exist.

45. decreasing in (–∞, –1] and increasing in {–1, ∞).

46. $a = -1,\ b = \dfrac{15}{2}$.

CHAPTER 6

APPLICATIONS OF DERIVATIVES

POINTS TO REMEMBER

- **Rate of Change :** If x and y are connected by $y = f(x)$ then $\frac{dy}{dx}$ represents the rate of change of y *w.r.t.* x.

- Equation of tangent to the curve $y = f(x)$ at the point $P(x_1, y_1)$ is given by $y - y_1 = \left.\frac{dy}{dx}\right|_P (x - x_1)$.

  Similarly equation of normal is $y - y_1 = -\frac{1}{\left.\frac{dy}{dx}\right|_P}(x - x_1)$.

  The angle of intersection between two curves is the angle between the tangents to the curves at the point of intersection. $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$, where m_1, m_2 are slopes of tangent at the point of intersection P.

- A function $f(x)$ is said to be strictly monotonic in (a, b) if it is either increasing or decreasing in (a, b).

- A function $f(x)$ is said to be strictly increasing in (a, b) if $\forall\ x_1, x_2$ in (a, b) s.t.

  $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$. Alternatively, $f(x)$ is increasing in (a, b) if $f'(x) > 0\ \forall\ x \in (a, b)$.

- A function $f(x)$ is said to be strictly decreasing in (a, b) if $\forall\ x_1, x_2$ in (a, b) s.t. $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$. Alternatively, $f(x)$ is strictly decreasing in (a, b) if $f'(x) < 0\ \forall\ x \in (a, b)$.

- A function $f(x)$ is said to have local maximum value at $x = c$, if there exists a neighbourhood $(c - \delta, c + \delta)$ of c, s.t. $f(x) < f(c)\ \forall\ x \in (c - \delta, c + \delta)$, $x \neq c$. Similarly, local minimum value can be defined.

- Local maximum and local minimum values of $f(x)$ may not be maximum and minimum value of $f(x)$.

- **Critical Point :** A point c is called critical point of $y = f(x)$ if either $f'(c) = 0$ or $f'(c)$ does not exist.

- If $f(x)$ is defined in $[a, b]$ and $f'(x) = 0$ gives $x = x_1, x_2, x_3, \ldots\ x_n$ then

  Max. $\{f(a), f(x_1), f(x_2), \ldots\ f(x_n), f(b)\}$ is called global maximum value. Similarly, Global minimum value can be defined.

VERY SHORT ANSWER TYPE QUESTIONS

1. If $f(x) = \cos x$, $x \in [0, 2\pi]$ then write the interval in which $f(x)$ is decreasing.

2. Write the interval in which $f(x) = x^2$ is decreasing.

3. For what value of λ, $f(x) = \sin 2x - 3\lambda x$ is strictly increasing.

4. Write the maximum value of $f(x) = \dfrac{1}{x^2 - 2x + 3}$ in [0, 2].

5. Find the max. and min. value of $f(x) = |3 \sin x + 5|$.

6. Write the slope of the normal to the curve $f(x) = 3x^2 - 7x + 1$ at $x = 1$.

7. If normal to the curve at a point P on $y = f(x)$ is parallel to y–axis, then write the value of $f'(x)$ at P.

8. On the curve $f(x) = \dfrac{3}{2}x^2$, find the points at which tangent is parallel to the chord joining the points $A\left(-1, \dfrac{3}{2}\right)$ and $B(2, 6)$.

9. Write the least value of $f(x) = x + \dfrac{1}{x}, (x > 0)$.

10. $y = 2x - 1$ is normal to the curve $y = x^2$ at which point?

11. If $y = 3e^{2x}$ and $y = b\, e^{-2x}$ cut each other at right angles, find the value of b.

12. If the tangent to $y = 3x - x^2$ is parallel to the line $y = 0$ then find the point of contact of tangent with the curve.

13. At which point on $y = \dfrac{1}{2\sqrt{3}}x^2 + 5$, tangent makes an angle of 30° with the +ve directions of x-axis.

14. In which interval $f(x) = \tan x - x$ is increasing?

15. If the radius of the circle is decreasing at the rate of 3 cm/sec. then, write the rate at which area is changing when $r = 5$ cm.

16. If length and breadth of a rectangle are increasing at the rate of 5 cm/sec. and 2 cm/sec. respectively. Find the rate at which area is increasing if length = 12 cm and breadth = 10 cm.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

17. Sand is pouring out from a pipe at the rate of 12 cm^3/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one sixth of the radius of the base. How fast is the height of sand cone increasing when the height is 4 cm.

18. Find the points of local maxima/minima for $f(x)$. If $f(x) = \sin x - \cos x$ where $0 < x < 2\pi$. Also find the local maximum and minimum values.

19. Find the interval(s) in which $f(x) = x^4 - \frac{x^2}{32}$ is increasing or decreasing?

20. Find the interval in which $f(x) = 2 \log (x - 2) - x^2 + 4x + 1$, $(x > 2)$ is increasing or decreasing?

21. For the curve $y = 2x^3 - 3x^2$, find the points on the curve at which the tangent passes through (0, 0).

22. Prove that the function : $f(x) = x^{50} + \sin x - 1$ is strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

23. Show that $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$ is an increasing function in $\left[\frac{3\pi}{8}, \frac{5\pi}{8}\right]$.

24. Find the intervals in which $f(x) = x^3 + 3x^2 - 105x + 25$ is increasing or decreasing?

25. Separate the interval $[0, \pi]$ into the interval in which $f(x) = x - \sin 2x$ is increasing or decreasing?

26. Find the point on the curve $y = x^3 - 3x^2 + 9x + 6$ at which slope of the tangent to the curve is minimum. Also, find the minimum slope.

27. Find the absolute maximum value of $f(x) = x + \sin 2x$ in $[0, \pi]$.

28. Show that the surface area of a closed cuboid with a square base and given volume is minimum when it is a cube.

29. Show that the equation of the tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $P(x_1, y_1)$ is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$.

30. Find the equation of tangent to $y^2 = 4ax$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$.

31. Show that $\frac{x}{2} + \frac{y}{3} = 1$ touches the curve $y = 3e^{-\frac{x}{2}}$ at the point where the curve crosses y-axis.

32. If $x = y^2$ and $xy = r$ cut each other at right angles then find the value of r^2.

LONG ANSWER TYPE QUESTION (6 MARKS)

33. A point on the hypotenuse of a right triangle is at a distance 'a' and 'b' from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$.

34. If the length of three sides of a trapezium other than base are equal to 10 cm, then find the area of trapezium when it is maximum.

35. Show that $f(x) = \sin^4 x + \cos^4 x$, $x \in [0, \pi/2]$ is increasing on $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ and decreasing on $[0, \pi/4]$.

36. Find the equation of tangent to the curve $y = (x^3 - 1)(x - 2)$ at the points where the curve cuts the x–axis.

37. Show that the semi-vertical angle of a cone of maximum volume and given height is $\tan^{-1}\sqrt{2}$.

38. Prove that the radius of the right circular cylinder of greatest curved surface which can be inscribed in a given cone is half of that of the cone.

39. A rectangular sheet of tin 45 cm × 24 cm is to be made into a box without top by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum?

40. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces so that the combined area of the square and the circle is minimum?

41. For a given curved surface of a right circular cone when volume is maximum, prove that semi-vertical angle is $\sin^{-1}\left(\frac{1}{3}\right)$.

42. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

43. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

44. A jet of an enemy is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). What is the nearest distance between the soldier and the jet?

H.O.T.S.

45. Show that $f(x) = \frac{1}{x^2}$ is neither increasing nor decreasing in $(-\infty, \infty)$.

46. Find the least value of a such that $f(x) = x^2 + ax + 1$ is increasing on [1, 2].

47. In which interval $f(x) = 5x^{3/2} - 3x^{5/2}$, $x \geq 0$ is decreasing?

48. Find the point on curve $y = (x + 1)^2$ where tangent is parallel to the chord joining the points $A(-3, 7)$ and $B(2, 5)$.

49. Verify mean value theorem for

$$f(x) = \begin{cases} 2x + 1 & x \geq 2 \\ x^2 - 3x + 7 & x < 2 \end{cases} \text{ in } [0, 4] \text{ (if applicable).}$$

50. Using differentials, find the approximate value of $\sqrt{0.037}$.

51. If $y = \sin x$ and x changes from $\frac{22}{14}$ to $\frac{\pi}{2}$. Find the approximate change in the value of y.

52. A rectangular window is surmounted by an equilateral triangle. Given that the perimeter is 16m, find the width of the window so that the maximum amount of light may enter.

53. A particular moves along the curve $6y = x^3 + 2$. Find the points on the curve at which y-coordinate is changing 8 times as fast as x-coordinate.

ANSWERS

1. $[0, \pi]$
2. $(-\infty, 0]$
3. $\left(-\infty, -\frac{2}{3}\right)$
4. 1/2
5. Max value = 8, Min. Value = 2.
6. 1
7. 0
8. $\left(\frac{1}{2}, \frac{3}{8}\right)$
9. 2
10. $\left(-\frac{1}{4}, \frac{1}{16}\right)$
11. $\frac{1}{12}$.
12. $\left(\frac{3}{2}, \frac{9}{4}\right)$
13. $\left(1, \frac{1}{2\sqrt{3}} + 5\right)$
14. $(-\infty, \infty)$
15. Decreasing at the rate of 30π cm^2/sec.
16. 74 cm^2/sec

SHORT ANSWER TYPE QUESTIONS

17. $\frac{1}{48\pi}$ cm/sec

18. Local max. value = $\sqrt{2}$ at $x = \frac{3\pi}{4}$

    Local min. value = $-\sqrt{2}$ at $x = \frac{7\pi}{4}$.

19. Decreasing in $\left(-\infty, -\frac{1}{8}\right) \cup \left(0, \frac{1}{8}\right)$

    Increasing in $\left(-\frac{1}{8}, 0\right) \cup \left(\frac{1}{8}, \infty\right)$.

20. Increasing in (2, 3) and decreasing in $(3, \infty)$.

21. $\left(\frac{3}{4}, \frac{-27}{32}\right), (0, 0)$

24. Increasing in $(-\infty, -7) \cup (5, \infty)$, Decreasing in $(-7, 5)$.

25. Decreasing in $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$. Increasing in $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$.

26. (1, 13), Minimum slope = 6.

27. π.

30. $y = mx + \frac{a}{m}$

32. 1/8.

34. $75\sqrt{3}$ sq. units.

36. $3x + y - 3 = 0$, $7x - y - 14 = 0$.

39. 5 cm.

40. $\frac{112}{\pi + 4} m, \frac{28}{\pi + 4} m.$

44. $\sqrt{5}$

46. –2

47. $[1, \infty)$

48. $\left(-\frac{6}{5}, \frac{1}{25}\right)$

49. M.V. Theorem Not applicable at $x = 2$, because $f(x)$ is not derivable.

50. 0.1925

51. 0

52. $\frac{16}{6 - \sqrt{3}} m.$

53. $(4, 11), \left(-4, \frac{-31}{3}\right)$.

CHAPTER 7

INTEGRATION

POINTS TO REMEMBER

Integration is inverse process of Differentiation.

STANDARD FORMULAE

1. $\int x^n dx = \begin{cases} \dfrac{x^{n+1}}{n+1} + c & n \neq -1 \\ \log|x| + c & n = -1 \end{cases}$

2. $\int (ax+b)^n \, dx = \begin{cases} \dfrac{(ax+b)^{n+1}}{n+1} + c & n \neq -1 \\ \dfrac{1}{a}\log|ax+b| + c & n = -1 \end{cases}$

3. $\int \sin x \; dx = -\cos x + c.$

4. $\int \cos x \; dx = \sin x + c.$

5. $\int \tan x \,.\, dx = -\log|\cos x| + c = \log|\sec x| + c.$

6. $\int \cot x \; dx = \log|\sin x| + c.$

7. $\int \sec^2 x \,.\, dx = \tan x + c.$

8. $\int \operatorname{cosec}^2 x \,.\, dx = -\cot x + c.$

9. $\int \sec x \,.\, \tan x \,.\, dx = \sec x + c.$

10. $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} + c.$

11. $\int \sec x \, dx = \log|\sec x + \tan x| + c.$

12. $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + c.$

13. $\int \dfrac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c, |x < 1|.$

14. $\int \dfrac{1}{1+x^2} dx = \tan^{-1} x + c.$

15. $\int \dfrac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c, |x| > 1.$

16. $\int \dfrac{1}{a^2-x^2} dx = \dfrac{1}{2a}\log\left|\dfrac{a+x}{a-x}\right| + c.$

17. $\int \dfrac{1}{x^2-a^2} dx = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right| + c.$

18. $\int \dfrac{1}{a^2+x^2} dx = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + c.$

19. $\int \dfrac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\dfrac{x}{a} + c.$

20. $\int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c.$ 21. $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c.$

22. $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c.$

23. $\int \sqrt{a^2 + x^2} dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2} \log\left|x + \sqrt{a^2 + x^2}\right| + c.$

24. $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log\left|x + \sqrt{x^2 - a^2}\right| + c.$

25. $\int e^x dx = e^x + c.$

26. $\int a^x dx = \frac{1}{\log a} . a^x + c.$

INTEGRATION BY SUBSTITUTION

1. $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c.$

2. $\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n + 1} + c.$

3. $\int \frac{f'(x)}{[f(x)]^n} dx = \frac{(f(x))^{-n+1}}{-n + 1} + c.$

INTEGRATION BY PARTS

$$\int f(x) . g(x) dx = f(x) . \left[\int g(x) dx\right] - \int f'(x) . \left[\int g(x) dx\right] dx.$$

PROPERTIES OF DEFINITE INTEGRALS

$$\int_a^b f(x) dx = F(b) - F(a), \text{ where } F(x) = \int f(x) dx.$$

1. $\int_a^b f(x) dx = -\int_b^a f(x) dx.$ 2. $\int_a^b f(x) dx = \int_a^b f(t) dt.$

3. $\int_a^b f(x)dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$

4. $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx.$

5. $\int_{-a}^a f(x) = 0;$ if $f(x)$ is odd function.

6. $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx,$ if $f(2a-x) = f(x).$

$= 0$ if $f(2a - x) = -f(x).$

Integral as limit of sum :

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + \ldots.. + f\left(a + \overline{n-1}\,h\right)\right]$$

where $h = \dfrac{b-a}{h}.$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Evaluate the following integrals

(i) $\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx.$

(ii) $\int \frac{1}{1-\sin^2 x}\,dx.$

(iii) $\int \frac{\cos x}{1-\cos^2 x}\,dx.$

(iv) $\int_1^2 \frac{1}{x\sqrt{x^2-1}}\,dx.$

(v) $\int_0^{\pi/2} \log\frac{4+3\sin x}{4+3\cos x}\,dx.$

(vi) $\int \operatorname{cosec} x\left(\operatorname{cosec} x + \cot x\right)dx.$

(vii) $\int \frac{dx}{\sin\left(\cos^{-1} x\right)}.$

2. Evaluate the following integrals.

(i) $\int \frac{x^2+x-1}{\sqrt{x}}.$

(ii) $\int\left(\sqrt{ax} - \frac{1}{\sqrt{ax}}\right)^2 dx.$

(iii) $\int\left(\sin 3x - 3e^{4x} + \sec^2\frac{x}{2}\right)dx.$

(iv) $\int \frac{1+\cos 2x}{1-\cos 2x}\,dx.$

(v) $\int \frac{x}{\sqrt{2x+1}}\,dx.$

(vi) $\int \frac{\sec x \ \operatorname{cosec} x}{\log \tan x}\,dx.$

(vii) $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\,dx.$

(viii) $\int \left(e^{a \log x} + e^{x \log a}\right)dx.$

(ix) $\int \sqrt{1-\sin x}\ dx, \left(\frac{\pi}{2} < x < \pi\right).$

(x) $\int \frac{1}{x\left(2+3\log x\right)}\,dx.$

(xi) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx.$

(xii) $\int 2\log x \ dx.$

(xiii) $\int \frac{ax+b}{ax^2+2bx+c}\,dx.$

(xiv) $\int \frac{\sin x}{a+b\cos x}\,dx.$

(xv) $\int \left(c^x + x^c\right)dx.$

(xvi) $\int \frac{1}{3x + x\log x}\,dx.$

(xvii) $\int \frac{1}{16+25x^2}\,dx.$

(xviii) $\int \frac{1}{9x^2-4}\,dx.$

(xix) $\int \frac{1}{16-25x^2}\,dx.$

(xx) $\int \frac{1}{\sqrt{4x^2-9}}\,dx.$

3. Evaluate the following definite integrals :

(i) $\int_0^{\frac{\pi}{2}} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\,dx.$

(ii) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \ dx.$

(iii) $\int_0^1 \frac{1}{1+x^2}\ dx.$

(iv) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}\,dx.$

(v) $\int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2}\ dx.$

(vi) $\int_0^1 \frac{e^x}{1+e^{2x}}\ dx.$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

4. Evaluate the following integrals :

(i) $\int \frac{x \ \operatorname{cosec}\left(\tan^{-1} x^2\right)}{1+x^4}\,dx.$

(ii) $\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\,dx.$

(iii) $\int \frac{1}{\sin(x-a)\sin(x-b)}\,dx.$

(iv) $\int \frac{\cos(x+a)}{\cos(x-a)}\,dx.$

(v) $\int \cos x \cos 2x \cos 3x\ dx.$

(vi) $\int \cos^5 x\ dx.$

(vii) $\int \sin^2 x \cos^4 x\ dx.$

(viii) $\int \cot^3 x\ \operatorname{cosec}^4 x\ dx.$

(ix) $\int \frac{\sin x \cos x}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}}\,dx.$

(x) $\int \frac{1}{\sqrt{\cos^3 x \cos(x+a)}}\,dx.$

(xi) $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}\,dx.$

(xii) $\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\,dx.$

5. Evaluate :

(i) $\int \frac{x}{x^4 + x^2 + 1}\,dx.$

*(ii) $\int \frac{1}{x\left[6(\log x)^2 + 7\log x + 2\right]}\,dx.$

(iii) $\int \frac{dx}{1 + x - x^2}.$

(iv) $\int \frac{1}{\sqrt{9 + 8x - x^2}}\,dx.$

(v) $\int \frac{1}{\sqrt{(x-a)(x-b)}}\,dx.$

(vi) $\int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+a)}}\,dx.$

(vii) $\int \frac{5x - 2}{3x^2 + 2x + 1}\,dx.$

(viii) $\int \frac{x^2}{x^2 + 6x + 12}\,dx.$

(ix) $\int \frac{x + 2}{\sqrt{4x - x^2}}\,dx.$

(x) $\int x\sqrt{1 + x - x^2}\,dx.$

(xii) $\int (3x - 2)\sqrt{x^2 + x + 1}\ dx.$

(xiii) $\int \sqrt{\sec x + 1}\ dx.$

6. Evaluate :

(i) $\int \frac{dx}{x\left(x^7 + 1\right)}.$

(ii) $\int \frac{\sin x}{(1 + \cos x)(2 + 3\cos x)}\,dx.$

(iii) $\int \frac{x^2 + 1}{(1 + \cos x)(2 + 3\cos x)}\,dx.$

(iv) $\int \frac{x - 1}{(x+1)(x-2)(x+3)}\,dx.$

(v) $\int \frac{x^2 + x + 2}{(x-2)(x-1)}\,dx.$

(vi) $\int \frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^3+3\right)\left(x^2+4\right)}\,dx.$

(vii) $\int \frac{dx}{(2x+1)\left(x^2+4\right)}.$

(viii) $\int \frac{dx}{\sin x\,(1 - 2\cos x)}.$

(ix) $\int \frac{\sin x}{\sin 4x} dx.$

(x) $\int \frac{x^2 - 1}{x^4 + x^2 + 1} dx.$

(xi) $\int \sqrt{\tan x}\ dx.$

(xii) $\int \frac{x^2 + 9}{x^4 + 81} dx.$

7. Evaluate :

(i) $\int x^5 \sin x^3 dx.$

(ii) $\int \sec^3 x\ dx.$

(iii) $\int e^{ax} \cos(bx + c)\,dx.$

(iv) $\int \sin^{-1} \frac{6x}{1 + 9x^2} dx.$

(v) $\int \cos\sqrt{x}\ dx.$

(vi) $\int x^3 \tan^{-1} x\ dx.$

(vii) $\int e^{2x} \left(\frac{1 + \sin 2x}{1 + \cos 2x}\right) dx.$

(viii) $\int e^x \left(\frac{x - 1}{2x^2}\right) dx.$

(ix) $\int e^x \left(\frac{1 - x}{1 + x^2}\right)^2 dx.$

(x) $\int e^x \frac{(x^2 + 1)}{(x + 1)^2} dx.$

(xi) $\int e^x \frac{(2 + \sin 2x)}{(1 + \cos 2x)} dx.$

(xii) $\int \left\{\log(\log x) + \frac{1}{(\log x)^2}\right\} dx.$

8. Evaluate the following definite integrals :

(i) $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx.$

(ii) $\int_0^{\frac{\Pi}{2}} \cos 2x \log \sin x\ dx.$

(iii) $\int_0^1 x\sqrt{\frac{1 - x^2}{1 + x^2}}\, dx.$

(iv) $\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1 - x^2)^{3/2}} dx.$

(v) $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx.$

(vi) $\int_1^2 \frac{5x^2}{x^2 + 4x + 3} dx.$

(vii) $\int_0^{\frac{\pi}{2}} \frac{x + \sin x}{1 + \cos x} dx.$

9. Evaluate :

(i) $\int_1^3 \{|x - 1| + |x - 2| + |x - 3|\,dx\}.$

(ii) $\int_0^{\pi} \frac{x}{1 + \sin x} dx.$

(iii) $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)\, dx.$

(iv) $\int_0^{\frac{\pi}{2}} \log \sin x \; dx.$

(v) $\int_0^{\pi} \frac{x \sin x}{\left(1 + \cos^2 x\right)}\, dx.$

(vi) $\int_{-2}^{2} f(x)\, dx$ where $f(x) = \begin{cases} 2x - x^3 & \text{when } -2 \le x < 1 \\ x^3 - 3x + 2 & \text{when } -1 \le x < 1 \\ 3x - 2 & \text{when } 1 \le x < 2. \end{cases}$

(vii) $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx.$

(viii) $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x}\, dx.$

10. Evaluate the following integrals as limit of a sum

(i) $\int_1^3 x^2 dx.$

(ii) $\int_{-1}^{1} e^x dx.$

(iii) $\int_0^2 e^{-x} dx.$

(iv) $\int_0^2 (2x + 3)\, dx.$

11. Evaluate the following integrals.

(i) $\int_1^3 \left|x^2 - 2x\right|\, dx.$

(ii) $\int_0^1 \sin^{-1}\left(\frac{2x}{1 + x^2}\right)\, dx.$

(iii) $\int_{-1}^{1} \log\left(\frac{1 + \sin x}{1 - \sin x}\right)\, dx.$

(iv) $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\, dx.$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

12. Evaluate the following integrals :

(i) $\frac{x^5 + 4}{x^5 - x}\, dx.$

(ii) $\int \frac{dx}{(x - 1)^2 \left(x^2 + 4\right)}\, dx$

(iii) $\int \frac{2x^3}{(x + 1)(x - 3)^2}\, dx$

(iv) $\int \frac{x^4}{x^4 - 16}\, dx$

(v) $\int_0^{\frac{\pi}{2}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx.$

(vi) $\int \frac{1}{x^4 + 1} dx.$

(vii) $\int_0^{\infty} \frac{x \tan^{-1} x}{\left(1 + x^2\right)^2} dx.$

13. Evaluate the following integrals as limit of sums :

(i) $\int_2^4 (2x + 1) dx.$

(ii) $\int_0^2 \left(x^2 + 3\right) dx.$

(iii) $\int_1^3 \left(3x^2 - 2x + 4\right) dx.$

(iv) $\int_0^4 \left(3x^2 + e^{2x}\right) dx.$

(v) $\int_2^5 \left(x^2 + 3x\right) dx.$

H.O.T.S.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK EACH)

14. Evaluate the following integrals :

(i) $\int \left(\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}\right) dx$

(ii) $\int \tan^{-1} (\cot x) dx$

(iii) $\int \frac{1}{x \cos^2 (1 + \log x)} dx$

(iv) $\int \frac{x}{e^{x^2}} dx$

(v) $\int_{-1}^{1} e^{|x|} dx$

(vi) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^3 \sin^4 x \, dx$

(vii) $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\tan x}} dx$

SHORT ANSWER TYPE QUESTIONS (4 MARKS EACH)

15. Evaluate

(i) $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\, dx,\ x \in [0, 1]$ (ii) $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\, dx$

(iii) $\int \frac{\sqrt{x^2+1}\left[\log(x^2+1) - 2\log x\right]}{x^4}\, dx$ (iv) $\int \frac{x^2}{(x\sin x + \cos x)^2}\, dx$

(v) $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\, dx$ (vi) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$

(vii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin|x| - \cos|x|)\, dx$ (viii) $\int_0^{\pi} \frac{x\, dx}{a^2\cos^2 x + b^2\sin^2 x}$

(ix) $\int_1^{\pi} [x^2]\, dx$, where [x] is greatest integer function

(x) $\int_{-1}^{\frac{3}{2}} |x \sin \pi x|\, dx$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

16. Evaluate

(i) $\int_0^1 \cot^{-1}(1 - x + x^2)\, dx$ (ii) $\int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)}$

(iii) $\int_0^1 \frac{\log(1+x)}{1+x^2}\, dx$ (iv) $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)\, dx$.

ANSWERS

1. (i) $\frac{2}{3}x^{3/2} + 2\sqrt{x} + c$. (ii) $\tan x + c$.

(iii) $-\operatorname{cosec} x + c$ (iv) $\frac{\pi}{3}$.

(v) 0; (vii) $-\cot x - \operatorname{cosec} x + C$

(ix) $\sin^{-1} x + c$

2. (i) $\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c.$ (ii) $\frac{a}{2}x^2 + \frac{1}{a}\log|x| - 2x + c.$

(iii) $\frac{-\cos x}{3} - \frac{3}{4}e^{4x} + 2\tan\frac{x}{2} + c.$ (iv) $-\operatorname{cosec} x + c.$

(v) $\frac{1}{2}\left[\frac{(2x+1)^{3/2}}{3} - (2x+1)^{1/2}\right] + c$ (vi) $\log[\log(\tan x)] + \text{c}.$

(vii) $\tan x + c.$ (viii) $\frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c.$

(ix) $2\left[-\cos\frac{x}{2} - \sin\frac{x}{2}\right] + c.$ (x) $\frac{1}{3}\log(2 + 3\log x) + c.$

(xi) $-2\cos\sqrt{x} + c$ (xii) $2x(\log x - 1) + c.$

(xiii) $\frac{1}{2}\log\left|ax^2 + 2bx + c\right| + c$ (xiv) $-\frac{1}{b}\log|a + b\cos x| + c.$

(xv) $\frac{c^x}{\log c} + \frac{x^{c+1}}{c+1} + c_1.$ (xvi) $\log|3 + \log x| + c.$

(xvii) $\frac{1}{20}\tan^{-1}\frac{5}{4}x + c.$ (xviii) $\frac{1}{12}\log\left|\frac{3x-2}{3x+2}\right| + c.$

(xix) $\frac{1}{40}\log\left|\frac{4+5x}{4-5x}\right| + c.$ (xx) $\frac{1}{2}\log\left|2x + \sqrt{4x^2 - 9}\right| + c.$

3. (i) $\frac{\pi}{4}.$ (ii) 0;

(iii) $\frac{\pi}{4}.$ (iv) $\frac{\pi}{4}.$

(v) $\frac{\pi^{3/2}}{12}.$ (vi) $\tan^{-1} e - \frac{\pi}{4}.$

4. (i) $\frac{1}{2}\log\left[\operatorname{cosec}\left(\tan^{-1} x^2\right) - \frac{1}{x^2}\right] + c.$

(ii) $\frac{1}{2}\left(x^2 - x\sqrt{x^2 - 1}\right) + \frac{1}{2}\log\left|x + \sqrt{x^2 - 1}\right| + c.$

(iii) $\frac{1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c$

(iv) $x \cos 2a - \sin 2a \log |\sec(x-a)| + c.$

(v) $\frac{1}{48} [12x + 6 \sin 2x + 3 \sin 4x + 2 \sin 6x] + c.$

(vi) $\sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + c.$

(vii) $\frac{1}{32} \left[2x + \frac{1}{2} \sin 2x - \frac{1}{2} \sin 4x - \frac{1}{6} \sin 6x \right] + c.$

(viii) $-\left(\frac{\cot^6 x}{6} + \frac{\cot^4 x}{4} \right) + c.$

(ix) $\frac{1}{(a^2 - b^2)\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}} + c.$ [**Hint. :** put $a^2 \sin^2 x + b^2 \cos^2 x = t$]

(x) $-2 \operatorname{cosec} a \sqrt{\cos a - \tan x . \sin a} + c.$ [**Hint. :** Take $\sec^2 x$ as numerator]

(xi) $\tan x - \cot x - 3x + c.$

(xii) $\sin^{-1} (\sin x - \cos x) + c.$

5. (i) $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2 + 1}{\sqrt{3}} \right) + c.$ [**Hint :** put $x^2 = t$]

(ii) $\log \left| \frac{2 \log x + 1}{3 \log x + 2} \right| + C$ [**Hint :** put $\log x = t$]

(iii) $\frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{5} - 1 + 2x}{\sqrt{5} + 1 - 2x} \right| + c$ (iv) $\sin^{-1} \left(\frac{x-4}{5} \right) + c.$

(v) $2 \log \left| \sqrt{x-a} + \sqrt{x-b} \right| + c$

(vi) $-\cos\alpha \sin^{-1} \left(\frac{\cos x}{\cos\alpha} \right) - \sin\alpha . \log \left| \sin x + \sqrt{\sin^2 x - \sin^2 \alpha} \right| + c$

$\left[\textbf{Hint} : \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}} = \frac{\sin(x-\alpha)}{\sin^2 x - \sin^2 \alpha} \right]$

(vii) $\frac{5}{6} \log \left| 3x^2 + 2x + 1 \right| + \frac{-11}{3\sqrt{2}} \tan^{-1} \left(\frac{3x+1}{\sqrt{2}} \right) + c$

(viii) $x - 3\log\left|x^2 + 6x + 12\right| + 2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c$

(ix) $-\sqrt{4x - x^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right) + c$

(x) $\frac{-1}{3}\left(1 + x - x^2\right)^{\frac{3}{2}} + \frac{1}{8}(2x-1)\sqrt{1 + x - x^2} + \frac{5}{16}\sin^{-1}\left(\frac{2x-1}{\sqrt{5}}\right) + c$

(xi) $\left(x^2 + x + 1\right)^{\frac{3}{2}} - \frac{7}{2}\left[\left(x + \frac{1}{2}\right)\sqrt{x^2 + x + 1} + \frac{3}{8}\log\left|x + \frac{1}{2} + \sqrt{x^2 + x + 1}\right|\right] + c$

(xii) $-\log\left|\cos x + \frac{1}{2} + \sqrt{\cos^2 x + \cos x}\right| + c$ [**Hint. :** Multiply and divide by $\sqrt{\sec x + 1}$]

6. (i) $\frac{1}{7}\log\left|\frac{x^7}{x^7 + 1}\right| + c$

(ii) $\log\left|\frac{1 + \cos x}{2 + 3\cos x}\right| + c$

(iii) $\frac{3}{8}\log|x - 1| - \frac{1}{2(x-1)} + \frac{5}{8}\log|x + 3| + c$

(iv) $\frac{9}{10}\log|x + 3| + \frac{4}{15}\log|x - 2| - \frac{1}{6}|x + 1| + c$

(v) $x + 4\log\left|\frac{(x-2)^2}{x-1}\right| + c$

(vi) $x + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - 3\tan^{-1}\left(\frac{x}{2}\right) + c$ [**Hint. :** put $x^2 = t$]

(vii) $\frac{2}{17}\log|2x + 1| - \frac{1}{17}\log\left|x^2 + 4\right| + \frac{1}{34}\tan^{-1}\frac{x}{2} + c$

(viii) $-\frac{1}{2}\log|1 - \cos x| - \frac{1}{6}\log|1 + \cos x| + \frac{2}{3}\log|1 - 2\cos x| + c$

[**Hint. :** multiply N^r and D^r by $\sin x$ and put $\cos x = t$]

(ix) $\frac{-1}{8}\log\left|\frac{1 + \sin x}{1 - \sin x}\right| + \frac{1}{4\sqrt{2}}\log\left|\frac{1 + \sqrt{2}\sin x}{1 - \sqrt{2}\sin x}\right| + c$

(x) $\frac{1}{2}\log\left|\frac{x^2 - x + 1}{x^2 + x + 1}\right| + c$

(xi) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2 \tan x}}\right) + \frac{1}{2\sqrt{2}} \log\left|\frac{\tan x - \sqrt{2 \tan x} + 1}{\tan x + \sqrt{2 \tan x} + 1}\right| + c$

(xii) $\frac{1}{3\sqrt{2}} \tan^{-1}\left(\frac{x^2 - 9}{3\sqrt{2}}\right) + c$

7. (i) $\frac{1}{3}\left[-x^3 \cos x^3 + \sin x^3\right] + c$

(ii) $\frac{1}{2}\left[\sec x \tan x + \log|\sec x + \tan x|\right] + c$

[**Hint. :** Write $\sec^3 x = \sec x \cdot \sec^2 x$ and take $\sec x$ as first function]

(iii) $\frac{e^{ax}}{a^2 + b^2}\left[a \cos(bx + c) + b \sin(bx + c)\right] + c_1$

(iv) $2x \tan^{-1} 3x - \frac{1}{3} \log\left|1 + 9x^2\right| + c$ [**Hint. :** put $3x = \tan \theta$]

(v) $2\left[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}\right] + c$

(vi) $\left(\frac{x^4 - 1}{4}\right) \tan^{-1} x - \frac{x^3}{12} + \frac{x}{4} + c.$

(vii) $\frac{1}{2} e^{2x} \tan x + c.$

(viii) $\frac{e^x}{2x} + c.$

(ix) $\frac{e^x}{1 + x^2} + c.$

(x) $e^x \left(\frac{x - 1}{x + 1}\right) + c.$

(xi) $e^x \tan x + c.$

(xii) $x \log|\log x| - \frac{x}{\log x} + c.$ [**Hint. :** put $\log x = t \Rightarrow x = e^t$]

8. (i) $\frac{1}{20} \log 3.$ (ii) $-\pi/2$

(iii) $\frac{\pi}{4} - \frac{1}{2}.$ [**Hint. :** put $x^2 = t$] (iv) $\frac{\pi}{4} - \frac{1}{2} \log 2.$

(v) $\frac{\pi}{2}$.

(vi) $5 - 10\log\frac{15}{8} + \frac{25}{2}\log\left(\frac{6}{5}\right)$.

(vii) $\pi/2$.

$\left[\textbf{Hint. :} \left(\frac{x}{1+\cos x} + \frac{\sin x}{1+\cos x}\right) dx.\right]$

9. (i) 8.

(ii) π.

(iii) $\frac{\pi}{8}\log 2$.

(iv) $\frac{-\pi}{2}\log 2$.

(v) $\frac{1}{4}\pi^2$.

(vi) 95/12.

$\left[\textbf{Hint. :} \int_{-2}^{2} f(x)\,dx = \int_{-2}^{-1} f(x)\,dx + \int_{-1}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx\right]$

(vii) $\frac{\pi^2}{16}$.

(viii) $\frac{\pi^2}{2ab}$.

$\left[\textbf{Hint. :} \text{ Use } \int_0^a f(x) = \int_0^a f(a-x)\right]$

10. (i) $\frac{26}{3}$.

(ii) $e - \frac{1}{e}$.

(iii) $1 - \frac{1}{e^2}$.

(iv) 10.

11. (i) 2.

(ii) $\frac{\pi}{2} - \log 2$.

(iii) 0.

(iv) $\pi/2$.

12. (i) $x - 4\log|x| + \frac{5}{4}\log|x-1| + \frac{3}{4}\log|x+1| + \log\left|x^2+1\right| - \frac{1}{2}\tan^{-1}x + c$.

$\left[\textbf{Hint. :} \frac{x^5+4}{x^5-x} = 1 + \frac{x+4}{x(x-1)(x+1)\left(x^2+1\right)}\right]$

(ii) $\frac{-2}{25}\log|x-1| - \frac{1}{5(x-1)} + \frac{1}{25}\log\left(x^2+4\right)\frac{-3}{50}\tan^{-1}\frac{x}{2} + c$.

$\left[\textbf{Hint. :} \frac{1}{(x-1)^2\left(x^2+4\right)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{cx+D}{x^2+4}\right]$

(iii) $2x - \frac{1}{8}\log|x+1| + \frac{81}{8}\log|x-3| - \frac{27}{2(x-3)} + c.$

(iv) $x + \frac{1}{2}\log\left|\frac{x-2}{x+2}\right| - \tan^{-1}\left(\frac{x}{2}\right) + c.$

(v) $\pi/\sqrt{2}.$

(vi) $\frac{1}{2\sqrt{2}}\tan^{-1}\frac{(x^2-1)}{\sqrt{2x}} - \frac{1}{4\sqrt{2}}\log\left|\frac{x^2-\sqrt{2x}+1}{x^2+\sqrt{2\lambda}+1}\right| + c$

(vii) $\pi/8.$

13. (i) 14. (ii) $\frac{26}{3}.$

(iii) 26. (iv) $\frac{1}{2}(127 + e^8).$

(v) $\frac{141}{2}.$

14. (i) $\frac{\pi}{2}x + c$ (ii) $\frac{\pi}{2}x - \frac{x^2}{2} + c$

(iii) $\tan(1 + \log|x|) + c$ (iv) $-\frac{1}{2e^{x^2}} + c$

(v) $2e - 2$ (vi) 0

(vii) $\frac{\pi}{4}.$

15. (i) $\frac{2(2x-1)}{\pi}\sin^{-1}\sqrt{x} + \frac{2\sqrt{x-x^2}}{\pi} - x + c$

(ii) $-2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x-x^2} + c$

(iii) $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right] + c$

(iv) $\frac{\sin x - x\cos x}{x\sin x + \cos x} + c$

(v) $(x+a)\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + c$ (vi) $2\sin^{-1}\frac{\sqrt{3}-1}{2}$

(vii) 0

(viii) $\dfrac{\pi^2}{2ab}$

(ix) $\sqrt{2} - \sqrt{3} + 5$

(x) $\dfrac{3}{\pi} + \dfrac{1}{\pi^2}$.

16. (i) $\dfrac{\pi}{2} - \log 2$

(ii) $-\dfrac{1}{5}\log\left|\dfrac{\tan x - x}{2\tan x + 1}\right| + c$

(iii) $\dfrac{\pi}{8}\log 2$.

(iv) $\dfrac{\pi}{2}\log\dfrac{1}{2}$.

CHAPTER 8

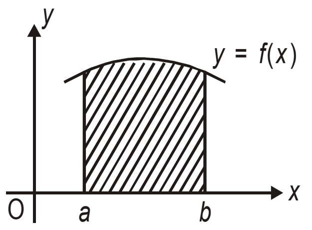
APPLICATIONS OF THE INTEGRALS

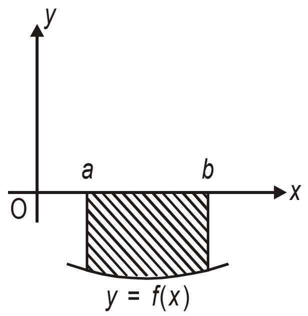
POINTS TO REMEMBER

AREA OF BOUNDED REGION

1. Area bounded by the curve $y = f(x)$, the x axis and between the ordinats, $x = a$ and $x = b$ is given by

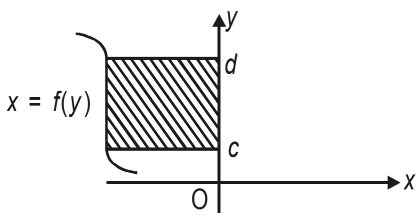
$$\text{Area} = \left| \int_a^b f(x)\, dx \right|$$

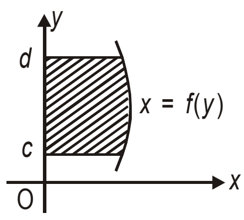




2. Area bounded by the curve $x = f(y)$ the y-axis and between abcissae, $y = c$ and $y = d$ is given by

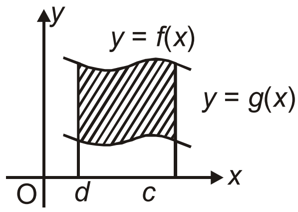
$$\text{Area} = \left| \int_c^d x\ dy \right| = \left| \int_c^d f(y)\ dy \right|$$





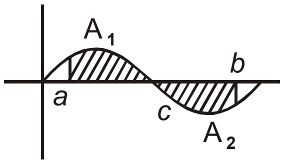
3. Area bounded by two curves $y = f(x)$ and $y = g(x)$ such that $0 \le g(x) \le f(x)$ for all $x \in [ab]$ and between the ordinate at $x = a$ and $x = b$ is given by

$$\text{Area} = \int_a^b \left[f(x) - g(x) \right] dx$$



4. If the curve $y = f(x)$ interest the axis (x-axis) then the area of shaded region is given by

Area = $|A_1| + |A_2|$.



Area = $\int_a^c f(x)\,dx + \left|\int_c^b f(x)\,dx\right|$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

1. Find the area enclosed by circle $x^2 + y^2 = a^2$.
2. Find the area of region bounded by $y^2 = 4x$, $x = 1$, $x = 4$ and x – axis in first quadrant.
3. Find the area enclosed by the ellipse

   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a > b.$$

4. Find the area of region in the first quadrant enclosed by x–axis the line $y = x$ and the circle $x^2 + y^2 = 32$.
5. Find the area of region $\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$
6. Prove that the curve $y = x^2$ and, $x = y^2$ divide the square bounded by $x = 0$, $y = 0$, $x = 1$, $y = 1$ into three equal parts.
7. Find smaller of the two areas enclosed between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line

   $bx + ay = ab$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

8. Find the common area bounded by the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.
9. Using integration find area of region bounded by the triangle whose vertices are

   (a) (–1, 0), (1, 3) and (3, 2)　　　(b) (–2, 2) (0, 5) and (3, 2)

10. Using integration find the area bounded by the lines.

(i) $x + 2y = 2$, $y - x = 1$ and $2x + y - 7 = 0$

(ii) $y = 4x + 5$, $y = 5 - x$ and $4y - x = 5$.

11. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.

12. Find the area of the region bounded by

$y = |x - 1|$ and $y = 1$.

13. Find the area enclosed by the curve $y = \sin x$ between $x = 0$ and $x = \frac{3\pi}{2}$ and x-axis.

14. Find the area bounded by semi circle $y = \sqrt{25 - x^2}$ and x-axis.

15. Find area of region given by $\{(x, y) : x^2 \leq y \leq |x|\}$.

16. Find area of smaller region bounded by ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and straight line $2x + 3y = 6$.

17. Find the area of region bounded by the curve $x^2 = 4y$ and line $x = 4y - 2$.

18. Using integration find the area of region in first quadrant enclosed by x-axis the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

19. Draw a sketch of the region $\{(x, y) : x^2 + y^2 \leq 4 \leq x + y\}$ and find its area.

20. Find smaller of two areas bounded by the curve $y = |x|$ and $x^2 + y^2 = 8$.

H.O.T.S.

21. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

22. Using integration find the area enclosed by the curve $y = \cos x$, $y = \sin x$ and x-axis in the interval $\left(0, \frac{\pi}{2}\right)$.

23. Sketch the graph $y = |x - 5|$. Evaluate $\int_0^6 |x - 5| \, dx$.

ANSWERS

1. πa^2 sq. units.
2. $\frac{28}{3}$ sq. units
3. πab sq. units
4. $4\pi - 8$ sq. units

5. $\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{8}\sin^{-1}\left(\frac{1}{3}\right)$ sq. units

7. $\frac{(\pi - 2)ab}{4}$ sq. units

8. $\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ sq. units

9. (a) 4 sq. units (b) 2 sq. units

10. (a) 6 sq. unit [**Hint.** Coordinate of verties are (0, 1) (2, 3) (4, – 1)]

(b) $\frac{15}{2}$ sq. [**Hint.** Coordinate of verties are (– 1, 1) (0, 5) (3, 2)]

11. $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq. units

12. 1 sq. units

13. 3 sq. units

14. $\frac{25}{2}\pi$ sq. units

15. $\frac{1}{3}$ sq. units

16. $\frac{3}{2}(\pi - 2)$ sq. units

17. $\frac{9}{8}$ sq. units

18. $\frac{\pi}{3}$ sq. unit

19. $(\pi - 2)$ sq. unit

20. 2π sq. unit.

21. $\frac{4}{3}(8 + 3\pi)$ sq. units

22. $\left(2 - \sqrt{2}\right)$ sq. units.

23. 5 sq. units.

CHAPTER 9

DIFFERENTIAL EQUATION

POINTS TO REMEMBER

- **Differential Equation :** Equation containing derivatives of a dependant variable with respect to an independent variable is called differential equation.
- **Order of a Differential Equation :** The order of a differential equation is defined to be the order of the highest order derivative occurring in the differential equation.
- **Degree of a Differential Equation :** The degree of differential equation is defined to be the degree of highest order derivative occurring in it after the equation has been made free from radicals and fractions. Solving a differential equation.

(i) Type $\frac{dy}{dx} = f(x).g(y)$: Variable separable method separate the variables and get $f(x)$ $dx = h(y)\ dy$. The $\int f(x)\,dx = \int h(y)\,dy + c$ is the required solution.

(ii) ***Homogenous differential equation :*** A differential equation of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ where $f(x, y)$ and $g(x, y)$ are both homogeneous functions of the same degree in x and y *i.e.,* of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is called a homogeneous differential equation. Substituting $y = vx$ and then $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get variable separable form.

(ii) ***Linear differential equation :*** Type I : $\frac{dy}{dx} + py = q$ where p and q are functions of x. Its solution is $y\ .\ (I.\ F.) = \int q(I.\ F.)dx$ where $I.F. = e^{\int p\,dx}$.

VERY SHORT ANSWER TYPE QUESTIONS

1. Write the order and degree of the following differential equations.

(i) $\frac{dy}{dx} + \cos y = 0.$

(ii) $\left(\frac{dy}{dx}\right)^2 + 3\frac{d^2y}{dx^2} = 4.$

(iii) $\frac{d^4y}{dx^4} + \sin x = \left(\frac{d^2y}{dx^2}\right)^5.$

(iv) $\frac{d^5y}{dx^5} + \log\left(\frac{dy}{dx}\right) = 0.$

*(v) $\sqrt{1+\frac{dy}{dx}} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$. (vi) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = K\frac{d^2y}{dx^2}$.

(vii) $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 = \sin x$.

2. Write the general solution of following differential equations.

(i) $\frac{dy}{dx} = x^5 + x^2 - \frac{2}{x}$. (ii) $(e^x + e^{-x})\,dy = (e^x - e^{-x})dx$

(iii) $\frac{dy}{dx} = x^3 + e^x + x^e$. (iv) $\frac{dy}{dx} = 5^{x+y}$.

(v) $\frac{dy}{dx} = \frac{1-\cos 2x}{1+\cos 2y}$. (vi) $\frac{dy}{dx} = \frac{1-2y}{3x+1}$.

3. What is the integrating factor in each of the following linear differential equations.

(i) $\frac{dy}{dx} + y\cos x = \sin x$. (ii) $\frac{dy}{dx} + \frac{y}{\cos^2 x} = \sin x \cos x$.

(iii) $x^2\frac{dy}{dx} + y = x^2 \cos x$. (iv) $x\frac{dy}{dx} + \log x \,.\, y = \tan x \,.\, e^x$.

(v) $\frac{dy}{dx} - \frac{3}{x}.y = \log x$. (vi) $\frac{dx}{dy} + (\tan y)x = \sec^2 y$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

4. (i) Verify that $y = e^{m\sin^{-1}x}$ is a solution of $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$.

(ii) Show that $y = \sin(\sin x)$ is a solution of diff. equation

$$\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} = -y\cos^2 x.$$

(iii) Show that $y = Ax + \frac{B}{x}$ is a solution of

$$x^2.\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$$

(iv) Show that function $y = a\cos(\log x) + b\sin(\log x)$ is the solution of

$$x^2.\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

(v) Find the differential equation of the family of curves $y = e^x (A \cos x + B \sin x)$, where A and B are arbitrary constants.

(vi) Find the differential equation of an ellipse with major and minor axes $2a$ and $2b$ respectively.

(vii) Form the differential equation corresponding to the family of curves $y = c(x - c)^2$.

(viii) Form the differential equation representing the family of curves $(y - b)^2 = 4(x - a)$.

5. Solve the following diff. equations.

(i) $\dfrac{dy}{dx} + y \cot x = \sin 2x.$ (ii) $x\dfrac{dy}{dx} + 2y = x^2 \log x.$

(iii) $\dfrac{dx}{dy} + \dfrac{1}{x} . y = \cos x + \dfrac{\sin x}{x}, \quad x > 0.$

(iv) $\cos^3 x \dfrac{dy}{dx} + \cos x = \sin x.$ (v) $y\, e^y\, dx = (x^3 + 2xe^y)\, dy.$

6. Solve each of the following differential equations :

(i) $y - x\dfrac{dy}{dx} = 2\left(y^2 + \dfrac{dy}{dx}\right).$ (ii) $\cos y\, dx + (1 + 2e^{-x}) \sin y\, dy = 0.$

(iii) $x\sqrt{1 - y^2}dy + y\sqrt{1 - x^2}dx = 0.$ (iv) $\sqrt{(1 - x^2)(1 - y^2)}\; dy + xy\; dx = 0.$

(v) $(xy^2 + x)\, dx + (yx^2 + y)\, dy = 0;\; y(0) = 1.$

(vi) $\dfrac{dy}{dx} = y \sin^3 x \cos^3 x + xy\, e^x.$

(vii) $\tan x \;\tan y\, dx + \sec^2 x \sec^2 y\, dy = 0$

7. Solve the following differential equations :

(i) $x^2\, y\, dx - (x^3 + y^3)\, dy = 0.$ (ii) $x^2 \dfrac{dy}{dx} = x^2 + xy + y^2.$

(iii) $\left(x^2 - y^2\right)dx + 2xy\; dy = 0, \quad y(1) = 1.$

(iv) $\left(y \sin \dfrac{x}{y}\right)dx = \left(x \sin \dfrac{x}{y} - y\right)dy.$ (v) $\dfrac{dy}{dx} = \dfrac{y}{x} + \tan\left(\dfrac{y}{x}\right).$

(vi) $\dfrac{dy}{dx} = \dfrac{2xy}{x^2 + y^2}$ (vii) $\dfrac{dy}{dx} = e^{x+y} + x^2 e^y.$

(xii) $\dfrac{dy}{dx} = \sqrt{\dfrac{1 - y^2}{1 - x^2}}.$

8. (i) Form the differential equation of the family of circles touching y-axis at (0, 0).

(ii) Form the differential equation of family of parabolas having vertex at (0, 0) and axis along the (i) positive y-axis (ii) +ve x-axis.

(iii) Form differential equation of all circles passing through origin and whose centre lie on x-axis.

9. Show that the differential equation $\frac{dy}{dx} = \frac{x + 2y}{x - 2y}$ is homogeneous and solve it.

10. Show that the differential equation :

$(x^2 + 2xy - y^2)\, dx + (y^2 + 2xy - x^2)\, dy = 0$ is homogeneous and solve it.

11. Solve the following differential equations :

(i) $\frac{dy}{dx} - 2y = \cos 3x.$

(ii) $\sin x \frac{dy}{dx} + y \cos x = 2 \sin^2 x \cos x$ given that $y = 1$ when $x = \frac{\pi}{2}$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

12. Solve the following differential equations :

(i) $(x^3 + y^3)\, dx = (x^2y + xy^2)dy.$ (ii) $x\, dy - y\, dx = \sqrt{x^2 + y^2}\, dx.$

(iii) $y \left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} dx - x \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} dy = 0.$

(iv) $x^2 dy + y(x + y)\, dx = 0$ given that $y = 1$ when $x = 1$.

(v) $xe^{\frac{y}{x}} - y + x \frac{dy}{dx} = 0$ given that $y = 0$ when $x = e$.

(vi) $(x^3 - 3xy^2)\, dx = (y^3 - 3x^2y)dy.$

HIGHER ORDER THINKING SKILLS (HOTS)

VERY SHORT ANSWER TYPE QUESTIONS (1 MARKS)

13. (i) Write the order and degree of the differential equation $\frac{dy}{dx} + \tan\left(\frac{dy}{dx}\right) = 0.$

(ii) What will be the order of the differential equation, corresponding to the family of curves $y = a \cos (x + b)$, where a is arbitrary constant.

(iii) What will be the order of the differential education $y = a + be^{x+c}$ where a, b, c are arbitrary constant.

(iv) Find the integrating factor for solving the differential education $\frac{dy}{dx} + y \tan x = \cos x$.

(v) Find the integrating factor for solving the differential equation $\frac{dx}{dy} + \frac{1}{1+y^2} x = \sin y$.

14. (i) Form the differential equation of the family of circles in the first quadrant and touching the coordinate axes.

(ii) Verify that $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ satisfies the differential education

$$\left(a^2 \div x^2\right) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0.$$

(iii) Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$ is given by $(x + y + 1) = A(1 - x - y - 2xy)$. Write A is parameter.

15. Solving the following differential equation

(i) $(\tan^{-1} y - x)\, dx = (1 + y^2)\, dx.$

(ii) $(x - y + 2) \frac{dy}{dx} = 1.$

(iii) $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0.$

(iv) $(x - \sin y)\, dy + \tan y\, dx = 0,\ y(0) = 0.$

LONG ANSWER TYPE QUESTIONS (6 MARKS EACH)

16. Solve the following differential equation

(i) $(x\,dy - y\,dx)\, y \sin\left(\frac{y}{x}\right) = (y\,dx + x\,dy)\, x \cos\left(\frac{y}{x}\right)$

(ii) $3e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$ given that $y = \frac{\pi}{4}$, when $x = 1$.

(iii) $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$ given that $y(0) = 0$.

ANSWERS

1.

(i) order = 1, degree = 1 (ii) order = 2, degree = 1

(iii) order = 4, degree = 1 (iv) order = 5, degree not defined.

(v) order = 2, degree = 2 (vi) order = 2, degree = 2

(vii) order = 3, degree = 2

2.

(i) $y = \frac{x^6}{6} + \frac{x^3}{6} - 2\log|x| + c$ (ii) $y = \log_e \left|e^x + e^{-x}\right| + c$

(iii) $y = \frac{x^4}{4} + e^x + \frac{x^{e+1}}{e+1} + c.$ (iv) $5^x + 5^{-y} = c$

(v) $2(y - x) + \sin 2y + \sin 2x = c.$ (vi) $2 \log |3x + 1| + \log_e |1 - 2y| = c.$

3.

(i) $e^{\sin x}$ (ii) $e^{\tan x}$

(iii) $e^{-1/x}$ (iv) $e^{\frac{(\log x)^2}{2}}$

(v) $\frac{1}{x^3}$ (vi) $\sec y$

4.

(v) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ [**Hint :** find $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ and eliminate A and B.]

(vi) $x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2} = y\frac{dy}{dx}$

(vii) $\left(\frac{dy}{dx}\right)^3 = 4y\left(x \times \frac{dy}{dx} - 2y\right)$ [**Hint :** divide y by $\frac{dy}{dx}$ and find c.]

(viii) $2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$

5.

(i) $y \sin x = \frac{2\sin^3 x}{3} + c$ (ii) $y = \frac{x^2(4\log_e x - 1)}{16} + \frac{c}{x^2}$

(iii) $y = \sin x + \frac{c}{x},\ x > 0$ (iv) $y = \tan x - 1 + ce^{-\tan x}$

(v) $x = -y^2e^{-y} + cy^2$

6.

(i) $cy = (x + 2)(1 - 2y)$ (ii) $(e^x + 2)\sec y = c$

(iii) $\sqrt{1 - x^2} + \sqrt{1 - y^2} = c$

(iv) $\frac{1}{2}\log\left|\frac{\sqrt{1 - y^2} - 1}{\sqrt{1 - y^2} + 1}\right| = \sqrt{1 - x^2} - \sqrt{1 - y^2} + c$

(v) $(x^2 + 1)(y^2 + 1) = 2$

(vi) $\log y = -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + xe^x - e^x + c$

(vii) $\log|\tan y| - \frac{\cos 2x}{y} = c$

7.

(i) $\frac{-x^3}{3y^3} + \log|y| = c$

(ii) $\tan^{-1}\left(\frac{y}{x}\right) = \log|x| + c$ [**Hint.** : Homogeneous Equation]

(iii) $x^2 + y^2 = 2x$

(iv) $y = ce^{\cos(x/y)}$ [**Hint. :** Put $\frac{x}{y} = v$] (v) $\sin\left(\frac{y}{x}\right) = cx$

(vi) $c\left(x^2 - y^2\right) = y$

(vii) $-e^{-y} = e^x + \frac{x^3}{3} + c$ [**Hint. :** Factorise R.H.S.]

(viii) $\sin^{-1} y = \sin^{-1} x + c$

8.

(i) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$ [**Hint. :** The family of circles is, $x^2 + y^2 + 2gx = 0$]

(ii) $2y = x\frac{dy}{dx}, \quad y = 2x\frac{dy}{dx}$ (iii) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

9. $\log\left|x^2 + xy + y^2\right| = 2\sqrt{3}\tan^{-1}\left(\frac{x + 2y}{\sqrt{3x}}\right) + c$

10. $\frac{x^3}{x^2 + y^2} = \frac{c}{x}(x + y)$

11.

(i) $y = \frac{3\sin 3x}{13} - \frac{2\cos 3x}{13} + ce^{2x}$

(ii) $y = \frac{2}{3}\sin^2 x + \frac{1}{3}\operatorname{cosec} x$

12.

(i) $-y = x\log\{c(x - y)\}$ (ii) $cx^2 = y + \sqrt{x^2 + y^2}$

(iii) $xy\cos\left(\frac{y}{x}\right) = c$ [**Hint. :** Put $y = vx$]

(iv) $3x^2y = y + 2x$ [**Hint. :** Put $y = vx$]

(v) $y = -x\log(\log|x|),\ x \neq 0$ (vi) $c(x^2 + y^2) = \sqrt{x^2 - y^2}.$

13. (i) Order = 1, Degree = not define

(ii) Order = 1 (iii) Order = 2

(iv) sec x (v) $e^{\tan^{-1}}y$

14. (i) $(x - y)^2\{1 + y^{12}\} = (x + y\,y')^2$

15. (i) $x = \tan^{-1} y - 1 + c \cdot e^{\tan^{-1} y}$

(ii) $x = y - 1 + ce^y$ (iii) $x + ye^{\frac{x}{y}} = c$

(iv) $y = \sin^{-1} 2x$

16. (i) $C\,xy = \sec\left(\frac{y}{x}\right)$

(ii) $(1 - e)^3 \tan y = (1 - e^x)^3$

(iii) $y = x^2$.

CHAPTER 10–11

VECTORS AND THREE DIMENSIONAL GEOMETRY

POINTS TO REMEMBER

- **Vector :** A directed line segment represents a vector.
- **Addition of vectors :** If two vectors are taken as two sides of a triangle taken in order then their sum is the vector represented by the third side of triangle taken in opposite order (triangle law).
- **Multiple of a vector by a scalar :** $\vec{a}$ is any vector and $\lambda \in R$ then $\lambda \vec{a}$ is vector of magnitude $|\lambda| |\vec{a}|$ in a direction parallel to $\vec{a}$.
- If $|\vec{a}| \neq 0$ then $\dfrac{\vec{a}}{|\vec{a}|}$ is unit vector in direction $\vec{a}$.
- **Scalar Product :** $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos\theta$ where θ is the angle between $\vec{a}$ and $\vec{b}$.
- Projection of $\vec{a}$ along $\vec{b}$ is $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$.
- $\vec{a} \cdot \vec{a} = |\vec{a}|^2$
- Vectors $\vec{a}$ and $\vec{b}$ are perpendicular iff $\vec{a} \cdot \vec{b} = 0$.
- **Cross Product :** $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin\theta\, \hat{n}$ where $\hat{n}$ is a unit vector perpendicular to $\vec{a}$ and $\vec{b}$, and θ is the angle between $\vec{a}$ and $\vec{b}$.
- Unit vector perpendicular to plane of $\vec{a}$ and $\vec{b}$ is $\pm \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$.
- Vector $\vec{a}$ and $\vec{b}$ are collinear if $\vec{a} \times \vec{b} = \vec{0}$.
- $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

  where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and

  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

- Area of a triangle whose two sides are $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}|\vec{a} \times \vec{b}|$.
- Area of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$.
- If $\vec{a}, \vec{b}$ represents the two diagonals of a parallelogram, then area of parallelogram $= \frac{1}{2}|\vec{a} \times \vec{b}|$.

THREE DIMENSIONAL GEOMETRY

- Distance between $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

- The coordinates of point R which divides line segment PQ where $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in ratio $m : n$ are $\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}, \frac{mz_2 + nz_1}{m + n}\right)$.
- If α, β, γ are the angles made by any line with coordinate axes respectively then l, m, n. Where $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$ are called the, direction cosines of the line and $l^2 + m^2 + n^2 = 1$. If a, b, c are the direction ratios then direction cosines are

$$l = \pm\frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \pm\frac{b}{\sqrt{a^2 + b^2 + c^2}}, n = \pm\frac{c}{\sqrt{a^2 + b^2 + c^2}}.$$

- Direction ratios of a line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) are $x_2 - x_1 : y_2 - y_1 : z_2 - z_1$.
- Vector equation of straight line :

  (i) Through a point A $(\vec{a})$ and parallel to vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

  (ii) Passing through two points $A(\vec{a})$ and $B(\vec{b})$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.

  (iii) Line passing through two given points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}, \text{ in cartesian form.}$$

- Angle θ between two lines with DC's l_1, m_1, n_1 and l_2, m_2, n_2 is given by

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

OR

with D.R's $<a_1, b_1, c_1>$ or $<a_2, b_2, c_2>$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}.$$

If lines are $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$.

then, $\cos\theta = \dfrac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$.

- Equation of plane :

  (i) Passing through $A(\vec{a})$ and perpendicular to $(\vec{n})$ is $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ Or $\vec{r} \cdot \vec{n} = d$ where $\vec{a} \cdot \vec{n} = d$.

  (ii) Passing through three given points is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0.$

  (iii) Having intercepts a, b, c on coordinate axes is $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1.$

- Angle between two planes $\vec{r} \cdot \vec{n_1} = d_1$ and $\vec{r} \cdot \vec{n_2} = d_2$ is $\cos\theta = \dfrac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$.

- Distance of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is $\dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$.

- Equation of plane passing through intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is $(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$.

- Equation of plane passing through intersection of two planes $\vec{r} \cdot \vec{n_1} = d_1$ and $\vec{r} \cdot \vec{n_2} = d_2$ is $\vec{r} \cdot (\vec{n_1} + \lambda \vec{n_2}) = d_1 + \lambda d_2$.

- Angle between a plane $\vec{r} \cdot \vec{n} = d$ and a line $\vec{r} = \vec{a} + \lambda \vec{m}$ is $\sin\theta = \dfrac{\vec{m} \cdot \vec{n}}{|\vec{m}||\vec{n}|}$.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. What is the horizontal and vertical components of a vector $\vec{a}$ of magnitude 5 making an angle of 150° with the direction of x-axis.

2. What is $a \in R$ such that $|a\vec{x}| = 1$, where $x = \hat{i} - 2\hat{j} + 2\hat{k}$?

3. Write when $|\vec{x} + \vec{y}| = |\vec{x}| + |\vec{y}|$.

4. What is the area of a parallelogram whose sides are given by $2\hat{i} - \hat{j}$ and $\hat{i} + 5\hat{k}$?

5. If A is the point (4, 5) and vector $\vec{AB}$ has components 2 and 6 along x-axis and y-axis respectively then write point B.

6. What is the point of trisection of PQ nearer to P if position of P and Q are $3\hat{i} + 3\hat{j} - 4\hat{k}$ and $9\hat{i} + 8\hat{j} - 10\hat{k}$.

7. What is the vector in the direction of $2\hat{i} + 3\hat{j} + 2\sqrt{3}\hat{k}$, whose magnitude is 10 units?

8. What are the angles which $3\hat{i} - 6\hat{j} + 2\hat{k}$ makes with coordinate axes.

9. Write a unit vector perpendicular to both the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$.

10. What is the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$?

11. If $|\vec{a}| = 2$, $|\vec{b}| = 2\sqrt{3}$ and $\vec{a} \perp \vec{b}$, what is the value of $|\vec{a} + \vec{b}|$?

12. For what value of λ, $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ is perpendicular to $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$?

13. What is $|\vec{a}|$, if $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 3$ and $2|\vec{b}| = |\vec{a}|$?

14. What is the angle between $\vec{a}$ and $\vec{b}$, if $|\vec{a} - \vec{b}| = |\vec{a} + \vec{b}|$?

15. What is the area of a parallelogram whose diagonals are given by vectors $2\hat{i} + \hat{j} - 2\hat{k}$ and $-\hat{i} + 2\hat{k}$?

16. Find $|\vec{x}|$ if for a unit vector $\hat{a}$, $(\vec{x} - \hat{a}).(\vec{x} + \hat{a}) = 12$.

17. If $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$, then what is the angle between $\vec{a}$ and $\vec{b}$.

18. If $\vec{a}$ and $\vec{b}$ are two unit vectors and $\vec{a} + \vec{b}$ is also a unit vector then what is the angle between $\vec{a}$ and $\vec{b}$?

19. If $\hat{i}, \hat{j}, \hat{k}$ are the usual three mutually perpendicular unit vectors then what is the value of $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{i} \times \hat{k}) + \vec{k}.(\hat{j} \times \hat{i})$?

20. What is the angle between $\vec{x}$ and $\vec{y}$ if $\vec{x}.\vec{y} = |\vec{x} \times \vec{y}|$?

21. Write a unit vector in xy-plane, making an angle of 30° with the +ve direction of x–axis.

22. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors with $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then what is the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$?

23. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $(\vec{a} + 2\vec{b})$ is perpendicular to $(5\vec{a} - 4\vec{b})$, then what is the angle between $\vec{a}$ and $\vec{b}$?

24. Write a unit vector which makes an angle of $\frac{\pi}{4}$ with x–axis and $\frac{\pi}{3}$ with z-axis and an acute angle with y-axis.

25. What is the ratio in which xy plane divides the line segment joining the points (–1, 3, 4) and (2, –5, 6)?

26. If x coordinate of the point P on the join of $Q(2, 2, 1)$ and $R(-5, 1, -2)$ is 4, then in what ratio P divides QR.

27. What is the distance of a point $P(a, b, c)$ from x-axis?

28. Write the equation of a line passing through (1, –1, 2) and perpendicular to plane $2x - 3y + 4z = 7$.

29. What is the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$?

30. What is the perpendicular distance of plane $2x - y + 3z = 10$ from origin?

31. What is the y-intercept of the plane $x - 5y + 7z = 10$?

32. Write the value of λ, so that the lines given below are perpendicular to each other

$$\frac{1-x}{3} = \frac{2y-1}{4} = \frac{z-1}{\lambda} \text{ and } \frac{x-2}{4} = \frac{y-5}{2} = \frac{3-z}{5}.$$

33. A (3, 2, 0), $B(5, 3, 2)$ and $C(5, 8, -10)$ are the vertices of ΔABC. D and E are mid points of AB and AC respectively. What are the direction cosines of DE?

34. What is the equation of the line, which passes through the point (–2, 4, –5) and parallel to $\frac{x+3}{5} = \frac{y-4}{5} = \frac{z+8}{-6}$?

35. What is the angle between the straight lines :

$$\frac{x+1}{2} = \frac{y-2}{2} = \frac{z+3}{4}, \frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}?$$

36. If the direction ratios of a line are proportional to 1, –3, 2 then what are the direction cosines of the line?

37. If a line makes angles $\frac{\pi}{2}$ and $\frac{\pi}{4}$ with x-axis and y-axis respectively then what is the acute angle made by the line with z axis?

38. What is the acute angle between the planes $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + 5 = 0$?

39. What is the distance between the planes $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + 5 = 0$.

40. What is the equation of the plane which cuts off equal intercepts of unit length on the coordinate axes.

41. What is the equation of the plane through the point (1, 4, – 2) and parallel to the plane $-2x + y - 3z = 7$?

42. Write the vector equation of the plane which is at a distance of 8 units from the origin and is normal to the vector $\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$.

43. What is equation of the plane if the foot of perpendicular from origin to this plane is (2, 3, 4)?

44. What is the angle between the line $\frac{x+1}{3} = \frac{2y-1}{4} = \frac{2-z}{-4}$ and the plane $2x + y - 2z + 4 = 0$?

45. If O is origin $OP = 3$ with direction ratios proportional to –1, 2, – 2 then what are the coordinates of P?

46. What is the distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + \hat{j} + 4\hat{k})$ from the plane $\vec{r}.(-\hat{i} + 5\hat{j} - \hat{k}) + 5 = 0$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

47. If ABCDEF is a regular hexagon then using triangle law of addition prove that :

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 3\overrightarrow{AD} = 6\overrightarrow{AO}$$

O being the centre of hexagon.

48. Points L, M, N divides the sides BC, CA, AB of a ΔABC in the ratios 1 : 4, 3 : 2, 3 : 7 respectively. Prove that $\overrightarrow{AL} + \overrightarrow{BM} + \overrightarrow{CN}$ is a vector parallel to $\overrightarrow{CK}$ where K divides AB in ratio 1 : 3.

49. If PQR and $P'Q'R'$ are two triangles and G, G' are their centroids, then prove that $\overrightarrow{PP'} + \overrightarrow{QQ'} + \overrightarrow{RR'} = \overrightarrow{3GG'}$.

50. PQRS is parallelogram. L and M are mid points of QR and RS. Express $\overrightarrow{PL}$ and $\overrightarrow{PM}$ in terms of $\overrightarrow{PQ}$ and $\overrightarrow{PS}$. Also prove that $\overrightarrow{PL} + \overrightarrow{PM} = \frac{3}{2}\overrightarrow{PR}$.

51. The scalar product of vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ *and* $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ.

52. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude. Show that $\vec{a} + \vec{b} + \vec{c}$ makes equal angles with $\vec{a}$, $\vec{b}$ and $\vec{c}$ with each angle as $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

53. If $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

54. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

55. If $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the angle between $\vec{a}$ and $\vec{b}$.

56. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = 3\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - \hat{k}$, find a vector $\vec{d}$ which is perpendicular to $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 1$.

57. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$ are the given vectors then find a vector $\vec{b}$ satisfying the equation.

$$\vec{a} \times \vec{b} = \vec{c}, \quad \vec{a} \cdot \vec{b} = 3.$$

58. Find a unit vector perpendicular to plane *ABC*. Position vectors of *A, B, C* are $3\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} - \hat{j} - 3\hat{k}$ and $4\hat{i} - 3\hat{j} + \hat{k}$ respectively.

59. Find the image of the point (3, – 2, 1) in the plane $3x - y + 4z = 2$.

60. The line $\dfrac{x-4}{1} = \dfrac{2y-4}{2} = \dfrac{k-z}{-2}$ lies exactly in the plane $2x - 4y + z = 7$. Find the value of *K*.

61. Find vector and cartesian equation of a line passing through a point with position vectors $2\hat{i} - \hat{j} + \hat{k}$ and which is parallel to the line joining the points with position vectors $-\hat{i} + 4\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$.

62. Find image (Reflection) of the point (7, 4, – 3) in the line $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$.

63. Find equations of a plane passing through the points (2, –1, 0) and (3, –4, 5) and parallel to the line $2x = 3y = 4z$.

64. Find distance of the point (– 1, – 5, – 10) from the point of intersection of line $\dfrac{x-2}{3} = \dfrac{y+1}{4} = \dfrac{z-2}{12}$ and the plane $x - y + z = 5$.

65. Find equation of the plane passing through the point (2, 3, – 4) and (1, –1, 3) and parallel to the x–axis.

66. Find equation of the plane which bisects the line joining the points (– 1, 2, 3) and (3, – 5, 6) at right angle.

67. Find the distance of the point (1, –2, 3) from the plane $x - y + z = 5$, measured parallel to the line $\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z}{-6}$.

68. Find the equation of the plane passing through the intersection of two plane $3x - 4y + 5z = 10$, $2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.

69. Find the equations of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ whose perpendicular distance from the point (1, 2, 3) is 1 unit.

70. Find equation of the plane passing through the point (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$.

71. Find cartesian as well as vector equation of the plane through the intersection of the plane $\vec{r} = (2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r}.(3\hat{i} - \hat{j} + 4\hat{k}) = 0$ which is at a unit distance from origin.

72. Find equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contain the line of intersection of the plane $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$.

73. Find equation of the plane containing the points (0, – 1, – 1) (– 4, 4, 4), (4, 5, 1). Also show that (3, 9, 4) lies on the required plane.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

74. The vector equations of two lines are :

$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k}(2\hat{i} + \hat{j} + 2\hat{k})$. Find the shortest distance between them.

75. Check the coplanarity of lines

$$\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 5\hat{k})$$

$$\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$$

If they are coplanar, find equation of the plane containing the lines.

76. Find shortest distance between the lines :

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad \text{and} \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

77. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Also find the point of intersection.

78. Find shortest distance between the lines whose vector equations are :

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} + (2s+1)\hat{k}$

79. Find the equations of the two lines through the origin such that each line is intersecting the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at an angle of $\frac{\pi}{3}$.

80. A plane passes through (1, – 2, 1) and is perpendicular to the planes $2x - 2y + 2z = 0$ and $x - y + 2z = 4$. Find the distance of that plane from origin.

81. Find the equation of the plane passing through the intersection of planes $2x + 3y - z = -1$ and $x + y - 2z + 3 = 0$ and perpendicular to the plane $3x - y - 2z = 4$. Also find the inclination of this plane with xy–plane.

82. Find the shortest distance and the vector equation of line of shortest distance between the lines given by

$$\vec{r} = (3\hat{i} + 8\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (-3\hat{i} - 7\hat{j} + 6\hat{k}) + \mu(-3\hat{i} - 2\hat{j} + 4\hat{k})$$

83. Show that the lines joining the points (7, 0, 6) and (2, 5, 1) intersects the line joining the points (2, 2, – 1) (3, 4, 2). Also find the point of intersection.

84. Find the equations of two planes through the points (4, 2, 1) and (2, 1, – 1) and making an angle of $\frac{\pi}{4}$ with the plane $x - 4y + z - 9 = 0$,

85. A variable plane is at a constant distance $3p$ from the origin and meet the coordinate axes in A, B, C. Show that the locus of centroid of ΔABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

86. A vector $\vec{n}$ of magnitude 8 units inclined to x–axis at 45°, y axis at 60° and an acute angle with z-axis. If a plane passes through a point $(\sqrt{2}, -1, 1)$ and is normal to $\vec{n}$, find its equation in vector form.

87. Find the foot of perpendicular from the point $2\hat{i} - \hat{j} + 5\hat{k}$ on the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$. Also find the length of the perpendicular.

SHORT ANSWER TYPE QUESTION (4 MARKS)

88. Evaluate $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$.

89. $\vec{a} = \hat{i} + \hat{j} + \hat{k}$; $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x - 2)\hat{j} - \hat{k}$. If c lies in the plane of $\vec{a}$ and $\vec{b}$ then find value of x.

90. Find the value of a for which the vector $\vec{r} = (a^2 - 4)\hat{i} + 2\hat{j} + (a^2 - 9)\hat{k}$ makes acute angle with coordinate axes.

91. Let $\hat{a}, \hat{b}, \hat{c}$ be unit vectors such that $\hat{a} \cdot \hat{b} = \hat{a} \cdot \hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$ then prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

92. Prove that angle between any two diagnals of a cube is $\cos^{-1}(1/3)$.

93. The cartesian equations of a line is $6x - 2 = 3y + 1 = 2z - 2$. Find direction ratios of the line. Also find vector and cartesian equations of a line parallel to this line and passing through (2, –1, –1).

LONG ANSWERS TYPE QUESTIONS (6 MARKS)

94. Three vectors of magnitude a, $2a$ and $3a$ meet in a point and their directions are along the diagonals of the adjacent faces of a cube determine their resultant.

95. A line makes angle α, β, γ and δ with four diagonals of a cube prove that

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}.$$

96. Show that the lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular if $aa' + cc' = -1$.

HIGHER ORDER THINKING SKILL QUESTIONS (HOTS)

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

97. What is the angle between $\vec{a}$ and $\vec{b}$ if $\vec{a} \cdot \vec{b} = 3$ and $\left|\vec{a} \times \vec{b}\right| = 3\sqrt{3}$.

98. What are the direction cosines of a vector equiangular with co-ordinate axes.

99. In a parallelogram $ABCD$ $\overrightarrow{AB} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\overrightarrow{AC} = \hat{i} + \hat{j} + 4\hat{k}$. what is the length of side BC.

100. Two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. What is a unit vector parallel to the diagonal which is co-initial with $\vec{a}$ and $\vec{b}$.

101. If $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{a} - \vec{b}\right| = 1$ then what is value of $\left|\vec{a} + \vec{b}\right|$.

102. For any vector $\vec{a}$ what is the value of $\hat{i} \cdot \left(\vec{a} \times \hat{j}\right) + \hat{j} \cdot \left(\vec{a} \times \hat{i}\right) + \hat{k} \cdot \left(\vec{a} \times \hat{k}\right)$?

103. If a line makes α, β and γ angles with co-ordinate axes then what is value of $\delta m^2 \gamma + \delta m^2 \beta + \delta m^2 \gamma$?

104. What is the equation of a line passing through point (–1, 2, 3) and equally include to the axes.

105. The foot of perpendicular from (1, 6, 3) on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ *is* $(1, 3, q)$ what is the value of q.

106. What is the distance between the line $\vec{r} = \hat{i} - \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$ from the plane $\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 3$.

ANSWERS

1. $\frac{5\sqrt{3}}{2}, \frac{5}{2}$.
2. $\pm\frac{1}{3}$
3. $\vec{x}$ and $\vec{y}$ are collinear.
4. 126 sq. units.
5. (6, 11)
6. (5, 4, –6)

7. $4\hat{i} + 6\hat{j} + 4\sqrt{3}\,\hat{k}$

8. $Cos^{-1}\frac{3}{7}, Cos^{-1}\frac{-6}{7}, Cos^{-1}\frac{2}{7}$

9. $\frac{3}{\sqrt{26}}\hat{i} + \frac{4}{\sqrt{26}}\hat{j} - \frac{1}{\sqrt{26}}\hat{k}$

10. 0

11. 4

12. $\lambda = -9$

13. $|\vec{a}| = 2$

14. 90°

15. $\frac{3}{2}$ sq. units.

16. $\sqrt{13}$

17. 90°

18. 120°

19. –1

20. $\frac{\pi}{4}$

21. $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$.

22. $\frac{-3}{2}$.

23. 60°

24. $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$.

25. 2 : 3 enternally

26. 2 : 5

27. $\sqrt{b^2 + c^2}$

28. $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-2}{4}$.

29. 90°

30. $\frac{10}{\sqrt{14}}$

31. –2

32. $\frac{-8}{5}$

33. $0, \frac{5}{13}, \frac{-12}{13}$.

34. $\frac{x+2}{5} = \frac{y-4}{5} = \frac{z+5}{-6}$.

35. $\frac{x+1}{1} = \frac{y-2}{1} = \frac{z+3}{1}$

36. $\pm\frac{1}{\sqrt{14}}, \mp\frac{3}{\sqrt{14}}, \pm\frac{2}{\sqrt{14}}$.

37. 60°

38. 60°

39. $\frac{1}{6}$ units.

40. $x + y + z = 1$.

41. $2x - y + 3z = -8$.

42. $\vec{r}\cdot\left(2\hat{i} + \hat{j} + 2\hat{k}\right) = 24$.

43. $2x + 3y + 5z = 29$.

44. 0°

45. (–1, 2, –2)

46. $\frac{10}{3\sqrt{3}}$

50. $\begin{Bmatrix} \overrightarrow{PL} = \overrightarrow{PQ} + \frac{1}{2}\overrightarrow{PS} \\ \overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ} + \overrightarrow{PS} \end{Bmatrix}$

51. –6.

53. $\begin{Bmatrix} \beta_1 = \frac{1}{2}\left(3\hat{i} - \hat{j}\right) \\ \beta_2 = \frac{1}{2}\left(\hat{i} + 3\hat{j} - 6\hat{k}\right) \end{Bmatrix}$

55. $Cos^{-1}\left(\frac{11}{14}\right)$.

56. $\frac{1}{4}\hat{i} + \frac{1}{4}\hat{j} + \frac{3}{4}\hat{k}$.

57. $\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{4}{3}\hat{k}$.

58. $\frac{1}{\sqrt{165}}\left(10\hat{i} + 7\hat{j} - 4\hat{k}\right)$.

59. (0, –1, –3)

60. 7.

61. $\overrightarrow{r} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(2\hat{i} - 2\hat{j} + 2\hat{k}\right)$.

62. (– 9, 2, 1).

63. $29x - 27y - 22z = 85$.

64. 13 units.

65. $7y + 4z - 5 = 0$.

66. $4x - 7y + 3z - 28 = 0$.

67. 1 unit

68. $x - 20y + 27z = 14$.

69. $x - 2y + 2z = 0$.

70. $5x + 2y - 3z = 17$.

71. $\overrightarrow{r}.\left(2\hat{i} + \hat{j} + 2\hat{k}\right) + 3 = 0,\ 2x + y + 2z + 3 = 0$.

Or

$\overrightarrow{r}.\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right) + 3 = 0,\ -x + 2y - 2z + 3 = 0$.

72. $51x + 15y - 50z + 173 = 0$.

73. $5x - 7y + 11z + 4 = 0$.

74. $\frac{3\sqrt{3}}{2}$

75. $\overrightarrow{r},\left(\hat{i} - 2\hat{j} + \hat{k}\right) = 1$

76. 14 units.

77. (–1, –1, –1).

79. $\frac{8}{\sqrt{29}}$

79. $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$; and $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$.

80. $2\sqrt{2}$

81. $7x + 13y + 4z - 9 = 0, \quad \sin^{-1}\left(\frac{4}{\sqrt{234}}\right)$

82. $S.D = 3\sqrt{30}, \quad \hat{r} = \left(3\hat{i} + 8\hat{j} + 3\hat{k}\right) + \mu\left(-6\hat{i} - 15\hat{j} + 3\hat{k}\right).$

83. (3, 4, 2).

84. $x + 2y - 2z = 6,\ 2x - 2y - z = 3.$

86. $\vec{r}.\left(\sqrt{2}\hat{i} + \hat{j} + \hat{k}\right) = 2.$

87. $\hat{i} + 2\hat{j} + 3\hat{k},\ \sqrt{14}.$

88. $2\left|\vec{a}\right|^2$

89. $x = -2$

90. $a \in (-3, -2) \cup (2, 3)$

93. $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z+1}{3},\ \vec{r} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$

94. $\frac{a}{\sqrt{2}}\left(4\hat{i} + 3\hat{j} + 5\hat{k}\right)$

97. 60°

98. $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

99. 3 units.

100. $\frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$

101. $\sqrt{3}$.

102. 0.

103. 2.

104. $\frac{x+1}{1} = \frac{y-2}{1} = \frac{z+3}{1}.$

105. $q = 5.$

106. 0.

CHAPTER 12

LINEAR PROGRAMMING

POINTS TO REMEMBER

- Linear Programming is the method used to obtain minimum or maximum value of the objective function restricted to some linear constraints.
- **Constraints :** The linear inequation or restrictions on the variables of a linear-programming problem are called constraints.
- **Objective Functions :** A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum) of a linear function of several variables. This linear function is called objective function Z, $Z = ax + by$, a and b are constraints.

  Types of linear programming problems are

  (i) Diet Problems

  (ii) Manufacturing Problems

  (iii) Transportation problems etc.
- **Feasible Region :** It is the common region determined by all the constraints of a linear programming problem.
- **To Find Feasible Region :** Draw the graph of all the linear inequations and shade common region determined by all the constraints.
- **Feasible Solutions :** Points within and on the boundary of the feasible region represents feasible solutions of the constraints.
- **Optimal Feasible Solution :** It occurs at the corner point and select the point which optimizes Z according to question.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

1. A man has Rs. 1500 to purchase two types of shares of two different companies S_1 and S_2. Market price of one share of S_1 is Rs 180 and S_2 is Rs. 120. He wishes to purchase a maximum to ten shares only. If one share of type S_1 gives a yield of Rs. 11 and of type S_2 Rs. 8 then how much shares of each type must be purchased to get maximum profit? And what will be the maximum profit?

2. A company manufacture two types of lamps say A and B. Both lamps go through a cutter and then a finisher. Lamp A requires 2 hours of the cutter's time and 1 hours of the finisher's time. Lamp B requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 100 hours and finishers has 80 hours of time available each month. Profit on one lamp A is Rs. 7.00 and on one

lamp B is Rs. 13.00. Assuming that he can sell all that he produces, how many of each type of lamps should be manufactured to obtain maximum profit?

3. Solve the following LPP problem graphically :

Maximise and Minimize $Z = 3x + 5y$

subject to $3x - 4y + 12 \geq 0$

$2x - 4y + 12 \geq 0$

$2x - 3y - 12 \geq 0$

$0 \leq x \leq 4$

$y \geq 2$

4. A company produces two types of belts A and B. Profits on these belts are Rs. 2 and Rs. 1.50 per belt respectively. A belt of type A requires twice as much time as belt of type B. The company can produce almost 1000 belts of type B per day. Material for 800 belts per day is available. Almost 400 buckles for belts of type A and 700 for type B are available per day. How much belts of each type should the company produce so as to maximize the profit?

5. Minimize $Z = 3x + 3y$, if possible, subject to the constraints

$x - y \leq 1;$ $x + y \leq 3,$ $x \geq 0;$ $y \geq 0.$

6. To Godowns X and Y have a grain storage capacity of 100 quintals and 50 quintals respectively. Their supply goes to three ration shop A, B and C whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintals from the godowns to the shops are given in following table :

To / From	***Cost of transportation (in Rs. per quintal***	
	X	***Y***
A	6.00	4.00
B	3.00	2.00
C	2.50	3.00

How should the supplies be transported to minimize the transportation cost?

7. An Aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 in made on each first class ticket and a profit of Rs. 300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However atleast four times as many passengers prefer to travel by second class than by first class. Determine, how many tickets of each type must be sold to maximize profit for the airline.

8. A diet for a sick person must contain atleast 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs. 5 and Rs. 4 per unit respectively. One unit of food A contains 200 unit of vitamins, 1 unit of minerals and 40 units of calories whereas one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the food A and B should be used to have least cost but it must satisfy the requirements of the sick person.

9. A dealor wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for almost 20 items. A fan and sewing machine cost Rs. 360 and Rs. 240 respectively. He can sell a fan at a profit of Rs. 22 and sewing machine at a profit of Rs. 18. Assuming that he can sell whatever he buys, how should he invest his money to maximise his profit?

10. If a young man rides his motorcycle at 25 km/h, he has to spend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/h, the petrol cost increases to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as L.P.P. and then solve it graphically.

11. A producer has 20 and 10 units of labour and capital respectively which he can use to produce two kinds of goods *X* and *Y*. To produce one unit of *X*, 2 units of capital and 1 unit of labour is required. To produce one unit of *Y*, 3 units of labour and one unit of capital is required. If *X* and *Y* are priced at Rs. 80 and Rs. 100 per unit respectively, how should the producer use his resources to maximise the total revenue?

12. A farmer has a supply of chemical fertilizers of type A which contains 10% nitrogen and 6% phosphoric acid and type *B* which contains 5% nitrogen and 10% phosphoric acid. After soil testing, it is found that at least 7 kg of nitrogen and the same quantity of phosphoric acid is required for a good crop. Fertilizer of type *A* costs Rs. 5 per kg and type *B* costs Rs. 8 per kg. How many kilograms of each type of fertilizers should be bought to meet the requirement at the minimum cost.

13. A factory owner purchases two types of machines *A* and *B* for his factory. The requirements and limitations for the machines are as follows :

Machine	***Area Occupied***	***Labour Force***	***Daily Output (In units)***
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

He has maximum area of 9000 m^2 available and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise the daily output.

14. A manufacturer makes two types of cups *A* and *B*. There machines are required to manufacture the cups and the time in minute required by each in as given below :

Types of Cup	***Machine***		
	I	***II***	***III***
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup *A* is 75 paise and on *B* is 50 paise, find how many cups of each type should be manufactured to maximise the profit per day.

15. Using graphical method, solve the following L.P.P.

Maximise $Z = 4x + 5y$

Subject to constraints

$$2x + y \leq 30$$

$$x + 2y \leq 24$$

$$x \geq 3$$

$$y \leq 9$$

$$y \geq 0$$

ANSWERS

1. Maximum Profit = Rs. 95 with 5 shares of each type.
2. Lamps of type A = 40, Lamps of type B = 20.
3. Max. value of Z = 42 at x = 4 and y = 6.

   Min. value of Z = 19 at x = 3 and y = 2.
4. Maximum Profit Rs. 1300, No. of belts of type A = 200 and type B = 600.
5. Maximum value is infinity as solution is unbounded.
6. From X to A, B and C 10 quintals, 50 quintals and 40 quintals respectively.

   From Y to A, B, C 50 quintals, NIL and NIL respectively.
7. No. of first class tickets = 40, No. of 2nd class tickets = 160.
8. Food A : 5 units, Food B : 30 units.
9. Fan : 8; Sewing machine : 12, Max. Profit = Rs. 392.
10. At 25 km/h he should travel 50/3 km, At 40 km/h, 40/3 km. Max. distance 30 km in 1 hr.
11. X : 2 units; Y : 6 units; Maximum revenue Rs. 760.
12. Type A : 50 kg, Type B : 40 kg Minimum cost Rs. 570.
13. Type A : 6; Type B : 0
14. Cup A : 15; Cup B : 30
15. x = 12, y = 6 Z_{max} = 96.

CHAPTER 13

PROBABILITY

POINTS TO REMEMBER

- **Conditional Probability :** Probability of event A given that event B has already occurred

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}.$$

- **Multiplication Rule of Probability :**

(i) $P(A \cap B) = P(B).P\left(\frac{A}{B}\right) = P(A).P\left(\frac{B}{A}\right).$

(ii) $P(A \cap B \cap C) = P(A).P\left(\frac{B}{A}\right).P\left(\frac{C}{AB}\right).$

- If (i) A and B are independent events then P (A $\cap$ B) = $P(A)$. $P(B)$

(ii) A, B and C are independent events then $P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$;

$P(A \cap B) = P(A).P(B)$,

$P(B \cap C) = P(B).P(C)$ and

$P(C \cap A) = P(C).P(A)$

- If A and B are Independent then

(i) A and $B^{¢}$ are independent

(ii) $A^{¢}$ and B are independent

(iii) $A^{¢}$ and $B^{¢}$ are independent.

- **Baye's Theorem :** If E_1, E_2 _ _ _ _ _ _ _ , E_n are mutually exclusive and exhaustive events and A be any event on sample space S, such that $P(A) \neq 0$, If A has already occurred then

$$P\left(\frac{E_i}{A}\right) = \frac{P(Ei).P\left(\frac{A}{E_i}\right)}{\sum_{i=1}^{n} P(E_i).P\left(\frac{A}{E_i}\right)}$$

i = 1, 2, _ _ _ _ _ _ _ n

- **Probability Distribution :** Let a random variable. Let a random variable x assumes values $x_1, x_2, x_3, \_\_\_\_\_ x_n$ with corresponding probabilities $p_1, p_2, p_3, \_\_\_\_ p_n$. Then different values of the random variable alongwith their corresponding probabilities form a probability distribution.

Mean of a probability distribution, $\equiv \mu = \sum_{i=1}^{n} p_i x_i$

Variance $\equiv \sigma^2 = \sum_{i=1}^{n} p_i x_i^2 - \mu^2.$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find $P(A \cap B)$if $P(A) = 2P(B) = \frac{6}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$.
2. Given that the numbers appearing on throwing a pair of dice together are different. Find the probability that the sum of numbers appearing on the dice is 6.
3. A and B are independent events. If $P(A) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{6}$. Find $P(B)$.
4. A problem is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem is solved?
5. If $P\left(\frac{A}{B}\right) = \frac{2}{5}$ find $P(A)$. It is given that A and B are independent events.
6. A and B are two independent events such that $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{3}$. Find $P(A \cap B')$
7. If $P(A) = 0.25$, $P(B) = 0.50$ and $P(A \cap B) = 0.14$. Find P (neither A nor B).
8. In a three letter word of English, find the probability that all the three letters are repeated.
9. Is the following probability distribution valid?

x	0	1	2	3
$p(x)$	0.3	–0.1	0.4	0.4

10. Find k from the following probability distribution :

x	–1	0	1	2
$p(x)$	k	$2k$	k	0.04

11. A boy throws a coin. If he throws a head, he gets Rs. 10. If he throws a tail, he gets Rs. 2. If he throws the coin only once, find his expectation.

12. Four bad eggs are accidently mixed with 10 good ones. Three eggs are drawn at random without replacements. Find the probability that all the three are bag eggs.

13. What is the probability that a non-leap year has 53 Sundays.

14. A policeman fires three shots on a dacoit. The probability that the dacoit will be killed by one fire is 0.6. What is the probability that the dacoit is still alive.

15. Find $P(x = 1)$ of the Binomial distribution $B\left(3, \frac{1}{6}\right)$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

16. A pair of dice is thrown and the sum of the numbers is observed to be even. What is the probability that both dice have come up with even numbers?

17. The probability that a students selected at random will pass in mathematics is $\frac{4}{5}$ and the probability that he will pas in mathematics and computer science is $\frac{1}{2}\cdot$ What is the probability that he will pass in computer science given that he has passed in Mathematics.

18. A bag contains 5 white, 3 red and 2 blue balls. Four balls are drawn at random one by one without replacement. Find the probability of drawing atleast one white ball.

19. *A* can hit a larget 4 times in 5 shots, *B* 3 times in 4 shots and *C* twice in 3 shots. They fire a volley. What is the probability that atleast two shots hit?

20. A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls. A bag is selected at random and a ball is drawn from it. Find the probability that the ball drawn in red.

21. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two card are drawn alrandom and found to be spade each. Find the probability that the lost cord was also spade.

22. A candidate may use bus, scooter or some other means to reach the examination centre with the probability $\frac{3}{10}, \frac{1}{10}$ and $\frac{3}{5}$ respectively. The probability that he will be late are $\frac{1}{4}$ and $\frac{1}{3}$ respectively if he travels by bus or scooter but will reach in time if he uses any other means. If he reached late at the centre, find the probability that he travelled by bus.

23. Find the probability distribution of the number of green balls drawn when 3 balls are drawn one by one without replacement from a bag containing 3 green and 5 white balls.

24. Three cards are drawn successively with replacement from a pack of well shuffted 52 cards. Determine the probability distribution of *X* if *X* denotes the number of cards of heart in three drawn cards.

25. In a game, a man wins a rupee for a six and looses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of amount he wins/looses.

26. A fair coin is tossed 10 times. Find the probability of almost 6 heads.

27. Find the probability distribution of number of doublets in three throws of a pair of dice.

28. In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

29. Five dice are thrown simultaneously. If getting 3, 4 or 5 on a single die is considered a success then find the probability of almost three successes.

30. Assuming that a family has two children :

    (i) Write the Sample space

    (ii) What is the probability that both the children are boys given that atleast one of them is a boy.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

31. In solving a question, probability that the students knows the answer is $\frac{3}{10}$, copies with probability $\frac{1}{5}$, guesses with probability $\frac{1}{10}$. Probability that the student does not attempt the question is $\frac{2}{5}$. Probability that the answer is correct given that he copied is $\frac{1}{4}$ and the probability of giving correct answer by guessing is $\frac{1}{5}$. Given that his answer is correct find the probability that he did it by guessing.

32. Probability of attempting a problem by *A*, *B* and *C* is $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$ respectively. Probability that *A, B* and *C* will solve it correctly is $\frac{1}{2}$, $\frac{3}{5}$ and $\frac{4}{5}$ respectively. If the problem is solved correctly, what is the probability that it was solved by (i) *A*; (ii) *B*; (iii) *C*?

33. Find the mean and variance of the number of kings if three cards are drawn at random without replacement from a pack of well shuffled 52 cards.

34. Find the probability distribution of number of sixes in throwing a die five times. Also find its mean and variance.

35. Suppose 15% of men and 36% of women have grey hair. Probability of dying hair by men is 21% and by women is 63%. A dyed hair person is selected at random. What is the probability that the selected person is (i) Male (ii) Female?

36. An unbiased coin is tossed six times. Find the probability of getting (i) almost 3 heads; (ii) atleast 2 heads; (iii) Mean and variance of no. of heads.

37. A pair of dice is thrown 7 times. Getting a total of 7 is considered success. Find the probability of (i) No success; (ii) 6 successes; (iii) atleast 6 successes; (iv) Almost 6 successes.

38. Three person review a book. Odds in favour of the book are 5:2; 4:3 and 3:4 respectively. Find the probability that majority are in favour of book.

39. A man takes steps forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of eleven steps, he is one step away from the starting point.

40. Four cards are drawn from a pack of well shuffled 52 cards. Find the probability that they are all of different suits.

ANSWERS

VERY SHORT ANSWER TYPE QUESTIONS

1. $\frac{6}{65}$
2. $\frac{2}{15}$
3. $\frac{1}{3}$
4. $\frac{3}{4}$
5. $\frac{2}{5}$
6. $\frac{1}{6}$
7. 0.39
8. $\frac{1}{676}$
9. No.
10. $k = 0.24$
11. Rs. 6
12. $\frac{1}{91}$
13. $\frac{1}{7}$
14. 0.064
15. $\frac{75}{216}$

SHORT ANSWER TYPE QUESTIONS

16. $\frac{1}{2}$
17. $\frac{5}{8}$
18. $\frac{41}{42}$
19. $\frac{5}{6}$
20. $\frac{19}{42}$
21. $\frac{11}{50}$
22. $\frac{9}{13}$

23.

X	:	0	1	2	3
$P(X)$	:	$\frac{5}{28}$	$\frac{15}{28}$	$\frac{15}{56}$	$\frac{1}{56}$

24.

X	:	0	1	2	3
$P(X)$	:	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

25. Expected to loose Rs. $\frac{91}{54}$

26. $\frac{53}{64}$

27.

X	:	0	1	2	3
$P(X)$	:	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

28. $\frac{11}{243}$

29. $\frac{13}{16}$

30. (i) $S = \{bb, bg, gb, gg\}$ (ii) $\frac{1}{3}$

LONG ANSWER TYPE QUESTIONS

31. $\frac{2}{37}$

16. (i) $\frac{3}{7}$ (ii) $\frac{12}{35}$ (iii) $\frac{8}{35}$

33.

X	:	0	1	2	3
$P(X)$	:	$\frac{4324}{5525}$	$\frac{1128}{5525}$	$\frac{72}{5525}$	$\frac{1}{5525}$

34.

X	:	0	1	2	3	4	5
$P(X)$	:	$\frac{3125}{7776}$	$\frac{3125}{7776}$	$\frac{1250}{7776}$	$\frac{250}{7776}$	$\frac{250}{7776}$	$\frac{1}{7776}$

Mean $= \frac{5}{6}$

Variance $= \frac{275}{36}$

35. (i) $\frac{5}{41}$ (ii) $\frac{36}{41}$

36. (i) $\frac{21}{32}$ (ii) $\frac{11}{32}$ (iii) Mean = 3; Variance = 1.5

37. (i) $\left(\frac{5}{6}\right)^7$ (ii) $35\left(\frac{1}{6}\right)^7$ (iii) $\left(\frac{1}{6}\right)^5$ (iv) $1-\left(\frac{1}{6}\right)^7$

38. $\frac{209}{343}$

39. $462\ (0.24)^5$

40. $\frac{2197}{20825}$.

PRACTICE QUESTIONS PAPER – I

MATHEMATICS

Time allowed : 3 hours **Maximum marks : 100**

General Instructions

1. All question are compulsory.
2. The question paper has three sections. Section A contains 10 questions of one mark each, Section *B* contains 12 questions of 4 marks each and Section C contains 7 questions of six marks each.
3. All questions are compulsory.
4. Interval choice are given in some questions, where one part is to be attempted out of two.
5. Calculators are not allowed.

SECTION A

1. What are the direction cosines of the vector $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$.
2. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ find $\vec{a} . \vec{b}$.
3. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} . \vec{b} = \sqrt{3}$, find the angle between $\vec{a}$ and $\vec{b}$.
4. Write the value of x and y if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

5. If $A \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ then find the value of x if $|2A| = x\ |A|$.
6. Write the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.
7. Evaluate $\int \frac{3x^2 + 4x - 5}{\left(x^3 + 2x^2 - 5x + 1\right)^2} dx$.
8. Find the derivative of $\sin^2 (2x + 3)$ *w.r.t. x.*
9. If the binary operation *, defined on Q is defined as

   $a * b = 2a + b - ab$, for all $a, b \in Q$, find the value of 5 * 4.
10. Write a 2 × 2 matrix $A = [a_{ij}]$ whose elements are $a_{ij} = i - j$.

SECTION B

11. Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1$ is congruent to $T_2\}$.

    Show that R is an equivalence relation.

12. Prove that $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

OR

Solve the following for x.

$$\tan^{-1}\left(\frac{2x}{x^2 - 1}\right) + \cos^{-1}\left(\frac{x^2 - 1}{x^2 + 1}\right) = \frac{2\pi}{3}.$$

13. Using the properties of determinent prove that $\begin{vmatrix} a+l & m & n \\ l & a+m & n \\ l & m & a+n \end{vmatrix} = a^2(a + l + m + n)$.

OR

If $A = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$.

Verify that $A^2 - 4A - 5I = 0$

14. For what value of k, is the function

$$f(x) = \begin{cases} \dfrac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ k & x = 0 \end{cases}$$

continuous at $x = 0$?

15. If $y = \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} + \log\sqrt{1 - x^2}$ then show that $\frac{dy}{dx} = \frac{\sin^{-1} x}{\left(1 - x^2\right)^{3/2}}$.

16. Find the equation of tangent to the curve $x = \sin 3t$, $y = \cos 2t$ at $t = \frac{\pi}{4}$.

17. Evaluate $\int_0^{\pi} \frac{x \sin x}{\left(1 + \cos^2 x\right)} dx$.

18. Solve the differential equation

$$\left(x^2 - 1\right)\frac{dy}{dx} + 2xy = \frac{2}{x^2 - 1}.$$

OR

Solve the differential equation. $(x^2 - y^2)\, dx + 2xy\, dy = 0$.

19. Solve the following differential equation

$$\cos^2 x \frac{dy}{dx} + y = \tan x.$$

20. If $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ are three vectors such that $\overrightarrow{a} \cdot \overrightarrow{b} = \overrightarrow{a} \cdot \overrightarrow{c}$ and $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{a} \times \overrightarrow{c}$. $\overrightarrow{a} \neq 0$, then show that $\overrightarrow{b} = \overrightarrow{c}$.

21. Find the angle between the line $\frac{x-2}{3} = \frac{2y-5}{4} = \frac{3-z}{-6}$ and the plane $x + 2y + 2z - 5 = 0$.

OR

Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance from $3\sqrt{2}$ from the point (1, 2, 3).

22. Five dice are thrown simultaneously. If the occurrence of an even number on a single die is considered a success. Find the probability of getting almost 3 successes.

SECTION C

23. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

Find AB use the result to solve the following system of linear equation

$$2x - y + z = -1$$

$$-x + 2y - z = 4$$

$$x - y + 2z = -3$$

24. A window is in the form of a rectangle Surmounted by a semi-circular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

OR

Find the interval in which the function $f(x) = (x + 1)^3 (x - 3)^3$ is (i) increasing (ii) decreasing.

25. Evaluate $\int \frac{1}{\sin^4 x + \cos^4 x} dx$.

26. Find the area of the region

$$\left\{(x, y) : 25x^2 + 9y^2 \leq 225 \text{ and } 5x + 3y \geq 15\right\}.$$

27. Find the equation of the plane passing through the point (–1, –1, 2) and perpendicular to each of the following planes.

$$2x + 3y - 3z = 2 \quad \text{and} \quad 5x - 4y + z = 6.$$

OR

Find the equation of the plane passing through the points (3, 4, 1) and (0, 1, 0) and parallel to the line $\frac{x + 3}{2} = \frac{y - 3}{7} = \frac{z - 2}{5}$.

28. There are two bags I and II containing 3 red and 4 white balls and 2 red and 3 white balls respectively. A bag is selected at random and a ball is drawn from it. If it is found to be a red ball. Find the probability that it is drawn from the first bag.

29. One kind of Cake requires 200 gm of flour and 25 gm of fat, and another kind of cake requires 100 gm of flour and 50 gm of fat. Find the maximum number of Cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the Cakes. Formulate the above as a linear programming problem and solve graphically.

MARKING SCHEME FOR PAPER – I

SECTION A

1. $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$.

2. $\vec{a} \cdot \vec{b} = 1$.

3. $\frac{\pi}{3}$

4. $x = 3$, $y = 3$.

5. 4.

6. $\frac{\pi}{3}$

7. $-\frac{1}{x^3 + 2x^2 - 5x + 1} + c$

8. $2 \sin (4x + 6)$.

9. –6

10. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

SECTION B

11. (i) **Reflexive**

As every triangle is congruent to itself

i.e., $(T_1, T_1) \in R$

$\therefore$ R is reflexive. 1

(ii) **Symmetric**

Now $(T_1, T_2) \in R \Rightarrow T_1 \cong T_2 \Rightarrow T_2 \cong T_1$

$\therefore$ $(T_2, T_1) \in R$

$\therefore$ R is symmetric 1

(iii) **Transitive**

Now $(T_1, T_2), (T_2, T_3) \in R$

$T_1 \cong T_2$ and $T_2 \cong T_3$

$\Rightarrow \quad T_1 \cong T_3$

$\therefore (T_1, T_3) \in R$

$\therefore$ R is transitive.

From (i), (ii) and (iii), R is an equivalence relation 2

12. $L.H.S. = \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4},\frac{2}{9}}\right) = \tan^{-1}\frac{1}{2}$ 2

$$\tan^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1-(1/2)^2}{1+(1/2)^2}\right) = \frac{1}{2}\cos^{-1}\frac{3}{5} = R.H.S.$$ 1+1 = 2

OR

$$\cos\left(\frac{x^2-1}{x^2+1}\right) = \pi - 2\tan^{-1}x$$ 1

$\Rightarrow \quad \pi - 2\tan^{-1}x - 2\tan^{-1}x = \frac{2\pi}{3}$ 1

$\Rightarrow \quad 4\tan^{-1}x = \frac{\pi}{3} \quad \Rightarrow \quad \tan^{-1}x = \frac{\pi}{12}$ 1

$\Rightarrow \quad x = \tan\frac{\pi}{12}.$ 1

13. Let $\Delta = \begin{vmatrix} a+l & m & n \\ l & a+m & n \\ l & m & a+n \end{vmatrix}$

Applying $c_1 \to c_1 + (c_2 + c_3)$ we get

$$\Delta = \begin{vmatrix} a+l+m+n & m & n \\ a+l+m+n & a+m & n \\ a+l+m+n & m & a+n \end{vmatrix}$$ 1

$$= (a+l+m+n)\begin{vmatrix} 1 & m & n \\ 1 & a+m & n \\ 1 & m & a+n \end{vmatrix}$$

Appllying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ we get

$$\Delta = (a + l + m + n)\begin{vmatrix} 1 & m & n \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix}$$ 2

$= (a + l + m + n)\ [a(a - 0)] = a^2\ (a + l + m + n)$ 1

OR

$$A^2 = \begin{bmatrix} 9 & 8 & 9 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix},$$ 2

$$4A = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}, \quad 5I = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$ 1

For verifying $A^2 - 4A - 5I = 0$. 1

14. Here $f(0) = k$ 1/2

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1 - \cos 4x}{8x^2}$$

$$= \lim_{x \to 0} \frac{2 \sin^2 2x}{2 \cdot 4x^2} = \underset{x \to 0}{\text{Lt}} \frac{\sin^2 2x}{4x^2}$$ 1

$$= \left[\underset{x \to 0}{\text{Lt}} \frac{\sin 2x}{2x} \right]^2$$ 1

$= (1)^2 = 1$ 1/2

For continuity at $x = 0$

$$\lim_{x \to 0} f(x) = f(0) = k$$ 1/2

$\therefore \quad k = 1$.

15. $$y = \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} + \log \sqrt{1 - x^2}$$

$$\frac{dy}{dx} = \frac{x^2 \sin^{-1} x}{(1 - x^2)^{3/2}} + \frac{x}{(1 - x^2)} + \frac{\sin^{-1} x}{\sqrt{1 - x^2}} - \frac{x}{1 - x^2}$$ 2

$$= \frac{x^2 \sin^{-1} x}{(1 - x^2)^{3/2}} + \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$$

$$= \frac{\sin^{-1} x}{\sqrt{1 - x^2}} \left[\frac{x^2}{1 - x^2} + 1 \right]$$ 1

$$= \frac{\sin^{-1} x}{\sqrt{1 - x^2}} \left[\frac{x^2 + 1 - x^2}{1 - x^2} \right]$$ 1/2

$$= \frac{\sin^{-1} x}{\left(1 - x^2\right)^{3/2}}$$ 1/2

16. At $t = \frac{\pi}{4}, \quad x = \frac{1}{\sqrt{2}}, \quad y = 0$ 1

$$\frac{dx}{dt} = 3 \cos 3t, \quad \frac{dy}{dt} = -2 \sin 2t$$ 1/2 + 1/2

$$\therefore \quad \frac{dy}{dx} = \frac{-2 \sin 2t}{3 \cos 3t}$$ 1/2

$$\therefore \quad \left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{2\sqrt{2}}{3}.$$ 1/2

Equation of tangecil $3y = 2\sqrt{2x} - 2$ or $3y - 2\sqrt{2x} + 2 = 0$.

17. $$I = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx.$$

Getting $2I = \int_0^{\pi} \frac{\pi \sin x}{1 + \cos^2 x} dx$ 1½

Put $\cos x = t \quad \Rightarrow \quad -\sin dx = dt \quad \Rightarrow \quad x = 0,\ t = 1,\ x = \pi \quad \Rightarrow \quad \pi = -1$

$$\therefore \quad 2I = \pi \int_0^{\pi} \frac{dt}{1 + t^2}$$ 1½

$$2I = \pi^2/_2$$

$$I = \frac{\pi^2}{4}.$$ 1

18. Writing the differential equation as

$$\frac{dy}{dx} + \frac{2x}{x^2 - 1} . y = \frac{2}{\left(x^2 - 1\right)^2}$$ 1/2

Integrating factor $= e^{\int \frac{2x}{x^2 - 1} dx}$

$\therefore$ The solution is 1

$$y \cdot (x^2 - 1) = \int \frac{2}{x^2 - 1} dx$$ 1

$\Rightarrow$ $$y \cdot (x^2 - 1) = \log\left|\frac{x - 1}{x + 1}\right| + c$$ 1½

OR

Writing $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$, which is homogeneous 1/2

Putting $y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1/2

$\therefore$ getting $\frac{2v}{1 + v^2} dv = \frac{-dx}{x}$ 1

$\therefore$ $\log |1 + v^2| = -\log |x| + \log |c|$

$$= \log\left|\frac{c}{x}\right|$$ 1

$\Rightarrow$ $x^2 + y^2 = cx$

$\Rightarrow$ $c = 2$ when $x = 1$ and $y = 1$ 1

$\therefore$ $x^2 + y^2 = 2x$

19. Writing $\frac{dy}{dx} + \sec^2 xy = \tan x . \sec^2 x$ 1/2

I.F. = $e^{\tan x}$. 1

$\therefore$ Solution is $y . e^{\tan x} = \int e^{\tan x}. \tan x . \sec^2 x \, dx + c$ 1/2

$$y . e^{\tan x} = e^{\tan x}(\tan x - 1) + c$$

or $y = (\tan x - 1) + c . e^{-\tan x}$. 2

20. $\vec{a} . \vec{b} = \vec{a} . \vec{c} \Rightarrow \vec{a}(\vec{b} - \vec{c}) = 0$

$\Rightarrow$ either $\vec{a} = 0$ or $\vec{b} - \vec{c} = 0$ or $\vec{a} \perp (\vec{b} - \vec{c})$ ...(i) 1½

$\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = 0$

$\Rightarrow$ either $\vec{a} = 0$ or $\vec{b} - \vec{c} = 0$ or $\vec{a} \parallel (\vec{b} - \vec{c})$ ...(ii) 1½

Give that $\vec{a} \neq 0$ and a vector can not be parallel or perpendicular to the same vector

$\therefore \quad \vec{b} - \vec{c} = 0 \quad$ [from (i) and (ii)]

$\Rightarrow \quad \vec{b} = \vec{c}$. 1

21. Equation of line is

$$\frac{x-2}{3} = \frac{y - 5/2}{2} = \frac{z-3}{6}$$

$\therefore$ DR´s at line are 3, 2,6 1

DR's of the normal to the plane $x + 2y + 2z - 5 = 0$

are 1, 2, 2 1

Let θ be the angle between line and plane then

$$\therefore \quad \sin\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} + \sqrt{a_2^2 + b_2^2 + c_2^2}}$$ 1/2

$$= \frac{|3.1 + 2.2 + 6.2|}{\sqrt{49}\,.\,\sqrt{9}}$$ 1

$$= \frac{19}{21}$$

$\therefore$ Angle between line and plane $\theta = \sin^{-1}\frac{19}{21}$. 1/2

OR

Getting $x = 3\lambda - 2$, $y = 2\lambda - 1$, $z = 2\lambda + 3$ 1

Distance D from (1, 2, 3) = $3\sqrt{2}$

$$\therefore \quad (3\sqrt{2})^2 = (3\lambda - 3)^2 + (2\lambda - 3)^2 + 2\lambda^2$$ 1

$$18 = 17\lambda^2 - 30\lambda + 18$$

$\Rightarrow \lambda(17\lambda - 30) = 0$

$\Rightarrow \quad \lambda = 0 \quad$ or $\quad \lambda = \frac{30}{17}$. 1

$\therefore$ points one $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ or $(-2, -1, 3)$. 1

22. P(sucess) = P(even number)

$= P(2, 4, 6)$

$$= \frac{3}{6} = \frac{1}{2}$$ 1

$$\therefore\ P(\text{Failure}) = 1 - \frac{1}{2} = \frac{1}{2}$$ 1/2

P(At most 3 sucesses) = P(x ≤ 3)

$= 1 - P(x > 3)$ 1/2

$= 1 - P(4 \text{ or } 5 \text{ successes})$

$= 1 - [P(4) + P(5)]$ 1/2

$$= 1 - \left[5c_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right) + 5c_5 \left(\frac{1}{2}\right)^5\right]$$ 1/2

$$= 1 - \left(\frac{5}{32} + \frac{1}{32}\right)$$

$$= 1 - \frac{6}{32}$$

$$= \frac{26}{32} \text{ or } \frac{13}{16}.$$ 1

23. $$AB = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$= 4 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 4I$$ 1½

i.e., $A \cdot \frac{1}{4} B = I$

$$\therefore \quad A^{-1} = \frac{1}{4} B = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$ ½ + ½

The given system of equation is

$2x - y + z = -1$

$-x + 2y - z = 4$

$x - y + 2z = -3$

In matrix form

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$$ 1

$\therefore$ $Ax = C$

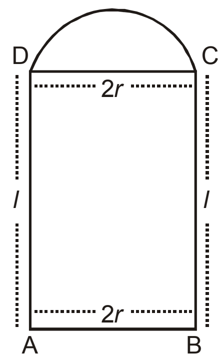
or $X = A^{-1} C$ 1/2

or $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 \\ 8 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$ 1½

$\therefore$ $x = 1$, $y = 2$, $z = -1$. 1/2

24. Let r be the radius of the semi circle.



For correct figure one mark.

$\therefore$ Breadth of rectangle portion = $2r$

Let 'l' be the length of the rectangular portion

$\therefore$ Perimeter of window = $2r + l + \pi r + l$

$= 2r + 2l + \pi r$ 1/2

As perimeter of window is 10 m.

$\therefore$ $2r + 2l + \pi r = 10$

$2l = 10 - 2r - \pi r$ ...(i) 1/2

Area of window $A = 2r \times l + 1/2\pi r^2$

$$= 10r - 2r^2 - \frac{1}{2}\pi r^2$$

$$\frac{dA}{dr} = 10 - 4r - \pi r$$ 1

Fox maximum Area $dA/dr = 0$

$= 10 - 4r - \pi r = 0$

$$r = \frac{10}{4 + \pi}$$

and $d^2A/dr^2 = -4 - \pi < 0$

$\therefore$ A is maximum at $r = \dfrac{10}{4+\pi} m$

From (i) $2l = 10 - (2+\pi)\dfrac{10}{4+\pi} = \dfrac{20}{4+\pi} m$

$\therefore \quad l = \dfrac{10}{4+\pi}$ 1/2

Hence dimentions one

length $= \dfrac{10}{4+\pi} mt.$; breadth $= \dfrac{20}{4+\pi} mt.$ 1/2

OR

We are given $f(x) = (x+1)^3 (x-3)^2$

Getting $f^1(x) = 6(x+1)^2 . (x-3)^2 (x-1)$ ...(i) 2

(i) For $f(x)$ to be increasing $f'(x) > 0$ 1/2

or $6(x+1)^2 (x-3)^2 (x-1) > 0$

or $x - 1 > 0$

$x > 1$ 1

$\therefore$ $f(x)$ is increasing function in $(1, \infty)$. 1/2

(ii) For $f(x)$ to be decreasing $f'(x) < 0$

i.e., $6(x+1)^2 (x-3)^2 (x-1) < 0$ 1/2

i.e., $x - 1 < 0$

or $x < 1$ 1

$\therefore$ $f(x)$ is decreasing function in $(-\infty, 1)$.

25. Let $I = \displaystyle\int \frac{dx}{\sin^4 x + \cos^4 x}$

$= \displaystyle\int \frac{\sec^2 x \left(1 + \tan^2 x\right)}{\tan^4 x + 1} dx$ 1

Let $\tan x = t \quad \Rightarrow \quad \sec^2 x \, dx = dt$

$\therefore \quad I = \displaystyle\int \frac{1+t^2}{1+t^4} dt$

$$= \int \frac{1 + \frac{1}{t^2}}{\left(t - \frac{1}{t}\right)^2 + 2} dt$$ 2

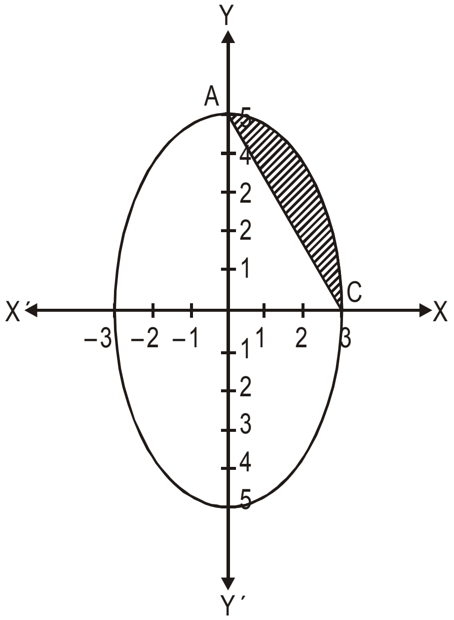
Let $t - \frac{1}{t} = z$ then $\left(1 + \frac{1}{t^2}\right) dt = dz$

$$I = \int \frac{dz}{z^2 + (\sqrt{2})^2}$$

$$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{z}{\sqrt{2}}\right) + c$$

$$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan^2 n - 1}{\sqrt{2} \tan x}\right) + c.$$ 2

26. $25x^2 + 9y^2 = 225 \quad \Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{25} = 1$



Correct Fig. 1

Finding the point of intersection $x = 0$, $x = 3$

$$\text{Area} = \frac{1}{3}\int_0^3 \sqrt{225 - 25x^2}\, dx - \frac{1}{3}\int_0^3 (15 - 5x)\, dx$$ 2

$$= \frac{5}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 + \frac{5}{3}\frac{(3 - x)^2}{2}\Bigg]_0^3$$ 1

$$= \frac{15\pi}{4} - \frac{15}{2} = \frac{15}{4}(\pi - 2) \text{ sq. unit.}$$ 1

27. Equation of plane through (–1, –1, 2) is

$a(x + 1) + b(y + 1) + c(z - 2) = 0$ ...(i) 1

$\therefore$ $2a + 3b - 3c = 0$ and $5a - 4b + c = 0$ 2

Solving to get $a : b : c = 9 : 17 : 23$ 2

$\therefore$ equation of plane is $9x + 17y + 23z = 20$

OR

Equation of plane through (3, 4, 1) is

$a(x - 3) + b(y - 4) + c(z - 1) = 0$ 1

$\therefore$ we get $3a + 3b + c = 0$ and $2a + 7b + 5c = 0$ 2

Solving to get $a : b : c = 8 : -13 : 15$ 2

$\therefore$ equation of plane $8x - 13y + 15z = 0$ 1

28. Let event E_1 : the first bag is selected

event E_2 : The second bag is selected/

and event A : A red ball is drawn.

then $P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{2}$ 1

$P(A/E_1) = \frac{3}{7}$ and $P(A/E_2) = \frac{2}{5}$. 2

$$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2)}$$ 1

$$= \frac{\frac{1}{2}.\frac{3}{7}}{\frac{1}{2}.\frac{3}{7} + \frac{1}{2}.\frac{2}{5}}$$

$$= \frac{\frac{3}{14}}{\frac{3}{14} + \frac{1}{15}} = \frac{\frac{3}{14}}{\frac{29}{70}} = \frac{15}{29}.$$ 2

29. Let number of cakes of first kind be x and that of second kind be y.

L.P.P. is maximize $Z = x + y$.

Subject to $200x + 100y \leq 5000$

or $\qquad 2x + y \leq 50$ ...(i)

$\qquad 25x + 50y \leq 1000$

or $\qquad x + 2y \leq 40$ ...(ii)

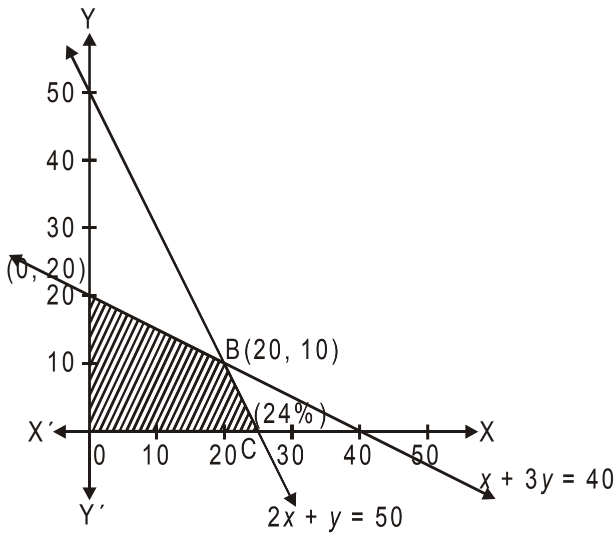
$\qquad x \geq 0, \quad y \geq 0$ ...(iii)

For (i), (ii), (iii) 2

Correct Graph 2 marks

Maximum at B(20, 10)

$\therefore$ Maximum Number of Cakes = 20 + 10 = 30.



PRACTICE QUESTIONS PAPER – II

MATHEMATICS

Time Allowed : 3 hours ***Maximum Marks : 100***

General Instructions

1. Question paper has three sections. Section A contains 10 questions of one mark each, Section *B* contains 12 questions of 4 marks each and Section C contains 7 questions of six marks each.
2. All questions are compulsory.
3. Interval choice are given in some questions, where one part is to be attempted out of two.
4. Calculators are not allowed.

SECTION A

1. Write the order and degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = x\,\frac{d^2y}{dx^2}.$$

2. Find the integrating factor of the differential equation

$$\frac{dy}{dx} - 3y\cot x = \sin 2x.$$

3. Evaluate $\int \frac{a - b\cos x}{\sin^2 x}\,dx.$

4. The radius of a circle is increasing at the rate of 2 cm/s, find the rate at which its circumference is increasing?

5. If $\begin{vmatrix} 5x+2 & 9 \\ 2x+5 & 3 \end{vmatrix} = 0$, find x.

6. If $\begin{pmatrix} x+3x & y \\ 7-x & 5 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 1 & 5 \end{pmatrix}$, find the values of x and y.

7. Find the cofactor of '4' in the following determinants.

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

8. If the sum of two unit vectors $\hat{a}$ and $\hat{b}$ is a unit vector, then what is the angle between $\hat{a}$ and $\hat{b}$.

9. If the projection of $\vec{b}$ on $\vec{a}$ is $\vec{a} \cdot \vec{b}$, then what is the value of $|\vec{a}|$.

10. If the vectors $-2\hat{i} + 3\hat{j} + y\hat{k}$ and $x\hat{i} - 6\hat{j} + 2\hat{k}$ are collinear, then find the values of x and y.

SECTION B

11. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x, -\frac{1}{2} \le x \le 1.$

OR

Solve for x : $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}.$

12. If $A' = \begin{pmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix}$, then find $(A + 2B)'$.

13. Find the derivative of the function given by $f(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$, and hence find $f'(1)$.

14. If $y = \sin^{-1}\left(\frac{4x + 3\sqrt{1-x^2}}{5}\right)$, find $\frac{dy}{dx}$.

15. Find the equation of normal to the curve $x^{2/3} + y^{2/3} = 8$ at the point $x = 8$.

OR

The length x of a rectangle is decreasing at the rate of 4cm/minute and the width y is increasing or the rate of 5 cm/minute. When $x = 8$ cm and $y = 6$cm, find the rate of change of the area of the rectangle.

16. Evaluate : $\int \frac{2x}{(x^2+1)(x^2+3)} dx.$

17. Evaluate : $\int \frac{1+\sin 2x}{1+\cos 2x} \cdot e^{2x} \cdot dx.$

18. Evaluate : $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx.$

19. Solve the differential equation : $(x - y)(dx + dy) = dx - dy$, given that $y = -1$ when $x = 0$.

OR

Solve the differential equation :

$$2ye^{x/y} dx + (y - 2x \cdot e^{x/y}) dy = 0, \text{ given that when } x = 0, y = 1.$$

20. If $\overrightarrow{\alpha} = 3\hat{i} - \hat{j}$, $\overrightarrow{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, express $\overrightarrow{\beta}$ in the form $\overrightarrow{\beta_1} + \overrightarrow{\beta_2}$ where $\overrightarrow{\beta_1}$ is parallel to $\overrightarrow{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicular to $\overrightarrow{\alpha}$.

OR

If $\overrightarrow{a}$ and $\overrightarrow{b}$ are unit vector inclined at an angle θ, then prove that $\cos\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} + \overrightarrow{b}\right|$.

21. Find the vector equation of a line passing through (1, 2, 3) and parallel to the planes

$$\overrightarrow{r}\cdot\left(\hat{i} - \hat{j} + 2\hat{k}\right) = 5 \text{ and } \overrightarrow{r}\cdot\left(3\hat{i} + \hat{j} + \hat{k}\right) = 6.$$

22. Twelve cards numbered 1 to 12 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the card drawn is more than 4, what is the probability that it is an even number.

SECTION C

23. Consider the binary operation * : $R \times R \to R$ and 0 : $R \times R \to R$ defined as $a * b = |a - b|$ and $a \text{ o } b = a$, $\forall\ a, b \in R$.

Show that * is commutative but not associative, o is associative but not commutative. Further show that $a * (b \text{ o } c) = (a * b) \text{ o } (a * c)$, $\forall\ a, b, c \in R$.

Also verify if $a \text{ o } (b * c) = (a \text{ o } b) * (a \text{ o } c)$ or not.

24. If $A = \begin{pmatrix} 2 & 1 & 1 \\ 8 & 1 & 2 \\ 5 & 1 & -1 \end{pmatrix}$, find A^{-1}. Using A^{-1}, solve the system of linear equations :

$$2x + 8y + 5z = 5,\ x + y + z = -2,\ x + 2y - z = 2.$$

If $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$. Where I is the identity matrix of order 3. Hence find A^{-1}.

25. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is 8m^3. If building of tank costs Rs. 70 per sq. meter for the base and Rs. 45 per sq. meter for sides. What is the cost of least expansive tank?

OR

Find the intervals in which the function given by

$$f(x) = \tan^{-1}(\sin x + \cos x),\ 0 < x < 2\pi.$$

is (a) strictly increasing (b) strictly decreasing.

26. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 6x$ and the parabola $y^2 = 3x$.

27. Two cards are drawn simultaneously (without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of spades.

28. Show that the point $A(1, 1, 1)$, $B(2, 2, -6)$, $C(3, 1, -5)$ and $D(-1, 1, 7)$ are coplaner.

29. A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour each on machine A and B and 3 hours on machine C. The profit on selling one chair is Rs. 30, while by selling one table the profit is Rs. 60. The total time available per week on machine A is 70 hours, on B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximise profit? Formulate the problem as a L.P.P. and solve it graphically.