DIRECTORATE OF EDUCATION

GNCT of Delhi, Delhi Government

SUPPORT MATERIAL
(2021-2022)

Class : XII

MATHEMATICS

Under the Guidance of

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MESSAGE

I would like to congratulate the members of Core Academic Unit and the subject experts of the Directorate of Education, who inspite of dire situation due to Corona Pandemic, have provided their valuable contributions and support in preparing the Support Material for classes IX to XII.

The Support Material of different subjects, like previous years, have been reviewed/ updated in accordance with the latest changes made by CBSE so that the students of classes IX to XII can update and equip themselves with these changes. I feel that the consistent use of the Support Material will definitely help the students and teachers to enrich their potential and capabilities.

Department of Education has taken initiative to impart education to all its students through online mode, despite the emergency of Corona Pandemic which has led the world to an unprecedented health crises. This initiative has not only helped the students to overcome their stress and anxiety but also assisted them to continue their education in absence of formal education. The support material will ensure an uninterrupted learning while supplementing the Online Classes.

(H. Rajesh Prasad)

UDIT PRAKASH RAI, IAS
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MESSAGE

The main objective of the Directorate of Education is to provide quality education to all its students. Focusing on this objective, the Directorate is continuously in the endeavor to make available the best education material, for enriching and elevating the educational standard of its students. The expert faculty of various subjects undertook this responsibility and after deep discussions and persistent efforts, came up with Support Material to serve the purpose.

Every year the Support Material is revised/ updated to incorporate the latest changes made by CBSE in the syllabus of classes IX to XII. The contents of each lesson/chapter are explained in such a way that the students can easily comprehend the concept and get their doubts solved.

I am sure, that the continuous and conscientious use of this Support Material will lead to enhancement in the educational standard of the students, which would definitely be reflected in their performance.

I would also like to commend the entire team members for their contributions in the preparation of this incomparable material.

I wish all the students a bright future.

(UDIT PRAKASH RAI)

Dr. RITA SHARMA
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(School/Exam)

सत्यमेव जयते

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D.O. No. PA/Addl.DE/Sch/31
Dated: 29.06.2021

MESSAGE

It gives me immense pleasure to present the revised edition of the Support Material. This material is the outcome of the tireless efforts of the subject experts, who have prepared it following profound study and extensive deliberations. It has been prepared keeping in mind the diverse educational level of the students and is in accordance with the most recent changes made by the Central Board of Secondary Education.

Each lesson/chapter, in the support material, has been explained in such a manner that students will not only be able to comprehend it on their own but also be able to find solution to their problems. At the end of each lesson / chapter, ample practice exercises have been given. The proper and consistent use of the support material will enable the students to attempt these exercises effectively and confidently. I am sure that students will take full advantage of this support material.

Before concluding my words, I would like to appreciate all the team members for their valuable contributions in preparing this unmatched material and also wish all the students a bright future.

(Rita Sharma)

DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
(2021-2022)

MATHEMATICS
Class : XII
(English Medium)

NOT FOR SALE

PUBLISHED BY : DELHI BUREAU OF TEXTBOOKS

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भारत का संविधान

भाग 4क

नागरिकों के मूल कर्तव्य

अनुच्छेद 51क

मूल कर्तव्य – भारत के प्रत्येक नागरिक का यह कर्तव्य होगा कि वह –

1. संविधान का पालन करे और उसके आदर्शों, संस्थाओं, राष्ट्र ध्वज और राष्ट्रगान का आदर करें।
2. स्वतंत्रता के लिए हमारे राष्ट्रीय आंदोलन को प्रेरित करने वाले उच्च आदर्शों को हृदय में संजोए रखे और उनका पालन करे।
3. भारत की प्रभुता, एकता और अखंडता की रक्षा करे और उसे अक्षुण्ण रखे।
4. देश की रक्षा करे।
5. भारत के सभी लोगों में समरसता और समान भ्रातृत्व की भावना का निर्माण करे।
6. हमारी सामाजिक संस्कृति की गौरवशाली परंपरा का महत्त्व समझे और उसका निर्माण करे।
7. प्राकृतिक पर्यावरण की रक्षा और उसका संवर्धन करे।
8. वैज्ञानिक दृष्टिकोण और ज्ञानार्जन की भावना का विकास करे।
9. सार्वजनिक संपत्ति को सुरक्षित रखे।
10. व्यक्तिगत एवं सामूहिक गतिविधियों के सभी क्षेत्रों में उत्कर्ष की ओर बढ़ने का सतत् प्रयास करे।
11. माता–पिता या संरक्षक द्वारा 6 से 14 वर्ष के बच्चों हेतु प्राथमिक शिक्षा प्रदान करना (86वां संशोधन)।

CONSTITUTION OF INDIA

Part IV A (Article 51 A)

Fundamental Duties

Fundamental Duties : It shall be the duty of every citizen of India —

1. to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;
2. to cherish and follow the noble ideals which inspired our national struggle for freedom;
3. to uphold and protect the sovereignty, unity and integrity of India;
4. to defend the country and render national service when called upon to do so;
5. to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;
6. to value and preserve the rich heritage of our composite culture;
7. to protect and improve the natural environment including forests, lakes, rivers and wild life, and to have compassion for living creatures.
8. to develop the scientific temper, humanism and the spirit of inquiry and reform;
9. to safeguard public property and to adjure violence;
10. to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement.
11. who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

भारत का संविधान

उद्देशिका

हम, भारत के लोग, भारत को एक **(सम्पूर्ण प्रभुत्व–सम्पन्न समाजवादी पंथनिरपेक्ष लोकतंत्रात्मक गणराज्य)** बनाने के लिए, तथा उसके समस्त नागरिकों को :

सामाजिक, आर्थिक और राजनैतिक **न्याय,**

विचार, अभिव्यक्ति, विश्वास, धर्म

और उपासना की **स्वतंत्रता,**

प्रतिष्ठा और अवसर की समता

प्राप्त करने के लिए,

तथा उन **सब में,**

व्यक्ति की गरिमा और **(राष्ट्र की एकता**

और अखंडता) सुनिश्चित करने वाली बंधुता

बढ़ाने के लिए

हम दृढ़संकल्प होकर इस संविधान को आत्मार्पित करते हैं।

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a **(SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC)** and to secure to all its citizens :

JUSTICE, social, economic and political,

LIBERTY of thought, expression, belief, faith and worship,

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the **(unity an integrity of the Nation)**;

WE DO HEREBY GIVE TO OURSELVES THIS CONSTITUTION.

Class-XII (2021-22)
(Mathematics 2020-21)
Term-I

One Paper **Max Marks:40**

No.	Units	Marks
I.	Relations and Functions	08
II.	Algebra	10
III.	Calculus	17
V.	Linear Programming	05
	Total	40
	Internal Assessment	10
	Total	50

Unit-I: Relations and Functions

1. Relations and Functions

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. Inverse Trigonometric Functions

Definition, range, domain, principal value branch.

Unit-II: Algebra

1. Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Noncommutativity of multiplication of matrices, Invertible matrices; (Here all matrices will have real entries).

2. Determinants

Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. Continuity and Differentiability

Continuity and differentiability, derivative of composite functions, chain rule, derivative of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives

Applications of derivatives: increasing/decreasing functions, tangents and normals, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real life situations).

Unit-V: Linear Programming

1. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems. Graphical method of solution for problems in two variables, feasible and infeasible regions (bounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

INTERNAL ASSESSMENT:	**10 MARKS**
Periodic Test:	5 Marks
Mathematics Activities: Activity file record + Term end assessment of one activity & Viva	5 Marks

Note: For activities NCERT Lab Manual may be referred.

Term-II

One Paper **Max Marks 40**

No.	Units	Marks
III.	Calculus	8
IV.	Vectors and Three-Diminsional Geometry	14
VI.	Probability	8
	Total	40
	Internal Assessment	10
	Total	50

Unit-III: Calculus

1. Integrals

Integration as an inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

Fundamental Theorem of Calculus (without proof).Basic properties of definite integrals and evaluation of definite integrals.

2. Applications of the Integrals

Applications in finding the area under simple curves, especially lines, parabolas; area of circles/ellipses (in standard form only) (the region should be clearly identifiable).

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$\int \frac{px}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2 - a^2} dx$$

3. Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree of the type: $\frac{dy}{dx} = f(y, x)$. Solutions of linear differential equation of the type:

$\frac{dy}{dx}+py=q,$ where p and q are functions of x or constant.

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three - dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Distance of a point from a plane.

Unit-VI: Probability

1. Probability

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution.

INTERNAL ASSESSMENT:	**10 MARKS**
Periodic Test:	5 Marks
Mathematics Activities: Activity file record + Term end assessment of one activity & Viva	5 Marks

Note: For activities NCERT Lab Manual may be referred.

Assessment of Activity Work:

In first term any 4 activities and in second term any 4 activities shall be performed by the student from the activities given in the NCERT Laboratory Manual for the respective class (XI or XII) which is available on the link : http://www.ncert.nic.in/exemplar/labmanuals.html a record of the same may be kept by the student. A term end test on the activity is to be conducted.

The weightage are as under:

- The activities performed by the student in each term and record keeping : 3 marks
- Assessment of the activity performed during the term end test and Viva-voce : 2 marks

Prescribed Books:

1. Mathematics Part I - Textbook for Class XII, NCERT Publication
2. Mathematics Part II - Textbook for Class XII, NCERT Publication
3. Mathematics Exemplar Problem for Class XII, Published by NCERT
4. Mathematics Lab Manual class XII, published by NCERT

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CHAPTER 1

RELATIONS AND FUNCTIONS

A relation R in a set A is a subset of $A \times A$.

Thus, R is a relation in a set $A = \subseteq A \times A$.

If $(a, b) \varepsilon R$ then we say that a is related to b and write, $a R b$.

If $(a, b) \notin R$ then we say that a is not related to b and write, $a R b$.

If number of elements in set A and set B are p & q respectively, Means $n(A) = p$, $n(B) = q$, then

No. of Relation of $A \times A = 2^{p^2}$

No. of Relation of $B \times B = 2^{q^2}$

No. of Relation of $A \times B$ = No. of Relation of $B \times A = 2^{pq}$

No. of NON-EMPTY Relation of $A \times A = (2^{p^2} - 1)$,

No. of NON-EMPTY Relation of $B \times B = (2^{q^2} - 1)$,

No. of NON-EMPTY Relation of $A \times B$ = No. of Relation of $B \times A = (2^{pq} - 1)$

Q. If $A = \{a, b, c\}$ & $B = \{1, 2\}$ find the number of Relation R on (i) $A \times A$ (ii) $B \times B$ (iii) $A \times B$

Ans: As $n(A) = 3$, $n(B) = 2$, so

No. of Relation R on $A \times A = 2^{3 \times 3} = 2^9 = 512$

No. of Relation R on $B \times B = 2^{2 \times 2} = 2^4 = 16$

No. of Relation R on $B \times B = 2^{3 \times 2} = 2^6 = 64$

DIFFERENT TYPES OF RELATION

- **Empty Relation Or Void Relation**

  A relation R in a set A is called an empty relation, if no element of A is related to any element of A and we denote such a relation by ϕ.

  Example: Let $A = \{1, 2, 3, 4\}$ and let R be a relation in A, given by $R = \{(a, b): a + b = 20\}$.

- **Universal Relation**

  A relation R in a set A is called an universal relation, if each element of A is related to every element of A. Example: Let $A = \{1, 2, 3, 4\}$ and let R be a relation in A, given by $R = \{(a, b): a + b > 0\}$.

- **Identity Relation**

  A relation R in a set A is called an Identity relation, where $R = \{(a, a), a \varepsilon A\}$.

  Example: Let $A = \{1, 2, 3, 4\}$ and let R be a relation in A, given by $R = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$.

- **Reflexive Relation**

  A relation R in a set A is called a Reflexive relation, if $(a, a) \varepsilon R$, for all $a \varepsilon A$.

  Example: Let $A = \{1, 2, 3, 4\}$ and let R be a relation in A, given by

  $R = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$.

  $R = \{(1, 2), (2, 2), (3, 3), (4, 4), (1, 2)\}$

  $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (2, 3), (1, 3), (3, 1)\}$.

- **Symmetric Relation**

  A relation R in a set A is called a Symmetric relation, if $(a, b) \varepsilon R$, then $(b, a) \varepsilon R$ for all a, $b \varepsilon A$.

  Example: Let $A = \{1, 2, 3, 4\}$ and let R be a relation in A, given by

  $R = \{(1, 1), (2, 2), (3, 3)\}$.

  $R = \{(1, 2), (2, 1), (3, 3)\}$.

  R = $\{(1, 1), (2, 2), (3, 3), (4, 4), (2, 3), (1, 3), (3, 1), (3, 2)\}$.

- **Transitive Relation**

  A relation R in a set A is called *a* Transitive relation,

  if $(a, b) \varepsilon R$ & $(b, c) \in R$ then $(a, c) \varepsilon R$ for all $a, b, c \varepsilon A$

  OR

  $(a, b) \in R$ &$(b, c) \notin R$ for all $a, b, c \in A$

  Example: Let $A = \{1, 2, 3, 4\}$ and let R be a relation in A, given by

  $R = \{(1, 1), (2, 2), (3, 3)\}$. (According to second condition)

  $R = \{(1, 2), (2, 1), (1, 1), (2, 2)\}$. (According to First condition)

  $R = \{(2, 3), (1, 3), (3, 1), (3, 3), (2, 2), (1, 1)\}$.

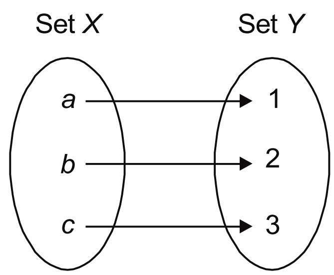
- **Equivalence Relation**

  A relation R in a set A is said to be an equivalence relation if it is reflexive, symmetric and transitive.

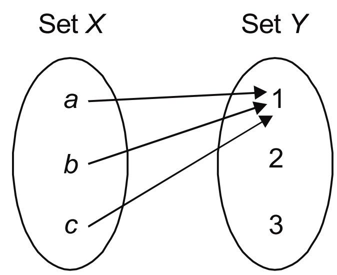
FUNCTIONS

Functions can be easily defined with the help of concept mapping. Let X and Y be any twon non-empty sets "A function from X to Y is a rule or correspondence that assigns to each element of set X, one and only on element of set Y". Let thte correspondence be 'f' then mathematically we write $f : X \to Y$ where $y = f(x)$, $x \varepsilon$ X and $y \varepsilon$ Y. We say that 'y' is the image of 'x' under f (or x is the pre image of y).

- A Mapping $f : X \to Y$ is said to be a function if each element in the set X has its image in set Y. If is also possible that there are few elements in set Y which are not the images of andy element in set X.
- Every element in set X should have one and only one image. That means it is impossible to have more than one image for a specific element in set X.
- Functions cannot be multi-valued (A mapping that is multi-valued is called a relation from X & Y) e.g.



Function



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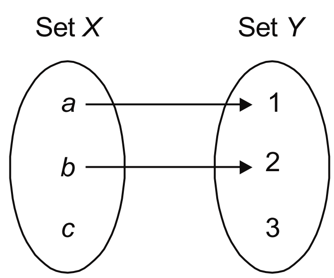
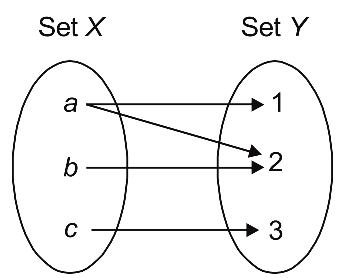


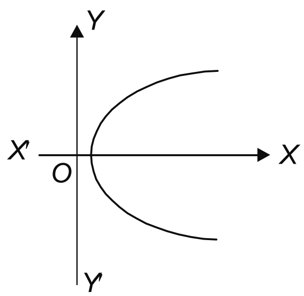
Not Function



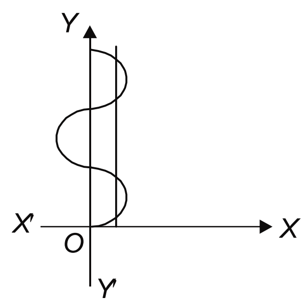
Not Function

TESTING FOR A FUNCTION BY VERTICAL LINE TEST

A relation $f : A \to B$ is a function or not, it can be checked by a graph of the relation. If it is possible to draw a vertical line which cuts the given curve at more than one point then the given relation is not a function and when this vertical line means line parallel to Y-axis cuts the curve at only one point then it is a function. Following Figures represents which is not a function and which is a function.

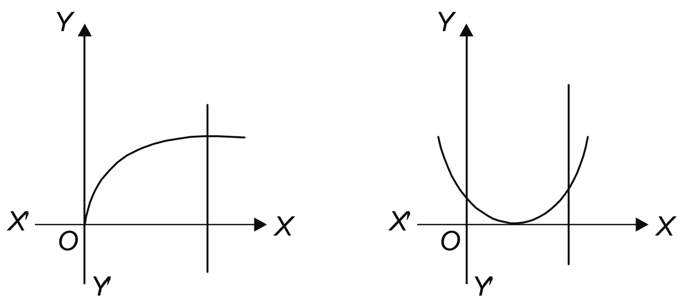


Not Function



Not a Function

Functions



Number of function : Let X and Y be two finite sets having m and n elements respectively. Thus each element of set X can be associated to any one of n elements of set Y. So, total number of functions from set X to set Y is nm.

Real valued function: If R, be the set of real numbers and A, B are subsets of R, then the function $f : A \to B$ is called a real function or real valued function.

DOMAIN, CO-DOMAIN AND RANGE OF FUNCTION

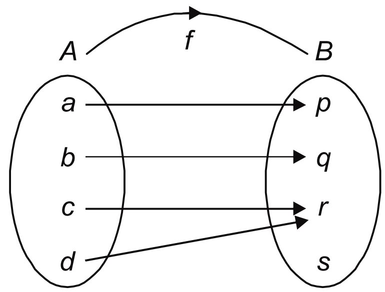
If a function f is defined from a set A to set B then ($f : A \to B$) set A is called the domain of f and set B is called the co-domain of f.

The set of all f-images of the elements of A is called the range of f.

In other words, we can say

Domain = All possible values of x for which $f(x)$ exists.

Range = For all values of x, all possible values of $f(x)$.



From the figure, we observe that

Domain = $A = \{a, b, c, d\}$,

Range = $\{p, q, r\}$,

Co-Domain = $\{p, q, r, s\} = B$

EQUAL FUNCTIONS

Two function f and g are said to be equal functions, if and only if

(i) Domain of f = Domain of g

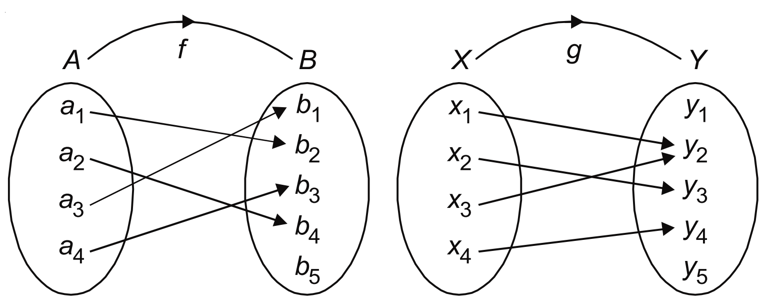
(ii) Co-domain of f = Co-domain of g

(iii) $f(x) = g(x)$ for all $x \in$ their common domain

TYPES OF FUNCTIONS

One-one function (injection): A function $f : A \to B$ is said to be a one-one function or an injection, if different elements of A have different images in B.

e.g. Let $f : A \to B$ and $g : X \to Y$ be two functions represented by the following diagrams.



Clearly, $f : A \to B$ is a one-one function. But $g : X \to Y$ is not one-one function because two distinct element x_1 and x_3 have the same image under function g.

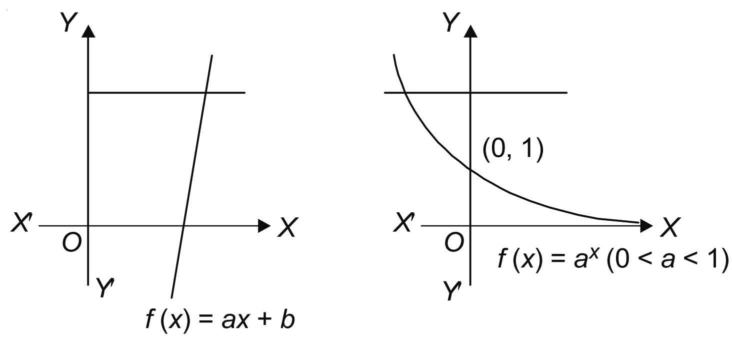
Method to check the injectivity (One-One) of a function

(i) Take two arbitrary elements x, y (say) in the domain of f.

(ii) Solve $f(x) = f(y)$. If $f(x) = f(y)$ gives $x = y$ only, then $f : A \to B$ is a one-one function (or an injection). otherwise not.

If function is given in the form of ordered pairs and if two ordered pairs do not have same second element then function is one-one.

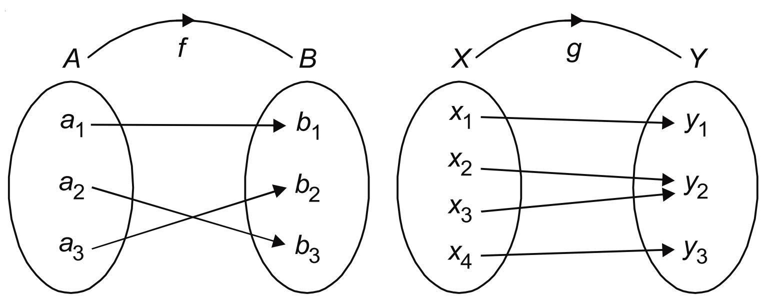
If the graph of the function $y = f(x)$ is given and each line parallel to x-axis cuts the given curve at maximum one point then function is one-one. (Strictly Increasing Or Strictly Decreasing Function). e.g.



Number of one-one functions (injections): If A and B are finite sets having m and n elements respectively, then number of one-one functions from A to $B = {}^nP_m$ if $n \geq m$ and 0 if $n < m$.

If $f(x)$ is not one-one function, then its Many-one function.

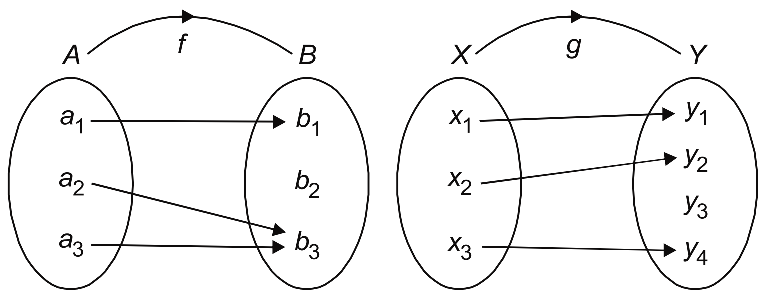
Onto function (surjection): A function $f : A \to B$ is onto if each element of B has its pre-image in A. In other words, Range of f = Co-domain of f. e.g. The following arrow-diagram shows onto function.



Number of onto function (surjection): If A and B are two sets having m and n elements respectively such that $1 \leq n \geq m$, then number of onto functions from A to B is $\sum_{r=1}^{n} (-1)^{n-r\,n} C_r . r^m$

Into function: A function $f : A \rightarrow B$ is an into function if there exists an element in B having no pre-image in A.

In other words, $f : A \rightarrow B$ is an into function if it is not an onto function e.g., The following arrow diagram shows into function.



Method to find onto or into function:

(i) Solve $f(x) = y$ by taking x as a function of y *i.e.*, $g(y)$ (say).

(ii) Now if $g(y)$ is defined for each $y \varepsilon$ co-domain and $g(y) \in$ domain then $f(x)$ is onto and if any one of the above requirements is not fulfilled, then $f(x)$ is into.

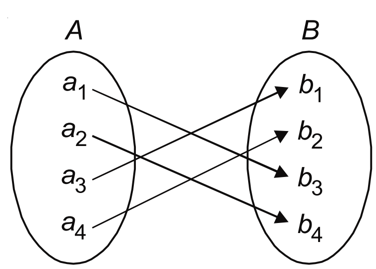
One-one onto function (bijection): A function $f : A \rightarrow B$ is a bijection if it is one-one as well as onto. In other words, a function $f : A \rightarrow B$ is a bijection if

(i) It is one-one i.e., $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in$ A.

(ii) It is onto *i.e.*, for all $y \in B$, there exists $x \in A$ such that $f(x)$ = y.

Clearly, f is a bijection since it is both injective as well as surjective.

BIJECTIVE FUNCTION



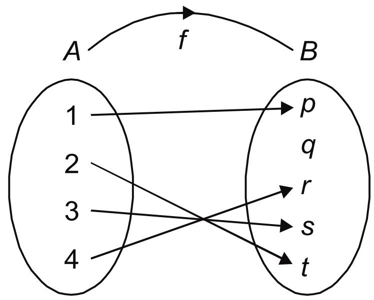
One-one (injective) Function

- No two elements of A have same image in B

  (i) $n(A) \leq n(B)$

  (ii) $f(x_1) = f(x_2)$

  $\Rightarrow x_1 = x_2$

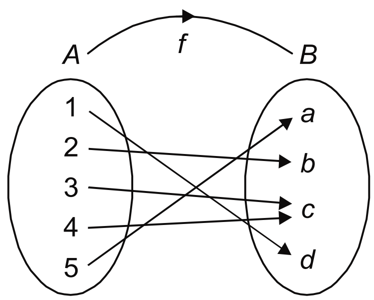


Onto (Surjective) Function

- All the elements of *B* have atleast one pre-image in *A*.

  (i) $n(A) \geq n(B)$

  (ii) Range = Codomain



Bijective Function

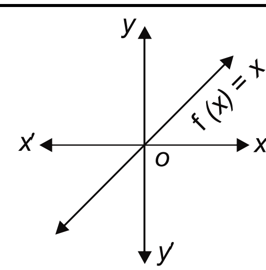
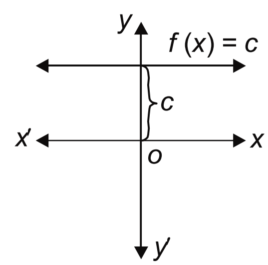
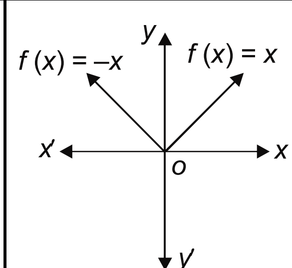
- A function which is both one-one & onto.

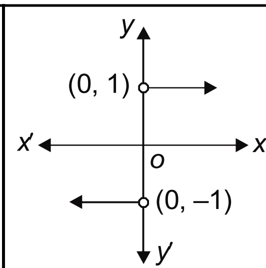
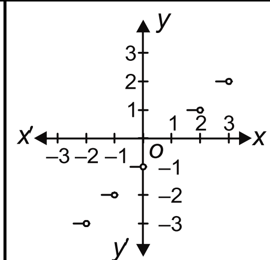
  (i) $n(A) = n(B)$

  (ii) Range = Codomain

- If $n(A) = a = n(B)$, no. of bijections = a!.

TYPE OF FUNCTIONS

	Name of Function	Definition	Domain	Range	Graph
1.	Identity Function	The function $f : R \to R$ defined by $f(x) = x$ A $x \in R$	R	R	
2.	Constant Function	The function $f : R \to R$ defined by $f(x)$ c A $x \in R$	R	$\{c\}$	
3.	Polynomial Function	The function $f : R \to R$ defined by $f(x) = P_0 + P_1x + P_2x^2 + ... + P_nx^n$, where $n \in N$ and P_0, $P_1, P_2......,P_n \in R$ A $x \in R$			
4.	Rational Function	The function f defined by $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomial functions, $Q(x) \neq 0$			
5.	Modulus Function	The function $f : R \to R$ defined by $f(x) = \lvert x \rvert = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ A $x \in R$	R	$[0, \infty)$	

6.	Signum Function	The function $f : R \to R$ defined by $f(x) = \begin{cases} \frac{\lvert x \rvert}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \\ 0, & x = 0 \end{cases}$	R	{–1, 0, 1}	
7.	Greatest Integer Function	The function $f : R \to R$ defined by $f(x) = [x] = \begin{cases} x, & x \in Z \\ \text{integer less than} \\ \text{equal to } x, & x \notin Z \end{cases}$	*R*	*Z*	
8.	Linear Function	The function $f : R \to R$ defined by $f(x) = mx + c$, $x \in R$ where *m* and *c* are constants	R	R	

ONE MARK QUESTIONS

1. Consider the set $A = \{1, 2, 3\}$, then write smallest equivalence relation on A.
2. Consider the set *A* containing *n* elements then, write the totoal number of injective functions from A onto itself is.
3. Let Z be the set of integers and *R* be the relation defined in Z such that *a R b* if $(a - b)$ is divisible by 3, then *R* partitions the set *Z* into how many Pairwise disjoint subsets.
4. Letthe relation *R* be defined in *N* by *a R b* if $2a + 3b = 30$. Find *R*.
5. If $f(x) = (4 - (x - 7)^3)$, then find $f^{-1}(x)$.
6. Let *R* be the equivalence relation in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$, then Find the equivalence class [0].
7. Let $A = \{1, 2, 3, ..., n\}$ and $B = \{a, b\}$. Find the number of surjections from *A* to *B*.
8. Consider the non-empty set consisting of children in a family and a relation *R* defined as arb if a is sister of *b*, Then check *R* is Transitive or not.

9. If $f : R \to R$ be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then find (fof)($x$).

10. Let $f(x) = \frac{x = 1}{x+1}$; $x \neq -1$, then find $f^{-1}(x)$.

11. If If $f : R \to R$ be given by $f(x) = (p - x^p)^{\frac{1}{p}}$, $p \neq 0$, where $p \neq 0$, then find $f(f(x))$.

12. Find the maximum number of equivalence relations on the set $A = \{1, 2, 3\}$.

13. Let $f : [0, 1] \to [0, 1]$ be defined by $f(x) = \begin{cases} x, \text{ if } x \text{ is rational} \\ 1 - x, \text{ if } x \text{ is irrational} \end{cases}$, then find (fof)($x$).

14. If $f = \{(5, 2), (6, 3)),g = \{(2, 5), (3, 6)\}$, write fog.

15. Let $f : R \to R$ be the function defined by $f(x) = 4x - 3 \forall x \in R$. Then write f^{-1}.

16. Let f : $R \to R$ be defined by $f(x) = 3x - 4$, then find $f^{-1}(x)$.

17. If $f(x) = x^2 - \frac{1}{x^2}$, then find $f\left(\frac{1}{x^2}\right) + f(x^2)$.

18. Show that the function $f : R \to R$ defined by $f(x) = x^2$ is not injective.

19. Show that the function $f : N \to N$ given by $f(x) = 3x$ is not Surjective.

20. Find the value of (fog) (x) – (gof)(x), = e^x and $g(x) = \log x$.

21. Find the largest Equivalence Relation on $A = \{a, b, c\}$.

22. Set A has 3 elements and the set B has 4 elements. Find the number of injective mappings that can be defined from A to B.

23. Let $f, g : R \to R$ be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$, $\forall x \in R$, respectively. Then, find gof.

24. Let $f : R \to R$ be the function defined by $f(x) = 2x - 3 \ \forall \ x \in R$. write f^{-1}.

25. If $A = \{a, b, c, d\}$ and the function $f = \{(a, b), (b, d), (c, a), (d, c)\}$, write f^{-1}.

26. If $f : R \to R$ is defined by $f(x) = x^2 - 3x + 2$, write $f(f(x))$.

27. Find the maximum number of reflexive relation on the set $A = \{a, b\}$.

28. If A is the set of students of a school then write, which of following relations arte Universal, Empty or neither of the two.

R_1 = {(a, b) : a, b are ages of students and $|a - b| > 0$}

R_2 = {(a, b) : a, b are weights of students, and $|a - b| < 0$}

R_3 = {(a, b) : a, b are students studying in same class}

29. If $f : A \to B$ is Bijective function such that $n(A) = 10$, then find $n(B)$.

30. IF $f : R \to B$ given by $f(x) = \sin x$ is onto function, then write set B.

31. Let $A = \{a, b, c\}$. How may relation can be defined on $A \times A$? How many of these are reflexive?

32. If $f(x) = \dfrac{1-x}{1+x}$ then find $(fof)(x)$.

33. Find the inverse of the function, $f(x) = \dfrac{4x}{3+3x}$.

34. Let A = {1, 3, 5}, then find the number of equivalence relations in A containing (1, 3).

35. If $f(x) = \dfrac{4x+3}{6x-4}, x \neq \dfrac{2}{3}$ then find inverse of f such that $(fof)(x) = x$.

36. If $A = \{a, b, c, d\}$ and $f = \{(a, b), (b, d), (c, a), (d, c)\}$, show that f is one-one from A to A. Find f^{-1}.

37. Let $A = \{1, 2, 3\}$. Find whether the function $f : A \to A$ defined as $f = \{(1, 3), (3, 2), (2, 1)\}$ has inverse. If yes find f^{-1}.

38. Find $(fog)(x)$ and $(gof)(x)$, if $f(x) = x^2 + 2$ and $g(x) = 1 - \dfrac{1}{1-x}$.

39. Let $f : R \to R$ defined by $f(x) = 2x - 3$ and $g : R \to R$ defined by $g(x) = \dfrac{x+3}{2}$ Show that fog = I_R = gof.

40. Let $f, g : R \to R$ be defined by $f(x) = x^2 + 1$, then find the pre-images of 17 and – 3.

41. Let $f, g : R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x \; \forall \; x \in R$. Then, find fog and gof.

42. Show that the relation R on defined as $R = \{(a, b) : a \leq b^3\}$ is not transitive.

43. If the function $f : R - \{1, -1\} \to A$ defined by $f(x) = \dfrac{x^2}{1 - x^2}$, is Surjective, the find A.

44. Give an example to show that the union of two equivalence rtelations on a set A need not be an equivalence relation on A.

45. How many reflexive relations are possible in a set A whose $n(A) = 4$. Also find How many symmetric relations are possible on a set B whose $n(B) = 3$.

46. If $f : R \to R$, $g : R \to R$, given by $f(x) = [x]$,$g(x) = |x|$, then find

(a) $\text{fog}\left(\dfrac{2020}{2021}\right) + \text{gof}\left(\dfrac{-2021}{2020}\right)$

(b) $\text{gof}\left(\dfrac{2020}{2021}\right) - \text{fog}\left(\dfrac{2021}{2020}\right)$

47. Let W denote the set of words in the English dictionary. Define the relation R by $R = \{(x, y) \in$ W x W such that x and y have at least one letter in common}. Show that this relation R is reflexive and symmetric, but not transitive.

48. Show that the relation R in the set of all real numbers, defined as $R = \{(a, b): a \leq b^2\}$ is neither reflexiv Nor symmetric.

49. Consider a Bijective function $f : R_+ \to (7, \infty)$ given by $f(x) = 16x^2 + 24x + 7$, where R + is the set of all positive real numbers. Find the inverse function of f.

50. If $f : R \to [-1, 1]$, $f(x) = \sin \times$ & g: $[-1, 1] \to$ Range of g, $g(x) = 1 - x^2$, then show that $(\text{gof})(x) = \left[f\left(\dfrac{\pi}{2} - x\right)\right]^2$.

51. Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective

(i) $\{(x, y)$: x is a person, y is the mother of $x\}$

(ii) $\{(a, b)$: a is a person, b is an ancestor of a}.

52. Show that the function $f : R \to R$ decined by $f(x) = \frac{x}{x^2+1}; \forall\ x \in R$, is neither one-one nor onto.

53. Let R be the set of real numbers and $f : R \to R$ be the function defined by $f(x) = 4x + 5$. Show that f is invertible and find f^{-1}.

54. Show that the relation R in the set $A = \{3, 4, 5, 6, 7\}$ given by $R = \{(a, b) : |a - b|$ is divisible by 2} is an equivalence relation. Show that all the elements of {3, 5, 7} are related to each other and all the elements of {4, 6} are related to each other, but no element of {3, 5, 7} is related to any element of {4, 6}.

55. If the function $f : R \to R$ be defined by $f(x) = 2x - 3$ and $g : R \to R$ by $g(x) = x^3 + 5$, then find the value of $(fog)^{-1}(x)$.

56. Check whether the relation R in the set Z of integers defined as $R = \{(a, b) : a + b$ is ″divisible by 2″} is reflexive, symmetric, transitive or Equivalence.

57. (Part *a, b, c, e, f, g*) Show that that following Relations R are equivalence relation in A. (for Remaining Parts) Check whether the following Relations are Reflexive, Symmetric or Transitive.

(a) Let A be the set of all triangles in a plane and let R be a relation in A, defined by

$$R = \{(T_1, T_2): T_1 \text{ is congruent } T_2\}$$

(b) Let A be the set of all triangles in a plane and let R be a relation in A, defined by

$$R = \{(T_1, T_2): T_1 \text{ is similar } T_2\}$$

(c) Let A be the set of all lines in xy-plane and let R be a relation in A, defined by

$$R = \{(L_1, L_2): L_1 \text{ is parallel to } L_2\}$$

(d) Let A be the set of all lines in xy-plane and let R be a relation in A, defined by

$$R = \{(L_1, L_2): L_1 \text{ is perpendicular to } L_2\}$$

(e) Let A be the set of all integers and let R be a relation in A, defined by

$$R = \{(a, b): (a - b) \text{ is even}\}$$

(f) Let A be the set of all integers and let R be a relation in A, defined by

$$R = \{(a, b): |a - b| \text{ is a multiple of } 2\}$$

(g) Let A be the set of all integers and let R be a relation in A, defined by

$$R = \{(a, b): |a - b| \text{ is a divisible by } 3\}$$

(h) Let A be the set of all real numbers and let R be a relation in A defined by

$$R = \{(a, b): a \leq b\}$$

(i) Let A be the set of all relal numbers and let R be a relation in A defined by

$$R = \{(a, b): a \leq b^2\}$$

(j) Let A be the set of all real numbers and let R be a relation in A defined by

$$R = \{(a, b): a \leq b^3\}$$

(k) Let A be the set of all natural numbers and let R be a relation in A defined by

$R = \{(a, b)$: a is a factor $b\}$

OR

$R = \{(a, b)$: b is divisible by $a\}$

(l) Let A be the set of all real numbers and let R be a relation in A defined by

$$R = \{(a, b): (1 + ab) > 0\}$$

58. Let S be the set of all real numbers. Show that the relation $R = \{(a, b) : a^2 + b^2 = 1\}$ is symmetric but neither reflexive nor transitive.

59. Check wheter relation R defined in R as $R = \{(a, b) : a^2 - 4ab + 3b^2 = 0, a, b, \in R\}$ is reflexive, symmetric and transitive.

60. Show that the function $f : (-\infty, 0) \to (-1, 0)$ defined by $f(x) = \dfrac{x}{1 + |x|}$, $x \in (-\infty, 0)$ is one-one and onto.

FIVE MARKS QUESTIONS

61. For real numbers x and y, define $x\ R\ y$ if and only if $x - y + \sqrt{2}$ is an irrational number. Then check the reflexivity, Symmetricity and Transitivity of the relation R.

62. Determine whether the relation R defined on the set of all real numbers as

$R = \{(a, b): a, b\ \varepsilon R$ and $a - b + \sqrt{3} \in S\}$

Where S is the set of all irrational numbers) is reflexive, symmetric or transitive.

63. Let N be the set of all natural numbers and let R be a relation on $N \times N$, defined by show that R is an equivalence relation.

(i) $(a, b) R(c, d) \Leftrightarrow a + d = b + c$

(ii) $(a, b) R(c, d) \Leftrightarrow ad = bc$

(iii) $(a, b) R(c, d) \Leftrightarrow \frac{1}{a} + \frac{2}{d} = \frac{2}{b} + \frac{1}{c}$

(iv) $(a, b) R(c, d) \Leftrightarrow ad(b + c) = bc(a + d)$

64. Let $A = R - \{1\}$. If $f : A \to A$ is a mapping defined by $f(x) = f(x) = \frac{x-2}{x-1}$, show that f is bijective.

Find f^{-1}. Also find

(a) x, if $f^{-1}(x) = \frac{5}{6}$ (b) $f^{-1}(2)$

65. Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where S is the range of is invertible. Also find the inverse of f. Hence find $f^{-1}(31)$.

CASE STUDIES

66. A person without family is not complete in this world because family is an integral part of all of us. Human beings are considered as the social animals living in group called as family. Family plays many important roles throughout the life.

Mr. D.N. Sharma is an Honest person who is living happily with his family. He has a son Vidya and a Daughter Madhulika. Mr. Vidya has 2 sons Tarun and Gajender and a daughter Suman while Mrs. Madhulika has 2 sons Shashank & Pradeep and 2 daughters Sweety and Anju. They all Lived together and everyone shares equal responsibilities within the family. Every member of the family emotionally attaches to each other in their happiness and sadness. They help each other in their bad times which give the felling of security.

A family provides love, warmth and secuity to its all members throughout the life which makes it a complete family. A good and healthy family makes a good society and ultimately a good society involves in making a good country.

On the basis of above information, answer the following questions:

Consider Relation R in the set A of members of Mr. D.N. Sharma and his family at a particular time

(i) If $R = \{(x, y) : x$ and y live in the same locality$\}$, then R is

(a) Reflexive only

(b) Reflexive and Symmetric

(c) Equivalence Relation

(ii) $R = \{(x, y) : x$ is exactly 7 cm taller than $y\}$, then R is

(a) Reflexive only

(b) Symmetric only

(c) Transitive only

(d) Neither Reflexive, symmetric nor Transitive

(iii) $R = \{(x, y) : x$ is wife of $y\}$, then R is

(a) Reflexive only

(b) Symmetric only

(c) Transitive only

(d) Neither Reflexive, symmetric nor Transitive

(iv) $R = \{(x, y) : x$ is father of $y\}$, then R is

(a) Equivalence Relation

(b) Symmetric only

(c) Transitive only

(d) Neither Reflexive, symmetric nor Transitive

(v) $R = \{(x, y) : x$ is Brother of $y\}$, then R is

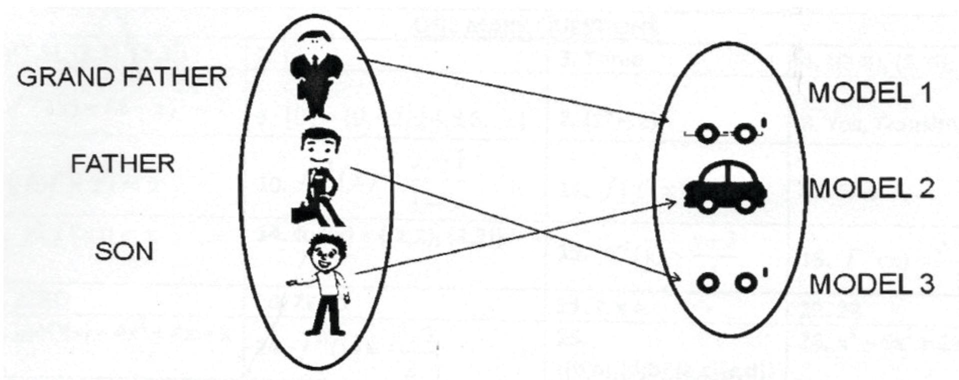
(a) Reflexive only

(b) Symmetric only

(c) Transitive only

(d) Neither Reflexive, symmetric nor Transitive

67. Let A be the Set of Male members of a Family, A = {Grand father, Father, Son} and B be the set of their 3 Cars of different Models, B = {Model 1, Model 2, Model 3}



On the basis of The above Information, answer the following questions:

(i) How many Relations are possible on $A \times B$?

(a) 3 (b) 9

(c) 8 (d) 512

(ii) How many Functions are possible on $A \times B$?

(a) 3 (b) 9

(c) 27 (d) None of these

(iii) How many One-one Functions (Injective) are possible on $A \times B$?

(a) 3 (b) 6

(c) 9 (d) 12

(iv) How many Onto Functions (surjective) are possible on $A \times B$?

(a) 6 (b) 9

(c) 27 (d) 81

(v) How many Bijective functions are possible on $A \times B$?

(a) 1 (b) 3

(c) 6 (d) 9

ANSWERS

ONE MARK QUESTIONS

1. {(1,1), (2,2), (3,3)}
2. n!
3. Three
4. {(3,8), (6,6), (9,4), (12, 2)}
5. $f^{-1}(x) = (4-x)^{\frac{1}{3}} + 7$
6. [0] = {0, ±2,±4,±6...}
7. $(2^n - 2)$
8. Yes, Transitive
9. $(\text{fof})(x) = x$
10. $f^{-1}(x) = \dfrac{x+1}{1-x}$
11. $f(f(x)) = x$
12. Five
13. $f(f(x)) = x$
14. $\text{fog}(x)$ ={(2,2),(3,3)}
15. $f^{-1}(y) = \dfrac{y+3}{4}$
16. $f^{-1}(x) = \dfrac{x+4}{3}$
17. Zero
20. Zero
21. AxA
22. 24
23. $(\text{gof})(x) = 4x^2 + 4x - 1$
24. $f^{-1}(y) = \dfrac{y+3}{2}$
25. $\{(b,a),(d,b),(a,c),(c,d)\}$
26. $x^4 - 6x^3 + 10x^2 - 3x$
27. Four
28. R1:Universal relation
R2: Empty relation
R3: Neither universal
Nor empty
29. $n(B) = 10$
30. $B = [1, 1]$
31. No. of Relations = 512
32. $(\text{fof})(x) = x$
33. $f^{-1}(y) = \dfrac{4y}{4-3y}$
34. Tow
35. $f^{-1}(x) = \dfrac{4x+3}{6x-4}$

TWO MARKS QUESTIONS

36. $f^{-1} = \{(b,a),(d,b),(a,c)\}$
37. $f^{-1} = \{(3,1),(2,3),(1,2)\}$
38. $(\text{fog})(x) = \left(\dfrac{x}{x-1}\right)^2 + 2$

$(\text{gof})(x) = \dfrac{x^2+2}{x^2+1}$

40. Pre-Image of 17 = 4, – 4

41. $(fog)(x) = \begin{cases} 0, x \geq 0 \\ 4x, x < 0 \end{cases}$

$(gof)(x) = 0, A\ x \in R$

43. A = R – [– 1, 0)

45. Reflexive Relations = 4096

46. (a) 2

(b) 0

49. $f^{-1}(y) = \dfrac{\sqrt{y+2}-3}{4}$

51. (a) Yes it's a Function,

Not Injective but Surjective

(b) No, its not a function

53. $f^{-1}(y) = \dfrac{y-5}{4}$

55. $(fog)^{-1}(x) = \left(\dfrac{x-7}{2}\right)^{\frac{1}{3}}$

56. Equivalence Relation

57. (d) Symmetric (h) Reflexive and Transitive

(i) Neither Reflexive, Symmetric nor Transitive

(j) Neither Reflexive, Symmetric nor Transitive

(k) Reflexive & Transitive

(l) Reflexive and Symmetric

FIVE MARKS QUESTIONS

61. Reflexive only

62. Reflexive only

64. $f^{-1}(x) = \dfrac{x-2}{x-1}$

(a) $x = 7$ (b) $f^{-1}(2) = 0$

65. $f^{-1}(x) = \dfrac{\sqrt{x-6}-3}{2}$ and $f^{-1}(31) = 1$

CASE STUDIES BASED QUESTION

66. (i) option (d) (ii) option (d) (iii) option (c)

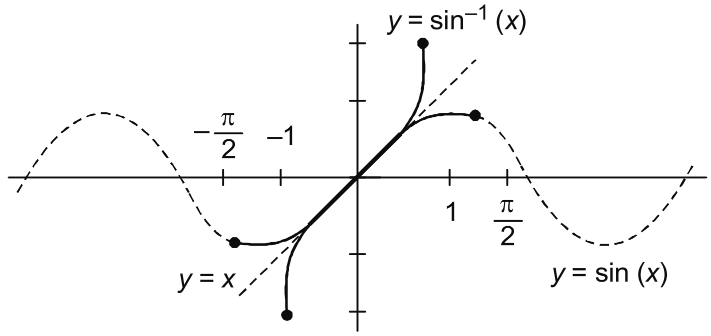
(iv) option (d) (v) option (c)

67. (i) option (d) (ii) option (c) (iii) option (d)

(iv) option (a) (v) option (c)

CHAPTER 2

INVERSE TRIGONOMETRIC FUNCTIONS



Function	Domain	Range
$y = \sin^{-1} x$	[–1, 1]	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	[–1, 1]	$[0, \pi]$
$y = \tan^{-1} x$	R	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	R	$(0, \pi)$
$y = \sec^{-1} x$	R – (–1, 1)	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \text{coses}^{-1} x$	R – (–1, 1)	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

- $\sin^{-1} (\sin x) = x$, when $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- $\cos^{-1} (\cos x) = x$, when $x \in [0, \pi]$

- $\tan^{-1}(\tan x) = x$, when $x \in \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$
- $\cot^{-1}(\cot x)$, when $x \in (0, \pi)$
- $\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x$, when $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
- $\sec^{-1}(\sec x) = x$, when $x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$
- $\sin(\sin^{-1} x) = x$, when $x \in [-1, 1]$
- $\cos(\cos^{-1} x) = x$, when $x \in [-1, 1]$
- $\tan(\tan^{-1} x) = x$, when $x \in R$
- $\cot(\cot^{-1} x) = \mathrm{x}$, when $x \in R$
- $\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x$, when $x \in R - (-1, 1)$
- $\sec(\sec^{-1} x) = x$, when $x \in R - (-1, 1)$
- $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$, when $x \in [-1, 1]$
- $\tan^{-1} x + \cot^{-1} = \frac{\pi}{2}$, $x \in R$
- sec–1 x +$\operatorname{cosec}^{-1} x = \frac{\pi}{2}$, when $x \in R - (-1, 1)$
- $\sin^{-1}(-x) = -\sin^{-1}(x)$, when $x \in [-1, 1]$
- $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$, when $x \in [-1, 1]$
- $\tan^{-1}(-x) = -\tan^{-1}(x)$, when $x \in R$
- $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$, when $x \in R$
- $\mathrm{coseec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)$, when $x \in R - (-1, 1)$

- $\tan^{-1} x + \tan^{-1} y = \tan{-1} \left(\frac{x+y}{1-xy}\right)$, when $xy < 1$

- $\tan^{-1} x - \tan^{-1} y = \tan{-1} \left(\frac{x=y}{1+xy}\right)$ when $xy > -1$

- $2 \tan^{-1} (x) = \tan{-1} \left(\frac{2x}{1-x^2}\right)$, when $|x| < 1$

- $2 \tan^{-1} (x) = \sin{-1} \left(\frac{2x}{1+x^2}\right)$, when $|x| \leq 1$

- $2 \tan^{-1} (x) = \cos{-1} \left(\frac{1-x^2}{1+x^2}\right)$, when $x > 0$

ONE MARK QUESTIONS

1. Find tha principal value of $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right)$.

2. Find the principal value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$.

3. Find the principal value of $\cos^{-1}\left(\cos\frac{14\pi}{3}\right)$.

4. Write the condition on xy, for which the result $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ is true.

5. Write the condition on xy, for which the result $\tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ is true.

6. If $\cos(\cos^{-1}\frac{1}{3} + \sin^{-1} x) = 0$, then find the value of x.

7. If $\sin\left(\sin^{-1}\frac{3}{5} + \cos^{-1} x\right) = 1$, then find the value of x.

8. Express $\cot^{-1}(-x)$ for all $x \in R$ in terms of $\cot^{-1}(x)$.

9. Find the domain of the function $\cos^{-1}(2x - 1)$.

10. Find the domain of the function $f(x) = \text{Sin}^{-1}\sqrt{x-1}$.

11. Find the value of $\cot\left(\cos^{-1}\frac{7}{25}\right)$.

12. Find the minimum value of n for which $\tan^{-1}\frac{n}{\pi} > \frac{\pi}{4}$, $n \in N$.

13. Find the value of x, If $3\tan^{-1} x + \cot^{-1} x = \pi$.

14. Find the principal value in each of the following:

(a) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

(b) $\cos{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

(c) $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

(d) $\sin^{-1}\left(\sin\frac{2\pi}{5}\right)$

(e) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{11\pi}{6}\right)$

15. If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$ then find the value of $\cot^{-1} x + \cot^{-1} y$.

16. Find the value of the expression sin [cot–1 (cos($\tan^{-1}$ 1))].

17. If $\text{Sin}^{-1} x + \text{Sin}^{-1} y = \frac{\pi}{2}$, then find the value of $\text{Cos}^{-1} x + \text{Cos}^{-1} y$.

18. Find the value of $\sin(2\sin^{-1}(0.6))$.

19. If $\tan^{-1} x = \frac{\pi}{10}$ = for some $x \in R$, then find the value of $\cot^{-1} x$.

20. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then find the value of $(x + y + z + xyz)$.

21. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then find the value of $(x^3 + y^3 + z^3 - 3xyz)$.

22. Find x, such that $\cos^{-1}(x) + \cos^{-1}(x^2) = 0$

23. Find x, if $\sin{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

24. Find the range of $f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x$.

25. If the function $f(x) = \cot^{-1}\sqrt{x(x+3)}\ \cos^{-1}\sqrt{x^2+3x+1}$ is defined on the set A, Find A.

TWO MARKS QUESTIONS

26. Match the following:

If $\cos^{-1} a + \cos^{-1} b = 2\pi$ & $\sin^{-1} c + \sin^{-1} d = \pi$ then

	Column1		**Column 2**
A	abcd	P	0
B	$a^2 + b^2 + c^2 + d^2$	Q	1
C	$(d - a) + (c - b)$	R	2
D	$a^3 + b^3 + c^3 + d^3$	S	4

27. If $\cot^{-1}\sqrt{\cos a} + \tan^{-1}\sqrt{\cos a} = x$ then, Match the following:

	Column1		**Column 2**
A	Sin x	P	0
B	Cos $2x$	Q	1
C	Tan$(x/2)$	R	–1

28. If $\theta = \cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18$ then find the value of $\cot\theta$.

29. If $P = \tan^2(\sec^{-1} 2) + \cot^2(\operatorname{cosec}^{-1} 3)$, then find the value of $(P^2 + P + 11)$.

30. If $P = \operatorname{Sec}^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3)$, then find value of $(P^2 - 2P)$.

31. If $\tan^{-1} 4 + \tan^{-1} 5 = \cot^{-1} (P)$, then find the value of P.

32. Find the value of $\text{Sin}\ (\frac{1}{2}\text{Cot}^{-1}(\frac{3}{4}))$.

33. Solve for x: $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$

34. Find the value of x, such that $\text{Sin}^{-1} x = \frac{\pi}{6} + \text{Cos}^{-1} x$

35. Find x, if $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{2}$

36. If $\tan^{-1} 3 + \tan^{-1} x = \tan^{-1} 8$, find x.

37. If $\tan^{-1} (\cot x) = 2x$, find x.

38. Solve for x:

(a) $\cos^{-1}(\cos\frac{5\pi}{3}) + \sin^{-1}(\sin\frac{5\pi}{3}) = x$

(b) $\cos^{-1}(\cos\frac{3\pi}{4}) + \sin^{-1}(\sin\frac{3\pi}{4}) = x$

39. Find the value of $\sin (2 \tan^{-1} (.75))$.

40. Simplify the following:

(a) $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

(b) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \neq 0$

(c) $\cos\left[2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right]$

THREE MARKS QUESTIONS

41. If $P = \frac{\tan^{-1}1+\tan^{-1}2+\tan^{-1}3}{\cot^{-1}1+\cot^{-1}2+\cot^{-1}3}$, then find the value of P.

42. Find the value of cot $\left(\frac{\pi}{4} - 2\cot^{-1} 3\right)$.

43. Prove that:

(a) $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{36}{85}\right) = \tan^{-1}\left(\frac{77}{85}\right)$

(b) $\cot^{-1}(7) + \cot^{-1}(8) + \cot^{-1}(18) = \cot^{-1}(3)$

(c) $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$

(d) $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$

(e) $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2}, x \varepsilon \left(0, \frac{\pi}{4}\right)$

(f) $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$

(g) $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \frac{\pi}{4}$

(h) $\cos^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right)$

(i) $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$

(j) $\tan\left((2\tan^{-1}\left(\frac{2}{3}\right)\right) = \frac{12}{13}$

(k) $\sin\left((2\tan^{-1}\left(\frac{2}{3}\right)\right) = \frac{12}{13}$

44. (a) If $y = \cot^{-1}\left(\sqrt{\cos x}\right) - \tan^{-1}\left(\sqrt{\cos x}\right)$ then Prove that $\sin y = \tan^2\left(\frac{x}{2}\right)$

(b) Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$

(c) Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\tan^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\frac{a}{b}\right) = \frac{2\sqrt{a^2+b^2}}{b}$

(d) Prove that $\cos[\tan^{-1}\{\sin(\cos^{-1} x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$

45. (a) Find x such that: $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$

(b) If $a_1, a_2, a_3, \ldots a_n$ are in A.P. with common difference is 'd', then find

$$\tan\left(\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots\ldots + \tan^{-1}\left(\frac{d}{1+a_{n-1}\,a_n}\right)\right)$$

(c) Find x : $\left(\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \ldots\ldots + \tan^{-1}\left(\frac{1}{1+n.(n+1)}\right)\right) = \tan^{-1} x$

46. Solve for x:

(a) $\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$, $x > 0$

(b) $\sin(\cot^{-1}(x + 1)) = \cos(\tan^{-1} x)$

(c) $\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$

(d) $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$

(e) $\tan^{-1}\left(\frac{2-x}{2+x}\right) = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$, $x > 0$

(f) $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\, x)$, $x \neq 0$

47. (a) Prove that: $\cot^{-1}\left(\frac{xy+1}{x-y}\right)+\cot^{-1}\left(\frac{yz+1}{y-z}\right)+\cot^{-1}\left(\frac{zx+1}{z-x}\right)$ = 0, (0 < *xy*, *yz, zx* <1)

(b) Prove that : $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\frac{x}{2}, x\varepsilon\left(0,\frac{\pi}{2}\right)$

(c) Prove that : $\tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\varepsilon\left(0,\frac{\pi}{4}\right)$

(d) Prove that : $\tan^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right)=\frac{x}{4}-\frac{1}{2}, \cos^{-1} x$

(e) Prove that : $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)=\frac{\pi}{4}$

(f) Solve for x : $\sin^{-1}(6x)$ + $\sin^{-1}(6\sqrt{3}x)=\frac{\pi}{2}$

(g) Solve for x : $\sin^{-1}(6x)$ + $\sin^{-1}(6\sqrt{3}x)=\frac{-\pi}{2}$

FIVE MARKS QUESTIONS

48. Prove the Following:

(a) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, then prove that $x^2 + y^2 + z^2 + 2xyz = 1$

(b) If cos–1 $\cos^{-1}\left(\frac{x}{a}\right)+\cos^{-1}\left(\frac{y}{b}\right)$ = α, then prove that $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}$ cos α = $\sin^2 \alpha$

(c) If $\cos^{-1}\left(\frac{x}{2}\right)+\cos^{-1}\left(\frac{y}{3}\right)$ = θ, then prove that $9x^2 + 4y^2 - 12xy \cos\theta = 36 \sin2\theta$

(d) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then prove that $x + y + z = xyz$

(e) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, then prove that $xy + yz + zx = 1$

(f) If $\cot^{-1} x + \cot^{-1} y + \cot^{-1} z = \pi$, then prove that $xy + yz + zx + = 1$

(g) If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2}$ = $2xyz$

49. Prove the following:

(a) Prove that: $2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right)=\tan^{-1}\left(\frac{\sin\alpha.\cos\beta}{\cos\alpha+\sin\beta}\right)$

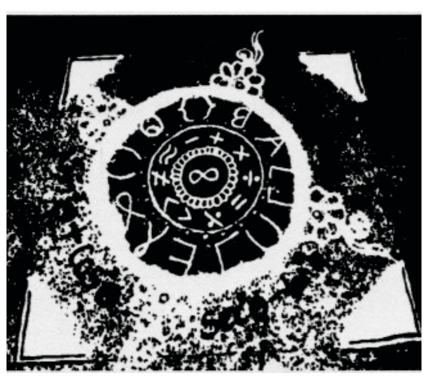
(b) Prove that : $2\tan^{-1}\left(\tan\frac{a}{2}\tan\frac{b}{2}\right)=\cos^{-1}\left(\frac{\cos a+\cos b}{1+\cos a\cos b}\right)$

(c) Prove that : $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)=\cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$

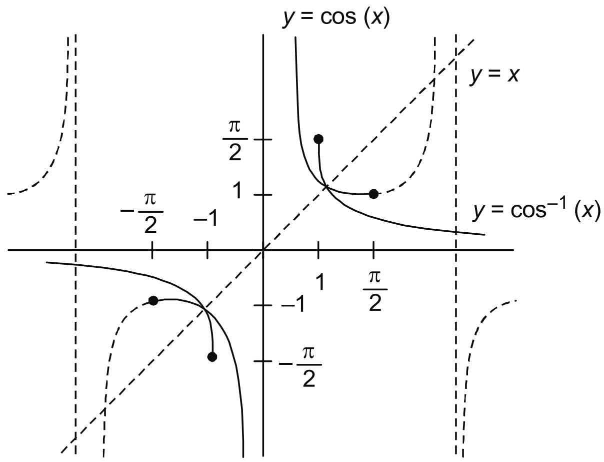
(d) Prove that : $\tan^{-1}\left[\frac{3\sin 2x}{5+3\cos 2x}\right]+\tan^{-1}\left[\frac{1}{4}\tan x\right] = x$

CASE STUDIES

50. On National Mathematics Day, December 22, 2020, Mathematics Teachers of DOE organized Mathematical Rangoli Competition for the students of all DOE schools to celebrate and remembering the contribution of Srinivase Ramanujan to the field of mathematics. The legendary Indian mathematician who was born on this date in 1887.



Team A of class XI students made a beautiful Rangoli on Trigonometric Identities as shown in the figur Above, While Team B of class XII students make the Rangoli on the graph of Trigonometric and inverse Trigonometric Functions. As shown in the following figure.



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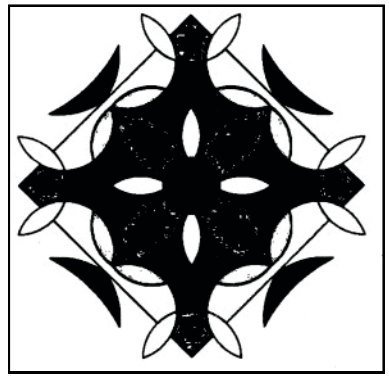


On the basis of above information, Teacher asked few questions from Team B. Now you try to answer Those questions which are as follows:

(i) What is the principla Branch of $\sin^{-1} x$?

(a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (b) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(c) $(0, \pi)$ (d) $[0, \pi]$

(ii) What is the Principal Branch of $\cos^{-1} x$?

(a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (b) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(c) $(0, \pi)$ (d) $[0, \pi]$

(iii) What is the one Branch of $\operatorname{cosec}^{-1} x$ other than Principal Branch?

(a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ (b) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{0\}$

(c) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$ (d) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

(iv) If the Principal Branch $\sec^{-1} x$ is $[0, \pi] - \{k\pi\}$, Then is the value of k?

(a) 0 (b) 1

(c) 0.5 (d) 0.25

(v) What is the Principal Branch of $\tan^{-1} x$?

(a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (b) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(c) $(0, \pi)$ (d) $[0, \pi]$

ANSWER

ONE MARK QUESTIONS

1. $\frac{5\pi}{6}$ 2. $\frac{2\pi}{5}$ 3. $\frac{2\pi}{3}$ 4. $xy < 1$

5. $xy > -1$ 6. $x = \frac{1}{3}$ 7. $x = \frac{3}{5}$ 8. $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$

9. $[0, 1]$ 10. $[1, 2]$ 11. $\frac{7}{24}$ 12. $n = 4$(Minimum)

13. $x = 1$ 14. $(a) \frac{\pi}{3}$ (b) π (c) $\frac{\pi}{6}$ (d) $\frac{2\pi}{5}$ (e) $\frac{-\pi}{12}$

15. $\frac{\pi}{5}$ 16. $\sqrt{\frac{2}{3}}$ 17. $\frac{\pi}{2}$ 18. $\frac{24}{25}$

19. $\frac{2\pi}{5}$ 20. (-4) 21. Zero (0) 22. $x = 1$

23. $x = 3$ 24. Range = $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ 25. $A = \{0, -3)$

TWO MARKS QUESTIONS

26. $A \to Q, B \to S, C \to S, D \to P$ 27. $A \to Q, B \to R, C \to Q$ 28. 3

29. $(P^2 + P + 11) = 143$ 30. $(P^2 - 2P) = 195$ 31. $P = \frac{-19}{9}$

32. $\frac{1}{\sqrt{5}}$ 30. $x = 0$ or -1 34. $\frac{\sqrt{3}}{2}$

35. $x = 1$ 36. $\frac{1}{5}$ 37. $\frac{\pi}{6}$

38. (a) 0 (b) π 39. $\frac{24}{25}$ 40. (a) $\frac{x}{2}$ (b) $\frac{\tan^{-1} x}{2}$ (c) x

THREE MARKS QUESTIONS

41. $P = 2$ 42. $(+7)$ 45. (a) $x = -1$ (b) $\frac{(n-1)d}{1+a_1 a_n}$ (c) $\frac{n}{n+2}$

46. (a) $x = \frac{3}{4}$ (b) $x = \frac{-1}{2}$ (c) $x = 0$ (d) $x = 0$ or $\frac{1}{2}$ (e) $x = \frac{2}{\sqrt{3}}$ (f) $x = \frac{\pi}{4}$

47. (f) $x = \frac{1}{12}$ (g) $x = \frac{-1}{12}$

CASE STUDY QUESTION

50. (i) option (a) (ii) option (d) (iii) option (b)
    (iv) option (c) (v) option (b)

CHAPTER 3

MATRICES

Matrices are defined as a rectangular arrangement of numbers or functions. Since it is a rectangular arrangement, it is 2-dimensional.

A two-dimensional matrix consists of the number of rows (m) and a number of columns (n). Horizontal ones are called Rows and Vertical ones are called Columns.

$$A = \begin{bmatrix} M & A & T \\ H & S & 1 \\ D & O & E \end{bmatrix} \begin{matrix} \longrightarrow \text{ROW 1} \\ \longrightarrow \text{ROW 2} \\ \longrightarrow \text{ROW 3} \end{matrix}$$

COLUMN 1 COLUMN 3

COLUMN 2

ORDER OF MATRIX

Tht order of matrix is a relationship with the number of elements present in a matrix.

The order of a matrix is denoted by $m \times n$, where m and n are the number of Rows and Columns Respectively and the number of elements in a matrix will be equal to the product of m and n.

TYPES OF MATRICES

ROW MATRIX

A matrix having only one row is called a row matrix.

Thus $A = [a_{ij}]_{mxn}$ is a row matrix if $m = 1$. So, a row matrix can be represented as $A = [a_{ij}]_{1xn}$. It is called so because it has only one row and the order of a row matrix will hence be $1 \times n$. For example,

$A = [1 \ \ 2 \ \ 3 \ \ 4]$ is row matrix of order 1×4. Another example of the row matrix is

$B = [0 \ \ 9 \ \ 4]$ which is of the order 1×3.

COLUMN MATRIX

A matrix having only one column is called a column matrix. Thus, $A = [a_{ij}]_{mxn}$ is a column matrix if $n = 1$. Hence, the order is m × 1. An example of a column matrix is:

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, B = \begin{bmatrix} M \\ A \\ T \\ H \end{bmatrix}$$

In the above example, A and B are 3 × 1 and 4 × 1 order matrices respectively.

SQUARE MATRIX

If the number of rows and the number of columns in a matrix are equal, then it is called a square matrix.

Thus, $A = [a_{ij}]_{mxn}$ is a square matrix if $m = n$; For example is a square matrix of order 3 × 3.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

For Additional Knowledge: The sum of the diagonla elements in a square matrix A is called the trace of matrix A, and which is denoted by tr(A);

$\text{tr}(A) = a_{11} + a_{22} + ... + a_{nn}$

ZERO OR NULL MATRIX

If in a matrix all the elements arte zero then it is called a zero matrix and it is generally denoted by O. Thus $A = [a_{ij}]_{mxn}$ is a zero-matrix if $a_{ij} = 0$ for all i and j; For example

$$A = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Here A and B are Null matrix of order 3 × 1 and 2 × 2 respectively.

DIAGONAL MATRIX

If all the non-diagonal elements of square matrix, are zero, then it is called a diagonal matrix. Thus, a square matrix $A = [a_{ij}]$ is a diagonal matrix if $a_{ij} = 0$, when $i \neq j$;

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

A, B & *C* are diagonal matrix of order 3 × 3, and *D* is a diagonal matrix of order 2 × 2.

Diagonal Matrix can also be denoted by

A = diagonal [2 3 4], *B* = diag [2 0 4], *C* = [0 0 4]

Important things to note:

(i) A diagonal matrix is always a square matrix

(ii) The diagonal elements are characterized by this genertal form: a_{ij}, where $i = j$. This means that a matrix can have only one diagonal.

SCALAR MATRIX

If all the elements in the diagonal of a diagonal of a diagonal matrix are equal, it is callled a scalar matrix. Thus, a square matrix $A = [a_{ij}]$ is a scalar matrix if

$A = [a_{ij}]$ is a scalar matrix if

$$A = [a_{ij}] = \begin{cases} 0; & i \neq j \\ k; & i = j \end{cases} \text{ where, } k \text{ is constant.}$$

For example *A* & *B* are scalar matrix of order 3 × 3 and 2 × 2 respectively.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$$

UNIT MATRIX OR IDENTITY MATRIX

If all the elements of a principal diagonal in a diagonal matrix are 1, then it is called a unit matrix. A unit matrix of order *n* is denoted by I_n. Thus, a square matrix $A = [a_{ij}]_{mxn}$ is an identity matrix if

$$A = [a_{ij}] = \begin{cases} 0; & i \neq j \\ 1; & i = j \end{cases}$$

For example I_3 & I_2 are identity matrix of order 3 × 3 and 2 × 2 respectively.

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- All identity matrices are scalar matrices
- All scalar matrices are diagonal matrices
- All diagonal matrices are square matrices

TRIANGULAR MATRIX

A square matrix is said to be a triangular matrix if the elements above or below the principal diagonal are zero. There are two types of Triangular Matrix:

UPPER TRIANGULAR MATRIX

A square matrix $[a_{ij}]$ is called an upper triangular matrix, if $a_{ij} = 0$, when $i > j$.

$$A = \begin{bmatrix} D & O & E \\ 0 & D & O \\ 0 & 0 & E \end{bmatrix}$$

is an upper triangular matrix of order 3 × 3.

LOWER TRIANGULAR MATRIX

A square matrix is called a lower triangular matrix, if $a_{ij} = 0$ when $i < j$.

$$A = \begin{bmatrix} D & 0 & 0 \\ O & D & 0 \\ E & O & E \end{bmatrix}$$

is a lower triangular matrix of order 3 × 3.

Transpose of a Matrix: Let A be any matrix, then on interchanging rows and columns of A. The new matrix so obtained is transposes of A donated A^T or A'.

[order of $A = m \times n$, then order of $A^T = n \times m$]

Properties of transpose matrices A and B are:

(a) $(A^T)^T = A$ (b) $(kA)^T = kA^T$ (k = constant)

(c) $(A + B)^T = A^T + B^T$ (d) $(AB)^T = B^T . A^T$

Symmetric Matrix and skew-Symmetric matrix

- A square matrix $A = [a_{ij}]$ is symmetric if $A^T = A$ i.e. $a_{ij} = a_{ji}$ $\forall$ i and j
- A square matrix $A = [a_{ij}]$ is skew-symmetric if $A^T = -A$ i.e. $a_{ij} = -a_{ij}$

  (All diagonal elements are zero in skew-symmetric matrix)

ONE-MARK QUESTIONS

1. Write the number of all possible matrices of order 2 × 2 with each entry 1, 2 or 3.
2. Find the order of the following Matrices. Also find the total number of elements in each matrix.

   (a) $A = \begin{bmatrix} M & 0 & 0 \\ A & T & 0 \\ H & S & L \end{bmatrix}$ (b) $B = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ (c) $C = \begin{bmatrix} 2 \\ 6 \\ 7 \end{bmatrix}$

   (d) $D = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 9 \\ 5 & 3 & 0 \\ 6 & 1 & 3 \end{bmatrix}$ (e) $E = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

3. Give an example of a 2 × 2 Non-zero matrices A, B & C such that

   (a) $AB = O$ but $BA \neq O$ (b) $AB = O$ and $BA = O$

   (c) $AB = AC$ but $B \neq C$

4. Give an example of a 3 × 3 matrix which is

   (a) Upper Triangular as well as Lower Triangular Matrix

   (b) Symmetric Matrix

   (c) Skew-Symmetric Matrix

   (d) Neither Symmetric nor Skew-Symmetric Matrix

   (e) Symmetric as well as skew-symmetric

5. (a) How many Matrices of order 2 × 2 are possible with entry 0 or 1. How many of these are diagonal matrices. List them.

   (b) How many Matricesof order 3 × 3 are possible with 0 or – 1. How many of these are Diagonal matrices?

   (c) If there are five one's i.e. 1, 1, 1, 1, 1 & four zeroes i.e. 0, 0, 0, 0, Thus how many symmetric matrices of ordr 3 × 3 are possible with these 9 entries?

6. Find 'x', if $A = \begin{bmatrix} 1 & x^2-2 & 3 \\ 7 & 5 & 7 \\ 3 & 7 & -5 \end{bmatrix}$ is symmetric Matrix.

7. Find x, if $A = \begin{bmatrix} 0 & x^2+6 & 1 \\ -5x & x^2-9 & 7 \\ -1 & -7 & 0 \end{bmatrix}$ is skew-symmetric Matrix.

8. Find $(x + y)$, If $A = \begin{bmatrix} 2y-7 & 0 & 0 \\ 0 & x-3 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a diagonal matrix.

9. Find the value of xy, If $A = \begin{bmatrix} 2 & 0 & y-x \\ x+y-2 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a diagonal matrix

10. For $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 3 & 8 \end{bmatrix}$, what will be the result if we apply

    (a) $R_2 \to R_2 + R_1$

    (b) $C_1 \to C1 - C_2$

11. If A is matrix of order $m \times n$ and B is a matrix such that AB' and $B'A$ are both defined, then find the order of matrix B.

12. If A is a skew-symmetric matrix, then $(A^2)^T = kA^2$, find the value of k.

13. If A is a Symmetric matrix, then $(A^3)^T = kA^3$, find the value of k.

14. If A is a square matrix such that $A^2 = I$, then find the value of $(A - I)^3 + (A + I)^3 - 7A$.

15. (a) If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ find 'k' for $A^2 = kA - 2I$

(b) If $[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$, then find x.

16. If $A = \begin{bmatrix} 2 & 3 & 5 \\ x-2 & 1 & 6 \\ 6-y & z & 4 \end{bmatrix}$ is an Upper-Triangular matrix, then find the Value of $(x + y - z)$.

17. If all entries of a square matrix of order 2 are either 3, – 3 or 0, then how many Non-zero matrices are possible?

18. If all the entries of a 3 × 3 Matrix A are either 2 or 6, then how many **DIAGONAL** matrices are possible?

19. If all the entries of a 3 × 3 Matrix B are either 0 or 1, then how many **SCALAR** matrices are possible?

20. If all the entries of a 3 × 3 Matrix C are either 0 or 1, then how many **IDENTITY** matrices are possible?

21. A matrix 'X' has 'p' number of elements, where p is a prime number, then how many orders X can have?

22. Let A and B are two matrices, such that the order of A is 3 × 4, if $A'B$ and BA' are both defined then find the order of B'.

23. If A = diag (3 – 5 7), B = diag (–1 2 4) then find $(A + 2B)$.

24. Find the value $(x + y)$ from the following matrix equation:

$$2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$$

25. In the following matrix equation use elementary operation $R_2 \to R_2 - 2R_1$ and write the equation thus obtained.

$$\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 8 & -3 \\ 9 & -4 \end{pmatrix}$$

26. If A is a square matrix, then show that

(a) $(A + A^T)$ is symmetric matrix.

(b) $(A - A^T)$ is symmetric matrix.

(c) (AA^T) is symmetric matrix.

27. Show that every square matrix can be expressed as the sum of a symmetric & a skew-symmetric matrix.

28. If A & B are two Symmetric matrices of same order, then show that

(a) $(AB - BA)$ is skew-symmetric Matrix.

(b) $(AB + BA)$ is symmetric Matrix.

29. (a) If $A = \begin{pmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$. Verify that $(A + B)\, C = AC + BC$.

(b) If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ then show that $A = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

30. If $A = \begin{bmatrix} i & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, show that $AB \neq BA$

31. Find a matrix X, for which $\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

32. If A and B are symmetric matrices, show that AB is symmetric, if $AB = BA$.

33. Match the following:

Possible Number of Matrtices (A_n) of order 3 × 3 with entry 0 or 1 which are

(a)	Condition		No. of matrices
(1)	A_n is diagonal Matrix	P	2^0
(2)	A_n is upper triangular Matrix	Q	2^1
(3)	A_n is identity Matrix	R	2^3
(4)	A_n is scalar Matrix	S	2^6

34. If $A = \begin{bmatrix} \text{Cos}\, x & -\text{Sin}\, x \\ \text{Sin}\, x & \text{Cos}\, x \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} \text{Cos}\, nx & -\text{Sin}\, nx \\ \text{Sin}\, nx & \text{Cos}\, nx \end{bmatrix}$ for all $n \in N$.

35. Express the following Matrices as a sum of a symmetric & skew-symmetric matrix.

(Note: Part (b) & (c) can be asked for one marker, **So Think About This!)**

(a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 5 & 7 \\ 5 & 7 & -5 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}$

36. Show that the Matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = 0$.

37. Find the values of x & y, if $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies the equation $A^2 + xA + yI = 0$.

38. Find $f(A)$, If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ such that $f(x) = x^2 - 4x + 7$

39. Find the inverse of $\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$ using Elementary Transformation.

40. Find the inverse of $\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ using Elementary Transformation.

THREE MARKS QUESTIONS

41. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3 × 3 matrices such that $Q = P^5 + I_3$, then

Prove that $\left(\frac{q_{21}+q_{31}}{q_{32}}\right) = 10$

42. Construct a 3 × 3 matrix $A = [a_{ij}]$ such that

(a) $a_{ij} = \begin{cases} i+j; \; i > j \\ \frac{i}{j}; \; i = j \\ i-j; \; i < j \end{cases}$

(b) $a_{ij} = \begin{cases} 2^i; \; i > j \\ i.j; \; i = j \\ 3^i; \; i < j \end{cases}$

(c) $a_{ij} = \begin{cases} i^2 + j^2; \; i \neq j \\ 0; \; i = j \end{cases}$

(d) $a_{ij} = \frac{|2i-3j|}{5}$

(e) $a_{ij} = \left[\frac{i}{j}\right]$, where [.] represents Greateds Integer Functio

43. If A = $\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, then prove by principle of Mathematical induction that A^n =

$\begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}$

44. If A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, evaluate $A^3 - 4A^2 + A$.

45. If $f(x) = \begin{bmatrix} \text{Cos}\,x & -\text{Sin}\,x & 0 \\ \text{Sin}\,x & \text{Cos}\,x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then prove that $f(x).\ f(y) = f(x + y)$.

Hence show that $f(x).\ f(-x) = I$, where I is the identy martix of order 3x3.

46. If $f(x) = \dfrac{1}{\sqrt{1-x^2}}\begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$, prove that $f(x).f(y) = f\left(\dfrac{x+y}{1+xy}\right)$. Heance show that $f(x)$.

$f(-x) = I$, Where $[x] < 1$.

FIVE MARKS QUESTIONS

47. Find x, y & z if $A^T = A^{-1}$ and $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$. Also Find How many triplets of (x, y,

z) are possible. (Note: $A.A^{-1} = A^{-1} A = I$)

48. (a) Find the inverse of $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ using Elementary Transformation.

(b) Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -2 & -5 \end{bmatrix}$ using Elementary Transformation.

49. (a) If A is a symmetric Matrix and B is skew-symmetric Matrix such that $A + B =$

$\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ then show that $AB = \begin{bmatrix} 1 & -2 \\ -1 & -4 \end{bmatrix}$.

(b) If $A = \begin{bmatrix} 4 & 1 \\ -9 & -2 \end{bmatrix}$ and $A^{50} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then show that $(a + b + c + d + 398) = 0$

CASE STUDIES

50. Two farmers Ramkishanand Gurcharan Singh cultivates only three varieties of rice namely Basmati, Permal and Naura. The Quantity of sale (in kg) of these varieties of rice by both the farmers in the month of Septembert and October are givenby the following matrices *A* and *B*.

$$\begin{array}{l} \\ A\text{(September sales)} = \end{array} \begin{array}{c} \quad\text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 1000 & 2000 & 3000 \\ 5000 & 3000 & 1000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Ramakrishan} \\ \text{Gurcharan Singh} \end{array}$$

$$\begin{array}{l} \\ B\text{ (October sales)} = \end{array} \begin{array}{c} \quad\text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Ramakrishan} \\ \text{Gurcharan Singh} \end{array}$$

Based on the above information answer the following:

(i) Find the combined sales in September and October for each farmer in eachvariety.

(a) $\begin{array}{l} \\ \text{Total sales} = \end{array} \begin{array}{c} \quad\text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 6000 & 3000 & 9000 \\ 7000 & 13000 & 2000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Ramakrishan} \\ \text{Gurcharan Singh} \end{array}$

(b) $\begin{array}{l} \\ \text{Total sales} = \end{array} \begin{array}{c} \quad\text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 6000 & 12000 & 9000 \\ 7000 & 13000 & 2000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Ramakrishan} \\ \text{Gurcharan Singh} \end{array}$

(c) $\begin{array}{l} \\ \text{Total sales} = \end{array} \begin{array}{c} \quad\text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 6000 & 12000 & 9000 \\ 25000 & 13000 & 2000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Ramakrishan} \\ \text{Gurcharan Singh} \end{array}$

(d) $\begin{array}{l} \\ \text{Total sales} = \end{array} \begin{array}{c} \quad\text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 6000 & 12000 & 9000 \\ 25000 & 13000 & 11000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Ramakrishan} \\ \text{Gurcharan Singh} \end{array}$

(ii) Find the decrease insales from September to October.

(a) Net Decrease in sales = $\begin{array}{c} \text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 4000 & 8000 & 9000 \\ 7000 & 13000 & 2000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Ramakrishan} \\ \text{Gurcharan Singh} \end{array}$

(b) Net Decrease in sales = $\begin{array}{c} \text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 4000 & 8000 & 3000 \\ 15000 & 13000 & 2000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Ramakrishan} \\ \text{Gurcharan Singh} \end{array}$

(c) Net Decrease in sales = $\begin{array}{c} \text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 4000 & 8000 & 3000 \\ 15000 & 7000 & 9000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Ramakrishan} \\ \text{Gurcharan Singh} \end{array}$

(d) Net Decrease in sales = $\begin{array}{c} \text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 4000 & 8000 & 3000 \\ 15000 & 13000 & 9000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Ramakrishan} \\ \text{Gurcharan Singh} \end{array}$

If Ramakrishan sell the variety of rice (per kg) i.e. Basmati, Permal and Naura at Rs. 30, Rs. 20 & Rs. 10 respectively, While Gurcharan Singh Sell the variety of rice (per kg) i.e. Basmati, Permal and Naura at Rs. 40, Rs. 30 & Rs. 20 respectively.

(iii) Find the Total Selling Price received by Ramakrishan in the month of September.

(a) Rs. 1,10,000 (b) Rs. 2,10,000

(c) Rs. 3,00,000 (d) Rs. 3,10,000

(iv) Find the Total Selling Price received by Gurcharan Singh in the month of September.

(a) Rs. 1,10,000 (b) Rs. 2,10,000

(c) Rs. 3,00,000 (d) Rs. 3,10,000

(v) Find the Total Selling Price received by Ramakrishan in the month of September & October.

(a) Rs. 4,00,000 (b) Rs. 5,00,000

(c) Rs. 5,10,000 (d) Rs. 6,10,000

ANSWERS

ONE MARK QUESTIONS

1. 81

2. (a) Order = 3 × 3; 9
   (b) Order = 2 × 2; 4
   (c) Order = 3 × 1; 3
   (d) Order = 4 × 3; 12
   (e) Order = 2 × 1; 2

3. Open ended question (So Any Suitable Answer)

4. Open ended Question (So Any Suitable Answer)

5. (a) 16;4 (b) 512;8 (c) 12

6. $x = \pm 3$ 7. $x = 3$ 8. $x + y = 17$ 9. $xy = 1$

10. (a) $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 4 & 13 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ -5 & 8 \end{bmatrix}$

11. m x n 12. $k = 1$ 13. $k = 1$ 14. A

15. (a) $k = 1$
    (b) $x = 2$

16. $x + y - z = 8$ 17. 80 18. Zero

19. 2 20. 1 21. Two 22. (4 × 3)

23. diag (1 – 1 15) 24. $x + y = 11$ 25. $\begin{pmatrix} 2 & 3 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 8 & -3 \\ -7 & 2 \end{pmatrix}$

TWO MARKS QUESTIONS

28. 3 31. $X = \begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$

33. (1) $\rightarrow$ R, (2) $\rightarrow$ S, (3) $\rightarrow$ P, (4) $\rightarrow$ Q

35. (a) $\begin{bmatrix} 1 & 2 & \frac{1}{2} \\ 2 & 5 & \frac{3}{2} \\ \frac{1}{2} & \frac{3}{2} & -5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{5}{2} \\ 0 & 0 & \frac{11}{2} \\ \frac{-5}{2} & \frac{-11}{2} & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 5 & 7 \\ 5 & 7 & -5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}$

37. $x = -2, y = 0$ 38. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 39. $\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$ 40. $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

THREE MARKS QUESTIONS

42. (a) $\begin{bmatrix} 1 & -1 & -2 \\ 3 & 1 & -1 \\ 4 & 5 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 9 & 27 \\ 4 & 4 & 27 \\ 8 & 8 & 9 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 5 & 10 \\ 5 & 0 & 13 \\ 10 & 13 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} \frac{1}{5} & \frac{4}{5} & \frac{7}{5} \\ \frac{1}{5} & \frac{2}{5} & 1 \\ \frac{3}{5} & 0 & \frac{3}{5} \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$

44. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 47. $x = \pm\frac{1}{\sqrt{6}}, y = \pm\frac{1}{\sqrt{6}}, z = \frac{1}{\sqrt{3}}$; 8 Triplets

48. $\begin{bmatrix} \frac{-3}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-3}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{-3}{5} \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

CASE STUDY QUESTION

50. (i) option (d) (ii) option (c) (iii) option (c)

(iv) option (d) (v) option (c)

CHAPTER 4

DETERMINANTS

A determinat of order 2 is written as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ where *a, b, c, d* are complex numbers (As complex number include real number). It denotes the complex number $ad - bc$.

Even though the value of determinants represented by modulus symbol but the value of a determinant may be positive, negative or zero.

In other words,

$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ (Product of diagnal elements - Product of non-diagonal elements).

Determinant of order 1 is the number itself.

We can expand the determinants along that row or column which contains maximum number of zeroes.

MINORS AND COFACTORS

MINOR OF AN ELEMENT

If we take an element of the determinant and delete/remove the row and column containing that element, the determinant of the elements left is called the minor of that element. It is denoted by M_{ij}. For example let us consider a determinant $|A|$

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} \Rightarrow$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} \Rightarrow M_{11} = \begin{vmatrix} e & f \\ q & r \end{vmatrix} \text{ (Minor of } a_{11} = M_{11}\text{)}$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} \Rightarrow M_{12} = \begin{vmatrix} d & f \\ p & r \end{vmatrix} \text{ (Minor of } a_{12} = M_{12}\text{)}$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} \Rightarrow M_{13} = \begin{vmatrix} d & e \\ p & q \end{vmatrix} \text{ (Minor of } a_{13} = M_{13}\text{)}$$

Hence, a determinant of order two will have 4 minors & a determinant of order three will have 9 minors.

COFACTORS OF AN ELEMENT a_{ij}

Cofactor of the element a_{ij} is $C_{ij} = (-1)^{i+j} M_{ij}$; where *i* and *j* denotes the row and column in which the particular element lies. (Means Magnitude of Minor and Cofactor of a_{ij} are equal).

- **Property:** If we multiply the elements of any row/column with their respective cofactors of the same row/column, then we get the value of the determinant. For example,

  $|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13} + C_{13}$

  $|A| = a_{31}C_{31} + a_{32}C_{32} + a_{33} + C_{33}$

- **Property:** If we multiply the elements of any row/column with their respective cofactors of the other row/column, then we get zero as a result.

  For example, $a_{11}C_{21} + a_{12}C_{22} + a_{13} + C_{23} = 0 = a_{11}C_{31} + a_{12}C_{32} + a_{13} + C_{33}$

Note that the value of determinant of order three in terms of Minor & Cofactor can be written as:

$|A| = a_{11}M_{11} + a_{12}M_{12} + a_{13} + M_{13}$ or $|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13} + C_{13}$

$|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13} + C_{13}$

Clearly, we see that, if we apply the appropriate sign to the minor of an element, we have its cofactor. The signs form a check board pattern.

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

PROPERTIES OF DETERMINANTS

- The value of a determinant remains unaltered, if the row and columns are inter changed.

  $$|A| = |A^T|$$

  $$\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

- If any two rows (or columns) of a determinant be interchanged, the value of determinant is changed in sign only. e.g.

  $$\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix} = \begin{vmatrix} b & y & q \\ a & x & p \\ c & z & r \end{vmatrix}$$

- If all the elements of a row (or column) are zero, then the determinant is zero.

  $$\begin{vmatrix} a & 0 & x \\ b & 0 & y \\ c & 0 & z \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ p & q & r \\ x & y & z \end{vmatrix} = 0$$

- If the all elements of a row (or column) are proportional (identical) to the elements of some other row (or column), then the determinant is zero.

  $$\begin{vmatrix} a & ka & x \\ b & kb & y \\ c & kc & z \end{vmatrix} = \begin{vmatrix} mp & mq & mr \\ p & q & r \\ x & y & z \end{vmatrix} = 0$$

- If all the elements of a determinant above or below the main diagonal consist of zeros (triangular matrix), then the determinant is equal to the product of diagonal elements.

  $$\begin{vmatrix} a & 0 & 0 \\ x & b & 0 \\ y & z & c \end{vmatrix} = \begin{vmatrix} a & x & y \\ 0 & b & z \\ 0 & 0 & c \end{vmatrix} = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = abc$$

- If all the elements of one row/column of a determinant are multiplied by *k* (A scalar), the value of the new determinant is k times the original determinant.

$$\begin{vmatrix} ka & p & x \\ kb & q & y \\ kc & r & z \end{vmatrix} = k\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

$$\begin{vmatrix} ka & kp & x \\ kb & kq & y \\ kc & kr & z \end{vmatrix} = k^2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

$$\begin{vmatrix} ka & kp & kx \\ kb & kq & ky \\ kc & kr & kz \end{vmatrix} = k^3\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

$|kA| = k^n\ |A|$, where *n* is the order of determinant.

- $\begin{vmatrix} a+x & p & m \\ b+y & q & n \\ c+z & r & o \end{vmatrix} = \begin{vmatrix} a & p & m \\ b & q & n \\ c & r & o \end{vmatrix} + \begin{vmatrix} x & p & m \\ y & q & n \\ z & r & o \end{vmatrix}$

- $\begin{vmatrix} a & p & m \\ b & q & n \\ c & r & o \end{vmatrix} + \begin{vmatrix} a+\alpha p+\beta m & p & m \\ b+\alpha q+\beta n & q & n \\ c+\alpha r+\beta o & r & o \end{vmatrix}$

A determinant remains unaltered under an operation of the form $C_i \to C_i + \alpha C_j + \beta C_k$ where $j, k \neq i$ or an operation of the form $R_i \to R_i + \alpha R_j + \beta R_k$ where $j, k \neq i$

AREA OF TRIANGLE

Area of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by

$$A = |\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_1 & y_2 & 1 \\ x_1 & y_3 & 1 \end{vmatrix}| \text{ (sq. units)}$$

ADJOINT OF A MATRIX

Let $A = [a_{ij}]_{m \times n}$ be a square matrix and C_{ij} be cofactor of a_{ij} in $|A|$.

Then, $(\text{adj}\,A) = [C_{ij}] \Rightarrow \text{adj}\,A = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$

- $A \cdot (\text{adj}A) = (\text{adj}A) \cdot A = |A|$
- $(\text{adj}\,AB) = (\text{adj}\,B) \cdot (\text{adj}\,A)$
- $|\text{adj}\,A| = |\text{A}|^{n-1}$, where n is the order of a matrix A.

SINGULAR MATRIX

A matrix A is singular if $|A| = 0$ and it is non-singular if $|A| \neq 0$

$|A| = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 5 \neq 0$ So A is non-singular matrix.

$|A| = \begin{vmatrix} 2 & 8 \\ 1 & 4 \end{vmatrix} = 8 - 8 = 0,$ So A is singular matrix.

INVERSE OF A MATRIX

A square matrix A is said to be invertible if there exists a square matrix B of the same order such that AB = BA = 1 then we write $A^{-1} = \text{B}$, (A^{-1} exists only if $|A| \neq 0$)

$$A^{-1} = \frac{1}{|A|}(\text{adj}\,A) = \frac{1}{|A|}\begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

- $(AB)^{-1} = B^{-1} \cdot A^{-1}$
- $(A^{-1})^{-1} = A$
- $(A^T)^{-1} = (A^{-1})T$
- $A \cdot A^{-1} = A^{-1}A = I$
- $|A^{-1}| = \frac{1}{|A|}$
- $|A \cdot \text{adj}A| = |A|^n$ (where n is the order of matrix A)

ONE MARK QUESTIONS

1. If A is a matrix of order 3 × 3, then find the value of $|3A|$.

2. What is the sum of the products of elements of any row of a matrix A with the co-factors of corresponding elements.

3. What is the sum of the products of elements of any row of a matrix A with the co factors of elements of other row.

4. If A, B, C are the angles of a triangle, then find the value of $\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$.

5. If $A = \begin{vmatrix} 2 & 2^5 & 2^{10} \\ 0 & 3 & 3^5 \\ 0 & 0 & 1 \end{vmatrix}$, then find the value $|A^{-1}|$.

6. Find the value of $\begin{vmatrix} \sin A & \cos A & \sin A + \cos B \\ \sin B & \cos A & \sin A + \cos B \\ \sin C & \cos A & \sin A + \cos B \end{vmatrix}$

7. If $= \begin{vmatrix} ax & x^2 & 1 \\ by & y^2 & 1 \\ cz & z^2 & 1 \end{vmatrix}, \begin{vmatrix} a & b & c \\ x & y & z \\ yz & xz & xy \end{vmatrix}$, then find the relation between A and B.

8. Find the value of k, if the area of a triangle with vertices (–3, 0), (3, 0) & (0, k) is 9 sq. units.

9. If A is a square matrix of order 3 × 3 such that $|A| = k$, find the value of $|-A|$.

10. A & B are square matrices of order 3 each, $|A| = 2$ & $|B^T| = 3$. Find $|-4AB|$.

11. If A is an invertible matrix of order 3 and $|\text{Adj } A| = 25$, then write the value of $|5A^{-1}|$.

12. If A is an invertible matrix of order 2 and $|A| = 4$, then write the value of $|A^{-1}|$.

13. For what value of k, the matrix $\begin{bmatrix} 2 & 10 \\ 3k+2 & -5 \end{bmatrix}$ is singular matrix.

14. Using determinants, find the area of triangle with vertices $A(2, 0)$, $B(4, 5)$, $C(6, 3)$.

15. For what value of k, the matrix $\begin{bmatrix} 2 & 5 \\ k & 10 \end{bmatrix}$ has no inverse.

16. If $A = \begin{vmatrix} 3 & 5 \\ 4 & 7 \end{vmatrix}$, find $A.(\text{Adj } A)$.

17. If $A = \begin{vmatrix} 2 & 3 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 6 \end{vmatrix}$, find $A.(\text{Adj } A)$.

18. If $A = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$, then for any natural number n, find the value of $|A^n|$.

19. For what value(s) of a, the points (a,0), (2,0) and (4,0) are collinear?

20. For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ write A^{-1}.

TWO MARKS QUESTIONS

21. Without expanding the determinants prove that $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$

22. Without expanding at any stage, prove that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0;$ where a, b and c are in A.P.

23. There are two values of a which makes determinant $\begin{vmatrix} 1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then find the sum of these numbers.

24. Let A be a 3 × 3 matrix such that $|A| = -2$, then find the value of $|2A^{-1}| + 2\,|A|$.

25. If $A=\begin{bmatrix} -4 & 3 \\ -7 & 5 \end{bmatrix}$ then find the value of k, such that $A^{700} = kA$

26. If $A=\begin{bmatrix} -4 & 3 \\ -7 & 5 \end{bmatrix}$ then find the value of k, such that $A^{345} = kI$

27. If $A=\begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix}, B=\begin{bmatrix} yr-zq & cq-br & bz-cy \\ zp-xr & ar-cp & cx-az \\ xq-yp & bp-aq & ay-bx \end{bmatrix}$. Find $|B|$ if $|A| = 4$

28. If $A=\begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix}, B=\begin{bmatrix} yr-zq & cq-br & bz-cy \\ zp-xr & ar-cp & cx-az \\ xq-yp & bp-aq & ay-bx \end{bmatrix}$. Find $|A|$ if $|B| = 25$

29. Find the value of E, if $\begin{vmatrix} x^2+3x & x-1 & x-3 \\ x+1 & 2-x & x-3 \\ x-3 & x+4 & 3x \end{vmatrix} = Ax^4+Bx^3+Cx^2+Dx+E$

30. Prove that $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} = 4!$

THREE MARKS QUESTIONS

31. If $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$ find the value of $\dfrac{p}{p-a}+\dfrac{q}{q-b}+\dfrac{r}{r-c}$.

32. Using properties of determinants prove that, $\begin{vmatrix} a^2+1 & ab & ab \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

33. Using properties of determinants, prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

34. If A is a square matrix of order 3, such that |Adj A| = 25, then find the value of

(a) $|A|$ (b) $|-2A^T|$

(c) $|4A^{-1}|$ (d) $|5A|$

(e) A.Adj A (f) $|A.\text{Adj } A|$

(g) $|A^3|$

35. If A is a square matrix of order 3, such that $|A|$ = 5, then find the value of

(a) $|3A|$ (b) $|-2A^T|$

(c) $|4A^{-1}|$ (d) |Adj A|

(e) $|A.\text{Adj } A|$ (f) $|A.\text{Adj } A|$

(g) $|A^3|$

36. If $A = \begin{bmatrix} 1 & 2020 & 2019 \\ 0 & 1 & 2018 \\ 0 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 0 \\ 2019 & 1 & 0 \\ 2018 & 2018 & 1 \end{bmatrix}$ then find the value of

(a) $|AB|$ (b) $|(AB)^{-1}|$

(c) $|A^2.B^3|$ (d) $|3(AB)^T|$

(e) |Adj (Ab)|

37. Find matrix X such that $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 7 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$

38. Using properties of determinants, prove the following

(a) $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$

(b) $\begin{vmatrix} 0 & 99 & 998 \\ -99 & 0 & -97 \\ -998 & 97 & 0 \end{vmatrix} = 0$

(c) $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

(d) $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

(e) $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$

(f) $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$

(g) $\begin{vmatrix} a & b & c \\ b & c & b \\ c & a & a \end{vmatrix} = 3abc - a^3 - b^3 - c^3 = -\frac{1}{2}(a+b+c[(a-b)^2+(b-c)^2+(c-a)^2]$

(h) $\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$

(i) $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

(j) $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

(k) $\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(1-x) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix} = 6x^2(1-x^2)$

39. Find matrix X such that

(a) $X\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & -2 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} X = \begin{bmatrix} 7 & -2 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} X \begin{bmatrix} 7 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

40. Without expanding at any stage, prove that

(a) $\begin{vmatrix} {}^nC_1 & {}^nC_2 & {}^{n+1}C_2 \\ {}^nC_2 & {}^nC_3 & {}^{n+1}C_3 \\ {}^nC_3 & {}^nC_4 & {}^{n+1}C_4 \end{vmatrix} = 0$ (b) $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

(c) $\begin{vmatrix} (2019^x + 2019^{-x})^2 & (2019^x + 2019^{-x})^2 & 1 \\ (2020^x + 2020^{-x})^2 & (2020^x + 2020^{-x})^2 & 1 \\ (2021^x + 2021^{-x})^2 & (2021^x + 2021^{-x})^2 & 1 \end{vmatrix} = 0$

(d) $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{3} & \sqrt{26} \\ \sqrt{15}+\sqrt{26} & 2\sqrt{15} & \sqrt{52} \\ 3+\sqrt{65} & 6 & \sqrt{130} \end{vmatrix} = 0$

41. If $\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$, then using properties of determinants, find the value of

$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}$, where x, y & $z \neq 0$

42. If a, b, c are real numbers such that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$ show that either

$a + b + c = 0$ or $a = b = c$.

43. If a, b, c are all distinct and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, find the value of (abc).

44. If $A = \begin{bmatrix} a & b & c \\ p & q & r \\ 1 & 1 & 1 \end{bmatrix}$, $B = A^2$, such that $(a-b)^2 + (p-q)^2 = 9, (b-c)^2 + (q-r)^2 = 16$

and $(c-a)^2 + (r-p)^2 = 25$ then find $|B|$.

FIVE MARKS QUESTIONS

45. Find the inverse of the matrix $\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{bmatrix}$ and hene solve the system of equations:

$$3x + 4y + 5z = 18$$

$$5x - 2y + 7z = 20$$

$$2x - y + 8z = 13$$

46. (a) A school wants to award its students for regularity and hardwork with a total cash award of Rs. 6,000. If three times the award money for hardwork added to that given for regularity amounts to Rs. 11,000 represent the above situation algebraically and find the award money for each value, using matrix method.

(b) A shopkeeper has 3 varieties of pen A, B and C. Rohan purchased 1 pen of each variety for total of Rs. 21. Ayush purchased 4 pens of A variety, 3 pens of B variety & 2 pen of C variety for Rs. 60. While Kamal purchased 6 pens of A variety, 2 pens of B variety & 3 pen of C variety for Rs. 70. Find cost of each variety of pen by matrix method.

47. Find A^{-1}, where $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence use the result to solve the following system of linear equations:

$$x + 2y + 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x + 3y + 4z = 11$$

48. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence, solve the system of linear equations :

$$x + 2y + 3z = 8$$

$$2x + 2y + 4z = 6$$

$$-3x + 2y + 4z = -6$$

49. Find A^{-1} using Elementary Transformation, where $A = \begin{bmatrix} 4 & -1 & 2 \\ -1 & 2 & -2 \\ 3 & -3 & 4 \end{bmatrix}$. Hence use the result to, solve the system of linear equations :

$$4x - y + 3z = 6$$

$$-x + 2y - 3z = -2$$

$$2x - 2y + z = 1$$

50. If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ find AB. Hence using the poroduct solve the system of eq.

$$x - y + z = 4, \quad x - 2y - 2z = 9, \quad 2x + y + 3z = 1$$

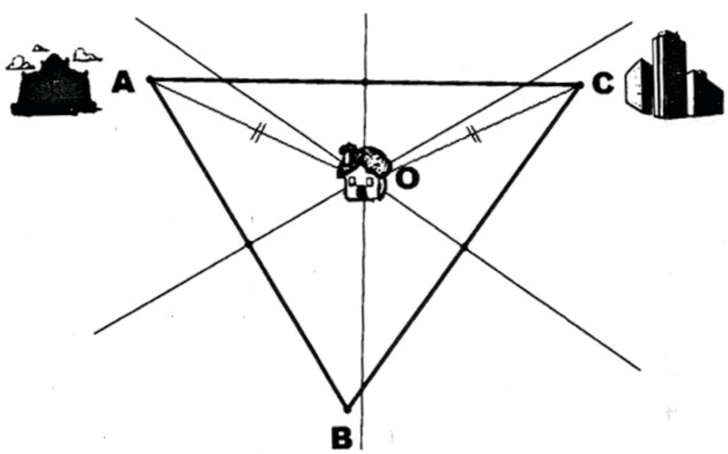
51. Find the product of matrices AB, where $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$ and use the result to solve following system of equations :

$$x - 2y - 3z = 1, \quad -2x + 4y + 5z = -1, \quad -3x + 7y + 9z = -4$$

52. If $A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 3 & 5 \end{pmatrix}$ Find A^{-1} & use it to solve the following system of equations :

$$x - y + z = 1$$
$$-x + 2y + 3z - 4 = 0$$
$$x + y + 5z = 7$$

53. A family wanted to buy a home, but they wanted it to be close to both the children's school and the parents' workplace. By looking a map, trhey could find a point that is equidistant from both the workplace and the school by finding the circumcenter of the triangular region.



If the coordinates are A (12,5), B (20, 5) and C (16, 7) on the basis of this answer the following :

(a) Using the concept of Determinants, Find the equation of AC.

(a) $x-2y=0$ (b) $x+2y=22$

(c) $x-2y=2$ (d) $x-2y=2$

(b) Using the concept of Determinants, Find the equation of BC.

(a) $x-2y=30$ (b) $x-2y=10$

(c) $x+2y=30$ (d) $x+2y=20$

(c) What will be the area of TRIANGULAR REGION ABC (in sq. Units).

(a) 2 (b) 4

(c) 6 (d) 8

(d) If O (16, 2) is the circum-centre of ΔABC, then find the Area of ΔAOC (in sq. Units).

(a) 10 (b) 8

(c) 6 (d) 4

(e) If any point P (2, k) is collinear with point A (12, 5) and O (16, 2), then find the value of (2k –15).

(a) –10 (b) 10

(c) 2 (d) 0

54. For keeping Fit, X people believes in morning walk, Y people believes in yoga and Z people join gym total no of people are 70. Further 20%, 30% and 40% people are suffereing from any disease who believe in morning walk, yoga and gym respectively. Total no. such people is 21. If morning walk cost Rs 0 cosr Rs. 400/ month and total expenditure is Rs 23000.

Solve the above problem using Matrices and Answer the following

(a) If we foremulater this problem, then which of the equation is NOT possible :

(a) $X+Y+Z=70$ (b) $2X+3Y+4Z=21$

(c) $2X+3Y+4Z=210$ (d) $5Y+4Z-230=0$

(b) If matrix A $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 0 & 5 & 4 \end{bmatrix}$, represents the coefficent of X, Y and Z in above 3 corrected Equation, then

(a) $A^{-1} = \frac{-1}{6}\begin{bmatrix} -8 & 1 & 1 \\ -8 & 4 & -2 \\ 10 & -5 & 1 \end{bmatrix}$

(b) $A^{-1} = \frac{-1}{6}\begin{bmatrix} -8 & 1 & 1 \\ -8 & 4 & -2 \\ 10 & -5 & -1 \end{bmatrix}$

(c) $A^{-1} = \frac{-1}{6}\begin{bmatrix} -8 & 1 & 1 \\ -8 & 4 & -2 \\ 10 & 5 & -1 \end{bmatrix}$

(d) $A^{-1} = \frac{-1}{6}\begin{bmatrix} -8 & 1 & 1 \\ -8 & -4 & -2 \\ 10 & -5 & -1 \end{bmatrix}$

(c) On solving above system of equations using matrix method, find the numer of person who prefer morning walk.

(a) 10 (b) 20

(c) 30 (d) 40

(d) On solving above system of equations using matrix method, find the total number of person who prefer GYM.

(a) 10 (b) 20

(c) 30 (d) 40

55. An amount of Rs 600 crores is spent by the government in three schemes. Scheme A is saving girl child from the cruel parents who don't want girl child and get the abortion before her birth. Scheme B is for saving of newlywed girls from death due to sowry. Scheme C is planing for good health for senior citizen. Now twice the amount spent on scheme C together with amount spent on Scheme A is Rs 700 crores. And three times the amount spent on Scheme A together with amount spent on scheme B and Scheme C is Rs. 1200 crores.

If we assume invest (In crores) Rs. X, Rs. Y and Rs. Z in scheme A, B and C respec tively.

Solve the above using matrices and answer the following :

(a) If we formulate this problem, then which of the equation is NOT correct :

(a) $X + Y + Z - 600 = 0$ (b) $X + 2Z = 700$

(c) $X + 2Y = 700$ (d) $3X + Y + Z - 1200 = 0$

(b) If matrix $= \begin{bmatrix} 1 & 1 & 3 \\ 1 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$, represents the coefficient of X, Y and Z in above 3 corrected Equation, then

(a) $A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}$ (b) $A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

(c) $A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & 1 & 1 \end{bmatrix}$ (d) $A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}$

(c) On solving above system of equations using matrix method, find the amount spent (in crores) in Scheme A.

(a) 100 (b) 200

(c) 300 (d) 400

(d) On solving above system of equations using matrix method, find the amount spent (in crores) in Scheme B.

(a) 100 (b) 200

(c) 300 (d) 400

(e) On solving above system of equations using matrix method, find the amount spent (in crores) in Scheme A.

(a) 100 (b) 200

(c) 300 (d) 400

ANSWER

ONE MARKS QUESTIONS

1. 27 |A| 2. |A| 3. 0 4. 0

5. $\frac{1}{6}$ 6. *0* 7. *A = B* 8. ± 3

9. (– k) 10. (– 384) 11. ± 25 12. $\frac{1}{4}$

13. (– 1) 14. 7 sq. units 15. k = 4 16. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

17. $\begin{pmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{pmatrix}$ 18. 1 (one) 19. a can be any real number

20. $\begin{pmatrix} -1 & 4 \\ -1 & 3 \end{pmatrix}$

TWO MARKS QUESTIONS

23. sum = 4 24. 0 (Zero)

25. 5 = –1 26. k = –1

27. 16 28. ± 5

29. (–39)

THREE MARKS QUESTIONS

31. *2*

34. (a) ± 5 (b) ∓ 40 (c) $\frac{\pm 64}{5}$ (d) ± 625

(e) $\pm 5I$ (f) ± 125 (g) ± 125

35. (a) 135 (b) – 40 (c) $\frac{64}{5}$ (d) *25*

(e) $5I$ (f) 125 (g) 125

36. (a) 6 (b) $\frac{1}{6}$ (c) 72 (d) 162

(e) 36

37. $X = \frac{1}{9}\begin{bmatrix} 2 & 31 \\ -1 & -11 \end{bmatrix}$

39. (a) $X = \begin{bmatrix} 16 & -25 \\ 1 & -1 \end{bmatrix}$ (b) $X = \begin{bmatrix} 11 & -7 \\ -5 & 4 \end{bmatrix}$ (c) $X = \frac{1}{9}\begin{bmatrix} 5 & -17 \\ -3 & 12 \end{bmatrix}$

41. 2 42. (–1) 44. 144

FIVE MARKS QUESTIONS

45. $x = 3, y = 1, z = 1$

46. (a) Award money given for Honesty = Rs 500,Regularity = Rs. 2000 anmd Hard work = Rs. 3500

46. (b) Cost Pen of Varietry A = Rs. 5, Variety B = Rs. 8, Variety C = Rs. 8

47. $x = 3, y = -2, z = 1$ 48. $x = 0, y = 1, z = 2$

50. $x = 3, y = -2, z = 1$ 51. $x = -4, y = -1, z = -1$

52. $x = -4, y = -3, z = 2$

CASE STUDY QUESTION

53. (a) option (iv) (b) option (iii) (c) option (iv) (d) option (i)
(e) option (ii)

54. (a) option (ii) (b) option (i) (c) option (ii) (d) option (iii)
(e) option (ii)

55. (a) option (iii) (b) option (ii) (c) option (iii) (d) option (i)
(e) option (ii)

CHAPTER 5

CONTINUITY AND DIFFERENTIABILITY

∀

POINTS TO REMEMBER

- A function $f(x)$ is said to be continuous at $x = c$ iff $\lim_{x \to c} f(x) = f(c)$

  i.e., $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$

- $f(x)$ is continuous in (a, b) iff it is continuous at $x = c$ $\forall c \in (a, b)$.

- $f(x)$ is continuous in $[a, b]$ iff

  (i) f(x) is continuous in (a, b)

  (ii) $\lim_{x \to a^+} f(x) = f(a)$

  (iii) $\lim_{x \to b^-} f(x) = f(b)$

- Modulus functions is Continuous on R

- Trigonometric functions are continuous in their respective domains.

- Exponential function is continuous on R

- Every polynomial function is continuous on R.

- Greatest integer function is continuous on all non-integral real numbers

- If $f(x)$ and $g(x)$ are two continuous functions at $x = a$ and if $c \in R$ then

  (i) $f(x) \pm g(x)$ are also continuous functions at $x = a$.

  (ii) $g(x) . f(x)$, $f(x) + c$, $cf(x)$, $|f(x)|$ are also continuous at $x = a$.

  (iii) $\frac{f(X)}{g(X)}$ is continuous at $x = a$ provided $g(a) \neq 0$.

- A function $f(x)$ is derivable or differentiable at $x = c$ in its domain iff

$\lim_{x \to c^-} \frac{f(X)-f(c)}{X-c} = \lim_{x \to c^+} \frac{f(X)-f(c)}{X-c}$, and is finite

The value of above limit is denoted by $f'(c)$ and is called the derivative of $f(x)$ at $x = c$.

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} + \frac{dv}{dx}$$

- $\frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v.\frac{du}{dx}$ (Product Rule)
- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$ (Quotient Rule)
- If y= f(u) and u=g(t) then $\frac{dy}{dt} = \frac{dy}{du} \times \frac{du}{dt} = f'(u)g'(t)$ (Chain Rule)
- If y =f(u), x=g(u) then,

  $$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{f'(u)}{g'(u)}$$

- **Rolle's theorem:** If $f(x)$ is continuous in [a, b] derivable in (a, b) and $f(a) = f(b)$ then there exists at least one real number c ∈ (a, b) such that $f'(c) = 0$.
- **Mean Value Theorem :** If $f(x)$ is continuous in $[a, b]$ and derivable in(a, b) then there exists at least one real number $c \in (a, b)$ such that

  $$f'(c) = \frac{f(b) - f(a)}{b - a}$$

  Every differentiable function is continuous but its converse is not true.

ONE MARK QUESTIONS

1. Let f(x) = sin x cos x. Write down the set of points of discontinuity of f(x).
2. Given f (x) = $\frac{1}{x+2}$, write down the set of points of discontinuity of f (f(x)).

3. Write the set of points of continuity of

$f(x) = |x-1| + |x+1|$

4. Write the number of points of discontinuity of $f(x) = [x]$ in [3, 7].

5. If $y = e^{\log(x^5)}$, find $\frac{dy}{dx}$.

6. If $f(x) = x^2 g(x)$ and $g(1) = 6$, $g'(x) = 3$, find the value of $f'(1)$.

7. If $y = a \sin t$, $x = a \cos t$ then find $\frac{dy}{dx}$

8. Find value of f(0), so that $\frac{-e^x+2^x}{x}$ may be continuous at x=0.

9. Find the values of x for which $f(x) = \frac{x^2+7}{x^3+3x^2-x-3}$ is discontinous.

10. If $y = \tan^{-1}x + \cot^{-1}x + \sec^{-1}x, \operatorname{cosec}^{-1}x$ then find dy/dx

11. If $y = \log_e e^{\sin x^2}$, find $\frac{dy}{dx}$

12. $y = \log_a x + \log_x a + \log_x x + \log_a a$, then $\frac{dy}{dx} = ?$

(a) $\frac{1}{x} + x \log a$ (b) $\frac{1}{x \log a} + x \log a$ (c) $\frac{\log a}{x} + \frac{x}{\log a}$ (d) None of these

13. If $y = 5^x \cdot x^5$, then find $\frac{dy}{dx}$

14. What is derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ w.r.t. $\operatorname{Sin}^{-1}(3x-4x^3)$?

15. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + ...\infty}}}$, then find $(2y-1)\frac{dy}{dx}$

TWO MARKS QUESTIONS

1. Differentiate $\sin(x^2)$ w. r. t. $e^{\sin x}$

2. $y = x^y$ then find $\frac{dy}{dx}$

3. If $y = x^x + x^3 + 3^x + 3^3$, find $\frac{dy}{dx}$

4. If $y = 2\sin^{-1}(\cos x) + 5\,\text{cosec}^{-1}(\sec x)$. Find $\frac{dy}{dx}$

5. If $y = e^{[\log(x+1) - \log x]}$ find $\frac{dy}{dx}$

6. Differentiate $\text{Sin}^{-1}[x\sqrt{x}]$ w. r. t. x.

7 Find the derivative of $|x^2+2|$ w.r.t. x

8. Find the domain of the continuity of $f(x) = \sin^{-1}x - [x]$

9. Find the derivative of $\cos(\sin x^2)$ w.r.t. x at $x = \sqrt{\frac{\pi}{2}}$

10. If $y=e^{3\log x + 2x}$, Prove that $\frac{dy}{dx} = x^2(2x+3)\,e^{2x}$.

11. Differentiate $\sin^2(\theta^2+1)$ w.r.t. θ^2

12. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$

13. If $x^2 + y^2 = 1$ verify that $\frac{dy}{dx}.\frac{dx}{dy} = 1$

14. Find $\frac{dy}{dx}$ when $y = 10^{x10^x}$

15. Find $y = x^x$ find $\frac{d^2y}{dx^2}$

16. Find $\frac{dy}{dx}$ if $y = \cos^{-1}(\sin x)$

17. If $f(x) = x + 7$, and $g(x) = x - 7$, $x \in R$, them find $\frac{d}{dx}$ (fog) (x).

18. Differntiate log (7 logx) w.r.t x

19. If $y = f(x^2)$ and $f'(x) = \sin x^2$. Find $\frac{dy}{dx}$

20. Find $\frac{dy}{dx}$ if $y = \sqrt{\sin^{-1}\sqrt{x}}$

THREE MARKS QUESTIONS

1. Examine the continuity of the following functions at the indicated points.

(I) $f(x)=\begin{cases}x^2\cos\left(\frac{1}{x}\right) & x\neq 0\\ 0 & x=0\end{cases}$ at $x=0$

(II) $f(x)=\begin{cases}x-[x] & x\neq 1\\ 0 & x=1\end{cases}$ at $x=1$

(III) $f(x)=\begin{cases}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & , x\neq 0\\ 0 & , x=0\end{cases}$ at $x=0$

(IV) $f(x)=\begin{cases}\dfrac{x-\cos(sin^{-1}x)}{1-\tan(sin^{-1}x)} & x\neq\frac{1}{\sqrt{2}}\\ \dfrac{-1}{\sqrt{2}} & x=\frac{1}{\sqrt{2}}\end{cases}$ at $x=\frac{1}{\sqrt{2}}$

2. For what values of constant K, the following functions are continuous at the indicated points.

(i) $f(x)=\begin{cases}\dfrac{\sqrt{1+\mathrm{K}x}-\sqrt{1-\mathrm{K}x}}{x} & x<0\\ \dfrac{2x+1}{x-1} & x>0\end{cases}$ at $x=0$

(ii) $f(x)=\begin{cases}\dfrac{e^x-1}{\log(1+2x)} & x\neq 0\\ K & x=0\end{cases}$ at $x=0$

(iii) $f(x)=\begin{cases}\dfrac{1-cos4x}{x^2} & x<0\\ K & x=0\\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & x>0\end{cases}$ at $x=0$

3. For what values a and b

$$f(x)=\begin{cases}\dfrac{x+2}{|x+2|}+a & if\ x<-2\\ a+b & if\ x=-2\\ \dfrac{x+2}{|x+2|}+2b & if x>-2\end{cases}$$

Is continuous at $x=-2$

4. Find the values of a, b and c for which the function

$$f(x) = \begin{cases} \dfrac{\sin[(a+1)x] + sinx}{x} & x < 0 \\ C & x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} & x > 0 \end{cases}$$

Is continuous at $x = 0$

5. $f(x) = \begin{cases} [x] + [-x] & x \neq 0 \\ \lambda & x = 0 \end{cases}$

Find the value of λ, *f* is continuous at $= 0$?

6. Let $f(x) = \begin{cases} \frac{1-sin^3x}{3co\ ^2x} & ; \quad x < \frac{\pi}{2} \\ a & ; \quad x = \frac{\pi}{2} \\ \frac{b(1-\sin\ )}{(\pi-2x)^2} & ; \quad x > \frac{\pi}{2} \end{cases}$

If $f(x)$ is continuous at $x = \frac{\pi}{2}$, find *a* and *b.*

7. If $f(x) = \begin{cases} x^3 + 3x + a & x \leq 1 \\ bx + 2 & x > 1 \end{cases}$

Is everywhere differentiable, find the value of *a* and *b*.

8. Find the relationship between a and b so that the function defined by

$$f(x) = \begin{Bmatrix} ax+1 \ , & x \leq 3 \\ bx+3 \ , & x > 3 \end{Bmatrix} is\ continous\ at\ x = 3.$$

9. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r.t $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ where $x \neq 0$.

10. If $y = x^{x^x}$, then find $\frac{dy}{dx}$.

11. Differentiate $(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$ w.r.t. x.

12. If $(x+y)^{m+n} = x^m . y^n$ then prove that $\frac{dy}{dx} = \frac{y}{x}$

13. If $(x-y). e^{\frac{x}{x-y}} = a$, prove that y$\left(\frac{dy}{dx}\right)$ + x = 2y

14. If $x = \tan\left(\frac{1}{a}\log y\right)$ then show that

$$(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$$

15. If $y = x\log\left(\frac{x}{a+bx}\right)$ prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

16. Differentiate $\sin^{-1}\left[\frac{2^{x+1}.3^x}{1+(36)^x}\right]$ w.r.t x.

17. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$, prove that

$\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$, Where –1 < x < 1 and –1 < y < 1 [HINT: put X^3sin A and Y^3 sin B]

18. If $f(x) = \sqrt{x^2+1}, g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x - 3$ find $f'[h'(g'(x))]$.

19. If x = $\sec\theta - \cos\theta$ and $y = sec^n\theta - cos^n\theta$, then prove that $\frac{dy}{dx} = n\sqrt{\frac{y^2+4}{x^2+4}}$

20. If $x^y + y^x + x^x = m^n$, then find the value of $\frac{dy}{dx}$.

21. If $x = a\, cos^3\theta, y = a\, sin^3\theta$ then find $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{6}$

22. If $y = \tan^{-1}\left[\frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right]$where $0 < x < \frac{\pi}{2}$ find $\frac{dy}{dx}$

23. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,then show that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$.

24. Verify Rolle's theorem for the function

$$f(x) = e^x \sin 2x \left[0, \frac{\pi}{2}\right]$$

25. Verify mean value theorem for the function

$f(x) = \sqrt{x^2 - 4}$ $[2,4]$

26. If the Rolle's theorem holds for the function

$f(x) = x^3 + bx^2 + ax + 5$ on [1,3] with $c = \left(2 + \frac{1}{\sqrt{3}}\right)$

Find the value of *a* and *b.*

27. If = $\left[x + \sqrt{x^2 + 1}\right]^m$, show that $(x^2 + 1)y_2 + xy_1 - m^2y = 0$.

28. Differentiate $\sin^{-1}\left[\frac{3x+4\sqrt{1-x^2}}{5}\right]$ w.r.t. x.

29. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log \ )^2}$

30. If $f: [-5,5] \to R$ is a differentiable function and $f'(x)$ does not vanish anywhere, then prove that $f(-5) \neq f(5)$.

31. If $y^{1/m} + y^{-1/m}$ = 2x then prove that $(x^2 - 1)y_2 + xy_1 = m^2 y$.

ANSWERS

ONE MARK QUESTIONS

1. { }

2. $\{-2, -5/2\}$

3. R

4. Points of discontinuity of $f(x)$ are 4,5,6,7

Note- At $x = 3, f(x) = [x]$ is continuous because $\lim_{x\to 3^+} f(x) = 3 = f(3)$

5. $5x^4$

6. 15

7. $-\cot t$

8. -1+log2

9. x=-1,1,-3

10. 0

11. $2x \cos(x^2)$

12. (d)

13. $5^x (x^5 \log 5 + 5x^4)$

14. 2/3

15. (c) $\frac{\cos x}{(2y-1)}$

TWO MARKS QUESTIONS

1. $\frac{2x\cos(x^2)}{\cos x\, e^{\sin x}}$

2. $\frac{y^2}{x[1 - y\log x]}$

3. $x^x [1 + \log x] + 3x^2 + 3^x \log 3$

4. –7

5. $-\frac{1}{x^2}$

6. $\frac{3}{2}\sqrt{\frac{x}{1-x^3}}$

7 $\frac{2x(x^2+2)}{|x^2+2|}$

8. $(-1,0) \cup (0,1)$

9. 0

11. $\text{Sin}\,(2\theta^2 + 2), \theta \neq 0$

12. 0

14. $10^{x^{10^x}} x^{10^x} 10^x \log 10 \left[\frac{1}{x} + \log 10 \log x\right]$

15. $x^x\ [(1+\log x)^2 + \frac{1}{x}]$

16. -1

17. 1

18. $\frac{1}{x \log x}$

19. $2x \sin x^4$

20. $\frac{1}{4\sqrt{x}\sqrt{1-x}\sqrt{\sin^{-1}\sqrt{x}}}$, where $0 < x < 1$

THREE MARKS QUESTIONS

1. (I) Continuous

   (II) Discontinuous

   (III) Not Continuous at $x = 0$

   (IV) Continuous

2. (I) K = -1

   (II) K =$1/2$

   (III) K = 8

3. $a = 0,\ \ b = -1$

4. $a = \frac{-3}{2},\ b = R - \{0\},\ c = \frac{1}{2}$

5. $\lambda = -1$

6. $a = \frac{1}{2},\ b = 4$

7. $a = 3,\ b = 5$

8. 3a – 3b = 2

9. $-\frac{1}{2}$

10. $x^x x^{x^x}\left\{(1 + \log x)\log x + \frac{1}{x}\right\}$

11. $(x\cos x)^x[1 - x\ \tan x + (\log x\ \cos x)] + (x\sin x)^{1/x}\left[\frac{1+x\ cot\,x - log(x\ sin)^x}{x^2}\right]$

16. $\left[\frac{2^{x+1}3^x}{1+(36)^x}\right]\log 6$

18. $\frac{2}{\sqrt{5}}$

20. $\frac{dy}{dx} = \frac{x^x(1+\log x)+yx^{y-1}-y^x\log y}{x^y\log x + xy^{x-1}}$

21. $\frac{32}{27a}$

22. $-\frac{1}{2}$

26. $a = 11,\ b = -6$

28. $\frac{1}{\sqrt{1-x^2}}$

CHAPTER 6

APPLICATION OF DERIVATIVES

IMPORTANT POINTS TO REMEMBER

- **Rate of change:** Let $y = f(x)$ be a function then the rate of change of y with respect to x is given by $\frac{dy}{dx} = f'(x)$ where a quantity y varies with another quantity x.

  $\left\{\frac{dy}{dx}\right\}_{x=x_1}$ or $f'(x_1)$ represents the rate of change of y w.r.t. x at $x = x_1$.

- **Increasing and Decreasing Function**

  Let f be a real-valued function and let I be any interval in the domain of f. Then f is said to be

  a) Strictly increasing on I, if for all $x_1, x_2 \in I$, we have

  $$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$$

  b) Increasing on I, if for all $x_1, x_2 \in I$, we have

  $$x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$$

  c) Strictly decreasing in I, if for all $x_1, x_2 \in I$, we have

  $$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$$

  d) Decreasing on I, if for all $x_1, x_2 \in I$, we have

  $$x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$$

- **Derivative Test:** Let f be a continuous function on [a, b] and differentiable on (a, b). Then

a) f is strictly increasing on [a, b] if $f'(x) > 0$ for each $x \in (a, b)$.

b) f is increasing on [a, b] if $f'(x) \geq 0$ for each $x \in (a, b)$.

c) f is strictly decreasing on [a, b] if $f'(x) < 0$ for each $x \in (a, b)$.

d) f is decreasing on [a, b] if $f'(x) \leq 0$ for each $x \in (a, b)$.

e) f is constant function on [a, b] if $f'(x) = 0$ for each $x \in (a, b)$.

- **Tangents and Normals**

a) Equation of the tangent to the curve $y = f(x)$ at (x_1, y_1) is

$$y - y_1 = \left[\frac{dy}{dx}\right]_{(x_1, y_1)} (x - x_1)$$

b) Equation of the normal to the curve $y = f(x)$ at (x_1, y_1) is

$$y - y_1 = \frac{-1}{\left[\frac{dy}{dx}\right]_{(x_1, y_1)}} (x - x_1)$$

- **Maxima and Minima**

a) Let f be a function and c be a point in the domain of f such that either f ‘(x)=0 or f ‘(x) does not exist are called critical points.

b) First Derivative Test: Let *f* be a function defined on an open interval I. Let *f* be continuous at a critical point *c* in I. Then

i. *f ´*(*x*) changes sign from positive to negative as x increases through *c*, then *c* is called the point of the local maxima.

ii. *f ´(x)* changes sign from negative to positive as *x* increases through *c*, then *c* is a point of *local minima*.

iii. $f'(x)$ does not change sign as x increases through c, then c is neither a point of *local maxima* nor a point of *local minima*. Such a point is called a point of *inflexion.*

c) **Second Derivative Test :** Let f be a function defined on an interval I and let $c \in$ I. Let f be twice differentiable at c. Then

i. $x = c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$. The value $f(c)$ is local maximum value of f.

ii. $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$. The value $f(c)$ is local minimum value of f.

iii. The test fails if $f'(c) = 0$ and $f''(c) = 0$.

ONE MARK QUESTIONS

1. Find the angle θ, where $0< \theta<\frac{\pi}{2}$, which increases twice as fast as its sine.

2. Find the slope of the normal to the curve $x = a\cos^3\theta$ and $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$

3. A balloon which always remains spherical has a variable radius. Find the rate at which its volume is increasing with respect to its radius when the radius is 7cm.

4. Write the interval for which the function $f(x) = \cos x, 0 \leq x \leq 2\pi$ is decreasing

5. For what values of x is the rate of increasing of $x^3 - 5x^2 + 5x + 8$ is twice the rate of increase of x?

6. Find the point on the curve $y = x^2 - 2x + 3$ where the tangent is parallel to x-axis.

7. Write the maximum value of $f(x) = \frac{\log x}{x}$,if it exists.

8. Find the least value of $f(x) = ax + \frac{b}{x}$, where a>0, b>0 and x>0.

9. Find the interval in which the function $f(x) = x - e^x + \tan\left(\frac{2\pi}{7}\right)$ increases

10. Find the value of a for which the function $f(x) = x^2 - 2ax + 6, x > 0$ is strictly increasing.

11. Find the minimum value of sin x + cos x.

12. Show that f(x) = cos 2x is Decreasing on $\left(0, \frac{\pi}{2}\right)$

13. Find the absolute maximum of $x^{40} - x^{20}$ on the interval (0,1).

14. Find the angle between $y^2 = x$ and $x^2 = y$ at the origin.

15. Find the local minimum value of f′(x) if f(x) = 3 + | x |, $x \in$ R.

16. The distance covered by a particle in t sec.is given by $x = 3 + 8t - 4t^2$. What will be its velocity after 1 second.

17. If the rate of change of volume of a sphere is equal to the rate of change of its radius, then find r.

TWO MARKS QUESTIONS

1. Find the co-ordinates of the point on the curie $y^2 = 3 - 4x$, where tangent is parallel to the line $2x + y - 2 = 0$
2. The sum of the two numbers is 8, what will be the maximum value of the sum of their reciprocals.
3. Find the maximum value of $f(x) = 2x^3 - 24x + 107$ in the internal [1, 3]
4. If the rate of change of Area of a circle is equal to the rate of change its diameter. Find the radius of the circle.
5. The sides of on equilateral triangle are increasing at the rate of 2 cm/s. Find the rate at which the area increases, when side is 10 cm.
6. If there is an error of a% in measuring the edge of cube, then what is the percentge error in its surface?
7. If an error of k% is made in measuring the radius of a sphere, then what is the percentage error in its volume?
8. Find the point on the curve $y^2 = x$ where taget makes 45° angle with x–axis.
9. Find the slope of target to the curve $x = 3t^2 + 1$, $y = t^3 - 1$ at $x = 1$
10. If the curves $y = 2e^x$ and $y = ae^{-x}$ interect orthogonally, then find a.
11. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate.
12. Prove that the function $f(x) = \tan x - 4x$ is strictly decreanig on $\left[\frac{-\pi}{3}, \frac{\pi}{3}\right]$.
13. Find the point on the curve $y = x^2$, where the slope of the tangent is equal to the x co-ordinate of the point.
14. Use differentials to approximate the cube root of 66.
15. Find the maximum and minimum values of the function $f(x) = \sin(\sin x)$
16. Find the local maxima and minima of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$.
17. If $y = a \log x + bx^2 + x$ has its exteme values at $x = -1$ and $x = 2$, then find a and b.
18. Find the equation of the target to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).
19. If the radius of the circle increases from 5 into 5.1 cm, then find the increase in area.
20. Find the equation of the normal to the curve $y = 2x^3 + 3 \sin x$ at $x = 0$.

THREE MARKS QUESTIONS

1. In a competition, a brave child tries to inflate a huge spherical balloon bearing slogans against child labour at the rate of 900 cm^3 of gas per second. Find the rate at which the radius of the balloon is increasing, when its radius is 15 cm.

2. An inverted cone has a depth of 10 cm and a base of radius 5 cm. Water is poured into it at the rate of $\frac{3}{2}$c.c. per minute. Find the rate at which the level of water in the cone is rising when the depth is 4 cm.

3. The volume of a cube is increasing at a constant rate. Prove that the increase in its surface area varies inversely as the length of an edge of the cube.

4. A kite is moving horizontally at a height of 151.5 meters. If the speed of the kite is 10m/sec, how fast is the string being let out when the kite is 250 m away from the boy who is flying the kite ? The height of the boy is 1.5 m.

5. A swimming pool is to be drained for cleaning. If L represents the number of litres of water in the pool t seconds after the pool has been plugged off to drain and $L = 200(10-t)^2$. How fast is the water running out at the end of 5 sec. and what is the average rate at which the water flows out during the first 5 seconds?

6. A man 2m tall, walk at a uniform speed of 6km/h away from a lamp post 6m high. Find the rate at which the length of his shadow increases.

7. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi- vertical angle is $\tan^{-1}(0.5)$. water is poured into it at a constant rate of $5m^3/h$. Find the

rate at which the level of the water is rising at the instant, when the depth of Water in the tank is 4m.

8. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface area. Prove that the radius is decreasing at a constant rate.

9. A conical vessel whose height is 10 meters and the radius of whose base is half that of the height is being filled with a liquid at a uniform rate of $1.5m^3/min$. find the rate at which the level of the water in the vessel is rising when it is 3m below the top of the vessel.

10. Let x and y be the sides of two squares such that $y = x - x^2$. Find the rate of change of area of the second square w.r.t. the area of the first square.

11. The length of a rectangle is increasing at the rate of 3.5 cm/sec. and its breadth is decreasing at the rate of 3 cm/sec. Find the rate of change of the area of the rectangle when length is 12 cm and breadth is 8 cm.

12. If the areas of a circle increases at a uniform rate, then prove that the perimeter various inversely as the radius.

13. Show that $f(x) = x^3 - 6x^2 + 18x + 5$ is an increasing function for all $x \in R$. Find its value when the rate of increase of f(x) is least.

    [Hint: Rate of increase is least when $f'(x)$ is least.]

14. Determine whether the following function is increasing or decreasing in the given interval: $f(x) = \cos\left(2x + \frac{\pi}{4}\right), \frac{3\pi}{8} \leq x \leq \frac{5\pi}{8}$.

15. Determine for which values of x, the function y=$x^4 - \frac{4x^3}{3}$ is increasing and for which it is decreasing.

16. Find the interval of increasing and decreasing of the function $f(x) = \frac{\log x}{x}$

17. Find the interval of increasing and decreasing of the function $f(x) = \sin x - \cos x,\ 0 < x < 2\pi$.

18. Show that $f(x) = x^2 e^{-x}, 0 \leq x \leq 2$ is increasing in the indicated interval.

19. Prove that the function $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

20. Find the intervals in which the following functionis decreasing.

$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$

21. Find the interval in which the function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}, x > 0$ is strictly decreasing.

22. Show that the function $f(x) = \tan^{-1}(\sin x + \cos x)$,is strictly increasing the interval $\left(0, \frac{\pi}{4}\right)$.

23. Find the interval in which the function $f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is increasing or decreasing.

24. Find the interval in which the function given by

$$f(x) = \frac{3x^4}{10} - \frac{4x^3}{5} - 3x^2 + \frac{36x}{5} + 11$$

(i) strictly increasing

(ii) strictly decreasing

25. Find the equation of the tangent to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$at the point $(\sqrt{2}a, b)$.

26. Find the equation of the tangent to the curve$y = x^2 - 2x + 7$ which is

(1) Parallel to the line $2x - y + 9 = 0$

(2) Perpendicular to the line $5y - 15x = 13$

27. Find the co-ordinates of the point on the curve $\sqrt{x} + \sqrt{y} = 4$ at which tangent is equally inclined to the axes.

28. Find a point on the parabola $f(x) = (x - 3)^2$ where the tangent is parallel to the chord joining the points (3,0) and (4,1).

29. Find the equation of the normal to the curve $y = e^{2x} + x^2\ at\ x = 0$. also find the distance from origin to the line.

30. Show that the line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-x/a}$ at the point, where the curve intersects the axis of y.

31. At what point on the circle $x^2 + y^2 - 2x - 4y + 1 = 0$ the tangent is parallel to

(i) X – axis

(ii) Y – axis

32. Show that the equation of the normal at any point 'θ' on the curve$x = 3\cos\theta - cos^3\theta,\ y = 3\sin\theta - sin^3\theta$ is
4($y\ cos^3\theta - xsin^3\theta$)=3 sin 4 θ.

33. Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

34. For the curve $y = 5x - 2x^{3,}$ if x increases at the rate of 2 Units/sec. then how fast is the slope of the curve changing when x=3?

35. Find the condition for the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ to interest orthogonally.

36. Show that the curves $y = a^x$ and $y = b^x$, a > b > 0 intersect at an angle of $\tan^{-1}\left(\left|\frac{\log\left|\frac{a}{b}\right|}{1+\log \quad \log b}\right|\right)$

37. Find the equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3).

38. Find the equation of the normal at a point on the curve$x^2 = 4y$, which passes through the point (1, 2). Also find the equation of the corresponding tangent.

39. Find the point on the curve $9y^2 = x^3$ where the normal to the curve makes equal intercepts with the axes.

40. Show that the tangents to the curve $y = 2x^3 - 3$ at the point where x = 2 and x = -2 are parallel.

Use differentials to find the approximate value of (Ques. 41 to 46)

41. $(66)^{1/3}$

42. $\sqrt{401}$

43 $\sqrt{0.037}$

44 $\sqrt{25.3}$

45. $(3.968)^{3/2}$

46 $(26.57)^{1/3}$

47. Find the value of $log_{10}(10.1)$ given that $log_{10}\mathrm{e} = 0.4343$.

48. If the radius of a circle increases from 5 cm to 5.1 cm, find the increase in area.

49. If the side of a cube be increased by 0.1%, find the corresponding increase in the volume of the cube.

50. Find the approximate value of f(2.01) where $f(x) = x^3 - 4x + 7$.

51. Find the approximate value of $\frac{1}{\sqrt{25.1}}$, using differentials.

52. The radius of a sphere shrinks from 10 cm. to 9.8 cm. Find the approximately decrease in its volume.

53. Find the maximum and minimum values of $f(x) = \sin x + \frac{1}{2}\cos 2x$ in $\left[0, \frac{\pi}{2}\right]$.

54. Find the absolute maximum value and absolute minimum value of the following question $f(x) = \left(\frac{1}{2} - x\right)^2 + x^3$ in [-2, 2.5]

55. Find the maximum and minimum values of $f(x) = x^{50} - x^{20}$ in the interval [0, 1]

56. Find the absolute maximum and absolute minimum value of $f(x) = (x-2)\sqrt{x-1}$ in [1, 9]

57. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

FIVE MARKS QUESTIONS

1. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}\,r$.

2. If the sum of length of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

3. Show that semi-vertical angle of a cone of maximum volume and given slant height is $cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

4. The sum of the surface areas of cuboids with sides x, 2x and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum if x = 3 radius of the sphere. Also find the minimum value of the sum of their volumes.

5. Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

6. Show that the cone of the greatest volume which can be inscribed in a given sphere has an altitude equal to $\frac{2}{3}$ of the diameter of the sphere.

7. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

8. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle $\alpha\ is\ \frac{4}{27}\pi h^3 tan^2\alpha.$ Also show that height of the cylinder is $\frac{h}{3}$

9. Find the point on the curve $y^2 = 4x$ which is nearest to the point (2,1).

10. Find the shortest distance between the line $y - x = 1$ and the curve$x = y^2$.

11. A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces, so that the combined area of the square and the circle is minimum?

12. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius r is $\frac{2r}{\sqrt{3}}$.

13. Find the area of greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

Answers

ONE MARK QUESTIONS

1. $\frac{\pi}{3}$
2. 1
3. $196\pi \frac{cm^3}{cm}$
4. $[0, \pi]$
5. $3, \frac{1}{3}$
6. (1,2)
7. $\frac{1}{e}$
8. $2\sqrt{ab}$
9. $(-\infty, 0)$
10. $a \leq 0$
11. $-\sqrt{2}$

13. 0
14. $\frac{\pi}{2}$
15. –1
16. 0 unit
17. $\frac{1}{2\sqrt{\pi}}$ unit

TWO MARKS QUESTIONS

1. $\left(\frac{1}{2}, 1\right)$
2. $\frac{1}{2}$
3. 89
4. $\frac{1}{\pi}$ units
5. $10\sqrt{3}\, cm^2 / s$
6. 2a%
7. 3k %
8. ⇓$\left(\frac{1}{4}, \frac{1}{2}\right)$
9. 0
10. ½
11. (2, 4)

13. (0, 0)
14. ¬ 4.042
15. sin 1, – sin 1
16. Local maxima at x = 1
    Local minima at x = 6
17. a = 2, b = –½
18. $\frac{xx_0}{a^2} - \frac{yy_0}{b^2}$
19. π cm^2
20. x + 3y = 0

THREE MARKS QUESTIONS

1. $\frac{1}{\pi}\, cm / s$
2. $\frac{3}{8\pi}\, cm / min$
4. 8 m/sec.
5. 3000 L/s
6. 3 km/h

7. $\frac{35}{88}$ m/h

9. $\frac{6}{49\pi}$ m/min.

10. $1 - 3x + 2x^2$

11. 8 cm^2/sec

13. 25

14. Increasing

15. Increasing for all $x \geq 1$
Decreasing for all $x \leq 1$

16. Increasing on (o, e)
Decreasing on [e, ∞)

17. Increasing on
$\left(\text{O}, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$
Decreasing on $[\frac{3\pi}{4}.\frac{7\pi}{4}]$

20. $(-\infty, 1]\text{U}[2, 3]$

21. [1,∞]

23. increasing on [0, ∞)
Decreasing (-∞, 0]

24. (i) Strictly increasing
[-2,1] U [3, ∞)
(ii) Strictly decreasing
(-∞, -2] U [1, 3]

25. $\sqrt{2}bx - ay - ab = 0$

26. (1) $y - 2x - 3 = 0$
(2) $36y + 12x - 227 = 0$

27. (4, 4)

28. $\left(\frac{7}{2}, \frac{1}{4}\right)$

29. $2y + x - 2 = 0, \frac{2}{\sqrt{5}}$

31. (i) (1, 0) and (1, 4)
(ii) (3, 2) and (-1, 2)

34. decrease 72 units/sec.

35 $a^2 = b^2$

37. $2x + 3my - 3a\,m^4 - 2am^2 = 0$

38. $x + y = 3, \qquad y = x - 1$

39 $\left(4, \pm\frac{8}{3}\right)$

41. 4.042

42. 20.025

43. 0.1924

44. 5.03

45. 7.904

46. 2.984

47. 1.004343

48. $\pi\ cm^2$

49. 0.3%

50. 7.08

51. 0.198

52. $80\pi cm^3$

53. max. value =$\frac{3}{4}$, mim value =$\frac{1}{2}$

54. ab. Max.= $\frac{157}{8}$, ab. Min. = $\frac{-7}{4}$

55. max.value=0,

min.value = $\frac{-3}{5}\left[\frac{2}{5}\right]^{2/3}$

56. ab. Max = 14 at x = 9

ab. Min.= $\frac{-3}{4^{4/3}}$ at $x = \frac{5}{4}$

57. π

FIVE MARKS QUESTIONS

4. $18r^3 + \frac{4}{3}\pi r^3$

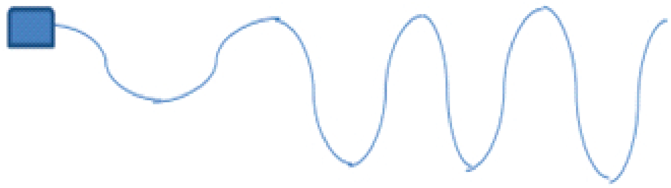
9. (1, 2)

10. $\frac{3\sqrt{2}}{8}$

11. $\frac{144}{\pi+4}m, \frac{36\pi}{\pi+4}m$

13. 2ab sq. Units.

APPLICATION OF DERIVATIVES
CASE STUDY QUESTION ON



Q .No 1. Raman is playing with a spring by throwing in the air which is moving along the function $f(x)=3x^4-4x^3-12x^2+5$

From the given study answer the following questions.

(i) Find the critical point of x which touches the x- axis :

(a) 0,1,2 (b) 0,-1,2

(c) 0,1,-2 (d) 1,2,-2

(ii) Find the interval in which spring is strictly increasing:

(a) $(-1,0) \cup (2,\infty)$ (b) $(1,0) \cup (2,\infty)$

(c) $(1,\infty)$ (d) $(0,\infty)$

(iii) Find the interval in which spring is strictly decreasing:

(a) $(-\infty,-1) \cap (0,2)$ (b) $(-\infty,-1) \cup (0,2)$

(c) $(-\infty,2)$ (d) $(-1,2)$

(iv) Find the value of x at which spring has local maxima

(a) 2 (b) -1

(c) 1 (d) 0

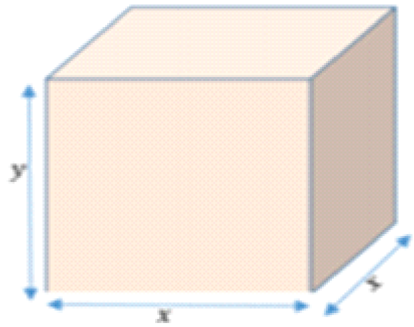
(v) What is the maximum height caused by the spring?

(a) 10 (b) 72

(c) 5 (d) 0

Attempt any four parts from each question. Each part caries one mark

Q.No 2. During the Deepawali festival season famousbrand of Sweets Company design a metal box for its products to be gift packed as shown below. The metal box with a square base and vertical sides is to contain 1024 cm^3. The material for the top and bottom costs ₹ 5 per cm^2 and the material for the sides costs ₹ 2.50 per cm^2



Based on the above information answer the following:

(i) If x represents the side of a square base and y the length of vertical sides, then the cost of making the top and bottom is

(a) ₹x2

(b) ₹$2x^2$

(c) ₹$5x^2$

(d) ₹$10x^2$

(ii) Area of the four sides is :

(a) $4y^2 cm^2$

(b) $4x^2 cm^2$

(c) $4xy\ cm^2$

(d) $4x^2 y^2\ cm^2$

(iii) If x represents the side of the square base and y the length of vertical sides, then the relation between the variables is :

(a) $\frac{1024}{x}$ (b) $\frac{1024}{x^2}$

(c) 1024x (d) $1024x^2$

(iv) Cost of making box C expressed as function of x is :

(a) $C=10x^2+\frac{1024}{x}$ (b) $C=5x^2+\frac{1024}{x}$

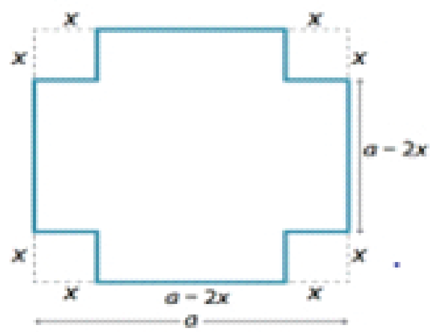
(c) $C=10x^2+\frac{10240}{x^2}$ (d) $C=5x^2+\frac{1024}{x^2}$

(v) The company decided to make the box with least cost. For this to happen the side of the square base x should be

(a) 2 cm (b) 5 cm

(c) 8 cm (d) 16 cm

Q.No 3. A Square sheet of cardboard with each side 'a' cm is to be used to make an open top box by cutting a small square of cardboard from each of the corners and bending up the sides.



(a) Volume V cm^3 of the box is given by :

(i) $V(x)=4ax^3-4x^2+a^2x$ (ii) $V(x)=4x^3-4ax^2+a^2x$

(iii) $V(x)=4a^3-4x^2+a^2$ (iv) $V(x)=4ax^3+4x^2+a^2x$

(b) Perimeter of the given net of box is given by:

(i) P(x)=8x-2a (ii) 2x

(iii) 8a-2x (iv) 4a

(c) The rate of change of Volume with respect to 'x' is given by:

(i) $12ax^2-8x+a^2$ (ii) $12ax^2-8ax+a^2$

(iii) -8x (iv) $12ax^2+8x+a^2$

(d) The local maximum volume of the box at x is given by:

(a) $\frac{a}{3}$ (b) $\frac{a}{6}$

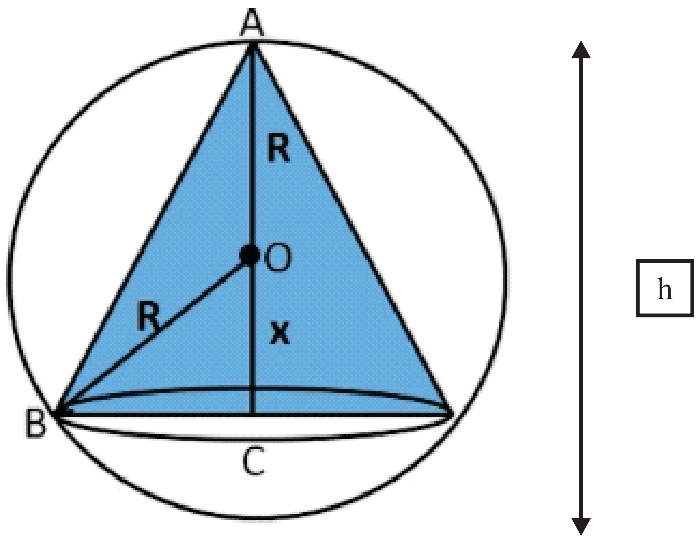
(c) $\frac{3}{a}$ (d) $\frac{6}{a}$

(e) The maximum volume of the box at a=3 is given by :

(i) 4 (ii) 6

(iii) 27 (iv) 2

Q. No 4. There is a right circular cone of maximum volume that can be inscribed in a sphere of radiu r.



Based on the above information answer the following questions.

(i) What is the volume of cone (V) ?

(a) $\frac{1}{3}\pi(-h^3+2h^2r)$

(b) $\frac{1}{2}\pi(-h^3+2h^2r)$

(c) $\frac{1}{4}\pi(-h^3+2h^2r)$

(d) $\frac{1}{5}\pi(-h^3+2h^2r)$

(iv) What is relation between h and r

(a) 2h=4r

(b) 3h=4r

(c) 2h=3r

(d) 3h=2r

(v) What is the value of OD ?

(a) r-h

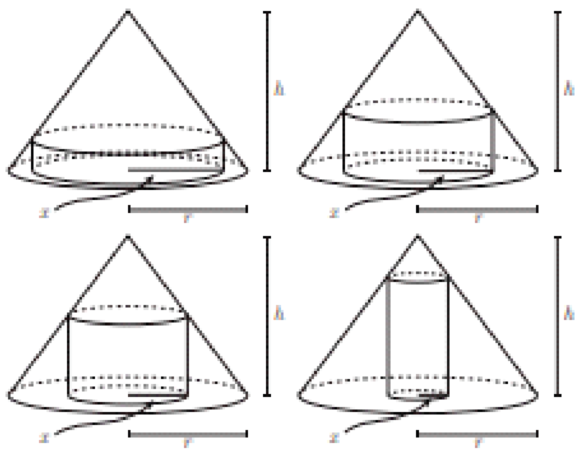
(b) h-r

(c) r- h/2

(d) h-r/2

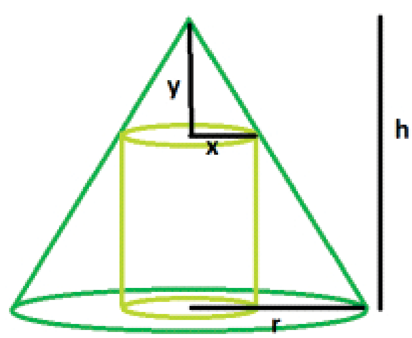
Question 5

In the following picture, you see that there can be various types of cylinders inscribed in a fixed cone of height h and radius r



(i) State whether true or false: The volume of all such cylinders will be same.

(ii) Let x be the radius of the cylinder and y be the distance from the top of the cone to the top of the inscribed cylinder. Then the relation between h, r, x and y is:



(a) $\frac{y}{h} = \frac{x}{r}$ (b) $\frac{h-y}{h} = \frac{x}{r}$

(c) $\frac{y}{h-y} = \frac{r-x}{r}$ (d) none of these

(iii) The function for volume of such a cylinder inscribed in a cone is:

(a) $V(x)=\pi r^2 h$

(b) $V(x)=\pi r^2 y$

(c) $V(x)=\pi x^2 y$

(d) $V(x)=\pi x^2 h(1-\frac{x}{r})$

(iv) A relation of curved surface of the cylinder in terms of radius of cylinder as one variable only is

(a) $C(x)=\pi rh$ (b) $C(x)=\frac{\pi rh}{r}(r-x)$

(b) $C(x) = 2\pi xy$ (b) $C(x) = 2\pi x^2 y$

(v) The curved surface area of the cylinder will be maximum when x is equal to

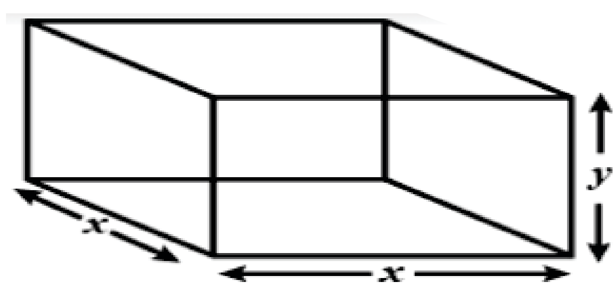
(a) $\frac{r}{3}$ (b) $\frac{r}{4}$

(c) $\frac{r}{2}$ (d) $\frac{r}{5}$

Question 6-

Shashank wants to make project for State Level science exhibition. For this he wants to make metal box with square base and vertical sides to contain 1024 cm^3 of water. Material for top and bottom costs ₹ 5 per cm^2 and material for sides costs ₹ 2.50 per cm^2.

Based on the information above answer the following:



(i) What will be the relation between x and y?

(a) $xy^2=1024$ (b) $x^2+4xy=1024$

(c) $x^2 y=1024$ (d) $2x^2+4xy=1024$

(ii) What will be the toatal cost (C) of the material used to construct the box?

(a) $C=5x^2+20xy$ (b) x^2+4xy

(c) $10x^2+10xy$ (d) None of these

(iii) What will be total cost(C) of the box in terms of x ?

(a) $C=5x^2+\frac{10240}{x}$

(b) $C=10x^2+\frac{10240}{x}$

(a) $C=x^2+\frac{10240}{x}$

(a) $C=20x\ \frac{10240}{x}$

(iv) What should be the the dimensions of the box to minimize the cost ?

(a) X=16, y=8

(b) X=8, y=16

(c) X=8, y=8

(d) X=8, y=4

(v) What is the least cost of the box

(a) ₹1620

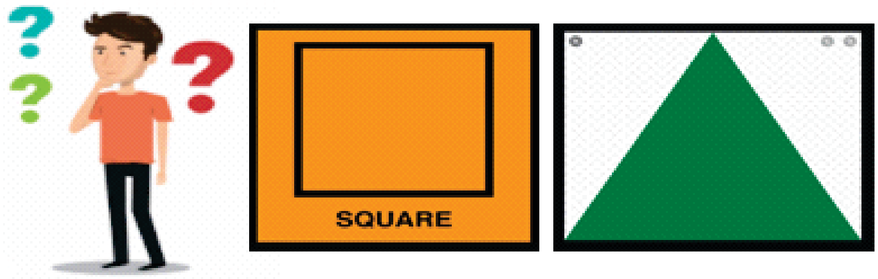
(b) ₹1024

(c) ₹1920

(d) ₹ 1780

Question 7–

Pradeep who is a student of a reputed public school is making a project which his maths teacher has given . He has a piece of wire of length 36 cm which he has cut into two pieces . He is turning one piece to form a square and other to form an equilateral triangle. In order to find the length of each piece so that sum of areas of two be minimum he is facing problem. He discussed and ask his friend Gajender to provide him the solution. Based on the above informationhelp Gajender to answer the following.



(i) The perimeter of square is x cm ; then each side of square is :

(a) x cm

(b) 9 cm

(c) $\frac{\pi}{4}$ cm

(d) 36 cm

(ii) Let the perimeter of square be x cm then the length of each side of an equilaterial triangle is :

(a) (36-x) cm

(b) $(12-\frac{x}{3})$ cm

(c) $\frac{\pi}{3}$ cm

(d) 12 cm

(iii) If x is the perimeter of the square then sum of area of two figures is:

(a) $x^2-2\sqrt{3}x+36\sqrt{3}$

(b) $\left(\frac{\sqrt{3}}{36}+\frac{1}{8}\right)x^2-2\sqrt{3}x$

(c) $\left(\frac{\sqrt{3}}{36}+\frac{1}{16}\right)x^2-2\sqrt{3}x+36\sqrt{3}$

(d) $\left(\frac{\sqrt{3}}{36}+\frac{1}{16}\right)x^2-4\sqrt{3}x+36\sqrt{3}$

(iv) What is the length of square piece if the sum of areas of two figures be minimum ?

(a) $\frac{144\sqrt{3}}{4\sqrt{3}+9}$ cm (b) $\frac{144}{4\sqrt{3}+9}$ cm

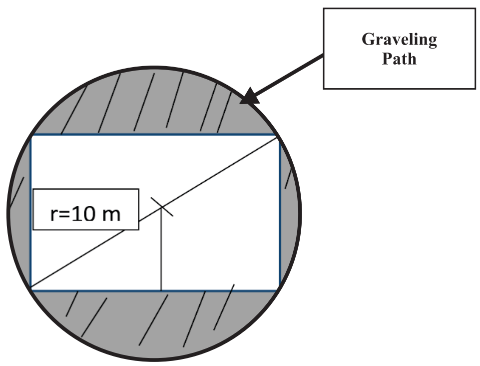
(c) $\frac{144\sqrt{3}}{4\sqrt{3}}$ cm (d) $\frac{72\sqrt{3}}{4\sqrt{3}+9}$ cm

(v) The side of an equilateral triangle is :

(a) $\frac{108}{4\sqrt{3}+3}$ cm (b) $\frac{108}{4\sqrt{3}+6}$ cm

(c) $\frac{54}{4\sqrt{3}+9}$ cm (b) $\frac{108}{4\sqrt{3}+9}$ cm

Question8- An architect design a garden for Central Greenery between flats for the Privatesociety at Faridabad. The garden Is in the shape of rectangle inscribed in a circle of radius 10m as shown in given figure. Based on the above information answer the following:



(i) 2x and 2y represents the length and breadth of the rectangular path Then relation between the variable is:

(a) $x^2-y^2=10$ (b) $x^2+y^2=10$

(c) $x^2-y^2=100$ (d) $x^2+y^2=100$

(ii) The area of the green grass A expressed as function of x is ;

(a) $2x\sqrt{100-x^2}$ (b) $4x\sqrt{100-x^2}$

(c) $2x\sqrt{100+x^2}$ (b) $4x\sqrt{100+x^2}$

(iii) The maximum value of area is :

(a) $100m^2$ (b) $200m^2$

(c) $400m^2$ (d) $1600m^2$

(iv) The value of length of rectangle ,if A is maximum is :

(a) $10\sqrt{2}$m (b) $20\sqrt{2}$m

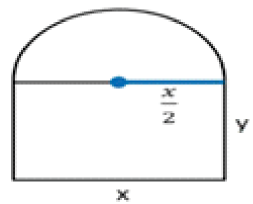
(c) 20m (b) $5\sqrt{2}$m

(v) The area of gravelling path is :

(a) $100(\pi+2)\ m^2$ (b) $100(\pi-2)\ m^2$

(c) $200(\pi+2)\ m^2$ (d) $200(\pi-2)\ m^2$

Question- 9 An architect design a window for MNC in the form of a rectanglesurmounted by a semi circle. The total perimeter of the window is 10 m. Based on the above information answer the following:



(i) The perimeter of window in terms of x and y is

(a) $2x+2y+\frac{\pi x}{2}$ (b) $x2+y+\frac{\pi x}{2}$

(c) $2x+y+\frac{\pi x}{2}$ (d) $x+2y+\pi x$

(ii) The value of y in terms of π and x:

(a) $10-\left(\frac{\pi+2}{4}\right)x$ (b) $5-\left(\frac{\pi+2}{2}\right)x$

(c) $5-\left(\frac{\pi+2}{4}\right)x$ (d) None of these

(iii) Area of the window through which light enters is :

(a) $xy+\frac{1}{2}\pi\left(\frac{\pi}{2}\right)^2$ (b) $xy+\pi\left(\frac{\pi}{2}\right)^2$

(c) $xy+\frac{1}{2}\pi x^2$ (d) $2xy+\frac{1}{2}\pi\left(\frac{\pi}{2}\right)^2$

(iv) For maximum light, x should be :

(a) $\frac{10}{\pi+4}$ (b) $\frac{20}{\pi+4}$

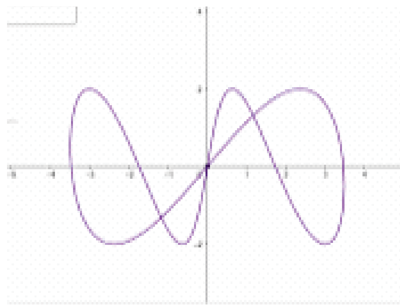
(c) $\frac{10}{\pi+2}$ (d) $\frac{20}{\pi+2}$

(v) For maximum light light height of the window is:

(a) $\frac{10}{\pi+4}$ (b) $\frac{20}{\pi+4}$

(c) $\frac{30}{\pi+4}$ (d) None of these

Question 10- A curve is given in parametric form as $x = a\cos^3\theta$ $y = a\sin^3\theta$



Sweety is confused in solving above problem to know few facts she talked to her friend Suman to answer the correct options for following questions help her to solve:

(i) Which one is slope of tangent to this curve ?

(a) $\tan\theta$ (b) $-\tan\theta$

(c) $\cot\theta$ (d) $-\cot\theta$

(ii) What is slope of tangent at $\theta=\pi \frac{\pi}{4}$?

(a) 0 (b) 1

(c) -1 (d) None of these

(iii) Which one is slope of normal to this curve ?

(a) $\tan\theta$ (b) $-\tan\theta$

(c) $\cot\theta$ (d) $-\cot\theta$

(iv) What is slope of normal at $\theta= \frac{\pi}{4}$

(a) 0 (b) 1

(c) -1 (d) None of these

(v) Equation of tangent at $\theta=\pi \frac{\pi}{4}$ is given by :

(a) $\sqrt{2}x+\sqrt{2}y+a=0$

(b) $\sqrt{2}x+\sqrt{2}y-a=0$

(c) $\sqrt{2}x+\sqrt{2}y+a=0$

(d) $\sqrt{2}x+\sqrt{2}y-a=0$

ANSWERS OF CASE STUDY BASED QUESTIONS APPLICATION OF DERIVATIVES

Q No 1

(i) b (ii) a (iii) b (iv) d (v) c

Q No 2

(i) d (ii) c (iii) b (iv) c (v) c

Q No 3

(a) ii) (b) iv (c) ii (d) ii (e) iv

Q No 4

(i) a (ii) c (iii) c (iv) b (v) b

Q No 5

(i) true (ii) a (iii) d (iv) b (v) c

Q No 6

(i) c (ii) c (iii) b (iv) b (v) c

Q No 7

(i) c (ii) b (iii) c (iv) a (v) d

Q No 8

(i) d (ii) b (iii) b (iv) a (v) b

Q No 9

(i) b (ii) c (iii) a (iv) b (v) a

Q No 10

(i) b (ii) c (iii) c (iv) b (v) d

CHAPTER 7

INTEGRALS

POINTS TO REMEMBER

- Integration or anti derivative is the reverse process of Differentiation.
- Let $\frac{d}{dx}F(x) = f(x)$ then we write $\int f(x)\,dx = F(x) + c$.
- These integrals are called indefinite integrals and c is called constant of integration.
- From geometrical point of view, an indefinite integral is the collection of family of curves each of which is obtained by translating one of the curves parallel to itself upwards or downwards along y-axis.

STANDARD FORMULAE

1. $\int x^n\,dx = \begin{cases} \frac{x^{n+1}}{n+1} + c & n \neq -1 \\ \log_e|x| + c & n = -1 \end{cases}$

2. $\int (ax+b)^n\,dx = \begin{cases} \frac{(ax+b)^{n+1}}{(n+1)a} + c & n \neq -1 \\ \frac{1}{a}\log|ax+b| + c & n = -1 \end{cases}$

3. $\int \sin x\,dx = -\cos x + c.$

4. $\int \cos x\,dx = \sin x + c$

5. $\int \tan x.\,dx = -\log|\cos x| + c = \log|\sec x| + c.$

6. $\int \cot x\,dx = \log|\sin x| + c.$

7. $\int \sec^2 x\,dx = \tan x + c$

8. $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

9. $\int \sec x \tan x\, dx = \sec x + c$

10. $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

11. $\int \sec x\, dx = \log|\sec x + \tan x| + c$

$$= \log\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + c$$

12. $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + c$

$$= \log\left|\tan\frac{x}{2}\right| + c$$

13. $\int e^x\, dx = e^x + c$

14. $\int a^x\, dx = \frac{a^x}{\log a} + c$

15. $\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + c, |x| < 1$

$$= -\cos^{-1} x + c$$

16. $\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + c$

$$= -\cot^{-1} x + c$$

17. $\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + c, |x| > 1$

$$= -\operatorname{cosec}^{-1} x + c$$

18. $\int \frac{1}{a^2-x^2}\, dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

19. $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

20. $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

21. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a} + c = -cos^{-1}\frac{x}{a} + c$

22. $\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \log\left|x + \sqrt{a^2+x^2}\right| + c$

23. $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x + \sqrt{x^2-a^2}\right| + c$

24. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

25. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x + \sqrt{a^2+x^2}\right| + c$

26. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + c$

RULES OF INTEGRATION

1. $\int [(f_1(x) \pm f_2(x) \pm \ldots\ldots \pm f_x(x)]\,dx = \int f_1(x)dx \pm \int f_2(x)dx \pm \ldots\ldots \pm \int f_x(x)dx$

2. $\int k.f(x)dx = k\int f(x)dx.$

3. $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$

INTEGRATION BY SUBSTITUTION

1. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$

2. $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c$

3. $\int \frac{f'(x)}{[f(x)]^n} dx = \frac{(f(x))^{-n+1}}{-n+1} + c$

INTEGRATION BY PARTS

$$\int f(x)\, g(x)\, dx = f(x) \int g(x)\, dx - \int \left[f'(x) \int g(x)\, dx \right]$$

DEFINITE INTEGRALS

$$\int_a^b f(x)dx = F(b) - F(a), \text{ where } F(x) = \int f(x)\, dx$$

DEFINITE INTEGRAL AS A LIMIT OF SUMS.

$$\int_a^b f(x)dx = \lim_{h \to o} h\, [f(a) + f(a+h) + f(a+2h) + \cdots + f(a+n-1h)]$$

$Where\ h = \frac{b-a}{h}$ $or \int_a^b f(x)dx = \lim_{h \to o}[h \sum_{r=1}^{n} f(a+rh)]$

PROPERTIES OF DEFINITE INTEGRAL

1. $\int_a^b f(x) = -\int_b^a f(x)dx$

2. $\int_a^b f(x)\, dx = \int_a^b f(t)\, dt.$

3. $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx.$

4. $(i) \int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx.$

(ii) $\int_{0}^{a} f(x)\,dx = \int_{0}^{a} f(a-x)dx$

5. $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx, \quad \textit{if } f(x) \textit{ is even function}$

6. $\int_{-a}^{a} f(x)dx = 0 \textit{ if } f(x) \textit{ is an odd function}$

7. $\int_{0}^{2a} f(x)dx = \begin{cases} 2\int_{0}^{a} f(x)dx, & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$

ONE MARK QUESTIONS

Evaluate the following integrals:

1. $\int\left(\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}\right)\,dx$

2. $\int_{-1}^{1} e^{|x|}\,dx$

3. $\int \frac{dx}{1-\sin^2 x}$

4. $\int_{-1}^{1} x^{99}\cos^4 x\,dx$

5. $\int \frac{1}{x\,\log x \log(\log x)}dx$

6. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log\left(\frac{1+x}{1-x}\right)dx$

7. $\int\left(e^{a\log x} + e^{x\log a}\right)dx$

8. $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}dx$

9. $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$

10. $\int \sqrt{10 - 4x + x^2} \; dx$

11. $\int_{-1}^{1} x^3 |x| \, dx$

12. $\int \frac{1}{\sin^2 x \; \cos^2 x} \; dx$

13. $\int_{-2}^{2} \frac{dx}{1+|x-1|}$

14. $\int e^{-\log x} \, dx$

15. $\int \frac{e^x}{a^x} \; dx$

16. $\int \frac{x}{\sqrt{x+1}} dx$

17. $\int \frac{x}{(x+1)^2} \; dx$

18. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \; dx$

19. $\int \cos^2 \; \alpha \, dx$

20. $\int \frac{1}{x \cos \alpha + 1} \; dx$

21. $\int \sec x \log(\sec x + \tan x) \; dx$

22. $\int \frac{1}{\cos \alpha + x \; \sin \alpha} dx$

23. $\int \frac{\sec^2 (\log x)}{x} \; dx$

24. $\int \frac{e^x}{\sqrt{4+e^{2x}}} \; dx$

25. $\int \frac{1}{x(2+3 \; \log x)} \; dx$

26. $\int \frac{1-\sin x}{x+\cos x} dx$

27. $\int \frac{1-\cos x}{\sin x}\, dx$

28. $\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x}\, dx$

29. $\int \frac{(x+1)}{x}(x+\log x)\, dx$

30. $\int_0^{\pi} |\cos x|\, dx$

31. $\int_0^2 [x]\, dx$ *where* $[x]$ *is greatest integers function.*

32. $\int \frac{1}{\sqrt{9-4x^2}}\, dx$

33. $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}\, dx$

34. $\int_{-2}^{1} \frac{|x|}{x}\, dx$

35. $\int_{-1}^{1} x\,|x|\, dx$

36. $\int x\sqrt{x+2}\, dx$

37. $\int_a^b f(x)\, dx + \int_b^a f(x)\, dx$

38. $\int \frac{\sin x}{\sin 2x}\, dx$

39. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\sin x|\, dx$

40. $\int \frac{1}{\sec x+\tan x}\, dx$

41. $\int \frac{\sin^2 x}{1+\cos x}\, dx$

42. $\int \frac{1-\tan x}{1+\tan x}\, dx$

TWO MARKS QUESTIONS

Evaluate :

1. $\int e^{[\log(x+1)-\log x]}dx$

2. $\int \frac{1}{\sqrt{x+1}+\sqrt{x+2}}dx$

3. $\int \sin x \sin 2x dx$

4. $\int \left[\frac{x}{a}+\frac{a}{x}+x^a+a^x\right]dx$

5. $\int_0^{\pi/2} \text{Log}\left(\frac{5+3\cos x}{5+3\sin x}\right)dx$

6. $\int \frac{a^x+b^x}{c^x}dx$

7. $\int \left(\sqrt{ax}-\frac{1}{\sqrt{ax}}\right)^2 dx$

8. $\int e^x 2^x dx$

9. $\neq 2^{2^{2^x}} 2^{2^x} 2^x dx$

10. $\int \frac{\sin\left(2\tan^{-1}x\right)}{1+x^2}dx$

11. $\neq x \log 2x \, dx$

12. $\int_0^{\pi/4} \sqrt{1+\sin 2x dx}$

13. $\int_0^{\pi/2} e^x \left(\sin x-\cos x\right)dx$

14. $\int_4^9 \frac{\sqrt{x}}{\left(30-x^{3/2}\right)^2}dx$

15. $\int_0^1 \frac{dx}{e^x+e^{-x}}dx$

16. $\neq \frac{\log|\sin x|}{\tan x}dx$

17. $\int \frac{\sin^4 x+\cos^4 x}{\sin^3 x+\cos^3 x}dx$

18. $\neq \sqrt{\tan x}\,(1+\tan^2 x)dx$

19. $\int \frac{\sin 2x}{\left(a+b\cos x\right)^2}dx$

20. $\int \frac{x^2-x+2}{x^2+1}dx$

THREE MARKS QUESTIONS

Evaluate :

1. (i) $\int \frac{x \, cosec\,(\tan^{-1} x^2)}{1+x^4}\,dx$

   (ii) $\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\,dx$

   (iii) $\int \frac{1}{\sin(x-a)\sin(x-b)}\,dx$

(iv) $\int \frac{\cos(x+a)}{\cos(x-a)}\,dx$

(v) $\int \cos 2x\ \cos 4x\ \cos 6x\ dx$

(vi) $\int \tan 2x \tan 3x \tan 5x\, dx$

(vii) $\int \sin^2 x \cos^4 x\ dx$

(viii) $\int \cot^3 x\, \mathrm{cosec}^4 x\, dx$

(ix) $\int \frac{\sin x \cos x}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}}\, dx$ [Hint: Put $a^2 \sin^2 x + b^2 \cos^2 x = t \text{ or } t^2$

(x) $\int \frac{1}{\sqrt{\cos^3 x\ \cos(x+a)}}\ dx$

(xi) $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}\ dx$

(xii) $\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\ dx$

Evaluate :

(i) $\int \frac{x}{x^4 + x^2 + 1}\, dx$

(ii) $\int \frac{1}{x[6(\log x)^2 + 7 \log x + 2]}\ dx$

(iii) $\int \frac{1}{\sqrt{\sin^3 x\ \cos^5 x}}\ dx$

(iv) $\int \frac{x^2 + 1}{x^4 + 1}\, dx$

(v) $\int \frac{1}{\sqrt{(x-a)(x-b)}}\, dx$

(vi) $\int \frac{5x-2}{3x^2+2x+1}\, dx$

(vii) $\int \frac{x^2}{x^2+6x+1}\, dx$

(viii) $\int \frac{x+2}{\sqrt{4x-x^2}} dx$

(ix) $\int x\sqrt{1+x-x^2}\, dx$

(x) $\int \frac{\sin^4 x}{\cos^8 x}\, dx$

(xi) $\int \sqrt{\sec x - 1}\, dx$ [Hint: Multiply and divided by $\sqrt{\sec x + 1}$]

Evaluate :

3. (i) $\int \frac{dx}{x(x^7+1)}$

(ii) $\int \frac{3x+5}{x^3-x^2-x+1}\, dx$

(iii) $\int \frac{\sin\theta\cos\theta}{\cos^2\theta - \cos\theta - 2}\, d\theta$

(iv) $\int \frac{dx}{(2-x)(x^2+3)}$

(v) $\int \frac{x^2+x+2}{(x-2)(x-1)}\, dx$

(vi) $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$

(vii) $\int \frac{dx}{(2x+1)(x^2+4)}$

(viii) $\int \frac{x^2-1}{x^4+x^2+1} dx$

(ix) $\int \sqrt{\tan x}\, dx$

(x) $\int \frac{dx}{\sin x - \sin 2x}$

4. Evaluate:

(i) $\int x^5 \sin x^3 \, dx$

(ii) $\int \sec^3 x \, dx$

(iii) $\int e^{ax} \cos(bx + c) \, dx$

(iv) $\int \sin^{-1}\left(\frac{6x}{1+9x^2}\right) dx$ [Hint: Put $3x=\tan\theta$]

(v) $\int \cos\sqrt{x}\, dx$

(vi) $\int x^3 \tan^{-1} x \, dx$

(vii) $\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right) dx$

(viii) $\int \left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right] dx$

(ix) $\int \sqrt{2ax - x^2}\, dx$

(x) $\int e^x \frac{(x^2+1)}{(x+1)^2} dx$

(xi) $\int x^3 \sin^{-1}\left(\frac{1}{x}\right) dx$

(xii) $\int \left\{\log(\log x) + \frac{1}{(\log x)^2}\right\} dx$ [Hint: Put $\begin{matrix} \log x = t \\ x = e^t \end{matrix}$]

(xiii) $\int (6x + 5)\sqrt{6 + x - x^2}\, dx$

(xiv) $\int \frac{1}{x^3+1} dx$

(xv) $\int \tan^{-1}\left(\frac{x-5}{1+5x}\right) dx$

(xvi) $\int \frac{dx}{5+4\cos x}$

5. Evaluate the following definite integrals:

(i) $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16 \sin 2x} dx$

(ii) $\int_0^{\pi/2} \cos 2x \log \sin x\, dx$

(iii) $\int_0^1 x\sqrt{\frac{1-x^2}{1+x^2}} dx$

(iv) $\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$

(v) $\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

(vi) $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right) dx$

(vii) $\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$

(viii) $\int_0^1 x \log\left(1+\frac{x}{2}\right)\, dx$

(ix) $\int_{-1}^{1/2} |x \cos \pi x|\, dx$

(x) $\int_{-\pi}^{\pi} (\cos a\, x - \sin b\, x)^2\, dx$

6. Evaluate:

(i) $\int_2^5 [|x-2| + |x-3| + |x-4|]\, dx$

(ii) $\int_0^{\pi} \frac{x}{1+\sin}\, dx$

(iii) $\int_{-1}^{1} e^{\tan^{-1} x} \left[\frac{1+x+x^2}{1+x^2}\right] dx$

(iv) $\int_0^{\pi} \frac{x\ \sin x}{1+\cos^2 x}\, dx$

(v) $\int_0^2 [x^2]\, dx$

(vi) $\int_0^{\pi/2} \frac{x \sin x\ \cos x}{\sin^4 x + \cos^4 x}\, dx$

(vii) $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x}\, dx$ $[\mathit{Hint: use}\ \int_0^a f(x)dx = \int_0^a f(a-x)dx$

7. Evaluate the following integrals:

(i) $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

(ii) $\int_{-\pi/2}^{\pi/2} \left(\sin|x| + \cos|x|\right) dx$

(iii) $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\, dx$

(iv) $\int_0^{\pi} \frac{x \tan x}{\sec x + \operatorname{cosec} x}\, dx$

(v) $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$

8. Evaluate

(i) $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\, dx \quad x \in [0, 1]$

(ii) $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\, dx$

(iii) $\int \frac{x^2 e^x}{(x+z)^2} dx$

(iv) $\int \frac{x^2}{(x \sin x + \cos x)^2}\, dx$

(v) $\int \sin^{-1} \sqrt{\frac{x}{a+x}}\, dx$

(vi) $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin\ 2x}}\, dx$

(vii) $\int \frac{\sin x}{\sin 4x}\, dx$

(viii) $\int_{-1}^{3/2} |x \ \sin \pi x| \ dx$

(ix) $\int \frac{\sin(x-a)}{\sin(x+a)} \ dx$

(x) $\int \frac{x^2}{(x^2+4)(x^2+9)} \ dx$

(xi) $\int \frac{\cos 5x + \cos 4x}{1 - 2\cos 3x} \ dx$

FIVE MARKS QUESTIONS

9. Evaluate the following integrals:

(*i*) $\int \frac{x^5+4}{x^5-x} \ dx$

(ii) $\int \frac{2e^t}{e^{3t} - 6e^{2t} + 11e^t - 6} \ dt$

(iii) $\int \frac{2x^3}{(x+1)(x-3)^2} \ dx$

(iv) $\int \frac{1 + \sin x}{\sin x \ (1 + \cos x)} \ dx$

(v) $\int_0^{\pi/2} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

(vi) $\int_0^1 x \sqrt{\frac{1-x^2}{1+x^2}} dx$

(vii) $\int_0^{\pi/2} \frac{\cos x}{1 + \cos x + \sin x}\, dx$

10. Evaluate the following integrals as limit of sums:

(i) $\int_2^4 (2x + 1)\, dx$

(ii) $\int_0^2 (x^2 + 3)\, dx$

(iii) $\int_1^3 (3x^2 - 2x + 4)\, dx$

(iv) $\int_0^4 (3x^2 + e^{2x})\, dx$

(v) $\int_0^1 e^{2-3x}\, dx$

(vi) $\int_0^1 (3x^2 + 2x + 1)\, dx$

11. Evaluate:

(i) $\int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)}$

(ii) $\int_0^1 \frac{\log(1 + x)}{1 + x^2}\, dx$

(iii) $\int_0^{\pi/2} (2\log \sin x - \log \sin 2x)\, dx$

12. $\int_{0}^{1} x(\tan^{-1} x)^2 \ dx$

13. $\int_{0}^{\pi/2} \log \sin x \ dx$

14. $Prove\ that \int_{0}^{1} \tan^{-1}\left(\frac{1}{1-x+x^2}\right) dx = 2 \int_{0}^{1} \tan^{-1} x \, dx$

Hence or otherwise evaluate the integral $\int \tan^{-1}(1-x+x^2)\, dx$.

15. $Evaluate \int_0^{\pi/2} \frac{sin^2 x}{\sin x + \cos x} dx.$

Answers

ONE MARKS QUESTIONS

1. $\frac{\pi}{2}x + c$
2. $2e - 2$
3. tan x + c
4. 0
5. $\log|\log|\log x|| + c$
6. 0
7. $\frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$
8. $\tan x + c$
9. 0
10. $\frac{(x-2)\sqrt{x^2-4x+10}}{2} + 3\log\left|(x-2) + \sqrt{x^2-4x+10}\right| + c$
11. 0
12. $\tan x - \cot x + c$
13. $3 \log_e 2$
14. $\log|x| + c$

15. $\frac{\left(\frac{e}{a}\right)^x}{\log\left(\frac{e}{a}\right)} + c$

16. $\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + c$

17. $\log|x+1| + \frac{1}{x+1} + c$

18. $2e^{\sqrt{x}} + c$

19. $x\cos^2\alpha + c$

20. $\frac{\log|x\ \cos\alpha + 1|}{\cos\alpha} + c$

21. $\frac{(\log|\sec x + \tan x|)^2}{2} + c$

22. $\frac{\log|\cos\alpha + x\sin\alpha|}{\sin\alpha} + c$

23. $\tan|\log x| + c$

24. $\log\left|e^x + \sqrt{4 + e^{2x}}\right| + c$

25. $\frac{1}{3}\log|2 + 3\log x| + c$

26. $\log|x + \cos x| + c$

27. $2\log\left|\sec\frac{x}{2}\right| + c$

28. $\frac{1}{e}\log|x^e + e^x| + c$

29. $\frac{(x+\log x)^2}{2} + c$

30. 2

31. 1

32. $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + c$

33. $\frac{b-a}{2}$

34. –1

35. 0

36. $\frac{2}{5}(x+2)^{5/2} -$
$\frac{4}{3}(x+2)^{3/2} + c$

37. 0

38. $\frac{1}{2}\log|\sec x + \tan x| + c$

39. $2-\sqrt{2}$

40. $\log|1 + \sin x| + c$

41. $x - \sin x + c$

42. $\log|\cos x + \sin x| + c$

TWO MARKS QUESTIONS

1. x + log x + c

2. $\frac{2}{3}\left[(x+2)^{3/2}-(x+1)^{3/2}\right]+c$

3. $\frac{-1}{2}\left[\frac{\sin 3x}{3}-\sin x\right]+c$

4. $\frac{1}{a}\frac{x^2}{2}+a\log|x|+\frac{x^{a+1}}{a+1}+\frac{a^x}{\log a}+c$

5. 0

6. $\frac{\left(\frac{a}{c}\right)^x}{\log\left|\frac{a}{c}\right|}+\frac{\left(\frac{b}{c}\right)^x}{\log\left|\frac{b}{c}\right|}+c$

7. $\frac{ax^2}{2}+\frac{\log|x|}{a}-2x+c$

8. $\frac{2^x e^x}{\log(2e)}+c$

9. $\frac{2^{2^{2^x}}}{(\log^2)^3}+C$

10. $\frac{-\left[\cos 2\left(\tan^{-1}x\right)\right]}{2}+C$

11. $\frac{x^2}{2}\log 2x-\frac{x^2}{4}+C$

12. 1

13. 1

14. $\frac{19}{99}$

15. $\tan^{-1}e-\frac{\pi}{4}$

16. $\frac{\log|\sin x|^2}{2}+C$

17. log|secx + tan x|
+ log |cosec x – cot x| + C

18. $\frac{2}{3}(\tan x)^{3/2}+C$

19. $-\frac{2}{b^2}\left[\log|a+b\cos x|+\frac{a}{a+b\cos x}\right]+C$

20. $x-\frac{1}{2}\log\left|x^2+1\right|+\tan^{-1}x+C$

THREE MARKS QUESTIONS

1. (i) $\frac{1}{2}\log\left[\operatorname{cosec}(\tan^{-1}x^2)-\frac{1}{x^2}\right]+c$

(ii) $\frac{1}{2}\left(x^2-x\sqrt{x^2-1}\right)+\frac{1}{2}\log\left|x+\sqrt{x^2-1}\right|+c$

(iii) $\frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+c$

(iv) $x\cos 2a-\sin 2a\ \log|\sec(x-a)|+c$

(v) $\frac{3}{8}x-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+c$

(vi) $\frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + c$

(vii) $\frac{1}{32}\left[2x + \frac{1}{2}\sin 2x - \frac{1}{2}\sin 4x - \frac{1}{6}\sin 6x\right] + c$

(viii) $-\left(\frac{\cot^6 x}{6} + \frac{\cot^4 x}{4}\right) + c$

(ix) $\frac{1}{a^2-b^2}\sqrt{a^2\sin^2 x + b^2\cos^2 x} + c$

(x) $-2\operatorname{cosec} a\sqrt{\cos a - \tan x \sin a} + c$

(xii) $\tan x - \cot x - 3x + c$

(vi) $\sin^{-1}[\sin x - \cos x] + c$

2. (i) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + c$

(ii) $\log\left|\frac{2\log x}{3\log x}\right| + c$

(iii) $\frac{-2}{\sqrt{\tan x}} + \frac{2}{3}\tan^{3/2}x + c$

(iv) $\frac{1}{\sqrt{2}}\tan^{-1}\left\{\frac{1}{\sqrt{2}}\left(x - \frac{1}{x}\right)\right\} + c$

(v) $2\log\left|\sqrt{x-a} + \sqrt{x-b}\right| + c$

(vi) $\frac{5}{6}\log|3x^2 + 2x + 1| + \frac{-11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + c$

(vii) $x - 3\log|x^2 + 6x + 12| + 2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c$

(viii) $-\sqrt{4x-x^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)+c$

(ix) $-\frac{1}{3}(1+x-x^2)^{3/2}+\frac{1}{8}(2x-1)\sqrt{1+x-x^2}+\frac{5}{16}\sin^{-1}\left(\frac{2x-1}{\sqrt{5}}\right)+c$

(x) $\frac{\tan^5 x}{5}+\frac{\tan^7 x}{7}+c$

(xi) $-\log\left|\cos x+\frac{1}{2}+\sqrt{\cos^2 x+\cos x}\right|+c$

3. (i) $\frac{1}{7}\log\left|\frac{x^7}{x^7+1}\right|+c$

(ii) $\frac{1}{2}\log\left|\frac{x+1}{x-1}\right|-\frac{4}{x-1}+c$

(iii) $\frac{-2}{3}\log|\cos\theta-2|-\frac{1}{3}\log|1+\cos\theta|+c$

(iv) $\frac{1}{14}\log\left|\frac{x^2+3}{(2-x)^2}\right|+\frac{2}{7\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+c$

(v) $x+4\log\left|\frac{(x-2)^2}{x-1}\right|+c$

(vi) $x+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)-3\ \tan^{-1}\left(\frac{x}{2}\right)+c$

(vii) $\frac{2}{17}\log|2x+1|-\frac{1}{17}\log|x^2+4|+\frac{1}{34}\tan^{-1}\frac{x}{2}+c$

(viii) $\frac{1}{2}\log\left|\frac{x^2-x+1}{x^2+x+1}\right|+c$

(ix) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+\frac{1}{2\sqrt{2}}\ \log\left|\frac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right|+c$

(x) $-\frac{1}{2}\log|\cos x - 1| - \frac{1}{6}\log|\cos x + 1| + \frac{2}{3}\log|1 - 2\cos x| + c$

4. (i) $\frac{1}{3}[-x^3 \cos x^3 + \sin x^3] + c$

(ii) $\frac{1}{2}[\sec x \tan x + \log|\sec x + \tan x|] + c$

(iii) $\frac{e^{ax}}{a^2+b^2}[a\cos(bx + c) + b\sin(bx + c)] + c$

(iv) $2x\ \tan^{-1} 3x - \frac{1}{3}\log|1 + 9x^2| + c$

(v) $2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + c$

(vi) $\left(\frac{x^4-1}{4}\right)\tan^{-1} x - \frac{x^3}{12} + \frac{x}{4} + c$

(vii) $\frac{1}{2}e^{2x}\tan x + c$

(viii) $\frac{x}{\log x} + c$

(ix) $\left(\frac{x-a}{2}\right)\sqrt{2ax - x^x} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + c$

(x) $e^x\left(\frac{x-1}{x+1}\right) + c$

(xi) $\frac{x^4}{4}\sin^{-1}\left(\frac{1}{x}\right) + \frac{x^2+2}{12}\sqrt{x^2 - 1} + c$

(xii) $x\log|\log x| - \frac{x}{\log x} + c$

(xiii) $-2(6 + x - x^2)^{\frac{3}{2}} + 8\left[\frac{2x-1}{4}\sqrt{6 + x - x^2} + \frac{25}{8}\sin^{-1}\left(\frac{2x-1}{5}\right)\right] + c$

(xiv) $\frac{1}{3}\log|x + 1| - \frac{1}{6}\log|x^2 - x + 1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c$

(xv) $x\tan^{-1} x - \frac{1}{2}\log|1 + x^2| - x\tan^{-1} 5 + c$

(xvi) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right) + c$

5. (i) $\frac{1}{20}\log 3$

(ii) $-\pi/4$

(iii) $\frac{\pi}{4}-\frac{1}{2}$

(iv) $\frac{\pi}{4}-\frac{1}{2}\log 2$

(v) $\frac{\pi}{2}$

(vi) $\pi/4$

(vii) $\pi/2$

(viii) $\frac{3}{4}+\frac{3}{2}\log\frac{2}{3}$

(ix) $\frac{3}{2\pi}-\frac{1}{\pi^2}$

(x) $2\pi+\frac{1}{2a}\sin 2a\pi-\frac{1}{2b}\sin 2b\pi$

6. (i) $\frac{1}{2}$

(ii) π

(iii) $e^{\pi/4}+e^{-\pi/4}$

(iv) $\frac{1}{4}\pi^2$

(v) $5-\sqrt{3}-\sqrt{2}$

(vi) $\frac{\pi^2}{16}$

(vii) $\frac{\pi^2}{2a}$

7. (i) $\frac{\pi}{12}$

(ii) 2

(iii) $\frac{\pi}{2}$

(iv) $\frac{\pi^2}{4}$

(v) $a\pi$

8. (i) $\frac{2(2x-1)}{\pi}\sin^{-1}\sqrt{x}+\frac{2\sqrt{x-x^2}}{\pi}-x+c$

(ii) $-2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x-x^2}+c$

(iii) $\frac{x-2}{x+2}e^x+c$

(iv) $\frac{\sin x-x\cos x}{x\sin x+\cos x}+c$

(v) $(x+a)\tan^{-1}\sqrt{\frac{x}{a}}-\sqrt{ax}+c$

(vi) $2\sin^{-1}\frac{\sqrt{3}-1}{2}$

(vii) $\frac{1}{8}\log\left|\frac{1-\sin x}{1+\sin x}\right|-\frac{1}{4\sqrt{2}}\log\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right|+c$

(viii) $\frac{3}{\pi}+\frac{1}{\pi^2}$

(ix) $(\cos 2a)(x+a)-(\sin 2a)\log|\sin(x+a)|+c$

(x) $-\frac{4}{5}\log|x^2+4|+\frac{9}{5}\log|x^2+9|+c$

(xi) $-\left(\frac{1}{2}\sin 2x+\sin x\right)+c$

9. (i) $x - 4\log|x| + \frac{5}{4}\log|x-1| + \frac{3}{4}\log|x+1| + \log|x^2 +$

(ii) $1| \frac{-1}{2}\tan^{-1} x + c$

(iii) $\log\left|\frac{(e^t-1)(e^t-3)}{(e^t-2)^2}\right| + c$

(iv) $2x - \frac{1}{8}\log|x+1| + \frac{81}{8}\log|x-3| - \frac{27}{2(x-3)} + c$

(v) $\frac{1}{4}\log\left|\frac{1-\cos x}{1+\cos}\right| + \frac{1}{2(1+\cos\ )} + \tan\frac{x}{2} + c$

(vi) $\frac{\pi}{\sqrt{2}}$

(vii) $\frac{\pi-2}{4}$

(viii) $\frac{\pi}{4} - \frac{1}{2}\log 2$

10. (i) 14

(ii) $\frac{26}{3}$

(iii) 26

(iv) $\frac{1}{2}(127 + e^8)$

(v) $\frac{1}{3}\left(e^2 - \frac{1}{e}\right)$

(vi) 3

11. (i) $\frac{1}{5}\log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + c$

(ii) $\frac{\pi}{8}\log 2$

(iii) $\frac{\pi}{2}\log\frac{1}{2}$

12. $\frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2}\log 2$

13. $\frac{-\pi}{2}\log 2$

14. $\log 2$

15. $\frac{1}{\sqrt{2}}\log\left|\sqrt{2} + 1\right|$

CHAPTER 8

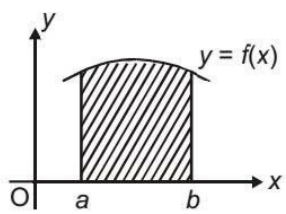
APPLICATIONS OF INTEGRALS

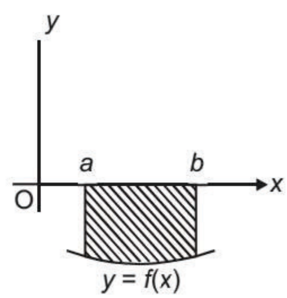
POINT TO REMEMBER

AREA OF BOUNDED REGION

- Area bounded by the curve $y = f(x)$, the x axis and between the ordinates, $x = a$ and $x = b$ is given by

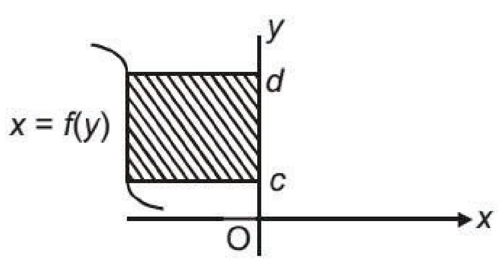
$$Area = \left|\int_a^b f(x)dx\right|$$

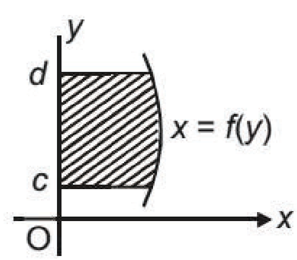




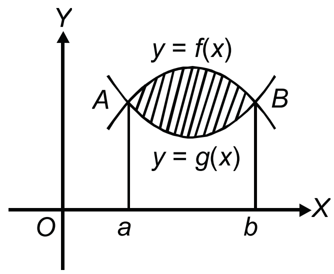
- Area bounded by the curve $x = f(y)$, the y-axis and between the abscissas, y = c and y = d is given by

$$Area = \left|\int_c^d f(y)dy\right|$$



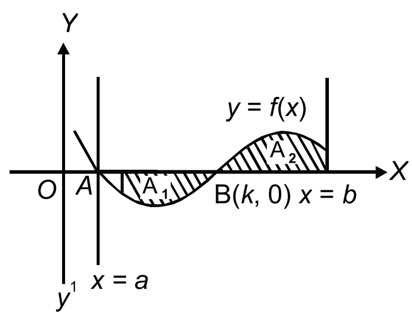


- Area bounded by two curves $y = f(x)$ and $y = g(x)$ such that $0 \le g(x) \le f(x)$ for all $x \in [a, b]$ and between the ordinates $x = a$ and $x = b$ is given by



Area = $\int_a^b [f(x) - g(x)]dx$

- Area of the following shaded region = $\left|\int_a^k f(x)dx\right| + \int_k^b f(x)dx$



ONE MARK QUESTIONS

1. Find the area bounded by $y = \sin 2x$, $0 \le x \le \frac{\pi}{4}$ and the coordinate axes.
2. Find the area bounded by $y = \cos 3x$, $0 \le x \le \frac{\pi}{6}$ and the coordinate axes.
3. Find the area bounded by the line $x + 2y = 8$, x – axis and the lines $x = 1$ and $x = 3$.
4. Find the area bounded by the curve $y = x^3$, x – axis and the lines $x = 0$ and $x = 4$.
5. Find the area bounded by the curve $xy = 2$, x – axis and the lines $x = 1$, $x = 2$.

6. Find the area of region bounded by the curve $y = x^2$, x – axis and the lines $x = -1$, $x = 1$.

7. Find the area under the curves $y = \sin^2 x$ between $x = 0$, $x = \pi$ and x – axis.

8. Find the area bounded by the curve $y = x^4$ x – axis and $x = 1$, $x = 3$.

TWO MARKS QUESTIONS

1. **Using integration:** Find the area of the circle $x^2 + y^2 = 16$.

2. Find the area of the parabols $y^2 = 4a\,x$ bounded by its latus nectum.

3. Find the area bounded by the curve $y = \sqrt{x} + 1$, $0 \le x \le 4$, x – axis and the lines $x = 0$, $x = 4$.

4. Find the area enclosed by the curve $y = 2\sqrt{1 - x^2}$, $x \in [0, 1]$ and the axes.

5. Find the area bounded by the curve $x = at^2$, $y = 2at$ between the ordinates corresponding to $t = 1$ and $t = 2$.

6. Find the area bounded by the region $\{(x, y): x^2 \le y \le |x|\}$.

7. Find the area bounded by the region $y = 9x^2$, $y = 1$ and $y = 4$.

8. Find the area bounded by the curve $y = 2x - x^2$ and x – axis

9. Find the area bounded by the are $y = \sin x$ between $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

10. Find the area bounded by the curve $y = 9 - x^2$, $0 \le x \le 3$ in first quadrant.

THREE/FIVE MARKS QUESTIONS

1. **Using integration:** Find the area bounded by the curve $4y = 3x^2$ and the line $3x - 2y + 12 = 0$.

2. Find the area bounded by the curve $x = y^2$ and the line $x + y = 2$.

3. Find the area of the triangular region whos verties are (1, 2), (2 – 2) and (4, 3).

4. Find the area bounded by the region $\{(x, y): x^2 + y^2 \le 1 \le x + \frac{y}{2}\}$

5. Find the area of the triangle formed by negative x – axis, the tangent and normal to the curve $x^2 + y^2 = 9$ at $\left(-1, 2\sqrt{2}\right)$.

6. Find the area of the region bounded by the lines $x - 2y = 1$, $3x - y - 3 = 0$ and $2x + y - 12 = 0$.

7. Find the area of region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

8. Prove that the curve $y = x^2$ and, $x = y^2$ divide the square bounded by $x = 0$, $y = 0$, $x = 1$, $y = 1$ into three equal parts.

8. Find the area of the smaller region enclosed between ellipse $b^2x^2 + a^2y^2 = a^2b^2$ and the line $bx + ay = ab$.

10. Find the common area bounded by the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

11. Using integration, find the area of the triangle whose sides are given by $2x + y = 4,\ 3x - 2y = 6\ and\ x - 3y + 5 = 0$.

12. Using integration, find the area of the triangle whose vertices are (–1, 0), (1, 3) and (3, 2).

13. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.

14. Find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

15. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$.

16. Using integration, find the area enclosed by the curve $y = \cos x$, $y = \sin x$ and x-axis in the interval $[0, \pi/2]$.

17. Using integration, find the area of the following region: $\left\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\right\}$

18. Using integration, find the area of the triangle formed by positive x-axis and tangent and normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$.

19. Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and y-axis.

20. Find the area of the region bounded by the curves $ay^2 = x^3$, the y-axis and the lines $y = a$ and $y = 2a$.

21. Find the area bounded by x-axis, the curve $y = 2x^2$ and tangent to the curve at the point whose abscissa is 2.

22. Using integration, find the area of the region bounded by the curve $y = 1 + |x + 1|$ and lines $x = -3, x = 3, y = 0$.

23. Find the area of the region $\{(x, y): y^2 \geq 6x, \ \ x^2 + y^2 \leq 16\}$

24. Find the area of the region enclosed between curves $y = \ |x - 1|$ and $y = 3 - |x|$.

25. If the area bounded by the parabola $y^2 = 16$ ax and the line y = 4 mx is $\frac{a^2}{12}\text{sq}$ unit then using integration find the value of m.

26. Given $\frac{dy}{dx}$ is directly proportional to the square of x and $\frac{dy}{dx} = 6$ at x = 2. Then find the equation of the curve, when x = 2 and y = 4. Also find the area of the region bounded by curve between lines y = 1 and y = 3.

27. Find the area between x-axis, curve $x = y^2$ and its normal at the point (1,1).

28. Using integration find the area bounded by the tangent to the curve $y = 3x^2$ at the point (1, 3), and the Lines whose equations are $y = \frac{x}{3}$ and x + y = 4

ANSWERS

ONE MARKS QUESTION

1. $\frac{1}{2}$ square units.

2. $\frac{1}{2}$ square units.

3. 7 square units.

4. 64 square units.

5. 2 log 2 square units.

6. 2/3 square units.

7. $\frac{\pi}{2}$ square units.

8. 48.4 square units.

TWO MARKS QUESTIONS

1. 16π square units.

2. $\frac{8}{3}a^2$ square units.

3. $\frac{28}{3}$ square units.

4. $\frac{\pi}{2}$ square units.

5. $\frac{56}{3}$ a^2 square units.

6. $\frac{1}{3}$ square units.

7. $\frac{28}{9}$ square units.

8. $\frac{4}{3}$ square units.

9. 2 square units.

10. 18 square units.

THREE/FIVE MARKS QUESTIONS

1. 27 square units.

2. $\frac{9}{2}$ square units.

3. $\frac{13}{2}$ square units.

4. $\left(\frac{\pi}{4}-\frac{2}{5}-\frac{1}{2}\sin^{-1}\frac{3}{5}\right)$ square units.

5. $9\sqrt{2}$ square units.

6. 10 square units.

7. $\left[\frac{\sqrt{2}}{6}+\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)\right]$

9. $\left(\frac{\pi-2}{4}\right)ab$ square units.

10. $\left(\frac{8\pi}{3}-2\sqrt{3}\right)$ square units.

11. 3.5 square units.

12. 4 square units.

13. $\left(\pi-\frac{1}{2}\right)$ square units.

14. $\frac{9}{8}$ square units.

15. $\frac{4}{3}$ $(8 + 3\pi)$ square units.

16. $(2-\sqrt{2})$ square units.

17. $\left(\frac{5\pi}{4}-\frac{1}{2}\right)$ square units.

18. $2\sqrt{3}$ square units.

19. $\frac{10}{3}$ square units.

20. $\frac{3}{5}a^2\left[(32)^{1/3}-1\right]$ square units.

21. $\frac{4}{3}$ square units.

22. 16 square units.

23. $\frac{32\pi - 4\sqrt{3}}{3}$ square units.

24. 4 square units.

25. m = 2.

26. $\frac{3}{4}(2)^{1/3}\left[(3)^{1/3} - 1\right]$ square units.

27. $\frac{11}{2}$ square units.

28. $\frac{56}{17}$ square units.

Chapter 9

DIFFERENTIAL EQUATIONS

KEY POINTS :

- **DIFFERENTIAL EQUATION :** is an equation involving derivatives of the dependent variable w.r.t independent variables and the variables themselves.
- **ORDINARY DIFFERENTIAL EQUATION (ODE) :** A.D.E. involving derivatives of the dependent variable w.r.t only one independent variable is an ordinary D.E.

  In class XII ODE is referred to as D.E.
- **PARTIAL DIFFERENTIAL EQUATION (PDE) :** A.D.E involving derivatives w.r.t more than one independet variables is called a partial D.E.
- **ORDER of a DE :** is the order of the highest order derivative occurring in the D.E.
- **DEGREE of a D.E. :** is the highest power of the highest order derivative occurring in the D.E provided D.E is a polynomial equation in its derivatives.
- **SOLUTION OF THE D.E :** A relation between involved variables, which satisfy the given D.E is called its solution.

Two Types of Solution of DE

General Solution	**Particular Solution**
The solution which contains as many arbitrary constants as the order of the D.E.	The solution free from arbitrary constants

- **FORMATION OF A DIFFERENTIAL EQUATION :** We differentiate the function successively as many times as the arbitary constants in the given function and then eliminate the arbitiary constants from these equations.
- **ORDER of A D.E :** Number of arbitrary constants in the general solution of a D.E.

Solution of a First Order First Degree D.E.

- Variable separable method
- Homogeneous D.E.
- Linear Differential equation

- **“VARIABLE SEPARABLE METHOD” :** is used to solve D.E. in which variables can be separated completely i.e, terms containing, x should remain with dx and terms containing y should remain with dy.

- **“HOMOGENEOUS DIFFERENTIAL EQUATION :** of the form $\frac{dy}{dx} = F(x, y)$ where *F(x, y)* is a homogeneous function of degree 0

  i.e. $F(\lambda x, \lambda y) = \lambda^0 F(x, y)$

  or $F(\lambda x, \lambda y) = F(x, y)$ for some non-zero constant λ.

  To solve this type put $y = vx$

  Solve homogenous D.E of the trype $\frac{dx}{dy} = G(x, y),$ we make substitution $x = vy$

- **LINEAR DIFFERENTIAL EQUATION :** A.D.E of the form $\frac{dy}{dx} + Py = Q$ where P and Q are constants or functions of x only is known as first order linear differential equation.

  Its solution

  $y.(IF) = \int Qx(I.F.)dx + C$ where

  I. F = Integrating factor = $e^{\int Pdx}$

  Another form of Linear Differential Equation is $\frac{dx}{dy} + P_1x = Q_1$, where P_1 and Q_1 are constants or functions of y only.

  Its solution is given as

  $x.(I.F) = \int Q_1 X(I.F.)\, dy + C,$ where $I.F. = e^{\int P_1 dy}$

ONE MARK QUESTIONS

1. Write the order and degree of the following D.E.'s

(i) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^2 = \frac{d^2y}{dx^2}$ (ii) $\frac{d^5y}{dx^5} + log\left(\frac{dy}{dx}\right) = 0$

(iii) $\sqrt{1+\frac{dy}{dx}} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$ (iv) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{kd^2y}{dx^2}$

(v) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} + x^{1/5} = 0$

2. (i) What will be the order of the D.E.

$$y = Ae^x + Be^{x+c}$$

(ii) What will be the order of the D.E. representing the family of circles with centre (o, a) and radius a.

(iii) Write the integrating factors of the following D Eqn.

(a) $\frac{dy}{dx} + y\cos x = \sin x$ (b) $\frac{xdy}{dx} + y\log x = x$

(c) $\frac{dy}{dx} + \frac{y}{x} = 1$

(iv) State whether the following statements are True or False.

(a) Integrating factor of the D.E.

$(1-x^2)\frac{dy}{dx} - xy = 1$ is $\sqrt{1-x^2}$

(b) Solution of D.E. xdy-ydx = 0 represents straight lines passing through origin.

(c) Number of arbitrary constants in the particular solution of a differential equation of order two is two.

(d) The differential equation of all non horizontal lines in a plane is $\frac{d^2x}{dy^2} = 0$

TWO MARKS QUESTIONS

1. Write the general solution of the following D.Eqns.

   (i) $\frac{dx}{dy} = x^5 + x^2 - \frac{2}{x}$ (ii) $\frac{dy}{dx} = \frac{1 - \cos 2x}{1 + \cos 2y}$

   (iii) $(e^x + e^{-x})\, dy = (e^x - e^{-x})dx$

2. Given that $\frac{dy}{dx} = e^{-2y}$ and y = 0 when x = 5.

   Find the value of x when y = 3.

3. Name the curve for which the slope of the tangent at any point is equal to the ratio of the abbcissa to the ordinate of the point.

4. Form the D.E for which y = a cos x + b sin x is a solution.

5. Solve $\frac{xdy}{dx} + y = e^x$.

THREE MARKS QUESTIONS

1. (i) Show that $y = e^{m \sin^{-1} x}$ is a solution of $(1 - x^2)\frac{d^2y}{dx^2} - \frac{xdy}{dx} - m^2 y = 0$

(ii) Show that $y = a \cos(\log x) + b \sin(\log x)$ is a solution of

$$\frac{x^2 d^2 y}{dx^2} + \frac{xdy}{dx} + y = 0$$

(iii) Verify that y = log $\left(x + \sqrt{x^2 + a^2}\right)$ satisfies the D.E.

$$\left(a^2 + x^2\right) y'' + xy' = 0$$

(iv) Form the D.E. having $y = \left(\sin^{-1} x\right)^2 + A \cos^{-1} x + B,$ where A and B are arbitrary constants, as its general solution.

(v) Form the D.E. of all circles that pass through (0, 0) and whose centre lie on y-axis.

2. Solve the following D Eqs.

(i) $xdy-(y+2x^2)dx=0$

(ii) $(1+y^2)\tan^{-1}x\,dx+2y(1+x^2)dy=0$

(iii) $x^2\frac{dy}{dx}=x^2+xy+y^2$

(iv) $\frac{dy}{dx}=1+x+y^2+xy^2$, $y=0$ when $x=0$

(v) $xdy-ydx=\sqrt{x^2+y^2}dx$ $y=0$ when $x=1$

3. Solve each of the following D Eqs.

(i) $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0,\ y(0)=0$

(ii) $(x+1)\frac{dy}{dx}=2e^{-y}-1, y(0)=0$

(iii) $e^x\tan ydx+(2-e^x)\sec^2 ydy=0,\ y(o)=\pi/4$

(iv) $(x^2-y^2)dx+2xydy=0$

(v) $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}, y=0$ when $x=1$

4. Solve the following D.E.s.

(i) Find the particular solution of

$2y\,e^{x/y}\,dx+(y-2xe^{x/y})\,dy=0, x=0$ if $y=1$

(ii) $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$

(iii) $(1+y^2)+\left(x-e^{\tan^{-1}}y\right)\frac{dy}{dx}=0$

(iv) Show that the Differential Equation $\frac{dy}{dx}=\frac{y^2}{xy-x^2}$ is homogenous and also solve it.

(v) $(x^2-1)\frac{dy}{dx}+2xy=\frac{1}{x^2-1},|x|\neq 1$

FIVE MARKS QUESTIONS

Q. 1 Solve $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

Q. 2 Solve $(x\,dy - ydx)y \sin\left(\frac{y}{x}\right) = (ydx + xdy)x \cos\left(\frac{y}{x}\right)$

Q. 3 Find the particular solution of the D.E. $(x - y)\frac{dy}{dx} = x + 2y$ given that $y = 0$ when $x = 1$.

Q. 4 Solve $dy = \cos x\,(2 - y\,\mathrm{cosec}\,x)dx,$ given that $y = 2$ when $x = \pi/2$

Q. 5 Find the particular solution of the D.E. $(1 + y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$ given that $y = 0$ when $x = 1$

CASE STUDY QUESTIONS

Q. 1 Suppose a person named Devdutt saved Rs 10,000 from his earnings for his daughter's marriage.

So, he deposits this amount in a bank where principal amount increases continously at the rate of 10% per year. Now based on the following information answer the following questions.

(i) The following D.E. represents the above situation if P denotes the principal at time t.

(a) $\frac{dP}{dt} = \frac{10}{100}P$ (b) $\frac{dt}{dP} = \frac{10}{100}P$

(c) $\frac{dP}{dt} = \frac{100}{10}P$ (d) None of these

(ii) The order and degree of the D.E. obtained in (i) is

(a) order : 2 degree : 1

(b) order : 1, degree : 2

(c) order : 1, degree : not defined

(d) order : 1, degree : 1

(iii) The most suitable method for solving D.E. obtained in part (i) is

(a) Ordinary method

(b) Method for Homogeneous D.E.

(c) Variables separable method

(d) None of these

(iv) Solution of the D.E. obtained in part (i) is given by

(a) $C = Pe^{t/10}$ (b) $P = Ce^{t/10}$ C = arbitrary constant

(c) $P = Ce^{10/t}$ (d) $C = Pe^{10/t}$

(v) In how many years will Rs 10,000 double itself.

(a) 10 $\log_{10} 2$ (b) 2 log e^{10}

(c) 10 log e^2 (d) 2 $\log_{10}$ 10

Answers

ONE MARK QUESTIONS

1. (i) $0 \to 2, D \to 1$ (ii) $0 \to 5, D \to$ Not defined (iii) $0 \to 2, D \to 2$

   (iv) $0 \to 2, D \to 2$ (v) $0 \to 2, D \to$ Not defined

2. (i) Order = 1 (ii) Order = 1

   (iii) $e^{\sin x}$ (b) $e^{\frac{(\log x)^2}{2}}$ (c) x

   (iv) True (b) True (c) False (d) True

TWO MARKS QUESTIONS

1. (i) $y = \frac{x^6}{6} + \frac{x^3}{3} - 2 \log |x| + C$ (ii) $2(y - x) + \sin 2y + \sin 2x = c$

   (iii) $y = \log_e |e^x + e^{-x}| + C$

2. $\frac{e^6 + 9}{2}$ 3. Rectangular hyperbola 4. $\frac{d^2y}{dx^2} + y = 0$

5. $y.x = e^x + c$

THREE MARKS QUESTIONS

1. (iv) $(1-x^2)y''-xy-2=0$ (v) $(x^2-y^2)y'=2xy$

2. (i) $y=2x^2+cx$ (ii) $\frac{1}{2}(\tan^{-1}x)^2+\log(1+y^2)=c$

   (iii) $\tan^{-1}\left(\frac{y}{x}\right)=\log|x|+c$ (iv) $y=tan\left(x+\frac{x^2}{2}\right)$

   (v) $y+\sqrt{x^2+y^2}=x^2$

3. (i) $(1+x^2)y=\frac{4x^3}{3}$ (ii) $(2-e^y)(x+1)=1$

   (iii) $\tan y=2-e^x$ (iv) $x^2+y^2=cx$

   (v) $(1+x^2)y=tan^{-1}x-\pi/4$

4. (i) $e^{x/y}=\frac{-1}{2}\log|y|+1$ (ii) $sin(y/x)=log|x|+c$

   (iii) $xe^{tan^{-1}y}=\frac{1}{2}e^{2\tan^{-1}y}+c$ (iv) $\frac{y}{x}-log|y|=c$

   (v) $(x^2-1)y=\frac{1}{2}log\left|\frac{x-1}{x+1}\right|+c$

FIVE MARKS QUESTIONS

1. $y=-cos\,x+\frac{2sin\,x}{x}+\frac{2cos\,x}{x^2}+\frac{x\,log\,x}{3}-\frac{x}{9}+\frac{c}{x^2}$

2. $xy\,cos\left(\frac{y}{x}\right)=c$

3. $\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right)-\frac{1}{2}\log|x^2+xy+y^2|=\frac{\sqrt{3}\pi}{6}$

4. $y\,sin\,x=\frac{-1}{2}cos\,(2x)+\frac{3}{2}$

5. $x=\frac{1}{2}e^{\tan^{-1}y}+\frac{1}{2}e^{-\tan^{-1}y}$

CASE STUDY QUESTIONS

1. (i) a (ii) d (iii) c
   (iv) b (v) c

CHAPTER–10

VECTORS

POINTS TO REMEMBER

- A quantity that has magnitude as well as direction is called a vector. It is denoted by a directed line segment.

- Two or more vectors which are parallel to same line are called collinear vectors.

- Position vector of a point P(a, b, c) w.r.t. origin (0, 0, 0) is denoted by $\overline{OP}$ where $\overline{\mathrm{OP}} = a\hat{i} + b\hat{j} + c\hat{k}$ and $|\overline{OP}| = \sqrt{a^2 + b^2 + c^2}$.

- If $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be any two points in space, then

  $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$ and

  $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z)^2}$

- Any vector $\vec{a}$ is called unit vector if $|\vec{a}| = 1$ It is denoted by $\hat{a}$

- If two vectors $\vec{a}$ and $\vec{b}$ are represented in magnitude and direction by the two sides of a triangle in order, then their sum $\vec{a} + \vec{b}$ is represented in magnitude and direction by third side of a triangle taken in opposite order. This is called triangle law of addition of vectors.

- If $\vec{a}$ is any vector and λ is a scalar, then $\lambda\,\vec{a}$ is vector collinear with $\vec{a}$ and $|\lambda\,\vec{a}| = |\lambda||\vec{a}|$.

- If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then $\vec{a} = \lambda\,\vec{b}$ where λ is some scalar.

- Any vector $\vec{a}$ can be written as $\vec{a} = |\vec{a}|\hat{a}$ where $\hat{a}$ is a unit vector in the direction of $\vec{a}$.

- If $\vec{a}$ and $\vec{b}$ be the position vectors of points A and B, and C is any point which divides $\overrightarrow{AB}$ in ratio m:n internally then position vector $\vec{c}$ of point C is given as $\vec{c} = \frac{m\vec{b}+n\vec{a}}{m+n}$. If C divides $\overrightarrow{AB}$ in ratio m:n externally, then $\vec{c} = \frac{m\vec{b}-n\vec{a}}{m-n}$.

- The angles α, β and γ made by $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ with positive direction of x, y and z-axis are called angles and cosines of these angles are called direction cosines of $\vec{r}$ usually denoted as $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

  Also $l = \frac{a}{|\vec{r}|}, m = \frac{b}{|\vec{r}|}, n = \frac{c}{|\vec{r}|}$ and $l^2 + m^2 + n^2 = 1$

- The numbers a, b, c proportional to l, m, n are called direction ratios.

- Scalar product or dot product of two vectors $\vec{a}$ and $\vec{b}$ is denoted as $\vec{a}.\vec{b}$ and is defined as $\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta$, θ is the angle between $\vec{a}$ and $\vec{b}$. $(0 \le \theta \le \pi)$.

- Dot product of two vectors is commutative i.e. $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

- $\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a} = \vec{o}, \vec{b} = \vec{o}$ or $\vec{a} \perp \vec{b}$.

- $\vec{a}.\vec{a} = |\vec{a}|^2$, so $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot k = 1$

- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3.$$

- Projection of $\vec{a}$ on $\vec{b} = \left|\frac{\vec{a}.\ \vec{b}}{|\vec{b}|}\right|$ and

  Projection vector of $\vec{a}$ along $\vec{b} = \left(\frac{(\vec{a}.\ \vec{b})}{|\vec{b}|}\right)\hat{b}$.

- Cross product or vector product of two vectors $\vec{a}$ and $\vec{b}$ is denoted as $\vec{a} \times \vec{b}$ and is defined as $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$. were θ is the angle between $\vec{a}$ and $\vec{b}$.$(0 \leq \theta \leq \pi)$. And $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}\ .\ \vec{b}$ and $\hat{n}$ from a right handed system.

- Cross product of two vectors is not commutative i.e., $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$, but $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$.

- $\vec{a} \times \vec{b} = \vec{0} \Leftrightarrow \vec{a} = \vec{0}, \vec{b} = \vec{0}$ or $\vec{a} \parallel \vec{b}$.

- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$.

- $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$ and $\hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j}$

- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$,then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- Unit vector perpendicular to both $\vec{a}$ and $\vec{b} = \pm\left(\frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}\right)$.

- $|\vec{a} \times \vec{b}|$ is the area of parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$

- $\frac{1}{2}|\vec{a} \times \vec{b}|$ is the area of parallelogram where diagonals are $\vec{a}$ and $\vec{b}$.

- If $\vec{a}, \vec{b}$ and $\vec{c}$ form a triangle, then area of the triangle

- $= \frac{1}{2}\left|\vec{a} \times \vec{b}\right| = \frac{1}{2}\left|\vec{b} \times \vec{c}\right| = \frac{1}{2}|\vec{c} \times \vec{a}|$.

- Scalar triple product of three vector$\vec{a}, \vec{b}$and $\vec{c}$ is defined as $\bar{a}.\left(\vec{b} \times \vec{c}\right)$ and is denotes as $\left[\vec{a}\vec{b}\,\vec{c}\right]$

- Geometrically, absolute value of scalar triple product $\left[\vec{a}\vec{b}\vec{c}\right]$ represents volume of a parallelopiped whose coterminous edges are $\vec{a}\,\vec{b}$ and $\vec{c}$.

- $\vec{a}\ \vec{b}\ \vec{c}$ are coplanar $\Leftrightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

- $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \left[\vec{b}\ \vec{c}\ \vec{a}\right] = \left[\vec{c}\ \vec{a}\ \vec{b}\right]$

- If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ and $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$ then

$$\left[\vec{a}\vec{b}\vec{c}\right] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

- Then scalar triple product of three vectors is zero if any two of them are same or collinear.

ONE MARK QUESTIONS

1. If $\overrightarrow{AB} = 3\hat{\imath} + 2\,\hat{\jmath} - \hat{k}$ and the coordinate of A are (4,1,1), then find the coordinates of B.

2. Let $\vec{a} = -2\hat{\imath} + \hat{\jmath},\ \vec{b} = \hat{\imath} + 2\hat{\jmath}$ and $\vec{c} = 4\hat{\imath} + 3\hat{\jmath}$. Find the values of x and y such that $\vec{c} = x\vec{a} + y\vec{b}$.

3. Find a unit vector in the direction of the resultant of the vectors $\hat{\imath} - \hat{\jmath} + 3\hat{k},\ 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$.

4. Find a vector of magnitude of 5 units parallel to the resultant of vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ and $\vec{b} = (\hat{\imath} - 2\hat{\jmath} - \hat{k})$

5. For what value of λ are the vector $\vec{a}\ and\ \vec{b}$ perpendicular to each other? Where $\vec{a} = \lambda\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{b} = 5\hat{\imath} - 9\hat{\jmath} + 2\hat{k}$

6. Write the value of p for which $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} + p\hat{\jmath} + 3\hat{k}$ are parallel vectors.

7. For any two vectors $\vec{a}\ and\ \vec{b}$ write when $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ holds.

8. Find the value of p if $(2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}) \times (\hat{\imath} + 3\hat{\jmath} + p\hat{k}) = \vec{0}$

9. Evaluate: $\hat{\imath}.(\hat{\jmath} \times \hat{k}) + (\hat{\imath} \times \hat{k}).\hat{\jmath}$

10. If $\vec{a} = 2\hat{\imath} - 3\hat{\jmath},\ \vec{b} = \hat{\imath} + \hat{\jmath} - \hat{k}, \vec{c} = 3\hat{\imath} - \hat{k}$, find $[\vec{a}\vec{b}\vec{c}]$

11. If $\vec{a} = 5\hat{\imath} - 4\hat{\jmath} + \hat{k},\ \vec{b} = -4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} - 2\hat{k}$, then evaluate $\vec{c}.(\vec{a} \times \vec{b})$

12. Show that vector $\hat{\imath} + 3\hat{\jmath} + \hat{k},\ 2\hat{\imath} - \hat{\jmath} - \hat{k},\ 7\hat{\jmath} + 3\hat{k}$ are parallel to same plane.

13. Find a vector of magnitude 6 which is perpendicular to both the vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $4\hat{\imath} - \hat{\jmath} + 3\hat{k}$.

14. If $\vec{a}\,.\vec{b} = 0$, then what can you say about $\vec{a}\ and\ \vec{b}$?

15. If $\vec{a}\ and\ \vec{b}$ are two vectors such that $|\vec{a} \times \vec{b}| = \vec{a}.\vec{b}$, then what is the angle between $\vec{a}\ and\ \vec{b}$?

16. Find the area of a parallelogram having diagonals $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and$\hat{\imath} -$ $\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$.

17. If $\hat{\imath}, \hat{\jmath}\ and\ \hat{k}$ are three mutually perpendicular vectors, then find the value of $\hat{\jmath}.(\hat{k} \times \hat{\imath})$.

18. P and Q are two points with position vectors $3\vec{a} - 2\vec{b}$ and $\vec{a} + \vec{b}$ respectively. Write the position vector of a point R which divides the segment PQ in the ratio 2:1 externally.

19. Find λ when scalar projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units.

20. Find "a" so that the vectors $\vec{p} = 3\hat{\imath} - 2\hat{\jmath}$ and $\vec{q} = 2\hat{\imath} + a\hat{\jmath}$ be orthogonal.

21. If $\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{c} = \lambda\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$ are coplanar, find the value of λ.

22. What is the point of trisection of PQ nearer to P if positions of P and Q are $3\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ and $9\hat{\imath} + 8\hat{\jmath} - 10\hat{k}$ respectively?

23. What is the angle between $\vec{a}$ and $\vec{b}$, if $\vec{a}.\vec{b} = 3$ and $|\vec{a} \times \vec{b}| = 3\sqrt{3}$.

24. Represent graphicaley a displacement of 50 km, 60° south of west.

25. If the vectors $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{b} = \hat{j}$ and $\hat{c}$ are such that $\vec{a}$, $\vec{c}$ and $\vec{b}$ from a right handed system, than find $\vec{c}$.

TWO MARK QUESTIONS

Q.1. A vector $\vec{r}$ is inclined to x – axis at 45° and y-axis at 60° if | $\vec{r}$ | = 8 units. find $\vec{r}$.

Q.2. if $|\vec{a}+\vec{b}| = 60$, $|\vec{a}-\vec{b}| = 40$ and $\vec{b} = 46$ find $|\vec{a}|$

Q.3. Write the projection of $\vec{b}+\vec{c}$ on $\vec{a}$ where

$\vec{a}=2\hat{i}-2\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$

Q.4. If the points (–1, –1, 2), (2, m, 5) and (3, 11, 6) are collinear, find the value of *m*.

Q.5. For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ write value of the following.
$\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})$

Q.6. If $(\vec{a}\times\vec{b})^2+(\vec{a}.\vec{b})^2$ = 144 and $|\vec{a}|$ = 4. Find the value of $|\vec{b}|$.

Q.7. If for any two vectors $\vec{a}$ and $\vec{b}$,

$(\vec{a}+\vec{b})^2+(\vec{a}-\vec{b})^2=\lambda\left[(\vec{a})^2+(\vec{b})^2\right]$ then write the value of λ .

Q.8. if $\vec{a}, \vec{b}$ are two vectors such that $|(\vec{a}+\vec{b})| = |\vec{a}|$ then prove that $2\vec{a}+\vec{b}$ is perpendicular to $\vec{b}$.

Q.9. Show that vectors $\vec{a}=3\hat{i}-2\hat{j}+\hat{k}$

$\vec{b}=\hat{i}-3\hat{j}+5\hat{k}, \vec{c}=2\hat{i}+\hat{j}-4\hat{k}$ form a right angle triangle.

Q.10. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}$ = 0 and $|\vec{a}|$ = 5, $|\vec{b}|$ = 12, $|\vec{c}|$ = 13, then find $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

Q.11. The two vectors $\hat{i}+\hat{j}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides AB and AC respectively of Δ ABC, find the length of median through A.

12. If position vectors of the points *A*, *B* and *C* are $\vec{a}$, $\vec{b}$ and $4\vec{a} - 3\vec{b}$ respectively, then find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$.

13. If position vectors of three points *A*, *B* and *C* are $-2\vec{a} + 3\vec{b} + 5\vec{c}$, $\vec{a} + 2\vec{b} + 3\vec{c}$ and $7\vec{a} - \vec{c}$ respectively. Then prove that *A*, *B* and *C* are collinear.

14. If the vector $\hat{i} + p\hat{j} + 3\hat{k}$ is rotated through an angle θ and is doubled in magnitude, then it becomes $4\hat{i} + (4p - 2)\hat{j} + 2\hat{k}$. Find the value of *p*.

15. If $\overrightarrow{AB} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} + 4\hat{k}$ are sides of the triangle *ABC*. Find the length of median through *A*.

16. Find scalar projection of the vector $7\hat{i} + \hat{j} + 4\hat{k}$ on the vector $2i + 6\hat{j} + 3\hat{k}$. Also find vector porojection

17. Let $\vec{a} = 3\hat{i} + x\hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ one mutually perpendicular and $|\vec{a}| = |\vec{b}|$. Find *x* and *y*.

18. If $\vec{a}, \vec{b}, \vec{c}$, are coplanar vectors, find the value of $[2\vec{a} - \vec{b} \quad 2\vec{b} - \vec{c} \quad 2\vec{c} - \vec{a}]$

19. If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $b = 2\hat{i} + 3\hat{j} - 5\hat{k}$. Find the angle between $\vec{a}$ and $\vec{a} \times \vec{b}$.

20. Using vector, prove that angle in a semi circle is 90°.

THREE MARKS QUESTIONS

1. The points A,B and C with position vectors $3\hat{\imath} - y\hat{\jmath} + 2\hat{k}$, $5\hat{\imath} - \hat{\jmath} + \hat{k}$ and $3x\hat{\imath} + 3\hat{\jmath} - \hat{k}$ are collinear. Find the values of x and y and also the ratio in which the point B divides AC.

2. If sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

3. Let $\vec{a} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}$, $\vec{b} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and satisfying $\vec{d}.\vec{c} = 21$

4. If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angleθ then proved that

   (i) $\cos\frac{\theta}{2} = \frac{1}{2}\left|\hat{a} + \hat{b}\right|$

   (ii) $\sin\frac{\theta}{2} = \frac{1}{2}|\hat{a} - \hat{b}|$

   (iii) $\tan\frac{\theta}{2} = \left|\frac{\hat{a} - \hat{b}}{\hat{a} - \hat{b}}\right|$

5. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of equal magnitude. Prove that $\vec{a}+\vec{b}+\vec{c}$ is equally in clined with vectors $\vec{a}, \vec{b}$ and $\vec{c}$. Also find angles.

6. For any vector $\vec{a}$ prove that $|\vec{a} \times \hat{\imath}|^2 + |\vec{a} \times \hat{\jmath}|^2 + \left|\vec{a} \times \hat{k}\right|^2 = 2|\vec{a}|^2$

7. Show that $\left(\vec{a} \times \vec{b}\right)^2 = |\vec{a}|^2\left|\vec{b}\right|^2 - \left(\vec{a}.\vec{b}\right)^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$

8. If $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of vertices A,B,C of a Δ ABC, show that the area of triangle ABC is $\frac{1}{2}\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right|$. Deduce the condition for points $\vec{a}, \vec{b}$ and $\vec{c}$ to be collinear.

9. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between b and c is $\pi/6$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

10. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

11. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k},\ \vec{c} = \hat{\jmath} - \hat{k}$ are given vectors, then find a vector $\vec{b}$ satisfying the equations $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a}.\vec{b} = 3$.

12. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. if the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$, prove that $[\vec{a}\vec{b}\vec{c}]^2 = \frac{1}{4}|\vec{a}|^2|\vec{b}|^2$

13. If the vectors $\vec{\alpha} = a\hat{\imath} + \hat{\jmath} + \hat{k},\ \vec{\beta} = \hat{\imath} + b\hat{\jmath} + \hat{k}$ and $\vec{\gamma} = \hat{\imath} + \hat{\jmath} + c\hat{k}$ are coplanar, then prove that $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$ where $a \neq 1, b \neq 1$ and $c \neq 1$

14. Find the altitude of a parallelepiped determined by the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ if the base is taken as parallelogram determined by $\vec{a}$ and $\vec{b}$ and if $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k},\ \vec{b} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\vec{c} = \hat{\imath} + \hat{\jmath} + 3\hat{k}$.

15. Prove that the four points $(4\hat{i} + 5\hat{j} + \hat{k}), -(\hat{j} + \hat{k}), (3\hat{i} + 9\hat{j} + 4\hat{k})$ and $4(-\hat{i} + \hat{j} + \hat{k})$ are coplanar.

16. If $|\vec{a}| = 3,\ |\vec{b}| = 4$ and $|\vec{c}| = 5$ such that each is perpendicular to sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$

17. Decompose the vector $6\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ into vectors which are parallel and perpendicular to the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$.

18. If $\vec{a}, \vec{b}$ and $\vec{c}$ are vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, a$\neq 0$, then show that $\vec{b} = \vec{c}$.

19. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non zero vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$. Prove that $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually at right angles and $|\vec{b}| = 1$ and $|\vec{c}| = |\vec{a}|$

20. Simplify $[\vec{a} - \vec{b},\ \vec{b} - \vec{c},\ \vec{c} - \vec{a}]$

21. If $[\vec{a}\vec{b}\vec{c}] = 2$, find the volume of the parallelepiped whose co-terminus edges are $2\vec{a} + \vec{b},\ 2\vec{b} + \vec{c},\ 2\vec{c} + \vec{a}$.

22. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$.

23. The magnitude of the vector product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of the vector $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ.

24. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, prove that $(\vec{a} - \vec{d})$ is parallel to $(\vec{b} - \vec{c})$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

25. Find a vector of magnitude $\sqrt{171}$ which is perpendicular to both of the vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$.

26. Prove that the angle betwen two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

27. If $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} + 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

28. Find a unit vector perpendicular to plane ABC, when position vectors of A,B,C are $\widehat{3\imath} - \hat{\jmath} + 2\hat{k}$, $\hat{\imath} - \hat{\jmath} - 3\hat{k}$ and $4\hat{\imath} - \widehat{3\jmath} + \hat{k}$ respectively.

29. Find a unit vector in XY plane which makes an angle 45° with the vector $\hat{\imath} + \hat{\jmath}$ at angle of 60° with the vector $3\hat{\imath} - 4\hat{\jmath}$.

30. Suppose $\vec{a} = \lambda\hat{\imath} - 7\hat{\jmath} + 3\hat{k}$, $\vec{b} = \lambda\hat{\imath} + \hat{\jmath} + 2\lambda\hat{k}$. If the angle between $\vec{a}$ and $\vec{b}$ is greater than 90°, then prove that λ satisfies the inequality$-7 < \lambda < 1$.

31. Let $\vec{v} = \widehat{2\imath} + \hat{\jmath} - \hat{k}$ and $\vec{w} = \hat{\imath} + 3\hat{k}$. If $\hat{u}$ is a unit vector, then find the maximum value of he scalar triple products $\vec{u}, \vec{v}, \vec{w}$.

32. If $\vec{a} = \hat{\imath} - \hat{k}, \vec{b} = x\hat{\imath} + \hat{\jmath} + (1-x)\hat{k}$ and $\vec{c} = y\hat{\imath} + x\hat{\jmath} + (1+x-y)\hat{k}$ then prove that $\left[\vec{a}\vec{b}\vec{c}\right]$ depends upon neither x nor y.

33. a, b and c are distinct non negative numbers, if the vectors a$\hat{\imath} + a\hat{\jmath} + c\hat{k}$, $\hat{\imath} + \hat{k}$ and $c\hat{\imath} + c\hat{\jmath} + b\hat{k}$ lie in a plane, then prove that c is the geometric mean of a and b.

34. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2), (1, b, b^2)\ and\ (1, c, c^2)$ are non-coplanar, then find the value of abc.

35. If $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 6\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar), then find the value of $\vec{a} + 2\vec{b} + 6\vec{c}$.

36. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$ then find the value of $(2\vec{a} - 5\vec{b}).(3\vec{a} + \vec{b})$.

37. Let $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. Find a vector $\vec{d}$ such that $\vec{a}.\vec{d} = 0$, $\vec{b}.\vec{d} = 2$ and $\vec{c}.\vec{d} = 4$.

<u>Answers</u>

ONE MARK QUESTIONS

1. (7, 3, 0)
2. $x = -1,\ y = 2$
3. $\frac{1}{\sqrt{21}}\left(4\hat{\imath} + 2\hat{\jmath} - \hat{k}\right)$
4. $\sqrt{\frac{5}{2}}(3\hat{i} + \hat{j})$
5. $\lambda = \frac{16}{5}$
6. $2/3$
7. $\vec{a}$ *and* $\vec{b}$ *are perpendicular*
8. $\frac{27}{2}$
9. 0
10. 4
11. –5
12. ....
13. $-2\hat{i} + 4\hat{j} + 4\hat{k}$
14. Either $\vec{a} = 0$ or $\vec{b} = 0$ or $\vec{a} \perp \vec{b}$
15. 45°
16. $5\sqrt{3}$ sq. Units
17. 1

18. $-\vec{a} + 4\vec{b}$

19. $\lambda = 5$

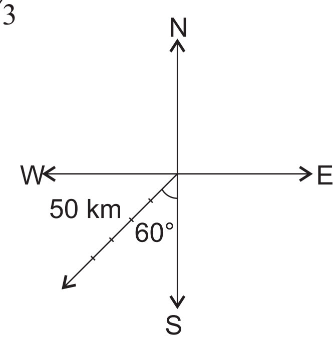
20. a = 3

21. $\lambda = 1$

22. $\left(5, \frac{14}{3}, -6\right)$

23. $\pi/3$

24.



25. $z\hat{j} - x\hat{k}$

TWO MARK QUESTIONS

1. $4(\sqrt{2}\hat{i} + \hat{j} + \hat{k})$
2. 22
3. 2
4. $m = 8$
5. 0
6. 3
7. $\lambda = 2$
8. —
9. —
10. –169
11. $2\sqrt{2}$

12. $\overrightarrow{AC} = 3(\vec{a} - \vec{b}), \overrightarrow{BC} = 4(\vec{a} - \vec{b})$

14. $p = \frac{2}{3}, 2$

15. $\sqrt{33}$

16. $\frac{32}{7}$ $\frac{32}{49}, (2\hat{i} + 6\hat{j} + 3\hat{k})$

17. $x = -\frac{31}{7}$ $y = \frac{41}{12}$

18. 0

19. $\frac{\pi}{2}$

THREE MARKS QUESTIONS

1. $x = 3,\ y = 3,\ 1:2$

3. $\vec{d} = 7\hat{\imath} - 7\hat{\jmath} - 7\hat{k}$

5. $\cos^{-1}\frac{1}{\sqrt{3}}$

11. $\vec{b} = \frac{5}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}$

14. $\frac{4}{\sqrt{38}}$ units

16. $5\sqrt{2}$

17. $(-\hat{\imath} - \hat{\jmath} - \hat{k}) + (7\hat{\imath} - 2\hat{\jmath} - 5\hat{k})$

20. 0

21. 18 cu. Units

22. 60°

23. $\lambda = 1$

25. $\hat{i} - 11\hat{j} - 7\hat{k}$

27. $\vec{\beta} = \left(\frac{3}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}\right) + \left(\frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath} + 3\hat{k}\right)$

28. $\frac{-1}{\sqrt{165}}(10\hat{\imath} + 7\hat{\jmath} - 4\hat{k})$

29. $\frac{13}{\sqrt{170}}\hat{i} + \frac{1}{\sqrt{170}}\hat{j}$

31. $\sqrt{59}$

34 -1

35. 0

36. $-\frac{11}{2}$

37. $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$

CHAPTER–11

THREE-DIMENSIONAL GEOMETRY

POINTS TO REMEMBER

- **Distance Formula:** Distance (d) between two points(x_1 , y_1 , z_1)and(x_2, y_2, z_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- **Section Formula:** line segment AB is divided by P (x, y, z) in ratio m:n

(a) Internally	**(b) Externally**
$\left(\frac{m\,x_2 + n\,x_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$	$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$

- **Direction ratio** of a line through $(x_1, y_1, z_1)$and (x_2, y_2, z_2) are$x_2 - x_1, y_2 - y_1, z_2 - z_1$
- **Direction cosines** of a line having direction ratios as a, b, c are:

$$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$$

- **Equation of line in space:**

Vector form	**Cartesian form**
(i) Passing through point $\vec{a}$ and parallel to vector $\vec{b}$; $\vec{r} = \vec{a} + \lambda\vec{b}$	(i) Passing through point (x_1, y_1, z_1) and having direction ratios a, b, c;

	$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
(ii)Passing through two points $\vec{a}$and$\vec{b}$; $\vec{r} = \vec{a} + \lambda\,(\vec{b} - \vec{a})$	(ii) Passing through two points $(x_1, y_1 z_1)$ and $(x_2, y_2 z_2)$; $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

- **Angle between two lines:**

Vector form	Cartesian form
(i) For lines $\vec{r} = \overrightarrow{a_1} + \lambda\,\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\,\overrightarrow{b_2}$, $\cos\theta = \frac{\lvert\vec{b}_1 \cdot \vec{b}_2\rvert}{\lvert\vec{b}_1\rvert\lvert\vec{b}_2\rvert}$ where 'θ' is the angle between two lines.	(ii) For lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ $\cos\theta = \frac{\lvert a_1a_2 + b_1b_2 + c_1c_2\rvert}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$
(iii) Lines are perpendicular if $\vec{b}_1 . \vec{b}_2 = 0$	(ii) Lines are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$
(iv) Lines are parallel if $\vec{b}_1 = k\vec{b}_2$; $k \neq 0$	(i) Lines are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- **Equation of plane:**

If p is length of perpendicular from origin to plane and $\hat{n}$ is unit vector normal to plane $\vec{r} . \hat{n} = p$	If p is length of perpendicular from origin to plane and l, m, n ared.c.s of normal to plane $lx + my + nz = p$

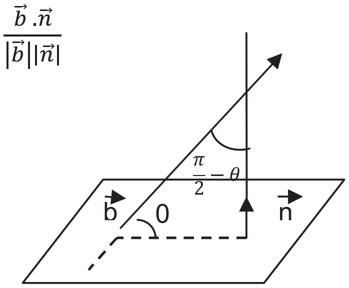
Passing through $\vec{a}$ and $\vec{n}$ is normal to plane :$(\vec{r}-\vec{a}).\vec{n}=0$	Passing through (x_1, y_1, z_1) and a, b, c are d.r.s of normal to plane: $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$
Passing through three non collinear points $\vec{a}, \vec{b}, \vec{c}$: $(\vec{r}-\vec{a}).\left[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\right]=0$	Passing through three non collinear points$(x_1, y_1, z_1)(x_2, y_2, z_2)(x_3, y_3, z_3)$: $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$
If a, b, c are intercepts on co-ordinate axes $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$	If x_1, y_1, z_1 are intercepts on coordinate axes $\frac{x}{x_1}+\frac{y}{y_1}+\frac{z}{z_1}=1$
Plane passing through line of intersection of planes $\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$ is $\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ (λ=real no.)	Plane passing through the line of intersection of planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is $(a_1x+b_1y+c_1z+d_1)+\lambda(a_2x+b_2y+c_2z+d_2)=0$

- **Angle between planes:**

Angle θ between planes $\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$ is $\cos\theta=\frac{\lvert\vec{n}_1 . \vec{n}_2\rvert}{\lvert\vec{n}_1\rvert\lvert\vec{n}_2\rvert}$	Angle θ between planes $a_1x+b_1y+c_1z=d_1$ and $a_2x+b_2y+c_2z=d_2$ is $\cos\theta=\frac{\lvert a_1a_2+b_1b_2+c_1c_2\rvert}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

Planes are perpendicular iff $\vec{n}_1 \cdot \vec{n}_2 = 0$	Planes are perpendicular iff $a_1a_2 + b_1b_2 + c_1c_2 = 0$
Planes are parallel iff $\vec{n}_1 = \lambda\, \vec{n}_2$; $\lambda \neq 0$	Planes are parallel if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

- **Angle between line and plane:**

Angle θ between line $\vec{r} = \vec{a} + \lambda\vec{b}$ and plane $\vec{r} \cdot \vec{n} = d$ is $\sin\theta = \cos(90^\circ - \theta)$ $= \dfrac{\vec{b} \cdot \vec{n}}{\lvert\vec{b}\rvert\lvert\vec{n}\rvert}$ 	Angle θ between line $\dfrac{x-x_1}{a_1} = \dfrac{y-y_1}{b_1} = \dfrac{z-z_1}{c_1}$ and plane $a_2x + b_2y + c_2z = d$ is $\sin\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

- **Distance of a point from a plane**

The perpendicular distance p from the point P with position vector $\vec{a}$ to the plane $\vec{r} \cdot \vec{n} = d$ is given by $p = \dfrac{\lvert\vec{a} \cdot \vec{n} - d\rvert}{\lvert\vec{n}\rvert}$	The perpendicular distance p from the point P (x_1, y_1, z_1) to the plane $Ax + By + Cz + D = 0$ is given by $p = \dfrac{\lvert Ax_1 + By_1 + Cz_1 + D\rvert}{\sqrt{A^2 + B^2 + C^2}}$

- **Coplanarity**

Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are coplanar iff $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 0$	Two lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplanar iff $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

- **Shortest distance between two skew lines**

The shortest distance between lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is $d = \left\lvert \frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{\lvert\vec{b}_1 \times \vec{b}_2\rvert} \right\rvert$	The shortest distance between $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is $d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{D}}$ Where $D = \{(a_1b_2 - a_2b_1)^2 + (b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2\}$

ONE MARK QUESTIONS

1. What is the distance of point (a, b, c) from x-axis?

2. What is the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$?

3. Write the equation of a line passing through (2, -3, 5) and parallel to line $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z+1}{-1}$.

4. Write the equation of a line through (1, 2, 3) and parallel to $\vec{r}.\left(\hat{\imath} - \hat{\jmath} + 3\hat{k}\right) = 5$.

5. What is the value of λ for which the lines $\frac{x-1}{2} = \frac{y-3}{5} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z}{2}$ are perpendicular to each other?

6. Write line $\vec{r} = (\hat{\imath} - \hat{\jmath}) + \lambda\left(2\hat{\jmath} - \hat{k}\right)$ into Cartesian form.

7. If the direction ratios of a line are 1, -2, 2 then what are the direction cosines of the line?

8. Find the angle between the planes $2x - 3y + 6z = 9$ and xy – plane.

9. Write equation of a line passing through (0, 1, 2) and equally inclined to co-ordinate axes.

10. What is the perpendicular distance of plane $2x - y + 3z = 10$ from origin?

11. What is the y-intercept of the plane $x - 5y + 7z = 10$?

12. What is the distance between the planes $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + 5 = 0$.

13. What is the equation of the plane which cuts off equal intercepts of unit length on the coordinate axes?

14. Are the planes $x + y - 2z + 4 = 0$ and $3x + 3y - 6z + 5 = 0$ intersecting?

15. What is the equation of the plane through the point (1, 4, -2) and parallel to the plane $-2x + y - 3z = 7$?

16. Write the vector equation of the plane which is at a distance of 8 units from the origin and is normal to the vector $(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$.

17. What is equation of the plane if the foot of perpendicular from origin to this plane is (2, 3, 4)?

18. Find the angles between the planes $\vec{r}.(\hat{\imath} - 2\hat{\jmath} - 2\hat{k}) = 1$ and $\vec{r}.(3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}) = 0$.

19. If O is origin OP = 3 with direction ratios proportional to -1, 2, -2 then what are the coordinates of P?

20. What is the distance between the line $\vec{r} = 2\hat{\imath} - 2\hat{\jmath} + 3\hat{k} + \lambda\,(\hat{\imath} + \hat{\jmath} + 4\hat{k})$ from the plane $\vec{r}.(-\hat{\imath} + 5\hat{\jmath} - \hat{k}) + 5 = 0$.

21. Write the line $2x = 3y = 4z$ in vector form.

22. The line $\frac{x-4}{1}=\frac{2y-4}{2}=\frac{k-z}{-2}$ lies exactly in the plane $2x-4y+z=7$. Find the value of k.

23. Write direction ratios and direction cosines of *z*-axis.

24. Write direction ratios and direction cosines of the line $\frac{x+1}{3}=\frac{y-1}{-1}$, z + 1 = 0.

25. The cartesian equations of a line are x = ay + b, z = cy + d. Find direction ratios of the line, also write its equation in vector form.

TWO MARK QUESTIONS

1. What is the angle between the line $\frac{x+1}{3}=\frac{2y-1}{4}=\frac{2-z}{-4}$ and the plane 2x + y – 2z + 4 = 0

2. Find the equation of a line passing though (2, 0, 5) and which is parallel to line 6x – 2 = 3y + 1 = 2z –2

3. Find the equation of the plane passing through the points (2, 3, –4) and (1, –1, 3) and parallel to the x – axis.

4. Find the distance between the planes 2x + 3y –4z + 5 = 0 and $\hat{r}.(4\hat{i}+6\hat{j}-8\hat{k})=11$

5. The equation of a line are 5x – 3 = 15y + 7 = 3 – 10 z. Write the direction cosines of the line

6. If a line makes angle α, β, γ with Co-ordinate axis then what is the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$

7. Find the equation of a line passing through the point (2, 0, 1) and parallel to the line whose equation is $\vec{r}=(2\lambda+3)\hat{i}+(7\lambda-1)\hat{j}+(-3\lambda+2)\hat{k}$

8. The plane 2x – 3y + 6 z – 11 = 0 makes an angle $\sin^{-1}\alpha$ with x – axis. Find the value of α.

9. If 4x + 4y – cz = 0 is the equation of the plane passing through the origin that contains the line $\frac{x+5}{2}=\frac{y}{3}=\frac{z-7}{4}$, then find the value of c.

10. Find the equation of the plane passing through the point (–2, 1, –3) and making equal intercept on the coordinate axes.

11. Write the sum of intercepts cut off by the plane $\hat{r}.(2\hat{i} + \hat{j} - \hat{k}) - 5 = 0$ on the three axis.

12. Find the condition that the lines x = ay + b, z = cy + d and x = a′y + b′, z = c′y + d′ may be perpendicular to each other.

13. If the line drawn from the point (2, 1, 3) meets a plane at right angles at the point (–1, 3, –3), find the equation of the plane.

14. If the products of distances of the point (1, 1, 1) from the origin and the plane $x - y + z + k = 0$ be 5 units, find the values of '*k*'.

THREE MARKS QUESTIONS

1. Find the equation of a plane containing the points (0, –1, –1), (–4, 4, 4) and (4, 5, 1). Also show that (3, 9, 4) lies on that plane.

2. Find the equation of the plane which is perpendicular to the plane $\bar{r}\,.\left(5\hat{\imath} + 3\hat{\jmath} + 6\hat{k}\right) + 8 = 0$ and which is containing the line of intersection of the planes $\bar{r}\,.\left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right) = 4$ and $\bar{r}\,.\left(2\hat{\imath} + \hat{\jmath} - \hat{k}\right) + 5 = 0$.

3. Find the distance of the point (3, 4, 5) from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$.

4. Find the equation of the plane passing through the intersection of two planes x + 2y + 3z – 5 = 0 and 3x – 2y – Z + 1 = 0 and cutting equal intercepts on x-axis and z-axis.

5. Find vector and Cartesian equation of a line passing through a point with position vector $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and which is parallel to the line joining the points with position vectors$-\hat{\imath} + 4\hat{\jmath} + \hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.

6. Find the equation of the plane passing through the point (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$.

7. Find equation of plane through line of intersection of planes $\overline{r}\,.\,(2\hat{\imath} + 6\hat{\jmath}) + 12 = 0$ and $\overline{r}\,.\,(3\hat{\imath} - \hat{\jmath} + 4\hat{k}) = 0$ which is at a unit distance from origin.

8. Find the image of point (3, –2, 1) in the plane $3x - y + 4z = 2$.

9. Find image (reflection) of the point (7, 4, –3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

10. Find equation of a plane passing through the points (2, –1, 0) and (3,–4, 5) and parallel to the line $2x = 3y = 4z$.

11. Find the distance of the point (–1, –5, –10) from the point of intersection of line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ and the plane $x - y + z = 5$.

12. Find the distance of the point (1, -2, 3) from the plane $x - y + z = 5$, measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$.

13. Find the equation of the plane passing through the intersection of two plane $3x - 4y + 5z = 10, 2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.

14. Find the equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ whose perpendicular distance from the point (1, 2, 3) is 1 unit.

15. Show that the lines line $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and line $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect each other. Find the point of intersection.

16. Find the shortest distance between the lines:

$\overline{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ and

$\overline{r} = (2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$.

17. Find the distance of the point (-2, 3, -4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

18. Find the equation of plane passing through the point (-1, -1, 2) and perpendicular to each of the plane

$\overline{r}\,.(2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}) = 2$ and $\overline{r}\,.(5\hat{\imath} - 4\hat{\jmath} + \hat{k}) = 6$

19. Find the equation of a plane passing through (-1, 3, 2) and parallel to each of the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

20. Show that the plane $\overline{r}\,.(\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) = 7$ contains the line

$\overline{r} = (\hat{\imath} + 3\hat{\jmath} + 3\hat{k}) + \lambda(3\hat{\imath} + \hat{\jmath})$.

21. Check the co planarity of lines

$\overline{r} = (-3\hat{\imath} + \hat{\jmath} + 5\hat{k}) + \lambda(-3\hat{\imath} + \hat{\jmath} + 5\hat{k})$.

$$\overline{r} = (-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}) + \mu(-\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$$

If they are coplanar, find equation of the plane containing the lines.

22. Find shortest distance between the lines:

$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{5-y}{2} = \frac{z-7}{1}$

23. Find the shortest distance between the lines:

$\overline{r} = (1-\lambda)\hat{\imath} + (\lambda - 2)\hat{\jmath} + (3 - 2\lambda)\hat{k}$

$$\overline{r} = (\mu + 1)\hat{\imath} + (2\mu - 1)\hat{\jmath} - (2\mu + 1)\hat{k}$$

24. A variable plane is at a constant distance 3 p from the origin and meets the coordinates axes in A, B and C. If the centroid of $\Delta\, ABC$ is (α, β, γ), then show that $\alpha^{-2} + \beta^{-2} + \gamma^{-2} = p^{-2}$

25. A vector $\vec{n}$ of magnitude 8 units is inclined to x-axis at 45°, y axis at 60° and an acute angle with z-axis. If a plane passes through a point $(\sqrt{2}, -1, 1)$and is normal to $\vec{n}$, find its equation in vector form.

26. Find the foot of perpendicular from the point $2\hat{\imath} - \hat{\jmath} + 5\hat{k}$ on the line $\overline{r} = (11\hat{\imath} - 2\hat{\jmath} - 8\hat{k}) + \lambda(10\hat{\imath} - 4\hat{\jmath} - 11\hat{k})$. Also find the length of the perpendicular.

27. A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonal of a cube. Prove that $cos^2\alpha + cos^2\beta + cos^2\gamma + cos^2\delta = \frac{4}{3}$

28. Find the equation of the plane passing through the intersection of planes $2x + 3y - z = -1$ and $x + y - 2z + 3 = 0$ and perpendicular to the plane $3x - y - 2z = 4$. Also find the inclination of this plane with xy-plane.

29. Find the length and the equations of the line of shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

30. Show that $\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{5}=\frac{y-2}{2}, z=2.$ do not intersect each other.

31. Find the direction ratios of a normal to the plane, which passes through the points (1, 0, 0), (0, 1, 0) and makes angle $\pi\backslash 4$ with the plane $x + y = 3$. Also find the equation of the plane.

32. Find the equations of the two planes passing through the points (0, 4, –3) and (6, –4, 3), if the sum of their intercepts on the three axes is zero.

33. Find the coordinates of the foot of perpendicular and perpendicular distance from the point $P(3, 2, 1)$ to the plane $2x - y + z + 1 = 0$. Also find the image of the point P in the plane.

34. If the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then find the value of k and hence fnd the equation of the plane containing these lines.

35. Find the equation of the line which intersects the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and passes through the point (1, 1, 1).

36. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angle of $\pi/3$.

ANSWERS

ONE MARK QUESTIONS

1. $\sqrt{b^2+c^2}$
2. 90°
3. $\frac{x-2}{3}=\frac{y+3}{4}=\frac{z-5}{-1}$
4. $\bar{r}=(\hat{\imath}+2\hat{\jmath}+3\hat{k})+\lambda(\hat{\imath}-\hat{\jmath}+3\hat{k})$
13. $x+y+z=1$
14. No
15. $-2x+y-3z=8$
16. $\bar{r}.(2\hat{\imath}+\hat{\jmath}+2\hat{k})=24$
17. $2x+3y+4z=29$

5. $\lambda = 2$

6. $\frac{x-1}{0} = \frac{y+1}{2} = \frac{z}{-1}$

7. $\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{2}{3}$

8. $\cos^{-1}(6/7)$

9. $\frac{x}{a} = \frac{y-1}{a} = \frac{z-2}{a}$,

$a \in R - \{0\}$

10. $\frac{10}{\sqrt{14}}$

11. -2

12. $\frac{1}{6}$

18. $\cos^{-1}\left(\frac{11}{21}\right)$

19. (–1, 2,–2)

20. $\frac{10}{3\sqrt{3}}$

21. $\vec{r} = \vec{0} + \lambda(6\hat{\imath} + 4\hat{\jmath} + 3\hat{k})$.

22. k = 7

23. <0, 0, 1>, 0, 0, 1

24. <3, –1, 0>, $\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}, 0$

25. <a, 1, c>, $\vec{r} = b\hat{i} + d\hat{k} + \lambda(a\hat{i} + \hat{j} + c\hat{k})$

TWO MARK QUESTIONS

1. 0° (line is parallel to plane)

2. $\frac{x-2}{1} = \frac{y}{2} = \frac{z-5}{3}$

3. 7y + 4z = 5

4. $\frac{21}{2\sqrt{29}}$ units

5. $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$

6. 2

7. $\vec{r} = (2\hat{i} + \hat{k}) + \lambda(2\hat{i} + 7\hat{j} - \hat{k})$

8. $\alpha = \frac{2}{7}$

9. C = 5

10. x + y + z = –4

11. $\frac{5}{2}$

12. aa′ + cc′ + 1 = 0

13. 3x – 2y + 6z = 27 = 0

14. K = 4, –6

THREE/FIVE MARK QUESTIONS

1. $5x - 7y + 11z + 4 = 0$

2. $\bar{r}.\left(-51\hat{\imath} - 15\hat{\jmath} + 50\hat{k}\right) = 173$

3. 6 units

4. 5x + 2y + 5z – 9 = 0

5. $\bar{r} = \left(2\hat{\imath} - \hat{\jmath} + \hat{k}\right) + \lambda\left(2\hat{\imath} - 2\hat{\jmath} + \hat{k}\right)$ and $\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-1}{1}$

6. $5x + 2y - 3z - 17 = 0$

7. $\bar{r}.\left(2\hat{\imath} + \hat{\jmath} + 2\hat{k}\right) + 3 = 0$ or $\bar{r}.\left(-\hat{\imath} + 2\hat{\jmath} - 2\hat{k}\right) + 3 = 0$

8. (0, -1, -3)

9. $\left(-\frac{51}{7}, -\frac{18}{7}, \frac{43}{7}\right)$

10. $29x$ –27y–22z=85

11. 13

12. 1 unit

13. $x - 20y + 27z = 14$

14. $x - 2y + 2z = 0$ *and* $x - 2y + 2z = 6$

15. $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$

16. $\frac{1}{\sqrt{6}}$

17. $\frac{17}{2}$ units

18. $\bar{r}.\left(9\hat{\imath} + 17\hat{\jmath} + 23\hat{k}\right) = 20$

19. $2x - 7y + 4z + 15 = 0$

21. $x - 2y + z = 0$

22. $2\sqrt{29}$ units

23. $\frac{8}{\sqrt{29}}$

25. $\bar{r}.\left(\sqrt{2}\hat{\imath} + \hat{\jmath} + \hat{k}\right) = 2$

26. $(1, 2, 3), \sqrt{14}$

28. $7x + 13y + 4z = 9, \cos^{-1}\left(\frac{4}{\sqrt{234}}\right)$

29. $SD = 14\, units,$

$$\frac{x-5}{2} = \frac{y-7}{3} = \frac{z-3}{6}$$

31. $< 1, 1, \pm\sqrt{2} >, x + y + \sqrt{2}z - 1 = 0, x + y - \sqrt{2}z - 1 = 0$

32. 6x + 3y – 2z = 18, 2x – 3y = 6z = 6

33. (1, 3, 0), $\sqrt{6}$, image (–1, 4, –1)

34. K = $\frac{9}{2}$, 5x – 2y – z – 6 =0

35. $\frac{x-1}{3}=\frac{y-1}{10}=\frac{z-1}{17}$

36. $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ and $\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$

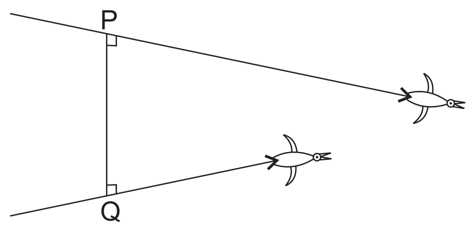
Case Study Based Questions

1. Two birds are flying in the space along straight path L_1 and L_2

(Neither parallel nor intersecting) where,

$L_1 = \frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$

$L_2 = \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-3}{1}$



On the basis of this answer the following

(a) Vector form of L_1 is

(i) $\vec{r} = (3\hat{i}+8\hat{j}+3\hat{k})+\lambda(3\hat{i}+\hat{j}+\hat{k})$

(ii) $r = (-3\hat{i}-8\hat{j}-3\hat{k})+\lambda(3\hat{i}-\hat{j}-\hat{k})$

(iii) $\vec{r} = (3\hat{i}+8\hat{j}+3\hat{k})+\lambda(3\hat{i}-\hat{j}+\hat{k})$

(iv) $\vec{r} = (3\hat{i}+8\hat{j}+3\hat{k})+\lambda(3\hat{i}-\hat{j}+2\hat{k})$

(b) If PQ $\perp$ L_1 and PQ $\perp$ L_2 and , then coordinates *P* are

(i) (3, 8, 3) (ii) (–3, 8, –3)

(iii) (3, –8, 3) (iv) (–3, –8, –3)

(c) Directions ratios of *PQ*

(i) 2, 1, 5 (ii) 2, 5, –1

(iii) 5, 1, 2 (iv) 5, 2, 1

(d) Distance *PQ* is

(i) $2\sqrt{15}$ units (ii) $2\sqrt{30}$ units

(iii) $3\sqrt{30}$ units (iv) $3\sqrt{20}$ units

(e) Equation of the path *PQ* is

(i) $\vec{r} = (3\hat{i} + 8\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 5\hat{j} - \hat{k})$

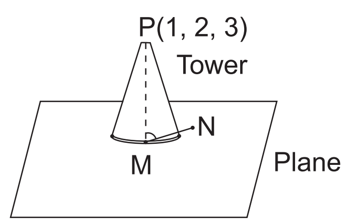
(ii) $\vec{r} = (3\hat{i} - 8\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 5\hat{j} - \hat{k})$

(iii) $\vec{r} = (3\hat{i} + 8\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 5\hat{j} + \hat{k})$

(iv) $\vec{r} = (3\hat{i} - 8\hat{j} - 3\hat{k}) + \lambda(2\hat{i} - 5\hat{j} - \hat{k})$

2. Let the point *P* (1, 2, 3) lies on the top of tower, which is standing (perpendicular) on the plane $x + 2y + 4z = 38$.

On the basis of the given informations answer the following.



(a) Equation of the line through *P*, perpendicular to the plane

(i) $\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

(ii) $\vec{r} = \hat{i} + 2\hat{j} + 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

(iii) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 4\hat{k})$

(iv) $\vec{r} = \hat{i} - 2\hat{j} - 3\hat{k} + \lambda(\hat{i} - 2\hat{j} - 4\hat{k})$

(b) Coordinates of foot of perpendicular *M* are

(i) (2, –4, 7)

(ii) (2, 4, 7)

(iii) (2, 4, –7)

(iv) (–2, 4, 7)

(c) Height of tower is

(i) $\sqrt{21}$ units

(ii) $2\sqrt{21}$ units

(iii) $\sqrt{17}$ units

(iv) $2\sqrt{17}$ units

(d) If plane assumed as plane mirror then image of *P* is

(i) (–2, –4, –7)

(ii) (4, 8, 14)

(iii) (2, 4, 7)

(iv) (3, 6, 11)

(e) If *N* (4, 5, 6) is any point on the plane, then angle between *PM* and *MN* is

(i) 60°

(ii) 45°

(iii) 30°

(iv) 90°

ANSWERS

1. (a) (iii) (b) (i) (c) (ii) (d) (iii) (e) (i)

2. (a) (iii) (b) (ii) (c) (i) (d) (iv) (e) (iv)

Chapter 12

LINEAR PROGRAMMING

KEY POINTS :

- **OPTIMISATION PROBLEM :** is a problem which seeks to maximize or minimize a function. An optimisation problem may involve maximization of profit, minimization of transportation cost etc, from available resources.
- **A LINEAR PROGRAMMING PROBLEM (LPP) :** LPP deals with the optimisation (maximisation/minimisation) of a linear function of two variables (say x and y) known as objective function subject to the condiotions that the variables are non negative and satisfy a set of linear inequalities (called linear constraints). A LPP is a special type of optimisation problem.
- **OBJECTIVE FUNCTION :** Linear function $z = ax + by$ where a and b are constants which has to be maximised or minimised is called a linear objective function.
- **DECISION VARIABLES :** In the objective function $z = ax + by$, x and y are called decision variables.
- **CONSTRAINTS :** The linear inequalities or restrictions on the variables of an LPP are called constraints.

  The conditions $x \geq 0, y \geq 0$ are called non-negative constraints.
- **FEASIBLE REGION :** The common region determined by all the constraints including non-negative constraints $x \geq 0, y \geq 0$ of an LPP is called the feasible region for the problem.
- **FEASIBLE SOLUTION :** Points within and on the boundary of the feasible region for an LPP represent feasible solutions.
- **INFEASIBLE SOLUTIONS :** Any point outside the feasible region is called an infeasible solution.
- **OPTIMAL (FEASIBLE) SOLUTION :** Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.
- **THEOREM 1 :** Let R be the feasible region (convex polygon) for an LPP and let $z = ax + by$ be the objective function. When z has an optimal value (maximum or minimum), where x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

- **THEOREM 2 :** Let R be the feasible region for a LPP. & let $z = ax + by$ be the objective function. If R is bounded, then the objective function z has both a maximum and a minimum value on R and each of these occur at a corner point of R.

  If the feasible region R is unbounded, then a maximum or minimum value of the objective function may or not exist. However, if it exists it must occur at a corner point of R.

- **MULTIPLE OPTIMAL POINTS :** If two corner points of the feasible region are optimal solutions of the same type i.e both produce the same maximum or minimum, then any point on the line segment joining these two points is also an optimal solution of the same type.

FIVE MARKS QUESTIONS

Q. 1 Solve the following LPP graphically.

Maximize $z = 3x + y$ subject to the constraints

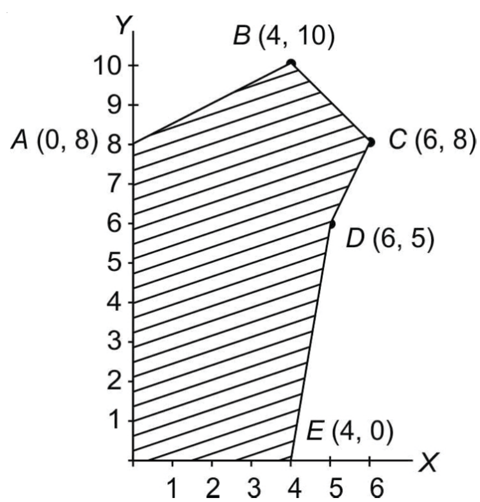
$x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x, y \geq 0$

Q.2 The corner points of the feasible region determined by the system of linear constraints are as shown below.



Answer each of the following :

(i) Let $z = 3x - 4y$ be the objective function. Find the maximum and minimum value of z and also the corresponding points at which the maximum and minimum value occurs.

(ii) Let $z = px + qy$ where $p, q > 0$ be the objective function. FInd the condition on p and q so that the maximum value of z occurs at B (4, 10) and C (6, 8). Also mention the number of optimal solutions in this case.

Q. 3 There are two types of fertilisers A and B. A consists of 10% nitrogen and 6% phosphoric acid and B consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If A costs Rs. 6 per kg and B costs Rs. 5 per kg. determine how much of each type of fertiliser should be used so that nutrient requirements are met at minimum cost. What is the minimum cost?

Q. 4 A man has Rs. 1500 to purchase two types of of shares of two different companies S1 and S2. Market price of one share of S1 is Rs. 180 and S2 is Rs 120. He wishes to purchase a maximum of ten shares only. If one share of type S1 gives a yield of Rs 11, and of type S2 yields Rs 8 then how much shares of each type must be purchased to get maximum profit? and what will be the maximum profit?

Q. 5 A company manufactures two types of lamps say A and B. Both lamps go through a cutter and then a finisher. Lamp A requiires 2 hours of the cutter's time and 1 hours of the finisher's time. Lamp B required 1 hr of cutter's 2 hrs of finisher's time. The cutter has 100 hours and finisher has 80 hours of time available each month. Profit on one lamp A is Rs. 7.00 and on one lamp B is Rs 13.00. Assuming that he can sell all that he produces how many of each type of lamp should be manufactured to obtain maximum profit and what will be the maximum profit?

Q.6 A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for atmost 20 items. A fan and sewing machine cost Rs 360 and Rs. 240 respectively. He can sell a fan at a profit of Rs. 22 and sewing machine at a profit of Rs. 18. Assuming that he can sell whatever he buys, how should he invest money to maximise his profit?

Q. 7 A producer has 20 and 10 units of labour and capital respectively which he can use to produce two kinds of goods X and Y. To produce one unit of X, 2 units of capital and 1 unit of labour is required. To produce one unit of Y, 3 units of labour and 1 unit of capital is required. If X and Y are priced at Rs. 80 and Rs100 per unit respectively, how should the producer use his resources to maximize revenue?

Q. 8 A factory owner purchases two types of machines A and B for his factory. The requirements and limitations for the machines are as follows :

Machine	Area Occupied	Labour Force	Daily Output (in units)
A	1000 m^2	12 men	50
B	1200 m^2	8 men	40

He has maximum area of 7600 m^2 available and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise the daily output?

Q.9 A manufacturer makes two types of cups A and B. Three machines are required to manufacture the cups and the time in minutes required by each in as given below :

Types of Cup	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paisa and on B is 50 paisa, find how many cups of each type should be manufactured to maximise the profit per day.

Q. 10 A company produces two types of belts A and B. Profits on these belts are Rs. 2 and Rs. 1.50 per belt respectively. A belt of type A requires twice as much time as belt of type B. The company can produce at most 1000 belts of type B per day. Material for 800 belts per day is available. At most 400 buckles for belts of type A and 700 for type B are available per day. How much belts of each type should the company produce so as to maximize the profit?

Q. 11 An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and as profit of Rs. 300 is made on each second class ticket. The airline reserves atleast 20 seats for first class. However at least four times as many passengers prefer to travel by second class than by first class. Determine how many tickets of each type must be sold to maximize profit for the airline.

Q. 12 A diet for a sick person must contains at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs. 5 and Rs. 4 per unit respectively. One unit of food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories whereas one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the food A and B should be used to have least cost but it must satisfy the requirements of the sick person.

Q.13 Anil wants to invest at most Rs. 12000 in bonds A and B. According to the rules, he has to invest at least Rs. 2000 in Bond A and at least Rs. 4000 in bond B. If the rate of interest on bond A and B are 8% and 10% per annum respectively, how should he invest this money for maximum interest? Formute the problem as LPP and solve graphically.

ONE MARKS QUESTIONS

1. The feasible region for a LPP is always a _______ polygon.
2. A corner point of a feasible region is a point in the region which is the _______ of two boundary lines.
3. Regions represented by the equations $x \geq 0, y \geq 0$ is which quadrant?
4. Half plane below the x-axis including the points on x-axis is represented by which inequality?
5. State T/F

   The solution set of the inequation $2x + y > 5$ is open half plane not containing the origin.
6. What do we call a feasible region of a system of linear inequalities if it can be enclosed within a circle?
7. If in a LPP, the objective function $z = ax + by$ has the same maximum value on two corner points A & B of the feasible region, then how many optimal solutions will that LPP have?
8. What do we call the linear inequalities or restrictions on the variables in LPP?
9. When the optimal value of the objective function in a LPP may or may not exist.
10. If the feasible region of LPP is bounded, then name the method which is used to find the optimal solution.

CASE STUDY QUESTIONS

Q. 1 A man rides his motorcycle at the speed of 50 km/hr. He has to spend Rs 2/km on petrol. But if he rides it at a faster speed of 80 km/hr, the petrol cost incxreases to Rs 3/km. He has atmost Rs 120 to spend on petrol and one hr's time. he wishes to find the maximum distance that he can travel.

Based on the above information answer the following questions.

(1) If he travels x km with the speed of 50 km/hr and y km with the speed of 80 km/hr, then which of the following is false.

(a) Maximise $x+y$ (b) $\frac{x}{50}+\frac{y}{80}\leq 1$

(c) $\frac{x}{60}+\frac{y}{40}\leq 1$ (d) All of these

(2) Maximum distance man can travel is given by

(a) $54\frac{2}{7}$ km (b) 50 km

(c) 40 km (d) 52 km

(3) If he covers maximum distance then how much distance he travels with the speed of 50 km/hr.

(a) 50 km (b) 40 km

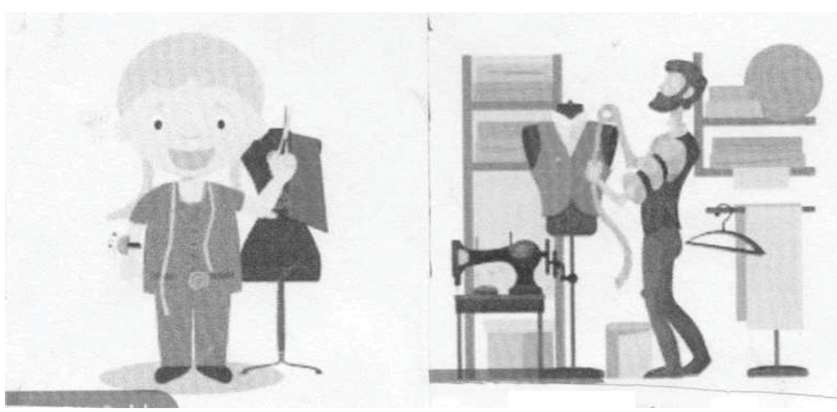
(c) $48\frac{6}{7}$ km (d) $11\frac{3}{7}$ km

(4) What is the average speed during the whole journey for covering the maximum distance.

(a) 80 km/hr (b) $55\frac{2}{7}$ km/hr

(c) $\frac{380}{7}$ km/hr (d) $53\frac{2}{7}$ km/hr

Q. 2 Two tailors A and B earn Rs 150 and Rs 200 per day respectively. A can sticth 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. it is desired to produce atleast 60 shirts and 32 pants at a minimum labour cost.



Tailor A **Tailor B**

Based on the above information answer the following.

(1) If x and y are the number of days A and B work respectively then the objective function for this LPP is

(a) min $z = 150x + 200y$ (b) $6x + 10y \geq 60$

(c) $x + y \geq 8$ (d) $4x + 4y \geq 32$

(2) The optimal solution for this LPP is

(a) (10, 0) (b) (0, 8)

(c) (5, 3) (d) (0, 6)

(3) Minimum labour cost will be

(a) Rs 400 (b) Rs 1250

(c) Rs 1600 (d) Rs 1350

(4) In this LPP, feasible region is

(a) Bounded (b) Un bounded

(c) None of these (d) All of these

(5) In a LPP, the feasible region is always a _________ polygon.

(a) Convexo voncave (b) Concavo convex

(c) Concave (d) Convex

Answers

FIVE MARKS QUESTIONS

1. Max z = 250 at x = 50, y = 100
2. (i) Max z = 12 at (4, 0) and min z = –32 at (0, 8)

   (ii) $p = q$, infinite solutions lying on the line segment joining the points B and C.
3. 100 kg of fertilizer A and 80 kg of fertilizer B, minimum cost Rs 1000
4. Maximum profit = Rs 95 with 5 shares of each type.
5. Lamps of type A = 40, Lamps of type B = 20 Max profit = Rs 540
6. Fans : 8, sewing machines : 12, max profit : Rs 392
7. X : 2 units, Y : 6 units, max revenue is Rs 760.
8. Type A : 4, Type B : 3
9. Cup A : 15, cup B : 30
10. Max profit Rs 1300, No of belts of type A = 200 and No of belts of type B = 600
11. No of first class ticket = 40, No of second class tickets = 160
12. Food A : 5 units, food B : 30 units
13. Maximum interest is Rs 1160 at (2000, 10000)

ONE MARK QUESTIONS

1. Convex
2. Intersection
3. Ist quadrant
4. $y \leq 0$
5. True
6. Bounded
7. Infinite
8. LInear constraints
9. If the feasible region is unbounded
10. Corner point method

CASE STUDY QUESTIONS

1. (i) (d) (ii) (a) (iii) (c) (iv) (c)
2. (i) (a) (ii) (c) (iii) (d) (iv) (d) (v) d

Chapter 13

PROBABILITY

KEY POINTS

Conditional Probability : If A and B are two events associated with the same sample space of a random experiment, then the conditional probability of the event A under the condition that the event B has already occurred, written as $P(A|B)$, is given by

$$P(A|\mathrm{B}) = \frac{(P \cap B)}{P(B)}, \quad P(B) \neq 0.$$

Properties :

(1) $P(S|F) = P(F|F) = 1$ where S denotes sample space

(2) $P((A \cup B)|F) = P(A|F) + P(B|F) - P((A \cap B)|F)$

(3) $P(E'|F) = 1 - P(E|F)$

Multiplication Rule : Let E and F be two events associated with a sample place of an experiment. Then

$$P(E \cap F) = P(E)\, P(F|E) \text{ provided } P(E) \neq 0$$

$$= P(F)\, P(E|F) \text{ provided } P(F) \neq 0.$$

If E, F, G are three events associated with a sample space, then

$$P(E \cap F \cap G) = P(E)\, P(F|E)\, P(G|(E \cap F))$$

Independent Events : Let E and F be two events, then if probability of one of them is not affected by the occurrence of the other, then E and F are said to be independent, i.e.,

(*a*) $P(F|E) = P(F), \quad P(E) \neq 0$

or (*b*) $P(E|F) = P(E), \quad P(F) \neq 0$

or (*c*) $P(E \cap F) = P(E)\, P(F)$

Three events A, B, C are mutually independent if

$$P(A \cap B \cap C) = P(A)\, P(B)\, P(C)$$

$$P(A \cap B) = P(A)\, P(B)$$

$$P(B \cap C) = P(B)\, P(C)$$

and $$P(A \cap C) = P(A)\, P(C)$$

Partition of a Sample Space : A set of events E_1, E_2, ..., E_n is said to represent a partition of a sample space S if

(*a*) $E_i \cap E_j = \phi$; $i \neq j$; $i, j = 1, 2, 3, ..., n$

(*b*) $E_1 \cup E_2 \cup E_3 ... \cup E_n = S$ and

(*c*) Each $E_i \neq \phi$ i.e. $P(E_i) > 0 \;\forall\; i = 1, 2, ..., n$

Theorem of Total Probability : Let $\{E_1, E_2, ..., E_n\}$ be a partition of the sample space S. Let A be the any event associated with S, then

$$P(A) = \sum_{j=1}^{n} P(E_j)\, P(A|E_j)$$

Baye's Theorem : If E_1, E_2, ..., E_n are mutually exclusive and exhaustive events associated with a sample space S, and A is any event associated with E_i's having non-zero probability, then

$$P(E_i|A) = \frac{P(A \mid E_i)P(E_i)}{\sum_{i=1}^{n} P(A \mid E_i)P(E_i)}$$

Random Variable : A (r.v.) is a real variable which is associated with the outcome of a random experiment.

Probability Distribution of a r.v. X is the system of numbers given by

X :	x_1	x_2	....	x_n
$P(X = x)$:	p_1	p_2	....	p_n

where $$p_i > 0, \quad i = 1, 2, ..., n, \quad \sum_{i=1}^{n} p_i = 1.$$

Mean of a r.v. X :

$$\mu = E(X) = \sum_{i=1}^{n} p_i\, x_i$$

Variance of a r.v. *X* :

$$\sigma^2 = V(X) = \sum_{i=1}^{n} (x_i - \mu)^2 p_i = \sum_{i=1}^{n} p_i x_i^2 - \left(\sum p_i x_i\right)^2$$

$$\Rightarrow \qquad \sigma^2 = \sum_{i=1}^{n} p_i x_i^2 - \mu^2$$

Standard Deviation of r.v. *X* : is

$$\sigma = \sqrt{\text{Var}(X)} = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 p_i}$$

Bernoulli Trials : Trials of a random experiment are called Bernoulli trails, if they satisfy the following conditions:

(*i*) There should be a finite no. of trials.

(*ii*) The trials should be independent.

(*iii*) Each trial has exactly two outcomes : success or failure.

(*iv*) The probability of success (or failure) remains the same in each trial.

Binomial Distribution : A r.v. X taking values 0, 1, 2, .., n is said to have a binomial distribution with parameters n and p, if its probability distribution is given by

$$P(X = x) = {}^nC_x\, p^x\, q^{n-x}\ ;\ x = 0, 1, 2, \ldots, n$$

where $\qquad q = 1 - p.$

ONE MARK QUESTIONS

1. The probabilities of A and B solving a problem independently are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If both of them try to solve the problem independently, what is the probability that the problem is solved ?

2. The probability that it will rain on any particular day is 50%. Find the probability that it rains only on first 4 days of the week.

3. Write the value of $P(A|B)$ if $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$.

4. A soldier fires three bullets on enemy. The probability that the enemy will be killed by one bullet is 0.7. What is the probability that the enemy is still alive ?

5. If $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\bar{A} \text{ or } \bar{B}) = \frac{1}{4}$. State whether A and B are independent.

6. A natural number x is chosen at random from the first hundred natural numbers. Find the probability such that $x + \frac{1}{x} < 2$.

7. A bag contains 50 tickets numbered 1, 2, 3, ..., 50 of which five are drawn at random and arranged in ascending order of magnitude ($x_1 < x_2 < x_3 < x_4 < x_5$). What is the probability that $x_3 = 30$.

8. Let A and B be two events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A|B) = 0.5$. Then find $P(A'|B')$.

9. If X follows binomial distribution with parameters $n = 5$, p and $P(X = 2) = 9P(X = 3)$, then find p.

10. The probability that a person is not a swimmer is 0.3. Find the probability that out of 5 persons 4 are swimmers.

11. Eight coins are tossed together. What is the probability of getting exactly 3 heads.

12. A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, then find the probability that both are dead.

TWO MARKS QUESTIONS

1. A and B are two events such that $P(A) \neq 0$, then find $P(B|A)$ if (*i*) A is a subset of B (*ii*) $A \cap B = \phi$.

2. A random variable X has the following probability distribution, find k.

X	0	1	2	3	4	5
$P(X)$	$\frac{1}{15}$	K	$\frac{15K-2}{15}$	K	$\frac{15K-1}{15}$	$\frac{1}{15}$

3. Out of 30 consecutive integers two are chosen at random. Find the probability so that their sum is odd.

4. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. Find the probability that the eldest child is a girl given that the family has atleast one girl.

5. If A and B are such that $P(A \cup B) = \frac{5}{9}$ and $P(\bar{A} \cup \bar{B}) = \frac{2}{3}$, then find $P(\bar{A}) + P(\bar{B})$.

6. Prove that if A and B are independent events, then A and B' are also independent events.

7. If A and B are two independent events such that $P(A) = 0.3$, $P(A \cup B) = 0.5$, then find $P(A|B) - P(B|A)$

8. Three faces of an ordinary dice are yellow, two faces are red and one face is blue. The dice is rolled 3 times. Find the probability that yellow, red and blue face appear in the first, second and third throw respectively.

9. Find the probability that a leap year will have 53 Fridays or 53 Saturdays.

10. A person writes 4 letters and addresses on 4 envelopes. If the letters are placed in the envelopes at random, then what is the probability that all the letters are not placed in the right envelopes.

11. A box has 100 pens of which 10 are defective. What is the probability that out of a sample of 5 pens drawn one by one with replacement atmost one is defective ?

12. In a class XII of a school, 40% of students study Mathematics, 30% of the students study Biology and 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, then find the probability that he will be studying Mathematics or Biology.

THREE MARKS QUESTIONS

Q.1. A problem in mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. What is the probability that the problem is solved ?

Q.2. If A and B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$ then find $P(A)$ and $P(B)$.

Q.3. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn. What is the probability that they both are diamonds.

Q.4. A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.

Q.5. In a game, a man wins a rupee for a six and looses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/looses.

Q.6. Suppose that 10% of men and 5% of women have grey hair. A grey haired person is selected at random. What is the probability that the selected person is male assuming that there are 60% males and 40% females ?

Q.7. Two dice are thrown once. Find the probability of getting an even number on the first die or a total of 8.

Q.8. Two aeroplanes X and Y bomb a target in succession. There probabilities to hit correctly are 0.3 and 0.2 respectively. The second plane will bomb only if first miss the target. Find the probability that target is hit by Y plane.

Q.9. The random variable X can take only the values 0, 1, 2. Given that $P(X = 0) = P(X = 1) = p$ and that $E(X^2) = E(X)$, find the value of p.

Q.10. Find the variance of the distribution

x	0	1	2	3	4	5
$P(x)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

FIVE MARKS QUESTIONS

Q.1. By examining the chest X-ray, the probability that TB is detected when a person is actually suffering is 0.99. The probability of a healthy person diagnosed to have TB is 0.001. In a certain city, 1 in 1000 people suffers from TB. A person is selected at random and is diagnosed to have TB. What is the probability that he actually has TB ?

Q.2. Three persons A, B and C apply for a job of Manager in a private company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce charges to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change doesn't take place, find the probability that it is due to the appointment of C.

Q.3. A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope, just two consecutive letters TA are visible. What is the probability that the letter came from TATANAGAR.

Q.4. The probability distribution of a random variable X is given as under :

$$P(X = x) = \begin{cases} kx^2 & \text{for } x = 1, 2, 3 \\ 2kx & \text{for } x = 4, 5, 6 \\ 0 & \text{Otherwise} \end{cases}$$

where k is a constant. Calculate

(*i*) $E(X)$ (*ii*) $E(3X^2)$ (*iii*) $P(X \geq 4)$

Q.5. Three critics review a book. Odds in favour of the book are 5 : 2, 4 : 3 and 3 : 4 respectively for the three critics. Find the probability that the majority are in favour of the book.

Q.6. Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6, 7. Let X denotes the larger of the two numbers obtained. Find the mean and variance of the probability distribution of X.

Q.7. An urn contains five balls. Two balls are drawn and are found to be white. What is the probability that all the balls are white ?

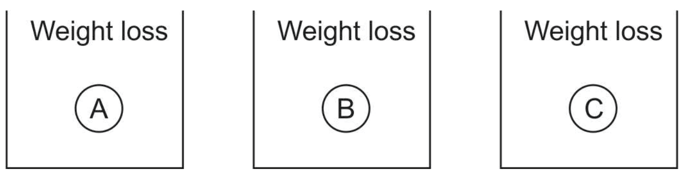
Q.8. Two cards are drawn from a well shuffled pack of 52 cards. Find the mean and variance for the number of face cards obtained.

Q.9. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be both clubs. Find the possibility of the lost card being of club.

Q.10. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II at random. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

CASE STUDY QUESTIONS

Q.1. A company sells three types of Nutritional foods *A*, *B*, *C* for a weightloss programme. These are sold as a mixture where the proportions are 4 : 4 : 2 respectively. The probability of loosing weight by these foods *A*, *B* and *C* are 75%, 80% and 60% respectively.



Based on the above information, answer the following questions :

(*a*) Calculate the probability of randomly chosen food to do weight loss.

(*i*) 73% (*ii*) 74% (*iii*) 75% (*iv*) 95%

(*b*) Calculate the probability that there is no weightloss when it is given that the person takes food *B*

(*i*) 80% (*ii*) 60% (*iii*) 20% (*iv*) 40%

(*c*) Calculate the probability that food was of type *C* given that there is reduction in weight.

(*i*) $\frac{32}{74}$ (*ii*) $\frac{12}{74}$ (*iii*) $\frac{8}{74}$ (*iv*) $\frac{74}{16}$

(*d*) The probability that there is no reduction in weight given the food *C* is

(*i*) 0.4 (*ii*) 0.6 (*iii*) 0.25 (*iv*) 0.2

(*e*) What is the probability of choosing food *B* given that there is no weight loss.

(*i*) 0.4 (*ii*) 0.8 (*iii*) 4/25 (*iv*) 0.3

Q.2. In a birthday party, a magician was being invited by a parent and he had 3 bags that contain number of red and white balls as follows :

Bag 1 : 3 red balls, Bag 2 : 2 white balls and 1 red ball

Bag 3 : 3 white balls

The probability that the bag *i* will be chosen by the magician and a ball is selected from it is $\frac{i}{6}$, $i = 1, 2, 3$.

Based on the above information, answer the following questions.

(*a*) What is the probability that a red ball is selected by the magician

(*i*) $\frac{13}{18}$ (*ii*) $\frac{5}{6}$ (*iii*) $\frac{1}{6}$ (*iv*) $\frac{5}{18}$

(*b*) What is the probability that a white ball is selected by the magician

(*i*) $\frac{5}{6}$ (*ii*) $\frac{13}{18}$ (*iii*) $\frac{1}{6}$ (*iv*) $\frac{5}{18}$

(*c*) Given that the magician selects the white balls, what is the probability that this ball was from Bag 2.

(*i*) $\frac{4}{13}$ (*ii*) $\frac{4}{5}$ (*iii*) 0 (*iv*) $\frac{1}{2}$

(*d*) Given that the magician selects the red ball, what is the probability that this ball was from Bag 1.

(*i*) $\frac{3}{5}$ (*ii*) $\frac{4}{5}$ (*iii*) $\frac{4}{13}$ (*iv*) $\frac{1}{2}$

(*e*) What is the probability of selecting either red or white ball from Bag 2.

(*i*) 0 (*ii*) $\frac{4}{5}$ (*iii*) $\frac{1}{5}$ (*iv*) 1

Q.3. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

Based on the above information answer the following :

(*i*) The conditional probability that an error is committed in processing given that Sonia processed the form is :

(*a*) 0.0210 (*b*) 0.04 (*c*) 0.47 (*d*) 0.06

(*ii*) The probability that Sonia processed the form and committed an error is :

(*a*) 0.005 (*b*) 0.006 (*c*) 0.008 (*d*) 0.68

(*iii*) The total probability of committing an error in processing the form is

(*a*) 0 (*b*) 0.047 (*c*) 0.234 (*d*) 1

(*iv*) The manger of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected has an error, the probability that the form is NOT processed by Vinay is :

(*a*) 1 (*b*) 30/47 (*c*) 20/47 (*d*) 17/47

(*v*) Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P(E_i \mid A)$ is

(*a*) 0 (*b*) 0.03 (*c*) 0.06 (*d*) 1

ANSWERS

ONE MARK QUESTIONS

1. $\frac{1}{2}$
2. $\left(\frac{1}{2}\right)^7$
3. 0.3
4. $(0.3)^3$
5. No
6. 0
7. $\frac{^{29}C_2 \times {^{20}C_2}}{^{50}C_5}$
8. $\frac{3}{8}$
9. $\frac{1}{10}$
10. 0.36
11. $\frac{7}{32}$
12. $\frac{3}{28}$

TWO MARKS QUESTIONS

1. (*i*) 1 (*ii*) 0
2. $\frac{4}{15}$
3. $\frac{15}{29}$
4. $\frac{4}{7}$
5. $\frac{10}{9}$
7. $\frac{1}{70}$
8. $\frac{1}{36}$
9. $\frac{3}{7}$
10. $\frac{23}{24}$
11. $\left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$
12. 0.6

THREE MARKS QUESTIONS

1. $\frac{3}{4}$
2. $P(A) = \frac{1}{5}$ and $P(B) = \frac{1}{6}$ or $P(A) = \frac{5}{6}$ and $P(B) = \frac{4}{5}$
3. $\frac{1}{17}$
4. 0.3678 or $^{11}C_5\,(0.4)^5\,(0.6)^5$
5. $-\frac{91}{54}$
6. $\frac{3}{4}$
7. $\frac{5}{9}$
8. $\frac{7}{22}$
9. $\frac{1}{2}$
10. $\frac{665}{324}$

FIVE MARKS QUESTIONS

1. $\frac{110}{221}$ 2. $\frac{7}{10}$ 3. $\frac{7}{11}$

4. (*i*) 4.31, (*ii*) 61.9, (*iii*) $\frac{15}{22}$ 5. $\frac{209}{343}$

6. $\bar{x} = \frac{17}{3}, \sigma^2 = \frac{14}{9}$ 7. $\frac{1}{2}$ 8. $\bar{x} = \frac{6}{13}, \sigma^2 = \frac{60}{169}$

9. $\frac{11}{50}$ 10. $\frac{16}{31}$

CASE STUDY QUESTIONS

1. (*a*) (*ii*) (*b*) (*iii*) (*c*) (*ii*) (*d*) (*i*)
   (*e*) (*iv*)
2. (*a*) (*iv*) (*b*) (*ii*) (*c*) (*i*) (*d*) (*i*)
   (*e*) (*iv*)
3. (*i*) (*b*) (*ii*) (*c*) (*iii*) (*b*) (*iv*) (*d*)
   (*v*) (*d*)

PRACTICE PAPER - I (2020-21)

CLASS XII

MATHEMATICS

[PREPARED BY TEAM MATHS – DOE]

Time Allowed : 3 Hours **Maximum Marks : 80**

General Instruction

1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
2. Part-A has objective Type Questions and Part-B has Descriptive Type Questions.
3. Both Part A and Part B have choices.

Part - A

1. It consists of two sections – I and II.
2. Section I comprises of 16 very short answers type questions.
3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part - B

1. It consists of three sections-III, IV and V.
2. Section III comprises of 10 questions of 2 marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section-III, 2 questions of Section IV and 3 questions of Section-V. You have to attempt only one of the alternative in all such questions.

PART - A

SECTION - I

All questions are compulsory. In case of internal choices attempt any one.

1. Find the principal value of $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)$.

2. Find the probability of obtaining an even prime number on each die, when a pair of dice is rolled.

3. Find the value of x is, if $\sin\left[\sin^{-1}\left(\frac{x}{5}\right)+\cos^{-1}\left(\frac{3}{5}\right)\right] = 1$

4. How many arbitrary constants are there in the particular solution of the differential equation $\frac{dy}{dx} = -4xy$; $y(0) = 1$.

5. Find the area bounded by $y = x^2$, the x-axis and the lines $x = -2$ and $x = 2$.

OR

Give an example of a function which is continuous everywhere but falls to be differentiable at exactly two points.

6. If $(\vec{a}\times\vec{b})^2 + (\vec{a}\,.\,\vec{b})^2 = 225$ and $|\vec{a}| = 5$ then find the value of $|\vec{b}|$.

7. Find the perpendicular distance of $P(1, 1, 1)$ from the plane $2x + 2y - z = 6$.

8. Find the integrating factor for solving the differential equation : $x\frac{dy}{dx} - 2y = e^x . x^3$.

9. Find the value of $\int_{-\pi}^{\pi} x^3 \sin x^2 x.dx$.

OR

If $\int e^x (x + 1)\, dx = f(x) = c$, then find $f(x)$.

10. Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$.

11. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

12. If $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$, find AA^{-1}.

OR

If A is a square Matrix such that $A^2 = A$, then find the value of $(m + n)$, if $(I + A)^3 - 7A = mI + nA$. (Where m and n are constants).

13. Write the direction cosines of x-axis.

14. Find the value $|\hat{i} - \hat{j}|^2$.

OR

Find the projection of $2\hat{i} - \hat{j} + 2\hat{k}$ on $\hat{i} - 2\hat{j} - 2\hat{k}$.

15. For what value of 'm' is the following a homogeneous differential equation:

$$\frac{dy}{dx} = \frac{y^m + \sqrt{x^2 + y^2}}{x}$$

OR

Find the sum of the order and degree of the differential equation : $\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2} = 3$.

16. What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i} + \hat{j} + 2\hat{k}$ makes with y-axis.

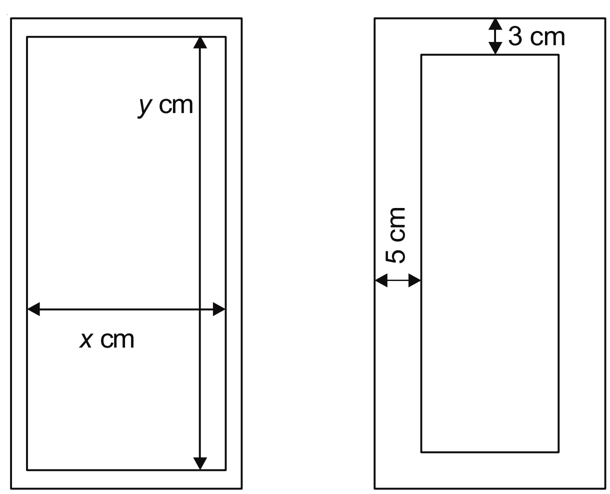
SECTION II

Both the case study based questions are compulsory.

Attempt any 4 sub parts from each questions (17 and 18).

Each question carries 1 mark.

17. A printed page must contain 60 cm^2 of printed material. There are to be margins of 5 cm on either side and margins of 3 cm on the top and bottom (Fig. 16.3).



Let x be the length of the line and let y be the height of the printed material.

Based on the above information answer the following :

(i) What will be the relation between x and y ?

(a) $xy = 15$ (b) $xy = 5$

(c) $xy = 3$ (d) $xy = 60$

(ii) What will be the total area (A) of the paper in terms of x and y ?

(a) $A = (x + 5)(y + 3)$ (b) $A = (x + 3)(y + 5)$

(c) $A = (x + 10)(y + 6)$ (d) $A = (2x + 10)(2y + 6)$

(iii) What will be the total area (A) of the paper in terms of x ?

(a) $A = 20 + x + \frac{100}{x}$ (b) $A = 6\left(20 + x + \frac{100}{x}\right)$

(c) $A = 60 + x + \frac{100}{x}$ (d) $A = 20 + 100x + \frac{1}{x}$

(iv) How long should the printed lines be in order to minimize the amount of paper used ?

(a) 6 cm (b) 8 cm

(c) 10 cm (d) 12 cm

(v) How long should the printed material in height in order to minimize the amount of paper

used ?

(a) 6 cm (b) 8 cm

(c) 10 cm (d) 5 cm

18. Two farmers Ramkrishan and Gurcharan Singh cultivates only three varieties of rice namely Basmati, Permal and Naura. The quantity of sale (in kg) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B.

$$\begin{array}{cc} & \begin{array}{ccc}\text{Basmati} & \text{Permal} & \text{Naura}\end{array} \\ A\ (\textit{September}\ \text{Sales}) = & \begin{bmatrix} 1000 & 2000 & 3000 \\ 5000 & 3000 & 1000 \end{bmatrix} \begin{array}{l}\text{Ramkrishan} \\ \text{Gurcharan Singh}\end{array} \end{array}$$

$$B\ (October\ \text{Sales}) = \begin{array}{c} \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramkrishan} \\ \text{Gurcharan Singh} \end{array}$$

Based on the above information answer the following :

(*i*) Find the combined sales in September and October for each farmer in each variety.

$$(a)\ \text{Total sales} = \begin{array}{c} \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ \begin{bmatrix} 6000 & 3000 & 9000 \\ 7000 & 13000 & 2000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramkrishan} \\ \text{Gurcharan Singh} \end{array}$$

$$(b)\ \text{Total sales} = \begin{array}{c} \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ \begin{bmatrix} 6000 & 12000 & 9000 \\ 7000 & 13000 & 2000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramkrishan} \\ \text{Gurcharan Singh} \end{array}$$

$$(c)\ \text{Total sales} = \begin{array}{c} \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ \begin{bmatrix} 6000 & 12000 & 9000 \\ 25000 & 13000 & 2000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramkrishan} \\ \text{Gurcharan Singh} \end{array}$$

$$(d)\ \text{Total sales} = \begin{array}{c} \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ \begin{bmatrix} 6000 & 12000 & 9000 \\ 25000 & 13000 & 11000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramkrishan} \\ \text{Gurcharan Singh} \end{array}$$

(*ii*) Find the decrease in sales from September to October.

$$(a)\ \text{Net Decrease in sales} = \begin{array}{c} \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ \begin{bmatrix} 4000 & 8000 & 9000 \\ 7000 & 13000 & 2000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramkrishan} \\ \text{Gurcharan Singh} \end{array}$$

$$(b)\ \text{Net Decrease in sales} = \begin{array}{c} \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ \begin{bmatrix} 4000 & 8000 & 3000 \\ 15000 & 13000 & 2000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramkrishan} \\ \text{Gurcharan Singh} \end{array}$$

$$(c)\ \text{Net Decrease in sales} = \begin{array}{c} \begin{array}{ccc} \text{Basmati} & \text{Permal} & \text{Naura} \end{array} \\ \begin{bmatrix} 4000 & 8000 & 3000 \\ 15000 & 7000 & 9000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramkrishan} \\ \text{Gurcharan Singh} \end{array}$$

$$(d) \text{ Net Decrease in sales} = \begin{array}{c} \text{Basmati} \quad \text{Permal} \quad \text{Naura} \\ \begin{bmatrix} 4000 & 8000 & 3000 \\ 15000 & 13000 & 9000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramkrishan} \\ \text{Gurcharan Singh} \end{array}$$

If Ramkrishan sell the variety of rice (per kg) i.e., Basmati, Permal and Naura at Rs. 30, Rs. 20, and Rs. 10 respectively, while Gurcharan Singh sell the variety of rice (per kg) i.e., Basmati, Permal and Naura at Rs. 40, Rs. 30 and Rs. 20 respectively.

(*iii*) Find the total selling price received by Ramkrishan in the month of September.

(*a*) Rs. 80,000 (*b*) Rs. 90,000

(*c*) Rs. 1,00,000 (*d*) Rs. 1,10,000

(*iv*) Find the total selling price received by Gurcharan Singh in the month of September.

(*a*) Rs. 1,10,000 (*b*) Rs. 2,10,000

(*c*) Rs. 3,00,000 (*d*) Rs. 3,10,000

(*v*) Find the total selling price received by Ramkrishan in the month of September and October.

(*a*) Rs. 4,00,000 (*b*) Rs. 5,00,000

(*c*) Rs. 5,10,000 (*d*) Rs. 6,10,000

Part - B

Section III

19. Check whether the relation R in the set Z of integers defined as

$R = \{(a, b) : a + b$ is "divisible by 2"$\}$ is reflexive, symmetric or transitive.

20. How many reflexive relations are possible in a set A whose $n(A) = 3$. Also find how many symmetric relations are possible in a set B whose $n(B) = 2$.

21. Find the vector and Cartesian equation of a line joining the points $B(4, 7, 1)$ and $C(3, 5, 3)$.

OR

Prove that the lines $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular if $pp' + rr' + 1 = 0$.

22. Find the area of the parallelogram whose adjacent sides are determined by the vectors

$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two then find the value of $|\vec{a} + \vec{b} + \vec{c}|$.

23. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.

24. Evaluate : $\int \frac{dx}{x^2 - 4x + 3}$

25. Find $\frac{dy}{dx}$, when $x = a(\cos\theta + \theta\sin\theta)$, and $y = a(\sin\theta - \theta\cos\theta)$.

26. Show that the relation R on defined as $R = \{(a, b) : a \le b^3\}$ is not transitive.

OR

Let $A = R - \{3\}$ and $B = R - \left\{\frac{2}{3}\right\}$, if $f : A \to B$, $f(x) = \frac{2x-4}{3x-9}$ then prove that f is Bijective function.

27. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$.

28. Find the area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$.

Section IV

All questions are compulsory. In case of Internal choices attempt any one.

29. Find the area of the ellipse $x^2 + 9y^2 = 36$ using integration.

30. There are three coins. One is a biased coin that comes up with tail 60% of the times, the second is also a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it showed heads. What is the probability that it was the unbiased coin ?

OR

Four defective bulbs are accidently mixed with six good ones. If it is not possible to just look at a bulb and tell whether or not it is defective, find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.

31. Two dice are thrown together and the total score is noted. The events. E, F and G are 'a total of 4', 'a total of 9 or more', and 'a total divisible by s', respectively. Calculate $P(E)$, $P(F)$, $P(G)$ and decide which pairs of events, if any, are independent.

32. Show that the function $f(x) = 2x - |x|$ is continuous but not differentiable at $x = 0$.

OR

If $y = \cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$, such that $x \in \left(0, \dfrac{\pi}{2}\right)$. Find the value of $\dfrac{dy}{dx}$.

33. Solve the differential equation : $x \log x \dfrac{dy}{dx} + y = \dfrac{2}{x} \log x$.

34. Find the equation of the tangent and Normal to the curve $x = a \sin^3 \theta$ and $y = \theta \cos^3 \theta$ a t

$\theta = \dfrac{\pi}{4}$.

35. Evaluate the integrals : $\displaystyle\int_{-1}^{2} | x^3 - x | \, dx$

OR

Evaluate the integrals, $\displaystyle\int \frac{(x+1)\,.dx}{(x+3)(x^2+4)}$

Section - V

36. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}$, find A^{-1} and the use the result to solve the following system of equations+

$$2x + y + 3z = 6$$
$$3x + 2y + z = 6$$
$$x + 3y + 2z = 6$$

OR

If $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 3 & 5 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :

$$x - y + z = 1$$

$$-x + 2y + 3z - 4 = 0$$

$$x + y + 5z = 7$$

37. Find the equation of the plane through the intersection of the planes $x + 3y + 6 = 0$ and $3x - y - 4z = 0$ and whose perpendicular distance from origin is unity.

OR

Find the distance of the point (3, 4, 5) from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$.

38. Solve the following linear programming problem (L.P.P) graphically.

Maximize $Z = 400x + 1000y$ subject to constraints:

$$x + y \leq 200$$

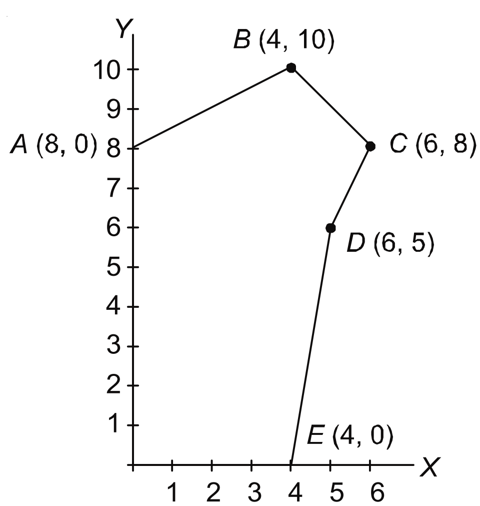
$$4x - y \leq 0$$

$$x \geq 20$$

$$x, y \geq 0$$

OR

The corner points of the feasible region determined by the system of linear constraints are as shown below:



Answer each of the following :

(*i*) Let $Z = 3x - 4y$ be the objective function. Find the maximum and minimum value of Z and

also the corresponding points at which the maximum and minimum value occurs.

(*ii*) Let $Z = px + qy$, where p, $q > 0$ be the objective function. Find the condition on p and q so

that the maximum value of Z occurs at $B(4, 10)$ and $c(6, 8)$. Also mention the number of

optimal solutions in this case.

Value Points of Practice Paper - I (2020-21)

Class XII (Mathematics)

[Prepared by Team Maths – DOE]

(SECTION - I)

1.	$\frac{\pi}{4}$	**1**
2.	$\frac{1}{36}$	**1**
3.	$x = 3$	**1**
4.	0	**1**
5.	$\frac{16}{3}$ sq. units	**1**
	OR	
	$F(x) = \|x - 1\| + \|x - 2\|$ (Or any correct Response)	
6.	3	**1**
7.	1 Unit	**1**
8.	$\frac{1}{x^2}$	**1**
9.	0	**1**
	OR	
	$f(x) = x \,.\, e^x$	

10. 11 **1**

12. $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **1**

OR

$m + n = 1$

13. (1, 0, 0) **1**

14. 2 **1**

OR

3

15. 1 **1**

OR

3

16. $\frac{1}{\sqrt{7}}$ **1**

SECTION - II

17. (*i*) **(*d*)** **1**

17. (*ii*) **(*c*)** **1**

17. (*iii*) **(*b*)** **1**

17. (*iv*) **(*c*)** **1**

17. (*v*) **(*a*)** **1**

18. (*i*) **(*d*)** **1**

18. (*ii*) **(*c*)** **1**

18. (*iii*) **(*c*)** **1**

18. (*iv*) **(*d*)** **1**

18. (*v*) **(*c*)** **1**

SECTION - III

19. **Reflexive :** Since, $a + a = 2a$, which is even ½

$\therefore$ $(a, a) \in R \ \forall \ a \in Z$ Hence R is reflexive

Symmetric : If $(a, b) \in R$, then $a + b + 2\lambda \Rightarrow b + a = 2\lambda \Rightarrow (b, a) \in R$, ½

Hence R is symmetric

Transitive : If $(a, b) \in R$ and $(b, c) \in R$ then

$(a + b) = 2\lambda$ .... (1) and $b + c = 2\mu$ ... (2)

Adding (1) and (2) we get $a + 2b + c = 2(\lambda + \mu) \Rightarrow a + c = 2(\lambda + \mu - b)$ **1**

$\Rightarrow a + c = 2k$, where $\lambda + \mu - b = k \Rightarrow (a, c) \in R$, Hence R is transitive

21. No. of Reflexive Relations = $2^6 = 64$ **1**

No. of Symmetric Relations = $2^3 = 8$ **1**

21. *Cartesian Equation* : $\dfrac{x-4}{-1} = \dfrac{y-7}{-2} = \dfrac{z-1}{2}$ **1**

Vector Equation : $\vec{r} = (4\hat{i} + 7\hat{j} + \hat{k}) + \lambda(-\hat{i} - 2\hat{j} + 2\hat{k})$ **1**

OR

The given lines can be written as :

$\dfrac{x-q}{p} = \dfrac{y-0}{1} = \dfrac{z-s}{r}$ and $\dfrac{x-q'}{p'} = \dfrac{y-0}{1} = \dfrac{z-s'}{r'}$ **1½**

As lines are perpendicular then $pp' + rr' + 1 = 0$.

22. $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = 20\hat{i} + 5\hat{j} - 5\hat{k}$$ **1**

Area of parallelogram = $\dfrac{1}{2}|\vec{a} \times \vec{b}| = \dfrac{1}{2}\sqrt{400+25+25} = \dfrac{15}{2}\sqrt{2}$ sq. units

OR

As, Each one of them being perpendicular tot he sum of other two

$2(\vec{a} \cdot \vec{b} + \vec{c} \cdot \vec{a} + \vec{b} \cdot \vec{c}) = 0$ ½

and

$|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) . (\vec{a} + \vec{b} + \vec{c})$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{c}.\vec{a} + \vec{b}.\vec{c})$

$= 9 + 16 + 25 + 0 = 50$ **1**

$|\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$ **½**

23. $f'(x) = 4x(x-1)(x-2)$ **1**

so, $f(x)$ is increasing for $x \in [0, 1] \cup [2, \infty]$

24. $I = \dfrac{dx}{(x-2)^2 - 1)^2} = \dfrac{1}{2} \log \left|\dfrac{x-2-1}{x-2+1}\right| + c$ **½ + 1**

$I = \dfrac{1}{2} \log \left|\dfrac{x-3}{x-1}\right| + c$ **½**

25. $\dfrac{dy}{d\theta} = a\theta \sin \theta, \ \dfrac{dx}{d\theta} = a\theta \cos \theta$ **1**

$\dfrac{dy}{dx} = \dfrac{a\theta \sin\theta}{a\theta \cos\theta} = \tan \theta$ **1**

26. As $(10, 3) \in R$, $(3, 2) \in R$ but $(10, 2) \notin R$ **1½**

so, R is not Transitive.

(Or any correct Response)

OR

Let, $f(x) = f(y) \Rightarrow \dfrac{2x-4}{3x-9} = \dfrac{2y-4}{3y-9} \Rightarrow x = y$ **½**

So, $f(x)$ is one-one function.

Let, $y = f(x) = \dfrac{2x-4}{3x-9} \Rightarrow x = \dfrac{9y-4}{3y-2}$

As, Range = Codomain **1**

thus, $f(x)$ is onto function.

So, $f(x)$ is Bijective Function. **½**

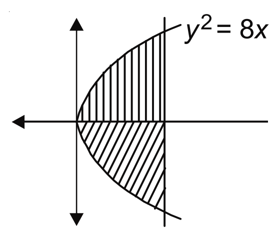
27. $I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$ **½**

$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx = \int_0^{\frac{\pi}{2}} I \; dx = \frac{\pi}{2}$ **1**

$I = \frac{\pi}{4}$ **½**

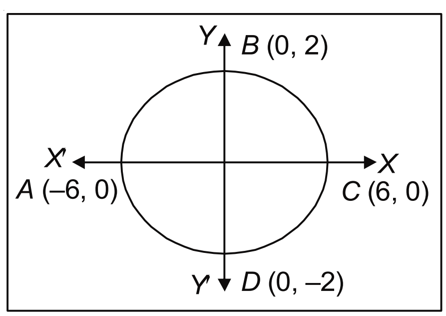
28. $A = 2\int_0^2 2\sqrt{2}\sqrt{x}\; dx = \frac{4\sqrt{2}}{3}\left(x^{\frac{3}{2}}\right)_0^2$ **1½**

$A = \frac{32}{3}$ sq. units. **½**



SECTION - IV

29. Area of Ellipse = 4 (Area of BOC) **½+½ (for figure)**



$A = 4\int_0^6 \frac{1}{3}\sqrt{6^2 - x^2}\;.dx$ **1**

$$A = \frac{4}{3}\left(\frac{x}{2}\sqrt{6^2 - x^2} + \frac{36}{2}\sin^{-1}\left(\frac{x}{6}\right)\right)_0^6$$

$A = 12\pi$ sq. units. **1**

30. Let the Events be

E_1 : Choosing 1st Coin

E_2 : Choosing 2nd Coin

E_3 : Choosing 3rd Coin

A : Getting Heads **1**

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$$P\left(\frac{A}{E_1}\right) = \frac{40}{100}, P\left(\frac{A}{E_2}\right) = \frac{75}{100}, P\left(\frac{A}{E_3}\right) = \frac{1}{2}$$ **1**

$$P\left(\frac{E_3}{A}\right) = \frac{\frac{1}{3}\cdot\frac{1}{2}}{\frac{1}{3}\left(\frac{40+75+50}{100}\right)} = \frac{50}{165} = \frac{10}{33}$$ **1**

OR

Let X represents the number of defective bulbs drawn.

$\therefore$ X can take values 0, 1, 2, 3, or 4

$$P(X = 0) = \left(\frac{6}{10}\cdot\frac{5}{4}\cdot\frac{4}{8}\cdot\frac{3}{7}\right) = \frac{360}{5040}$$

$$P(X = 1) = 4\left(\frac{6}{10}\cdot\frac{5}{9}\cdot\frac{4}{8}\cdot\frac{4}{7}\right) = \frac{1920}{5040}$$

$$P(X = 2) = 6\left(\frac{6}{10}\cdot\frac{5}{9}\cdot\frac{4}{8}\cdot\frac{3}{7}\right) = \frac{2160}{5040}$$

$$P(X = 3) = 4\left(\frac{6}{10}\cdot\frac{4}{9}\cdot\frac{3}{8}\cdot\frac{2}{7}\right) = \frac{576}{5040}$$

Probability distribution ½

X	***P*(*X*)**
0	$\frac{360}{5040}$
1	$\frac{1920}{5040}$
2	$\frac{2160}{5040}$
3	$\frac{576}{5040}$
4	$\frac{24}{5040}$
Total	1

$P(X = 4) = 1\left(\frac{4}{10}\cdot\frac{3}{9}\cdot\frac{2}{8}\cdot\frac{1}{7}\right) = \frac{24}{5040}$ **2½**

31. Two dice are thrown together i.e.,

$\therefore$ $n(S) = 36$, where S is the sample space.

Event 'E' is 'a total of 4'

$\therefore$ $E = \{(2, 2), (3, 1), (1, 3)\}$

Event 'F' is a 'total of 9 or more'

$\therefore$ $F = \{(3, 6, (6, 3), (4, 5), (5, 4), (4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)\}$

Event 'G' is 'a total divisible by 5'

$\therefore$ $G = \{(1, 4, (4, 1), (2, 3), (3, 2), (4, 6), (6, 4), (5, 5)\}$

Here, $(E \cap F) = \phi$ and $(E \cap G) = \phi$

Also, $(F \cap G) = \{(4, 6), (6, 4), (5, 5)\}$

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$ **1**

$P(F) = \frac{n(F)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

$P(G) = \frac{n(G)}{n(S)} = \frac{7}{36}$ **1**

$P(F \cap G) = \frac{3}{36} = \frac{1}{12}$

and $P(F) \,.\, P(G) = \frac{5}{18} \cdot \frac{7}{36} = \frac{35}{648}$

So, $P(F \cap G) \neq P(F) \,.\, P(G)$

Hence, there is no pair which is independent. **1**

32. The function $f(x) = 2x - |x|$ can be written as

$$f(x) = \begin{cases} 3x, & x \leq 0 \\ x, & x > 0 \end{cases}$$

Now, **1½**

$\lim\limits_{x \to 0} + f(x) = 0.$

$f(0) = 0$

So we've, LHL = RHL = $f(x = 0)$, Thus the function $f(x)$ is continuous at $x = 0$.

We observe LHD = 3, RHD = 1 thus LHD is not equal to RHD, thus $f(x)$ is

1½

not differentiable at $x = 0$.

OR

$y = \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2}$ **2**

$\frac{dy}{dx} = \frac{1}{2}$ **1**

33. $\frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x^2}$

Integrating factor = $\log x$ **1**

so, solution of given differential equation is,

$y \,.\, \log x = \int \frac{2}{x^2} \log x \, dx$ **1**

$y \,.\, \log x = \frac{-2}{x}(1 + \log x) + c$ **1**

34. $\frac{dy}{dx} = \frac{-b}{a}\cot\theta$

so, slope of tangent = $\frac{-b}{a}$ **1½**

Slope of Normal = $\frac{a}{b}$

Equation of Tangent : $\sqrt{2}\,(bx + ay) = ab$

Equation of Normal : $2\sqrt{2}\,(by - ax) = b^2 - a^2$ **1½**

35. $I = \int_{-1}^{2} |x^3 - x|\, dx = \int_{-1}^{0} (x^3 - x)\, dx + \int_{0}^{1} (x - x^3)\, dx + \int_{1}^{2} (x^3 - x)\, dx$ **1**

$$I = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} - \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$$

$I = \frac{11}{4}$ **2**

OR

Let, $\frac{x+1}{(x+3)(x^2+4)} = \frac{A}{(x+3)} + \frac{Bx+C}{x^2+4}$

On solving we get, $A = \frac{-2}{13}$, $B = \frac{2}{13}$, $C = \frac{7}{13}$

1½

$I = \frac{-2}{13} \log (x + 3) + \frac{1}{13} \log (x^2 + 4) + \frac{7}{26} \tan^{-1}\left(\frac{x}{2}\right) + C$ **1½**

SECTION - V

36. $A^{-1} = \frac{1}{18}\begin{bmatrix} 1 & 7 & -5 \\ -5 & 1 & 7 \\ 7 & -5 & 1 \end{bmatrix}$ **2**

Then $X = A^{-1} B = \frac{1}{18}\begin{bmatrix} 1 & 7 & -5 \\ -5 & 1 & 7 \\ 7 & -5 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 18 \\ 18 \\ 18 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ **2½**

Thus, $x = 1$, $y = 1$, $z = 1$ **½**

OR

$A^{-1} = \frac{1}{-4}\begin{bmatrix} 7 & 8 & -3 \\ 6 & 4 & -2 \\ -5 & -4 & 1 \end{bmatrix}$ **2**

$$X = (A^T)^{-1} B = (A^{-1})^T B = \frac{1}{-4}\begin{bmatrix} 7 & 6 & -5 \\ 8 & 4 & -4 \\ -3 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} = \frac{1}{-4}\begin{bmatrix} -4 \\ -4 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ **2½**

Thus, $x = 1$, $y = 1$, $z = 1$. ½

37. Let the required equation of plane passing through the intersection of planes

$x + 3y + 6 = 0$ and $3x - y - 4z = 0$ be

$(x + 3y + 6) + \lambda(3x - y - 4z) = 0$ ... (*i*)

Above equation can be written as

$x + 3y + 6 + 3\lambda x - \lambda y - 4\lambda z = 0$

or $x(1 + 3\lambda) + y(3 - \lambda) - 4\lambda z + 6 = 0$... (*ii*)

which is the general form of equation of plane.

Also, given that perpendicular distance of plane (*i*)

from origin, i.e., (0, 0, 0) is unity, i.e., one.

$$\therefore \left[\frac{(1+3\lambda).(0)+(3-\lambda)(0)-4\lambda(0)+6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}}\right] = 1$$ **2**

$$\left[\begin{array}{l} \because \text{distance of point } (x_1, y_1, z_1) \text{ from a} \\ \text{plane } ax + by + cz + d = 0 \text{ is given by} \\ d = \left|\dfrac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2+b^2+c^2}}\right| \\ \text{here, } a = 1 + 3\lambda,\ b = 3-\lambda,\ c = -4\lambda, \\ (x_1, y_1, z_1) = (0,0,0) \end{array}\right]$$

$$\text{or } \left|\frac{6}{\sqrt{1+9\lambda^2+6\lambda+9+\lambda^2-6\lambda+16\lambda^2}}\right| = 1$$

$$\text{or } \frac{6}{\sqrt{26\lambda^2+10}} = 1 \text{ or } 6 = \sqrt{26\lambda^2+10}$$

On squaring both sides, we get

$$36 = 26\lambda^2 + 10$$

or $26\lambda^2 = 26$ or $\lambda^2 = 1$ or $\lambda = \pm 1$.

Now, on putting $\lambda = \pm 1$ in Eqn. (*i*), we get

$x + 3y + 6 - 3x + y + 4z = 0$

or $-2x + 4y + 4z + 6 = 0$

or $x - 2y - 2z - 3 = 0$ [divide by – 2]... (*iv*)

Hence, required equations of the plane are

$2x + y - 2z + 3 = 0$ and $x - 2y - 2z - 3 = 0$

OR

$2x = y = z$

or $\dfrac{x}{1/2} = \dfrac{y}{1} = \dfrac{z}{1}$

Direction ratios of this line are $\left(\dfrac{1}{2}, 1, 1\right)$

$\therefore$ Line parallel to this line and passing through

(3, 4, 5) is $\dfrac{x-3}{1/2} = \dfrac{y-4}{1} = \dfrac{z-5}{1} = k$ (let)

or $x = 3 + \dfrac{k}{2}$, $y = 4 + k$, $z = 5 + k$

Substituting in the plane $x + y + z = 2$, we get

$3 + \dfrac{k}{2} + 4 + k + 5 + k = 2$

or $12 + \dfrac{5k}{2} = 2$

or $5k = -10 \times 2$

or $k = -4$

Point of intersection is :

$\left(3-\frac{4}{2}, 4-4, 5-4\right) = (1, 0, 1)$

$\therefore$ Distance = $\sqrt{(3-1)^2+(4-0)^2+(5-1)^2}$

= $\sqrt{4+16+16}$

= 6 units.

38. Corner points are $A(20, 180)$, $B(40, 160)$ and $C(20, 80)$

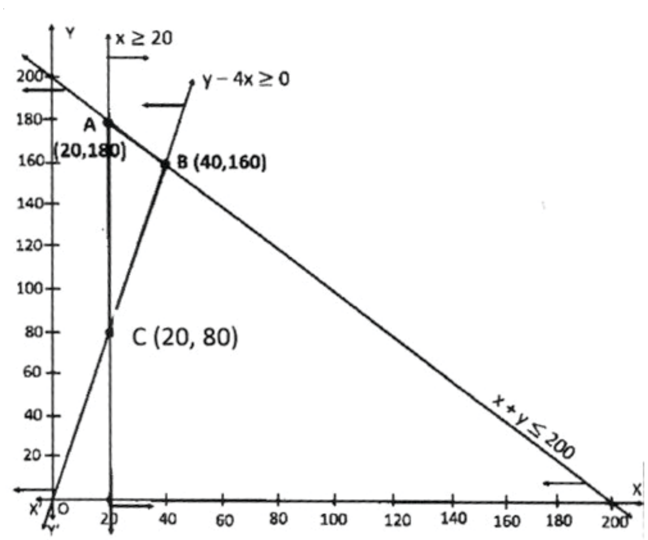
Z at A = 1, 88,000

Z at B = 1,76,000

Z at C = 88,000

So maximum Z = 1,88,000

At $x = 20$ and $y = 180$



OR

(*i*)

Corner points	$Z = 3x - 4y$
O(0, 0)	0
$A(0, 8)$	– 32
$B(4, 10)$	– 28
$C(6, 8)$	– 14
$D(6, 5)$	– 2
$E(4, 0)$	12

1½

Max $Z = 12$ at $E(4, 0)$ Min $Z = -32$ at $A(0, 8)$ **1**

(*ii*) Since maximum value of Z occurs at $B(4, 10)$ and $C(6, 8)$

$\therefore\ 4p + 10q = 6p + 8q \Rightarrow 2q = 2p \Rightarrow p = q$ **2**

Number of optimal solution are infinite **½**

Practice Paper - 2 (2020-21)

Class XII

MATHEMATICS

[Prepared by Team Maths – DOE]

Time Allowed : 3 Hours **Maximum Marks : 80**

General Instruction

1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
2. Part-A has objective Type Questions and Part-B has Descriptive Type Questions.
3. Both Part A and Part B have choices.

Part - A

1. It consists of two sections – I and II.
2. Section I comprises of 16 very short answers type questions.
3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part - B

1. It consists of three sections-III, IV and V.
2. Section III comprises of 10 questions of 2 marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section-III, 2 questions of Section IV and 3 questions of Section-V. You have to attempt only one of the alternative in all such questions.

PART - A

SECTION - I

(All questions are compulsory. In case of internal choices attempt any one)

1. Write the smallest reflexive relation on set $A = \{1, 2, 3, 4\}$.

OR

If $f : A \to B$ is an injection such that range of $f = \{a\}$. Determine the number of elements in A.

2. If R is a symmetric relation on a set A, then write a relation between R and R^{-1}.

3. Let $A = \{1, 2, 3\}$. Then, what is the number of equivalence relations containing (1, 2)?

OR

If $A = \{a, b, c\}$ and $B = \{-2, -1, 0, 1, 2\}$, write total number of one-one functions from A to B.

4. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, find the values of a and b.

5. If I is the identity matrix and A is a square matrix such that $A^2 = A$, then what is the value of $(I + A)^2 - 3A$?

OR

Write a square matrix which is both symmetric as well as skew-symmetric.

6. A matrix A of order 3 × 3 is such that $|A| = 4$. Find the value of $|2A|$.

7. Write a value of $\int e^x \sec x(1 + \tan x)\, dx$

OR

Write a value of $\int \frac{1-\sin x}{\cos^2 x}\, dx.$

8. Find the area bounded by the curves $y = \sin x$ between the ordinates $x = 0$, $x = \pi$ and the x-axis.

9. If $\sin x$ is an integrating factor of the differential equation $\frac{dy}{dx} + py = Q$, then write the value of P.

OR

Write the order of the differential equation associated with the primitive

$y = C_1 + C_2 e^x + C_3 e^{-2x + C_4}$, where C_1, C_2, C_3, C_4 are arbitrary constants.

10. Find a vector in the direction of $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, which has magnitude of 6 units.

11. For a what value of λ are the vectors $\vec{a} = 2i + \lambda i + k$ and $\vec{b} = \lambda - 2j + 3k$ perpendicular to each other ?

12. If a and $\vec{b}$ are mutually perpendicular unit vectors, write the value of $|a + \vec{b}|$.

13. The Cartesian equation of a line *AB* is $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$. Find the direction cosines of a line parallel to *AB*.

14. Write the distance between the parallel planes $2x - y + 3z = 4$ and $2x - y + 3z = 18$.

15. If $P(A) = 0.3$, $P(B) = 0.6$, $P(B/A) = 0.5$, find $P(A \cup B)$.

16. If *X* is a random-variable with probability distribution as given below:

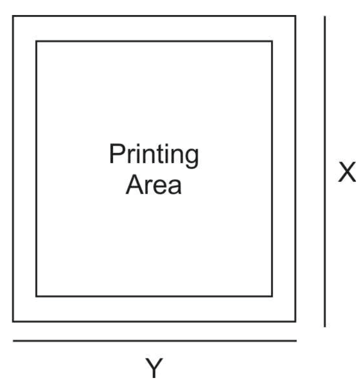
X :	0	1	2	3
$P(X = x)$:	k	$3k$	$3k$	k

Find the value of *k*.

SECTION II

(Both the case study based questions are compulsory. Attempt any 4 sub parts from each questions 17 and 18. Each part carries 1 mark)

17. Following is the pictorial description for a page.



The total area of the page is 150 cm^2. The combined width of the margin at the top and bottom is 3 cm and the side 2 cm.

Using the information given above, answer the following :

(i) The relation between x and y is given by

(a) $(x-3)y = 150$ (b) $xy = 150$

(c) $x(y-2) = 150$ (d) $(x-)(y-3) = 150$

(ii) The area of page where printing can be done, is given by

(a) xy (b) $(x+3)(y+2)$

(c) $(x-3)(y-2)$ (d) $(x-3)(y+2)$

(iii) The area of the printable region of the page, in terms of x, is

(a) $156 + 2x + 450/x$ (b) $156 - 2x + 450/x$

(c) $156 - 2x - 45/x$ (d) $156 - 2x - 450/x$

(iv) For what value of 'x', *the* printable area of the page is maximum ?

(a) 15 cm (b) 10 cm

(c) 12 cm (d) 15 units

(v) What should be dimension of the page so that it has maximum area to be printed ?

(a) Length = 1 cm, width = 15 cm (b) Length = 15 cm, width = 10 cm

(c) Length = 15 cm, width = 12 cm (d) Length = 150 cm, width = 1 cm

18. Anju and her sister Sweety are playing with a Die and a Coin. If Anju throws 5 or 6 with Die, then Sweety tosses coin three times. If Anju gets 1, 2, 3 or 4 with Die, then Sweety tosses coin twice. Using the information given above, answer the following:

(i) Probability of getting 5 or 6 in single throw of a Die is

(a) 1/3 (b) 2/3

(c) 1/6 (d) 5/6

(ii) Probability of getting 1, 2, 3 or in single throw of a Die is

(a) 1/3 (b) 2/3

(c) 1/6 (d) 5/6

(iii) If Anju gets 5 or 6, then probability of getting exactly one head by Sweety is

(a) 1/8 (b) 3/8

(c) 5/8 (d) 1/2

(*iv*) If Anju gets 1, 2, 3 or 4 then probability of getting exactly one head by Sweety is

(*a*) 1/8 (*b*) 3/8

(*c*) 5/8 (*d*) 1/2

(*v*) If Sweety obtained exactly one head, then probability of getting 1, 2, 3 or 4 by Anju is

(*a*) 2/3 (*b*) 5/11

(*c*) 8/11 (*d*) 1/3

Part - B

Section III

19. If $\frac{dy}{dx} = e^{-2y}$ and $y = 0$ when $x = 5$, then the value of x when $y = 3$.

20. If the points $A(-1, 3, 2)$, $B(-4, 2, -2)$ and $C(5, 5, \lambda)$ are collinear, find the value of λ.

21. Find a vector in the directional of $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 6 units.

22. Find the principal value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) + \cot^{-1}\left(\cot\frac{7\pi}{6}\right)$.

23. Evaluate

$$\int_0^{\frac{\pi}{2}} \log\left(\frac{3+5\cos x}{3+5\sin x}\right)dx.$$

OR

Evaluate $\int \frac{\log(\sin x)}{\tan x}\, dx$.

24. Find the slope of the normal to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$.

25. If $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ find $\frac{dy}{dx}$

26. Find matrices X and Y, if

$$X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} \text{ and } X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

OR

Find the value of k so that the points $A(5, 5)$ $B(k, 1)$ and $C(11, 7)$ are collinear.

27. A bag contains 15 tickets numbered from 1 to 15. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show even numbers.

OR

A die is tossed thrice. Find the probability of getting an odd number at least once.

28. Using integration, find the area of the region bounded by the line $y - 1 = x$, the x-axis and the ordinates $x = -2$ and $x = 3$.

SECTION IV

(All questions are compulsory. In case of internal choices attempt any one)

29. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a, b) : a, b \in Z, (a - b)$ is divisible by $5\}$. Prove that R is an equivalence relation.

30. Using integration, find the area of the region bounded by the following curves, after making a rough sketch:

$$y = 1 + |x + 1|, x = -3, x = 3, y = 0.$$

OR

Find the area bounded by the parabola $y^2 = 4x$ and the straight line $x + y = 3$.

31. Evaluate

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx$$

32. Find the intervals in which $f(x) = (x + 1)^3 (x - 3)^3$ is increasing or decreasing.

33. If $x = \tan\left(\frac{1}{a}\log y\right)$, show that $(1 + x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$.

34. If $x = ae^{\theta}(\sin\theta - \cos\theta)$ and $y = ae^{\theta}(\sin\theta + \cos\theta)$, find $\frac{dy}{dx}$

OR

If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$

35. Solve the following differential equation

$$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1};\ |x| \neq 1$$

SECTION V

(All questions are compulsory. In case of internal choices attempt any one)

36. Find the equation of the plane through the intersection of the planes $\vec{r}\cdot(\hat{i} + 3\hat{j}) - 6 = 0$ and $\vec{r}\cdot(3\hat{i} - \hat{j} - 4\hat{k}) = 0$, whose perpendicular distance from origin is unity.

OR

Find the distance of the point (2, 3, 4) from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$ measured parallel to the plane $3x + 2y + 2z - 5 = 0$.

37. Solve the following LPP Graphically :

Maximize $z = 1000x + 600y$

Subject to $x + y \leq 200,\ x \geq 20,\ y \geq 4x,\ x,\ y \geq 0.$

OR

Solve the following linear programming problem graphically :

Maximize $z = 6x + 5y$ subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x, y \geq 0$.

38. Solve the following system of equations $3x + 2y + z = 6$; $4x - y + 2z = 5$; $7x + 3y - 3z = 7$.

If $A = \begin{vmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{vmatrix}$ find A^{-1},

OR

Find AB, use this to solve the system of equations $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$.

Where $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

MARKING SCHEME (PRACTICE PAPER - 2)

PART - A

SECTION - I

1. {(1, 1), (2, 2), (3, 3), (4, 4)} OR 1

2. $R = R^{-1}$

3. 2 OR 60

4. 2, 4

5. I OR Null Matrix

6. 32

7. $e^x \sec x + C$ OR $\tan x - \sec x + C$

8. 2 sq units

9. $\cot x$ OR 3

10. $4\hat{i} - 2\hat{j} + 4\hat{k}$

11. 5/2

12. $\sqrt{2}$

13. $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

14. $\sqrt{14}$

15. 0.75

16. 1/8

SECTION-II

17. Following is the pictorial description for a page.

(*i*) B (*ii*) C (*iii*) D (*iv*) A (*v*) B

18. (*i*) B (*ii*) C (*iii*) D (*iv*) A (*v*) B

PART - B

SECTION III

19. Given that $\frac{dy}{dx} = e^{-2y} \Rightarrow \frac{dy}{e^{2y}} = dx$

$$\Rightarrow \int e^{2y}\, dy = \int dx \Rightarrow \frac{e^{2y}}{2} = x + C$$

When $x = 5$ and $y = 0$, then substituting these values in Eqn. (i), we get

$$\frac{e^0}{2} = 5 + C$$

$$\Rightarrow \frac{1}{2} = 5 + C \Rightarrow C = \frac{1}{2} - 5 = -\frac{9}{2}$$

Eqn. (i) becomes $e^{2y} = 2x - 9$

When $y = 3$, then $e^6 = 2x - 9 \Rightarrow 2x = e^6 + 9$

$$\therefore \quad x = \frac{(e^6 + 9)}{2}.$$

20. The equation of the line passing through $A(-1, 3, 2)$ and $B(-4, 2, -2)$ is

$$\frac{x+1}{-4+1} = \frac{y-3}{2-3} = \frac{z-2}{-2-2}$$

$$\Rightarrow \frac{x+1}{-3} = \frac{y-3}{-1} = \frac{z-2}{-4}$$

$$\Rightarrow \frac{x+1}{3} = \frac{y-3}{1} = \frac{z-2}{4}$$

... (i)

If the points $A(-1, 3, 2)$, $B(-4, 2, -2)$ and $C(5, 5, \lambda)$ are collinear, then the coordinates of C must satisfy equation (i). Therefore,

$$\frac{5+1}{3} = \frac{5-3}{1} = \frac{\lambda-2}{4}$$

$$\Rightarrow \frac{\lambda-2}{4} = 2$$

$\Rightarrow \qquad \lambda = 10.$

21. Firstly, we find a unit vector in the direction of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$

2

$$= \frac{2\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{(2)^2 + (-1)^2 + (2)^2}}$$

$$= \frac{2\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{9}}$$

$$= \frac{2}{3}\text{i} - \frac{1}{3}\text{j} + \frac{2}{3}\text{k}$$

Now, vector of magnitude of units = $6\left[\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right]$

$$= 4\hat{i} - 2\hat{j} + 4\hat{k}$$

22. We know that the principal value branch of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\cot^{-1} x$ is $(0, \pi)$.

2

$\therefore$ Principal value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) + \cot^{-1}\left(\cot\frac{7\pi}{6}\right)$

$$= \tan^{-1}\left[\tan\left(\pi + \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\pi + \frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left(\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{6}\right)$$

$$= \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}.$$

23. $$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{3+5\cos\left(\frac{\pi}{2}-x\right)}{3+5\sin\left(\frac{\pi}{2}-x\right)}\right) dx$$

$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$ **2**

$$= \int_0^{\frac{\pi}{2}} \log\left(\frac{3+5\sin x}{3+5\cos x}\right) dx = -I$$

$$2I = 0 \quad \Rightarrow \quad I = 0$$

OR

The differential of log (sin x) is tan x, which exists in denominator. So solve by substitution method.
2

Given integral is $\int \frac{\log(\sin x)}{\tan x}\, dx$

Putting $\log \sin x = t$

$\Rightarrow \quad \frac{1}{\sin x} . \cos x \, dx = dt$

$\Rightarrow \quad \cot x \, dx = dt$

$\Rightarrow \quad \frac{1}{\tan x}\, dx = dt$

$\left[\because \int \sin x\, dx = -\cos x + C\right]$

$\therefore$ We get $\int \frac{\log(\sin x)}{\tan x}\, dx = \int t\, dt$

$$= \frac{t^2}{2} + C$$

$$= \frac{(\log \sin x)^2}{2} + C$$

24. We have, $x = a\cos^3\theta,\ y = a\sin^3\theta$
2

$$\Rightarrow \quad \frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$$

$$\text{and} \quad \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$$

$$\text{Now,} \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$$

$\therefore$ Slope of the normal at any point on the curve $= -\dfrac{1}{\frac{dy}{dx}} = \dfrac{-1}{-\tan\theta} = \cot\theta$

Hence, $\left(\text{Slope of the normal at } \theta = \frac{\pi}{4}\right) = \cot\frac{\pi}{4} = 1.$

25. $$y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$$ **2**

$$= \tan^{-1}\left\{\frac{\sin\left(\frac{\pi}{2}+x\right)}{1-\cos\left(\frac{\pi}{2}+x\right)}\right\}$$

$$= \tan^{-1}\left\{\frac{2\sin\left(\frac{\pi}{2}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right\}$$

$$= \tan^{-1}\left\{\cot\left(\frac{\pi}{4}+\frac{x}{2}\right)\right\}$$

$$= \tan^{-1}\left\{\tan\frac{\pi}{2} - \left(\frac{\pi}{4} + \frac{x}{2}\right)\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right\}$$

$$= \frac{\pi}{4} - \frac{x}{2}$$

$$\therefore \qquad \frac{dy}{dx} = -\frac{1}{2}.$$

26. We have, $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ **2**

$$\Rightarrow \qquad (X + Y) + (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \qquad 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow \qquad X = \frac{1}{2}\begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow \qquad X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

and $\qquad (X + Y) - (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

$$\Rightarrow \qquad 2Y = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

$$Y = \frac{1}{2}\begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

OR

Let $A(5, 5)$, $B(k, 1)$ and $C(11, 7)$ be three vertices of a ΔABC. **2**

Then area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ k & 1 & 1 \\ 11 & 7 & 1 \end{vmatrix}$

$$= \frac{1}{2}[5(1-7) - 5(k-11) + 1(7k-11)]$$

$$= \frac{1}{2}[-30 - 5k + 55 + 7k - 11]$$

$$= \frac{1}{2}[2k + 14] = [k + 7] \text{ sq. units.}$$

But it is given that $A(5, 5)$, $B(k, 1)$ and $C(11, 7)$ are collinear.

$\therefore$ Area of $\Delta ABC = 0$

$\therefore$ $k + 7 = 0 \Rightarrow k = -7$

27. Let A and B be the events of drawings an even number ticket in the first and second drawn respectively.

In the first draw, there are 7 even numbers out of 15 numbers.

$\therefore$ $P(A) = \frac{7}{15}$

After first draw, there are 14 tickets left.

In the second drawn, one even number ticket is drawn out of 14 tickets.

$\therefore$ $P(B/A) = \frac{6}{14} = \frac{3}{7}$

$\therefore$ $P(A \cap B) = P(A) \,.\, P(B/A) = \frac{7}{15} \times \frac{3}{7} = \frac{1}{5}$

OR

When a die is thrown, there are 3 odd numbers on the die out of 6 numbers. **2**

$\therefore$ Probability of getting odd number $= \frac{3}{6} = \frac{1}{2}$

Probability of getting even number $= 1 - \frac{1}{2} = \frac{1}{2}$

Now probability of getting no odd number when the die is tossed thrice

= Probability of getting even number when the die is tossed thrice

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$\therefore$ Probability of getting an odd number at least once when the die is tossed thrice

$$= 1 - \frac{1}{8} = \frac{7}{8}.$$

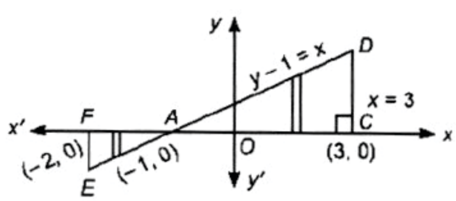
28. $y - 1 = x$ or $y = x + 1$ is the given line DE ...(i) **2**

$x = -2$ is the line EF.

$x = 3$ is the line CD.

Let A be a point of intersection of (i) and x-axis.

Limits are $x = -2$ and $x = -1$ for the area AEF and the limits for the area ACD are $x = -1$ and $x = 3$.



The required area = Shaded area

$$= |\Delta AFE| + |\Delta ACD|$$

$$= \left|\int_{-2}^{1} (x+1)\,dx\right| + \left|\int_{-1}^{3} (x+1)\,dx\right|$$

$$= \left|\left[\frac{x^2}{2} + x\right]_{2}^{-1}\right| + \left|\left[\frac{x^2}{2} + x\right]_{1}^{3}\right|$$

$$= \left|\left(\frac{1}{2} - 1\right) - (2 - 2)\right| + \left|\left(\frac{9}{2} + 3\right) - \left(\frac{1}{2} - 1\right)\right|$$

Section IV

29. $R = \{(a, b) : a - b \text{ is divisible by } 5,\ a, b \in Z\}$ **3**

For reflexive : $(a, a) \in R \Rightarrow a - a$ is divisible by 5, true. Hence R is reflexive.

For symmetric : $(a, b) \in R \Rightarrow a - b$ divisible by 5 $\Rightarrow b - a$ is divisible by 5 $\Rightarrow (b, a) \in R$.

Hence R is symmetric.

For transitive : Let for $(a, b), (b, c) \in R$

$(a,b) \in R \Rightarrow a - b$ divisible by 5

$(a, c) \in R \Rightarrow b - c$ divisible by 5

As $a - b$ divisible by 5 and $b - c$ divisible by 5. Hence $a - c$ is also divisible by 5.

i.e., $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$. Hence R is transitive.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

30. Given curve is **3**

$$y = 1 + |x + 1| = \begin{cases} 1 + x + 1, & \text{if } x + 1 \geq 0 \\ 1 - (x + 1), & \text{if } x + 1 < 0 \end{cases} \quad \text{... } (i)$$

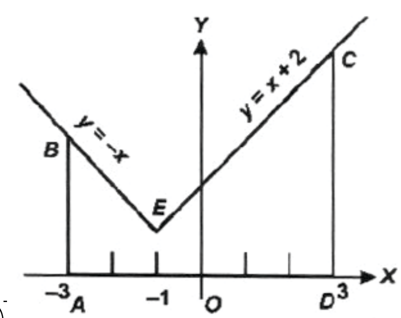
Given lines are $x = -3,\ x = 3,\ y = 0$ ... (*ii*)

The rough sketch of (*i*) has been shown in the figure.

$\therefore$ The required area = the area of the shaded region $ABECDA$

$$= \int_{-3}^{-1} y_{BE}\, dx + \int_{-1}^{3} y_{EC}\, dx$$

$$= \int_{-3}^{-1} (-x)\, dx + \int_{-1}^{3} (x + 2)\, dx$$



$$= \left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$$

$$= -\frac{1}{2}(1 - 9) + \left[\left(\frac{9}{2} + 6\right) - \left(\frac{1}{2} - 2\right)\right]$$

$= 4 + 12 = 16$ sq. units.

OR

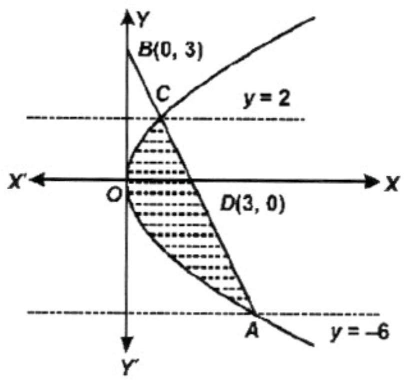
Given curves are $y^2 = 4x$ ... (*i*) **3**

and $x + y = 3$ ... (*ii*)

Curve (*i*) is a right handed parabola whose vertex is (0, 0) and axis is $y = 0$.

Line (*ii*) cuts *x*-axis at (3, 0) and *y*-axis at (0, 3). Here required area *OCDAO* is bounded by curves (*i*) and (*ii*) and abscissa at *A* and *C*.

Hence we will find the values of *y* from equations (*i*) and (*ii*).



Putting the value of *x* from equation (*ii*) in (*i*), we get

$$y^2 = 4(3 - y)$$

or, $$y^2 + 4y - 12 = 0$$

$\therefore$ $$y = -6, 2$$

$$\text{Required area } OCDAO = \int_{-6}^{2} (x_1 - x_2)\ dy$$

$$= \int_{-6}^{1} \left(x_{\text{line (i)}} - x_{\text{curve (ii)}} \right) dy$$

$$= \int_{-6}^{2} \left[(3 - y) - \frac{y^2}{4} \right] dy = \left[3y - \frac{y^2}{2} - \frac{y^3}{12} \right]_{-6}^{2}$$

$$= \left(6 - 2 - \frac{2}{3} \right) - \left(-18 - 18 + \frac{216}{12} \right) = \frac{10}{3} + 18 = \frac{64}{3} \text{ sq.}$$

units.

31. Let $$I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx \quad \text{... }(i) \quad \mathbf{3}$$

$$I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}\,dx$$

$$\left[\because \int_0^{\theta} f(x)\,dx = \int_0^{\theta} f(a-x)\,dx\right]$$

$$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}\,dx \quad \text{... }(ii)$$

On adding Eqs. (*i*) and (*ii*), we get

$$2I = \int_0^{\pi} \frac{(x + \pi - x)\sin x}{1+\cos^2 x}\,dx$$

$$I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$$

[$\because$ Let $t = \cos x$, $dt = -\sin x\,dx$]

$$\begin{bmatrix}\text{upper limit} \rightarrow \cos\pi = -1 \\ \text{Lower limit} \rightarrow \cos 0 = 1\end{bmatrix}$$

$$I = \frac{\pi}{2}\int_1^{-1} \frac{-dt}{1+t^2} = \frac{\pi}{2}\int_{-1}^{1} \frac{dt}{1+t^2}$$

$$\left[\because \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx\right]$$

$$= \frac{\pi}{2}\left[\tan^{-1} t\right]_{-1}^{1} = \frac{\pi}{2}\,[\tan^{-1}(1) - \tan^{-1}(-1)]$$

$$= \frac{\pi}{2}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \frac{\pi}{2}\left(\frac{\pi}{4} + \frac{\pi}{4}\right) = \frac{\pi}{2}\cdot\frac{\pi}{2}$$

$$= \frac{\pi^2}{4}.$$

32. We have,

$$f(x) = (x+1)^3 (x-3)^3 \quad \textbf{3}$$

$$\Rightarrow \quad f'(x) = (x-3)^3 \,.\, 3(x+1) + (x+1)^3 \,.\, 3(x-3)^2 \frac{d}{dx}(x-3)$$

$$\Rightarrow \quad f'(x) = 3(x+1)^2 (x-3)^3 + 3(x+1)^3 (x-3)^2$$

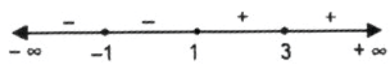
$$\Rightarrow \quad f'(x) = 3(x+1)^2 (x-3)^2 (x+1+x-3)$$

$$\Rightarrow \quad f'(x) = 6(x+1)^2 (x-3)^2 (x-1)$$

For $f(x)$ to be increasing, we must have

$$f'(x) \geq 0$$

$$\Rightarrow \quad 6(x+1)^2 (x-3)^2 (x-1) \geq 0$$



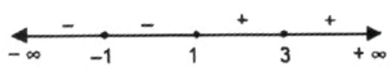
$$\Rightarrow \quad x \in [1, \infty]$$

So, $f(x)$ is increasing on $[1, \infty]$

For $f(x)$ to be decreasing, we must have

$$f'(x) \leq 0$$

$$\Rightarrow \quad 6(x+1)^2 (x-3)^2 (x-1) \leq 0$$



$$\Rightarrow \quad x \in (-\infty, 1)$$

So, $f(x)$ is decreasing on $(-\infty, 1)$.

33. Given that $\quad x = \tan\left(\frac{1}{a}\log y\right)$

$$\Rightarrow \quad \tan^{-1} x = \frac{1}{a} \log y$$

$$\Rightarrow \quad a \tan^{-1} x = \log y$$

Now, differentiating both sides w.r.t, x, we get

$$a \times \frac{1}{1+x^2} = \frac{1}{y} \cdot \frac{dy}{dx}$$

$$\left[\because \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}\right]$$

$$\Rightarrow \quad (1 + x^2)\frac{dy}{dx} = ay$$

[By cross multiplication]

Differentiating again on both sides w.r.t. x, we get

$$(1 + x^2) \cdot \frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx}\frac{d}{dx}(1 + x^2) = \frac{d}{dx}(ay)$$

$$\left[\because \frac{d}{dx}(u \cdot v) == u\frac{dv}{dx} + v\frac{du}{dx}\right]$$

$$\Rightarrow \quad (1 + x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot (2x) = a \cdot \frac{dy}{dx}$$

$$\Rightarrow \quad (1 + x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - a\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (1 + x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0. \text{ Hence proved.}$$

34. $x = ae^{\theta}(\sin\theta - \cos\theta)$ **3**

Diff. w.r.t. θ

$$\frac{dx}{d\theta} = a[e^{\theta}(\cos\theta + \sin\theta) + (\sin\theta - \cos\theta)e^{\theta}]$$

$$= ae^{\theta}[\cos\theta + \sin\theta + \sin\theta - \cos\theta]$$

$$= 2ae^{\theta}\sin\theta$$

Now, $y = ae^{\theta}(\sin\theta + \cos\theta)$

Diff. w.r.t. θ

$$\frac{dy}{d\theta} = a[e^{\theta}(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta)e^{\theta}]$$

$$= ae^{\theta} [\cos\theta - \sin\theta + \sin\theta + \cos\theta]$$

$$= 2ae^{\theta}\cos\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2ae^{\theta}\cos\theta}{2ae^{\theta}\sin\theta}$$

$$= \cot\theta.$$

OR

Given that $y^x = e^{y-x}$ **3**

Taking log on both sides, we get

$$\log y^x = \log e^{(y-x)}$$

$\Rightarrow$ $x \log y = (y - x) \log e$

$[\because \log e = 1]$

$\Rightarrow$ $x \log y = y - x$ ... (i)

$\Rightarrow$ $x = \dfrac{y}{1+\log y}$

Differentiating eqn. (i) both sides w.r.t. x, we get

$\Rightarrow$ $x \cdot \dfrac{d}{dx}(\log y) + \log y \dfrac{d}{dx}(x) = \dfrac{d}{dx}(y) - \dfrac{d}{dx}(x)$

$\Rightarrow$ $x \cdot \dfrac{1}{y} \cdot \dfrac{dy}{dx} + \log y \cdot 1 = \dfrac{dy}{dx} - 1$

$\Rightarrow$ $(1 + \log y) = \dfrac{dy}{dx}\left(1 - \dfrac{x}{y}\right)$

$\Rightarrow$ $\dfrac{dy}{dx} = \dfrac{y(1+\log y)}{(y-x)}$ ... (ii)

Put the value of x from Eqn. (i) in Eqn. (ii), we get

$$\frac{dy}{dx} = \frac{y + (1+\log y)}{y - \left(\dfrac{y}{1+\log y}\right)}$$

$$\left[\because x\log y = y - x \text{ or } x = \frac{y}{1+\log y}\right]$$

$\Rightarrow$
$$\frac{dy}{dx} = \frac{y+(1+\log y)^2}{(y+y\log y - y)}$$

35. Given differential equation is

$$(x^2-1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1}\ ;\ |x| \neq 1$$ **3**

Dividing both sides by $x^2 - 1$, we get

$$\frac{dy}{dx} + \frac{2xy}{x^2-1} = \frac{1}{(x^2-1)^2} \qquad \text{... }(i)$$

This is a linear differential equation and is of the form

$$\frac{dy}{dx} + Py = Q \qquad \text{... }(ii)$$

Comparing Eqns. (*i*) and (*ii*), we get

$$P = \frac{2x}{x^2-1} \text{ and } Q = \frac{1}{(x^2-1)^2}$$

Now, solution of above equation is given by

$$y \times \text{I.F.} = \int (Q \times I.F.)\, dx + c \qquad \text{... }(iii)$$

where, I.F. = Integrating factor and I.F. = $e^{\int P\,dx}$

$$\therefore \qquad \text{I.F.} = e^{\int \frac{2x}{x^2-1}dx} = e^{\log(x^2-1)} = x^2 - 1 \qquad [\text{Q } e^{\log x} = x]$$

$$\left[\because \int \frac{2x}{x^2-1}dx \text{ Put } x^2 - 1 = t \Rightarrow 2x\,dx = dt \therefore \int \frac{dt}{t} = \log|t| = \log|x^2-1| + c\right]$$

Putting I.F. = $x^2 - 1$ and $Q = \dfrac{1}{(x^2-1)^2}$ in Eqn. (*iii*), we get

$$y(x^2-1) = \int (x^2-1) \cdot \frac{1}{(x^2-1)^2}\, dx$$

$\Rightarrow$ $$y(x^2-1) = \int \frac{1}{x^2-1}\, dx$$

$\Rightarrow$ $$y(x^2-1) = \int \frac{dx}{x^2-(1)^2}$$

$$y(x^2-1) = \frac{1}{2} \log \left|\frac{x-1}{x+1}\right| + C.$$

SECTION - V

36. Let $P(2, 3, 4)$ be the given point and given equation of line be

$$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$$

Any random point T on the given lines is calculated as

$$\frac{x+3}{3} =$$

$\frac{y-2}{6} = \frac{z}{2} = \lambda$ [Say]

or $x = 3\lambda - 3,\ y = 6\lambda + 2,\ z = 2\lambda$

$\therefore$ Coordinates of T are $(3\lambda - 3, 6\lambda + 2, 2\lambda)$

Now, DR's of line PT are

$(3\lambda - 3 - 2, 6\lambda + 2 - 3, 2\lambda - 4) = (3\lambda - 5, 6\lambda - 1, 2\lambda - 4)$

Since, the line PT is parallel to the plane

$3x + 2y + 2z - 5 = 0$

$\therefore$ $a_1a_2 + b_1b_2 + c_1c_2 = 0$

[$\because$ Line is parallel to the plane, therefore, normal to the plane is

perpendicular to the line]

where, $a_1 = 3\lambda - 5,\ b_1 = 6\lambda - 1,\ c_1 = 2\lambda - 4$

and $a_2 = 3,\ b_2 = 2,\ c_2 = 2$

[$\because$ a_2, b_2, c_2 are DR's of plane whose equation

is $3x + 2y + 2z - 5 = 0$]

$\therefore$ We get,

$$3(3\lambda - 5) + 2(6\lambda - 1) + 2(2\lambda - 4) = 0$$

$$\Rightarrow \quad 9\lambda - 15 + 12\lambda - 2 + 4\lambda - 8 = 0$$

$$\Rightarrow \quad 25\lambda - 25 = 0$$

$$\Rightarrow \quad 25\lambda = 25$$

or $\lambda = 1$

$\therefore$ Coordinates of $T = (3\lambda - 3, 6\lambda + 2, 2\lambda) = (0, 8, 2)$
[Put $\lambda = 1$]

Finally, the required distance between points $P(2, 3, 4)$ and $T(0, 8, 2)$ is given by

$$PT = \sqrt{(0-2)^2 + (8-3)^2 + (2-4)^2}$$

[Q $(x_1, y_1, z_1) = (2, 3, 4)$ and $(x_2, y_2, z_2) = (0, 8, 2)$]

$$= \sqrt{4+25+4} = \sqrt{33} \text{ units.}$$

OR

We have, $\vec{n_1} = (\hat{i} + 3\hat{j}),\ d_1 = 6$ and $\vec{n_2} = (3\hat{i} - \hat{j} - 4\hat{k}),\ d_2 = 0$

Using the relation, $\vec{r} \cdot (\vec{n_1} + \lambda \vec{n_2}) = d_1 + d_2\lambda$

$$\Rightarrow \quad \vec{r} \cdot [(\hat{i} + 3\hat{j}) + \lambda(3\hat{i} - \hat{j} - 4\hat{k})] = 6 + 0 \cdot \lambda$$

$$\Rightarrow \quad \vec{r} \cdot [(1 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + \hat{k}(-4\lambda)] = 6$$
... (*i*)

On dividing both sides by $\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + (-4\lambda)^2}$, we get

$$\frac{\vec{r}\cdot[(1+3\lambda)\hat{i}+(3-\lambda)\hat{j}+\hat{k}(-4\lambda)]}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}} = \frac{6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}}$$

Since, the perpendicular distance from origin is unity.

$$\therefore \quad \frac{6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}} = 1$$

$\Rightarrow \quad (1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2 = 36$

$\Rightarrow \quad 1 + 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2 = 36$

$\Rightarrow \quad 26\lambda^2 + 10 = 36$

$\Rightarrow \quad \lambda^2 = 1$

$\therefore \quad \lambda = \pm 1$

Using Eqn. (*i*), the required equation of plane is

$\vec{r}\cdot[(1 \pm 3)\hat{i} + (3 \mp 1)\hat{j} + (\mp 4)\hat{k}] = 6$

$\Rightarrow \vec{r}\cdot[(1 + 3)\hat{i} + (3 - 1)\hat{j} + (-4)\hat{k}] = 6$

and$r\cdot[(1 - 3)\hat{i} + (3 + 1)\hat{j} + 4\hat{k}] = 6$

$\Rightarrow \quad \vec{r}\cdot(4\hat{i} + 2\hat{j} - 4\hat{k}) = 6$

and $\vec{r}\cdot(-2\hat{i} + 4\hat{j} + 4\hat{k}) = 6$

$\Rightarrow \quad 4x + 2y - 4z - 6 = 0$

and $\quad -2x + 4y + 4z - 6 = 0$-

37. Maximize $\quad z = 1000x + 600y$

Subject to $\quad x + y \leq 200, x \geq 20, y \geq 4x, x, y \geq 0$

Consider the linear constraint defined by the inequality

$$x + y \leq 200$$

First draw the graph of the line $x + y = 200$

x	100	80
y	100	120

Putting (0, 0) in the inequality $x + y \leq 200$, we have

$$0 + 0 \leq 200 \Rightarrow 0 \leq 200, \text{ which is true}$$

So the half plane of $x + y \leq 200$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$x \geq 20$$

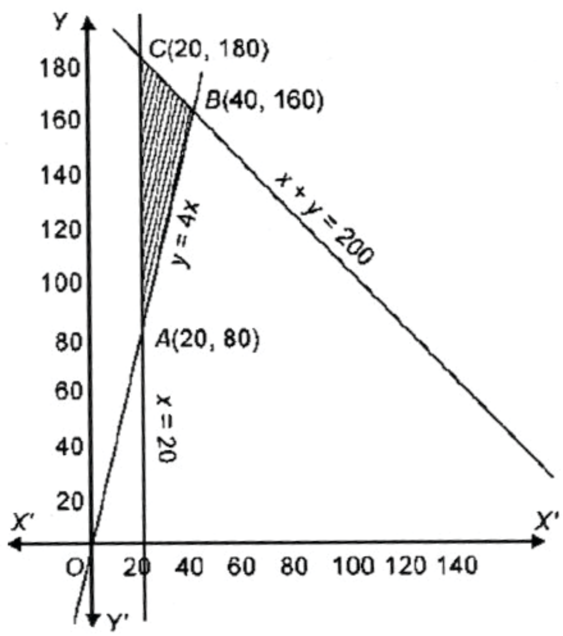
First draw the graph of the line $x = 20$

Putting (0, 0) in the inequality $x \geq 20$, we have

$$0 \geq 20, \text{ which is false}$$

So the half plane of $x \geq 20$ is away from origin.

Now consider the linear constraint defined by the inequality.



$$y \geq 4x$$

First draw the graph of the line $y = 4x$

x	10	20
y	40	80

Putting (10, 0) in the inequality $y \geq 4x$, we have

$$0 \geq 4 \times 10 \quad \Rightarrow \quad 0 \geq 40, \text{ which is false.}$$

So the half plane of $y \geq 4x$ is away from the point (10, 0)

Since $x, y \geq 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are A(20, 80), 8(40, 160) and C(20, 180). These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $z = 100x + 600y$

At A(20, 80) $z = 1000 \times 20 + 600 \times 80 = 20000 + 48000 = 68000$

At B(40, 160) $z = 1000 \times 40 + 600 \times 160 = 40000 + 96000 = 136000$

At C(20, 180) $z = 1000 \times 20 + 600 \times 180 = 20000 + 108000 = 128000$

Thus z is maximum at (40, 160) and maximum value = 136000

OR

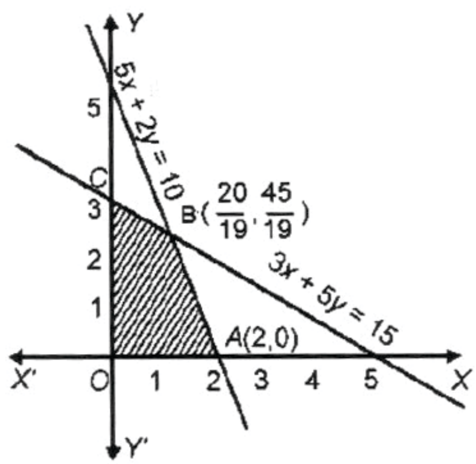
The given objective function is $z = 6x + 5y$

Consider the linear constraint defined by the inequality

$$3x + 5y \leq 15$$

First draw the graph of the line $3x + 5y = 15$

x	0	5
y	3	0



Putting (0, 0) in the inequality $3x + 5y \le 15$, we have

$$3 \times 0 + 5 \times 0 \le 15 \Rightarrow 0 \le 15, \text{ which is true}$$

So the half plane of $3x + 5y \le 15$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$5x + 2y \le 10$$

First draw the graph of the line $5x + 2y = 10$

x	0	2
y	5	0

Putting (0, 0) in the inequality $5x + 2y \le 10$, we have

$$5 \times 0 + 2 \times 0 \le 10 \Rightarrow 0 \le 10, \text{ which is true.}$$

So the half plane of $5x + 2y \le 10$ is towards the origin.

Since $x, y \ge 0$

So the feasible region lies in first quadrant.

The coordinate of the corner points of the feasible region are $O(0, 0)$

$A(2, 0)$, $B\left(\frac{20}{19}, \frac{45}{19}\right)$ and $C(0, 3)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\quad z = 6x + 5y$

At $O(0, 0)$ $\quad z = 6 \times 0 + 5 \times 0 = 0$

At $A(2, 0)$ $\quad z = 6 \times 2 + 5 \times 0 = 12 + 0 = 12$

At $B\left(\frac{20}{19}, \frac{45}{19}\right)$ $\quad z = 6 \times \frac{20}{19} + 5 \times \frac{45}{19} = \frac{120}{19} + \frac{225}{19} = \frac{345}{19}$

At C (0, 3) $\quad z = 6 \times 0 + 5 \times 3 = 0 + 15 = 15$

Thus z is maximum at $\left(\frac{20}{19}, \frac{45}{19}\right)$ and maximum value $= \frac{345}{19}$.

38. Given that

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$$

The given system of equations can be written as

$$AX = B$$

where, $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$

Solution of above system of equations is given by

$$X = A^{-1}B$$

... (*i*)

So, now we find A^{-1}

where $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

Now $|A| = 3(3 - 6) - 2(-12 - 14) + 1(12 + 7)$

$= 3(-3) - 2(-26) + 1(19)$

$= -9 + 52 + 19 = 62$

$|A| \neq 0$

hence unique solution.

and $\text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$

$\therefore \quad A_{11} = (-1)^2 \begin{vmatrix} -1 & 2 \\ 3 & -3 \end{vmatrix} = (-1)^2 \times (3 - 6) = -3$

$A_{12} = (-1)^3 \begin{vmatrix} 4 & 2 \\ 7 & -3 \end{vmatrix} = -1\,(-12 - 14) = 26$

$A_{13} = (-1)^4 \begin{vmatrix} 4 & -1 \\ 7 & 3 \end{vmatrix} = 1\,(12 + 7) = 19$

$A_{21} = (-1)^3 \begin{vmatrix} 2 & 1 \\ 3 & -3 \end{vmatrix} = -1\,(-6 - 3) = 9$

$$A_{22} = (-1)^4 \begin{vmatrix} 3 & 1 \\ 7 & -3 \end{vmatrix} = 1\,(-9-7) = -16$$

$$A_{23} = (-1)^5 \begin{vmatrix} 3 & 2 \\ 7 & 3 \end{vmatrix} = -1\,(9-14) = 5$$

$$A_{31} = (-1)^4 \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = 1\,(4+1) = 5$$

$$A_{32} = (-1)^5 \begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} = -1\,(6-4) = -2$$

$$A_{33} = (-1)^6 \begin{vmatrix} 3 & 2 \\ 4 & -1 \end{vmatrix} = 1\,(-3-8) = -11$$

$$\therefore \qquad \text{adj}\,(A) = \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$$

$$\Rightarrow \qquad A^{-1} = \frac{1}{62} \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

$$\left[\because A^{-1} = \frac{\text{adj}(A)}{|A|}\right]$$

Now, by using Eqn. (*i*), we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{62} \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

$$= \frac{1}{62}\begin{bmatrix} -18 & +45 & +35 \\ 156 & -80 & -14 \\ 114 & +25 & -77 \end{bmatrix} = \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix}$$

$$\therefore \quad x = \frac{62}{62}\ ; y = \frac{62}{62}; z = \frac{62}{62}$$

Hence, $x = 1$, $y = 1$ and $z = 1$.

OR

First find the product AB and then premultiply both sides of product AB by A^{-1} and obtain A^{-1}. Then, using the relation $X = A^{-1}C$ and simplify it to get the result.

First we find the product AB

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4+0 & 2-2-0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6I$$

$$\therefore \quad AB = 6I \qquad \text{... } (i)$$

Now, given system of equations can be written as

$$AX = C \text{ d } X = A^{-1}C \qquad \text{... } (ii)$$

where $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

Now again from Eqn. (*i*)

$$AB = 6I$$

$$\Rightarrow \quad A^{-1}AB = 6A^{-1}I$$

[Premultiplying by A^{-1} on both sides]

$\Rightarrow$ $B = 6A^{-1}$
[$\because$ $A^{-1} A = I$ and IB = B]

$$\therefore \quad A^{-1} = \frac{1}{6} B = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Now from Eqn. (*ii*), we get

$$X = A^{-1}C \text{ where } C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\therefore$ $x = 2,\ y = -1$ and $z = 4$.

PRACTICE PAPER - 3 (2020-21)

Class XII

Mathematics

[Prepared by Team Maths – DOE]

Time Allowed : 3 Hours **Maximum Marks : 80**

General Instruction

1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
2. Part-A has objective Type Questions and Part-B has Descriptive Type Questions.
3. Both Part A and Part B have choices.

Part - A

1. It consists of two sections – I and II.
2. Section I comprises of 16 very short answers type questions.
3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part - B

1. It consists of three sections-III, IV and V.
2. Section III comprises of 10 questions of 2 marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section-III, 2 questions of Section IV and 3 questions of Section-V. You have to attempt only onhe of the alternative in all such questions.

PART - A

SECTION - I

(All questions are compulsory. In case of internal choices attempt anyone)

1. If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ and $BA = [b_{ij}]$, find $b_{21} + b_{32}$. **1**

2. Find the direction ratios of the line where equation is given by **1**

$$5x - 3 = 15y + 7 = 3 - 10z$$

3. Find a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ where $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$. **1**

OR

If $|\vec{a}| = 5$, $|\vec{b}| = 6$ and $\vec{a} \cdot \vec{b} = 20$, find $|\vec{a} \times \vec{b}|$ **1**

4. A relation *R* on set {2, 3, 4} is defined by *R* = {(2, 3), (3, 2), (2, 2)}. Which order pairs should be added to it to make *R* is a smallest equivalence relation ? **1**

5. Evaluate : **1**

$$\int \sqrt{\frac{x}{1-x^3}} \, . \, dx$$

OR

If $\int_0^a \frac{1}{1+4x^2} dx = \frac{7}{8}$, then find '*a*'. **1**

6. Write the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + e^{\frac{dy}{dx}} = 0$$ **1**

7. Evaluate : **1**

$$\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right].$$

8. If $f(x) = x^2 + 2x + 3$ and $g(x) = 2x - 3$. Find $g \circ f(x)$. **1**

OR

If $f : R \to R$ be defined by $f(x) = x^2 + 1$, find the preimage of 26 and 17. **1**

9. If the rate of change of volume of a sphere is equal to the rate of change of its radius, then find the radius of the sphere. **1**

10. If A is any square matrix of order 3 × 3 and |adj A| = 25, then find |4A| and |adj adj A|. **1**

OR

If $\begin{bmatrix} 2 & 3 & -1 \\ \lambda & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ is non singular, find λ. **1**

11. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ -1 & 3 & 0 \end{bmatrix}$, then find the value of (adj A) A. **1**

12. If P(not A) = 0.7, $P(B)$ = 0.7 and $P(B/A)$ = 0.5, then find $P(A/B)$ and $P(A \cup B)$. **1**

13. Write the integrating factor of the differential equation. **1**

$$x \log x \frac{dy}{dx} + y = 2 \log x$$

14. Evaluate : **1**

$$\int_{-1}^{1} (x^5 + \tan^3 x + x + 1)\ dx$$

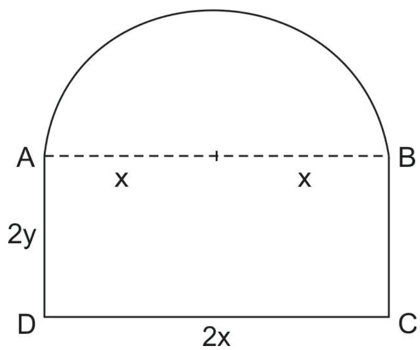
15. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$. **1**

16. Is function $f(x) = \begin{cases} 2x-3, & x \geq 1 \\ 1-2x, & x < 1 \end{cases}$ continuous at x = 1 ? **1**

SECTION II

(Both the case study based questions are compulsory. Attempt any 4 sub parts from each questions 17 and 18. Each part carries 1 mark)

17. Mr. X who is an architect designs a building for a small company. The design of the window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semicircular opening. This window is having a perimeter of 10 m as shown below:



Based on the above information answer the following : **1**

(*i*) If $2x$ and $2y$ represents the length and breadth of the rectangular portion of the window, then relation between variables is

(*a*) $4y - 2x = 10 - \pi$ (*b*) $4y = 10 - (2 - \pi)x$

(*c*) $4y = 10 - (2 + \pi)x$ (*d*) $4y - 2x = 10 + \pi$

(*ii*) The combined area A of the rectangular region and semicircular region of the window expressed as a function of x is **1**

(*a*) $A = 10x + \left(2 + \frac{1}{2}\pi\right)x^2$

(*b*) $A = 10x - \left(2 + \frac{1}{2}\pi\right)x^2$

(*c*) $A = 10x - \left(2 - \frac{1}{2}\pi\right)x^2$

(*d*) $A = 4xy + \frac{1}{2}\pi x^2$, where $y = \frac{5}{2} + \frac{1}{4}(2 + \pi)x$

(*iii*) The maximum value of area of the whole window, *A* is **1**

(*a*) $A = \frac{50}{4+\pi}$ m^2

(*b*) $A = \frac{50}{4+\pi}$ m^2

(*c*) $A = \frac{100}{4+\pi}$ m^2

(*d*) $A = \frac{50}{4-\pi}$ m^2

(*iv*) The owner of this small company is interested in maximizing the area of whole window so that max. light input is possible. For this to happen, the length of the rectangular portion of the window should be **1**

(*a*) $\frac{20}{4+\pi}$ m

(*b*) $\frac{10}{4+\pi}$ m

(*c*) $\frac{4}{10+\pi}$ m

(*d*) $\frac{100}{4+\pi}$ m

(*v*) In order to get the max. light input through the whole window, the area (in sq. m) of the only semi circular opening of the window is : **1**

(*a*) $\frac{100\pi}{(4+\pi)^2}$

(*b*) $\frac{50\pi}{4+\pi}$

(*c*) $\frac{50\pi}{(4+\pi)^2}$

(*d*) Same as the area of rectangular portion of the window.

18. There are three categories of students in a class of 60 students : **1**

A : Very hard working students

B : Regular but not hard working

C : Careless and irregular

It is known that 10 students are in category A, so in category B and rest in category C. It is also found that probability of students of category A, unable to get good marks in the final year examination is 0.002 of category B it is 0.02 and of category C, the probability is 0.20.

Based on the above information answer the following :

(*i*) If a student selected at random was found to be the one who could not get good marks in the examination, then the probability that this student is of category C is **1**

(*a*) $\frac{201}{231}$ (*b*) $\frac{200}{231}$

(*c*) $\frac{31}{231}$ (*d*) $\frac{21}{231}$

(*ii*) Assume that a student selected at random was found to be the one who could not get good marks in the examination, then the probability that this student is either of category *A* or of category B is **1**

(*a*) $\frac{31}{231}$ (*b*) $\frac{200}{231}$

(*c*) $\frac{201}{231}$ (*d*) $\frac{21}{231}$

(*iii*) The probability that the student is unable to get good marks in the examination is

1

(*a*) $\frac{231}{300}$ (*b*) $\frac{231}{3000}$

(*c*) $\frac{770}{1000}$ (*d*) 0.007

(*iv*) A student selected at random was found to be the one who could not get good marks in the examination, the probability that this student is NOT of category A is **1**

(*a*) 0 (*b*) $\frac{230}{231}$

(*c*) $\frac{21}{231}$ (*d*) 1

(*v*) A student selected at random was found to be the one who could not get good marks in the examination, The probability that this student is of category *A* is

(*a*) $\frac{1}{231}$ (*b*) $\frac{200}{231}$

(*c*) $\frac{230}{231}$ (*d*) None of these

PART - B

SECTION - III

19. Prove that **2**

$$\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$$

OR

Solve for x

$\sin^{-1} x + \sin^{-1} (1 - x) = \cos^{-1} x$.

20. If $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ and $A'A = I$. Find x, y and z. **2**

21. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $f(x) = x^2 - 7x - 2$, find $f(A)$. **2**

22. If $e^x + e^y = e^{x+y}$, then prove that $\frac{dy}{dx} + e^{y-x} = 0$. **2**

23. Evaluate : **2**

$$\int \frac{x}{1+x\tan x}.dx$$

OR

Evaluate : $\int e^{2x}\ (2\cos x - \sin x)\, dx$ **2**

24. The surface area of a spherical balloon is increasing at the rate of 2 cm^2/sec. Find the rate of change of its volume when its radius is 6 cm. **2**

25. Find the differential equation for $y = a\, e^{bx}$, a . b are arbitrary constants. **2**

26. Using differentials, find approximate value of $\sqrt{0.082}$.

27. If $\vec{a}$ is any vector in space, then prove that

$$\vec{a} = (\vec{a}\cdot\hat{i})\hat{i} + (\vec{a}\cdot\hat{j})\hat{j} + (\vec{a}\cdot\hat{k})\hat{k}$$ **2**

OR

Find the projection of $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$.

28. Are the vector $\vec{a} = -2\hat{i} - 2\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ coplanar? **2**

Section - IV

29. Using properties of determinants, show that

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+bc & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab + bc + ca)^3.$$ **3**

OR

Using elementary operators find A^{-1}, where **3**

$$A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

30. Let $A = R - \{3\}$ and $B = R - \{1\}$, consider the function $f : A \to B$ defined by $f(x) = \frac{x-2}{x-3}$.
Show that f is bijective. Hence find f^{-1}. **3**

31. Three numbers are selected at random (without replacement) from first six positive integers. Let X denotes the largest of the three numbers obtained. **3**

Find the probability distribution of X. Also fine the mean and variance of the distribution.

32. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Find the point of intersection also. **3**

33. Solve the differential equation $x^2\, dy + y(x + y)\, dx = 0$. Given that $y = 1$ when $x = 1$. **3**

OR

Solve the following differential equation **3**

$$x\frac{dy}{dx} + y - x + xy \cot x = 0.$$

Given that $y = 0$ when $x = 0$.

34. If $y^{1/m} + y^{-1/m} = 2x$ **3**

Then prove that $(x^2 - 1)\, y_2 + xy_1 = m^2y$.

35. Evaluate : **3**

$$\int_0^1 \frac{\log(1+x)}{1+x^2}\, dx$$

Section - V

36. Find the equations of the two planes passing through the points (0, 4, – 3) and (6, – 4, 3). If the sum of their intercepts on the three axes is zero. **5**

OR

Find the image of the points (2, 3, – 4) in the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 3$.

37. Using integration, find the area of the region bounded by the lines

$x - 3y + 5 = 0$, $3x - 2y = 6$, $2x + y = 4$. **5**

OR

Find the area of that part of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$. (using integration). **5**

38. Solve graphically maximize and minimize

$$Z = -x + 2y$$

subject to the constraints

$$x - y \leq 2,\ -x + 3y \leq 10,\ x + y \leq 6,\ x, y \geq 0.$$

OR

Two tailors *A* and *B* are paid ₹ 225 and ₹ 300 per day respectively. A can stitch 9 shirts and 6 pants while B can stitch 15 shirts and 6 pants per day. Form a Linear Programming Problem to minimize the labour cost to produce atleast 90 shirts and 48 pants. Solve the problem graphically.

ANSWERS

Practice Paper - 3

1. – 18.

2. <6, 2, – 3>

3. $\frac{1}{\sqrt{35}}(\hat{i} + 5\hat{j} + 3\hat{k})$ OR $10\sqrt{5}$.

4. (3, 3), (4, 4)

5. $\frac{2}{3}\sin^{-1}(x^{3/2}) + C$ OR $a = \frac{1}{2}$.

6. Order = 2, degree = Not defined.

7. 1.

8. $2x^2 + 4x + 3$ OR $\pm 5, \pm 4$.

9. $\frac{1}{2\sqrt{\pi}}$ units.

10. $\pm 320, 625$ OR $R - \{0\}$.

11. $\begin{vmatrix} 22 & 0 & 0 \\ 0 & 22 & 0 \\ 0 & 0 & 22 \end{vmatrix}$

12. $P(A/B) = \frac{3}{14}$, $P(A \cup B) = 0.85$.

13. $\log x$.

14. 2.

15. 1.

16. Yes.

17. (*i*) c (*ii*) b (*iii*) b (*iv*) a (*v*) c

18. (*i*) b (*ii*) a (*iii*) c (*iv*) b (*v*) a

19. OR $0, \frac{1}{2}$.

20. $x = \pm \frac{1}{\sqrt{2}}, y = \pm \frac{1}{\sqrt{6}}, z = \pm \frac{1}{\sqrt{3}}$

21. 0.

23. $\log (\cos x + x \sin x) + C$ OR $e^{2x} \cos x + C$.

24. 6 cm^3/sec.

25. $y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ OR $yy_2 = y_1^2$

26. 0.2867.

27. OR 5 units.

28. Yes.

29. OR $A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix}$

30. $f^{-1}(x) = \frac{3x-2}{x-1}$ $(x \neq 1)$

31.

X	3	4	5	6
$P(x)$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{10}{20}$

Mean = $\frac{21}{4}$, Var. = $\frac{63}{80}$

32. $\left(\frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}\right)$

33. $y + 2x = 3x^2y$ OR $y \,.\, x \sin x = -x \cos x + \sin x$

35. $\frac{\pi}{8} \log 2$.

36. $6x + 3y - 2z = 18$ OR $2x - 3y - 6z = 6$.

37. $\frac{7}{2}$ square units OR $\frac{4}{3}(8\pi - \sqrt{3})$ square units.

38. Max. = $\frac{20}{3}$ at $\left(0, \frac{10}{3}\right)$

Min. = – 2 at (2, 0).

OR

Min. Labour cost = ₹ 2025.

When A works for 5 days and B works for 3 days.

SAMPLE QUESTIONS PAPER

Class XII

Session : (2021-22)

Mathematics (Code-041

Time Allowed : 90 Min **Maximum Marks : 40**

General Instruction

1. This question paper contains three sections- A, B and C. Each part is compulsory.
2. Section - A has 20 MCQs, attempt any 16 out of 20.
3. Section - B has 20 MCQs, attempt any 16 out of 20.
4. Section - C has 10 MCQs, attempt any 8 out of 10.
5. All questions carry equal marks.
6. There is no negative marking.

SECTION - A

(In this section, attempt any 16 questions out of questions 1-20. Each question is of 1 mark weightage.)

1. $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to **1**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) –1 (d) 1

2. The value of k ($k < 0$) for which the function f defined as **1**

$$f(x) = \begin{cases} \frac{1-\cos kx}{x\sin x}, & x \neq 0 \\ \frac{1}{2}, & x = 0 \end{cases}$$

(a) ± 1 (b) – 1

(c) $\pm\frac{1}{2}$ (d) $\frac{1}{2}$

3. If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, \text{ when } i \neq j \\ 0, \text{ when } i = j \end{cases}$, then A^2 is: **1**

(a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. Value of k, for which $A = \begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is singular matrix is **1**

(a) 4 (b) – 4

(c) ± 4 (d) 0

5. Find the intervals in which the function of given by $f(x) = x^2 - 4x + 6$ is strictly increasing **1**

(a) $(-\infty, 2) \cup (2, \infty)$ (b) $(\infty, 2)$

(c) $(-\infty, 2)$ (d) $(-\infty, 2] \cup (2, \infty)$

6. Given that A is square matrix of order 3 and $|A| = -4$, then $|\text{adj } A|$ is equal to **1**

(a) – 4 (b) 4

(c) – 16 (d) 16

7. A relation R in set A = {1, 2, 3} is defined as R = {(1, 1), (1, 2), (2, 2), (3, 3)}. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A? **1**

(a) (1, 1) (b) (1, 2)

(c) (2, 2) (d) (3, 3)

8. If $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$, then value of $a + b - c + 2d$ is: **1**

(a) 8 (b) 10

(c) 4 (d) – 8

9. The point at which the normal to the curve $y = x + \frac{1}{x} > 0$ is perpendicular to the line $3x - 4y - 7 = 0$ is **1**

(a) (2, 5/2) (b) (± 2, 5/2)

(c) (– 1/2, 5/2) (d) (1/2, 5/2)

10. $\sin(\tan^{-1}x)$, where $|x| < 1$, is equal to **1**

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

11. Let the relation R in the $A = \{x \in z : 0 \leq x \geq 12\}$, given by $R = \{(a, b) : |a - b|$ is a multiple of 4}. Then [1], the equivalence class containing 1, is: **1**

(a) {1, 5, 9} (b) {0, 1, 2, 5}

(c) ϕ (d) A

12. If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx}$ is: **1**

(a) e^{y-x} (b) e^{x+y}

(c) $-e^{y-x}$ (d) $2e^{x-y}$

13. Given that matrices A and B are of order $3 \times n$ and $m \times 5$ respectively, then the order of matrix $C = 5A + 3B$ is **1**

(a) 3 × 5 (b) 5 × 3

(c) 3 × 3 (d) 5 × 5

14. If $y = 5 \cos x - 3 \sin x$, then $\frac{d^2y}{dx^2}$ is equal to **1**

(a) $-y$ (b) y

(c) $25y$ (d) $9y$

15. For matrix $A = \begin{bmatrix} 2 & 5 \\ -11 & 7 \end{bmatrix}$, (adj A) is equal to: **1**

(a) $\begin{bmatrix} -2 & -5 \\ 11 & -7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 5 \\ 11 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$

16. The points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to y-axis are **1**

(a) $(0, \pm 4)$ (b) $(4, \pm 0)$

(c) $(\pm 3, 0)$ (d) $(0, \pm 3$

17. Given that $A = [a_{ij}]$ is a square matrix of order 3 × 3 and $|A| = -7$, then the value of $\sum_{i=1}^{3} a_{i2} A_{i2}$, where A_{ij} denotes the cofactor of element a_{ij} is: **1**

(a) 7 (b) – 7

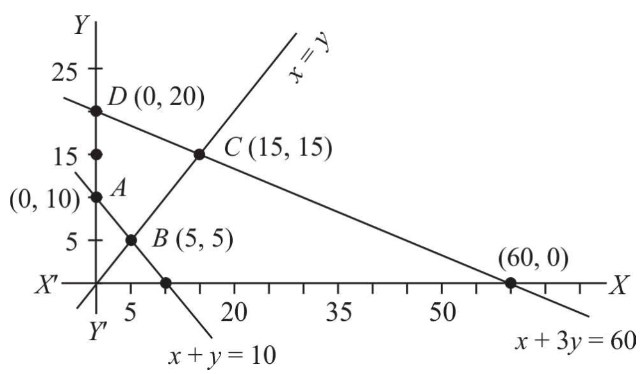
(c) 0 (d) 49

18. If $y = \log(\cos e^x)$, then $\frac{dy}{dx}$ is **1**

(a) $\cos e^{x-1}$ (b) $e^{-x} \cos e^x$

(c) $e^x \sin e^x$ (d) $-e^x \tan e^x$

19. Based on the given shaded region as the feasible region in the graph, at which point(s) is the objective function $Z = 3x + 9y$ maximum? **1**



(a) Point B (b) Point C

(c) Point D (d) Every point on the line segment CD

20. The least value of the function $f(x) = 2\cos x + x$ in the closed interval $\left[0, \frac{\pi}{2}\right]$ is **1**

(a) 2 (b) $\frac{\pi}{2} + \sqrt{3}$

(c) $\frac{\pi}{2}$ (d) The least value does not exist.

SECTION - B

(In this section, attempt any 16 questions out of questions 21-40. Each question is of 1 mark weightage.)

21. The function $f : R \to R$ defined as $f(x) = x^3$ is: **1**

(a) One-on but not onto (b) Not one-one but onto

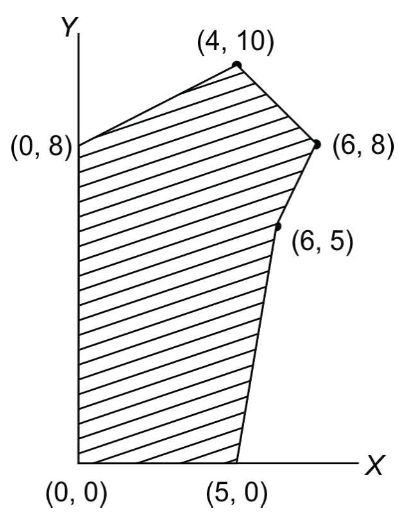
(c) Neither one-one nor onto (d) One-one and onto

22. If $x = a \sec \theta$, $b \tan \theta$, then $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$ is **1**

(a) $\frac{-3\sqrt{3}b}{a^2}$ (b) $\frac{-2\sqrt{3}b}{a}$

(c) $\frac{-3\sqrt{3}b}{a}$ (d) $\frac{-b}{3\sqrt{3}a^2}$

23. In the given graph, the feasible region for a LPP is shaded. The objective function $Z = 2x - 3y$, will be minimum at: **1**



(a) (4, 10) (b) (6, 8)

(c) (0, 8) (d) (6, 5)

24. The derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ w.r.t $\sin^{-1}x$, $-\frac{1}{2} < x < \frac{1}{\sqrt{2}}$, is **1**

(a) 2 (b) $\frac{\pi}{2}-2$

(c) $\frac{\pi}{2}$ (d) -2

25. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, then **1**

(a) $A^{-1} = B$ (b) $A^{-1} = 6B$

(c) $B^{-1} = B$ (d) $B^{-1} = \frac{1}{6}A$

26. The real function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is **1**

(a) Strictly increasing in $(-\infty, -2)$ and strictly decreasing in $(-2, \infty)$

(b) Strictly decreasing in $(-2, 3)$

(c) Strictly decreasing in $(-\infty, 3)$ and strictly increasing in $(3, \infty)$

(d) Strictly decreasing in $(-\infty, -2) \cup (3, \infty)$

27. Simplest form of $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)$, $\pi < x < \frac{3\pi}{2}$ **1**

(a) $\frac{\pi}{4}-\frac{\pi}{2}$ (b) $\frac{3\pi}{2}-\frac{\pi}{2}$

(c) $-\frac{x}{2}$ (d) $\pi-\frac{x}{2}$

28. Given that A is non-singular matrix of order 3 such that $A^2 = 2A$, then value of $|2A|$ is **1**

(a) 4 (b) 8

(c) 64 (d) 16

29. The value of b for which the function $f(x) = x + \cos x + b$ is strictly decreasing over R is : **1**

(a) $b < 1$ (b) No value of b exists

(c) $b \leq 1$ (d) $b \geq 1$

30. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b = 6\}$, then **1**

(a) $(2, 4) \in R$ (b) $(3, 8) \in R$

(c) $(6, 8) \in R$ (d) $(8, 7) \in R$

31. The point(s), at which the function f given by $f(x) = \begin{cases} \dfrac{x}{|x|}, x < 0 \\ -1, \geq x \end{cases}$ **1**

(a) $x \in R$ (b) $x = 0$

(c) $x \in R - \{0\}$ (d) $x = -1$ and 1

32. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a and b respectively are **1**

(a) –6, –12, –18 (b) –6, –4, –9

(c) –6, 4, 9 (d) –6, 12, 18

33. A linear programming problem is as follows:

Minimize $Z = 30x + 50y$

Subject to the contraints,

$$3x + 5y \geq 15$$
$$3x + 3y \geq 18$$
$$x \geq 0, y \geq 0$$

In the feasible region, the minimum value of Z occurs at **1**

(a) a unique point (b) no point

(c) infinitely many points (d) two points only

34. The area of a trapezium is defined by function f and given by $f(x) = (10 + x)\sqrt{100 - x^2}$, then the area when it is maximised is **1**

(a) 75 cm^2 (b) $7\sqrt{3} \text{ cm}^2$

(c) $75\sqrt{3} \text{ cm}^2$ (d) 5 cm^2

35. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to **1**

(a) A (b) $I + A$

(c) $I - A$ (d) I

36. If $\tan^{-1}x = y$, then **1**

(a) $-1 < y < 1$ (b) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(c) $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (d) $y \in \left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$

37. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Based on the given information, f is best defined as **1**

(a) Surjective function (b) Injective function

(c) Bijective function (d) function

38. For $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $14\,A^{-1}$ is given by **1**

(a) $14\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$

(c) $2\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$ (d) $2\begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix}$

39. The point(s) on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$ is/are **1**

(a) (–2, 19) (b) (2, –9)

(c) (±2, 19) (d) (–2, 19) and (2, –9)

40. Given that $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = 31$, then **1**

(a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 - \beta\gamma = 0$

(c) $3 - \alpha^2 - \beta\gamma = 0$ (d) $3 + \alpha^2 + \beta\gamma = 0$

SECTION - C

(In this section, attempt any 8 questions. Each question is of 1 mark weightage. Questions 46-50 are based on a case-study)

41. For an objective function $Z = ax + by$, where $a, b > 0$; the corner points of the feasible region determned by a set of constraints (linear inequalities) are (0, 20), (10, 20), (30, 30) and (0, 40). The condition on a and b such that the maximum Z occurs at both the points (30, 30) and (0, 40) is **1**

(a) $b - 3a = 0$ (b) $a = 3b$

(c) $a + 2b = 0$ (d) $2a - b = 0$

42. For which value of m is the line $y = mx + 1$ a tangent to the curve $y^2 = 4x$? **1**

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) 3

43. The maximum value of $[x(x-1)+1]^{1/3}$, $0 \le x \le 1$ is **1**

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) $\sqrt[3]{\frac{1}{3}}$

44. In a linear programming problem, the constraints on the decision variables x and y are $x - 3y \ge 0$, $y \ge 0$, $0 \le x \le 3$. The feasible region **1**

(a) is not in the first quadrant (b) is bounded in the first quadrant

(c) is unbounded in the first quadrant (d) does not exist

45. Let $A = \begin{bmatrix} 1 & \sin\alpha & 1 \\ -\sin\alpha & 1 & \sin\alpha \\ -1 & -\sin\alpha & 1 \end{bmatrix}$, where $0 \le \alpha \le 2\pi$, then **1**

(a) $|A| = 0$ (b) $|A| \in (2, \infty)$

(c) $|A| \in (2, 4)$ (d) $|A| \in [2, 4)$

Case Study

The fuel cost per hour for running a train is proportional to the square of the speed it generates in km per hour. If the fuel costs Rs 48 per hour at speed 16 km per hour and the fixed charges to run the train amount to Rs 1200 per hour. Assume the speed of the train as v km/h.

fig.

Based on the given information, answer the following question.

46. Given that the fuel cost per hour is k times the square of the speed the train generates in km/h, the value of k is **1**

(a) $\frac{16}{3}$ (b) $\frac{1}{3}$

(c) 3 (d) $\frac{3}{16}$

47. If the train has travelled a distance of 500 km, then the total cost of running the train is given by function **1**

(a) $\frac{15}{16}v+\frac{600000}{v}$ (b) $\frac{375}{4}v+\frac{600000}{v}$

(c) $\frac{5}{16}v^2+\frac{150000}{v}$ (d) $\frac{3}{16}v+\frac{6000}{v}$

48. The most economical speed to run the train is: **1**

(a) 18 km/h (b) 5 km/h

(c) 80 km/h (d) 40 km/h

49. The fuel cost for the train to travel 500 km at the most economical speed is **1**

(a) Rs 3750 (b) Rs 750

(c) Rs 7500 (d) Rs 75000

50. The total cost of the train to travel 500 km at the most economical speed is **1**

(a) Rs 3750 (b) Rs 75000

(c) Rs 7500 (d) Rs 15000

Answers

1. (d) $\sin\left[\frac{\pi}{3}-\left(\frac{-\pi}{6}\right)\right]=\sin\left(\frac{\pi}{2}\right)=1$

2. (b) $\lim_{x\to 0}\left(\frac{1-\cos kx}{x\sin x}\right)=\frac{1}{2}$

$$\Rightarrow \lim_{x \to 0}\left(\frac{2\sin\frac{2kx}{2}}{x\sin x}\right) = \frac{1}{2}$$

$$\Rightarrow \lim_{x \to 0} 2\left(\frac{k}{2}\right)^2 \left(\frac{\sin\frac{kx}{2}}{\frac{kx}{2}}\right)^2 \left(\frac{x}{\sin x}\right) = \frac{1}{2}$$

$\Rightarrow k^2 = 1 \Rightarrow k = \pm 1$ but $k < 0 \Rightarrow k = -1$

3. (*d*) $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. (*c*) As A is singular matrix

$\Rightarrow \quad |A| = 0$

$\Rightarrow \quad 2k^2 - 32 = 0 \Rightarrow k = \pm 4$

5. (*b*) $f(x) = x^2 - 4x + 6$

$f'(x) = 2x - 4$

Let $f'(x) = 0 \Rightarrow x = 2$

fig.

as $f'(x) > 0$ □ $x \in (2, \infty)$

$\Rightarrow f(x)$ is strictly increasing in $(2, \infty)$

6. (*d*) as $|\text{adj } A| = |A|^{n-1}$, where n is order of the square matrix A.

$= (-4)^2 = 16$

7. (*b*) (1, 2)

8. (*a*) $2a + b = 4$, $a - 2b = -3$, $5c - d = 11$, $4c + 3d = 24\}$

$\Rightarrow \quad a = 1 b = 2 c = 3 d = 4$

$\therefore \quad a + b - c + 2d = 8$

9. (*a*) $f(x) = x + \frac{1}{x}$, $x > 0 \Rightarrow f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$, $x > 0$

As normal to the curve $y = f(x)$ at some point (x, y) is $\perp$ to given line

$\Rightarrow \quad \left(\frac{x^2}{1-x^2}\right) \times \frac{3}{4} = -1 \; (m_1.m_2 = -1)$

$\Rightarrow \qquad x^2 = 4 \;\Rightarrow\; x = \pm 2$

But $x > 0$, $\therefore \quad x = 2$

Therefore point = $\left(2, \frac{5}{2}\right)$

10. (*d*) $\sin(\tan^{-1}x) = \sin\left\{\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right\} = \frac{x}{\sqrt{1+x^2}}$

11. (*a*) {1, 5, 9}

12. (*c*) $e^x + e^y = e^{x+y}$

$\Rightarrow \qquad e^{-y} + e^{-x} = 1$

Differentiating w.r.t. x:

13. (b) 3 × 5

14. (a) $y = 5\cos x - 3\sin x \;\Rightarrow\; \frac{dy}{dx} = -5\sin x - 3\cos x$

$\Rightarrow \frac{d^2y}{dx^2} = -5\cos x + 3\sin x = -y$

15. (c) adj $A = \begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$ (adj A)′ = $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$

16. (c) $\frac{x^2}{9} + \frac{y^2}{16} = 1 \quad \Rightarrow \quad \frac{2x}{9} + \frac{26}{16}\frac{dy}{dx} = 0$

$\Rightarrow$ Slope of normal at any point (x, y) to the curve $= -\frac{dx}{dy} = \frac{9y}{16}$

As tangent to the curve at the point (x, y) is parallel to y-axis

$\Rightarrow \quad \frac{9y}{16x} = 0 \;\Rightarrow\; y = 0$ and $x = \pm 3$

$\therefore$ Points = (±3, 0)

17. (b) $|A| = -7$

$\therefore \sum_{i=1}^{3} a_{i2}A_{i2} = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} = |A| = -7$

18. (d) $y = \log(\cos e^x)$

Differentiating w.r.t. x:

$$\frac{dy}{dx} = \frac{1}{\cos(e^x)} \cdot (-\sin e^x) \cdot e^x \text{ (chain rule)}$$

$$\Rightarrow \quad \frac{dy}{dx} = -e^x \tan e^x$$

19. (d) Z is maximum 180 at points C (15, 15) and D (0, 20).

Z is maximum at every point on the line segment CD

20. (d) $f(x) = 2\cos x + x,\ x \in \left[0, \frac{\pi}{2}\right]$

$f'(x) = -2\sin x + 1$

Let $f'(x) = 0 \Rightarrow x = \frac{\pi}{6} \in \left[0, \frac{\pi}{2}\right]$

$f(0) = 2$

$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \sqrt{3}$

$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \quad \Rightarrow$ Least value of $f(x)$ is $\frac{\pi}{2}$ at $x = \frac{\pi}{2}$

Section-B

21. (d) Let $f(x_1)$

$= f(x_2)$ such that $x_1 x_2 \in R$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

$\Rightarrow f$ is one-one

Let $y \in R$ (codomain). Then for any x, $f(x) = y$

if $x^3 = y$

i.e., $x = y^{1/3} \in R$ (domain)

i.e., every element $y \in R$ (codomain) has a

preimage $y^{1/3}$ in R (domain)

$\Rightarrow$ f is onto

$\therefore$ f is one-one and onto

22. (a) $x = a \sec\theta \Rightarrow \dfrac{dx}{d\theta} = a\sec\theta\tan\theta$

$y = b \tan\theta \Rightarrow \dfrac{dy}{d\theta} = b\sec^2\theta$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{b}{a}\operatorname{cosec}\theta$

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = \dfrac{-b}{a}\operatorname{cosec}\theta\cdot\cot\theta\cdot\dfrac{d\theta}{a^2}\cot^3\theta$

$\therefore \quad \left.\dfrac{d^2y}{dx^2}\right|_{\theta=\frac{\pi}{6}} = \dfrac{-3\sqrt{3}b}{a^2}$

23. (c) Z is minimum – 24 at (0, 8)

24. (a) Let $u = \sin^{-1}(2x\sqrt{1-x^2})$

and $v = \sin^{-1}x$, $-\dfrac{1}{\sqrt{2}} < x < \dfrac{1}{\sqrt{2}}$

using (1), we get:

$= \sin^{-1}(2\sin v\cos v) = \sin^{-1}(\sin 2v)$

$\Rightarrow \quad u = 2v$, $-\dfrac{\pi}{2} < 2v < \dfrac{\pi}{2}$

Differentiating u with respect to v, we get: $\dfrac{du}{dv} = 2$

25. (d) $AB = 6I \Rightarrow B^{-1} = \dfrac{1}{6}A$

26. (b) $f'(x) = 6(x^2 - x - 6) = 6(x - 3)(x + 2)$

As $f'(x) < 0$ $x \in (-2, 3)$

$\Rightarrow$ $f(x)$ is strictly decreasing in (–2, 3)

27. (a) $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)$

$$= \tan^{-1}\left(\frac{-\sqrt{2}\cos\frac{x}{2}+\sqrt{2}\sin\frac{x}{2}}{-\sqrt{2}\cos\frac{x}{2}-\sqrt{2}\sin\frac{x}{2}}\right), \ \pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{2}$$

$$= \tan^{-1}\left(\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right) = \tan^{-1}\left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right) = \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]$$

$$= \frac{\pi}{4}-\frac{x}{2}, -\frac{\pi}{4} > \frac{\pi}{4}-\frac{x}{2} > -\frac{\pi}{2}$$

28. (c) $A^2 = 2A$

$\Rightarrow$ $|A^2| = |2A|$

$\Rightarrow$ $|A^2| = 2^3\,|A|$ as $|kA| = k^n|A|$ for a square matrix of order n

$\Rightarrow$ either $|A| = 0$ or $|A| = 8$

But A is non-singular matrix

$\therefore$ $|A| = 8^2 = 64$

29. (b) $f'(x) = 1 - \sin x$ $\Rightarrow$ $f(x)\ 0 \quad x \in R$

$\Rightarrow$ No value of b exists.

30. (c) $a = b - 2$ and $b > 6$

$\Rightarrow$ $(6, 8) \in R$

31. (a) $f(x) = \begin{cases} \frac{x}{-x} = -1, x < 0 \quad -1, \quad x \geq 0 \end{cases}$

$\Rightarrow$ $f(x) = -1 \quad x \in R$

$\Rightarrow$ $f(x)$ is continuous $x \in R$ as it is a constant function.

32. (b) $kA = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$\Rightarrow$ $k = -6$, $a = -4$ and $b = -9$

33. (c)

Corner points of feasible region	$Z = 30x + 50y$
(5, 0)	150
(9, 0)	270
(0, 3)	150
(0, 6)	300

34. (c) $f'(x) = -\dfrac{2x^2 - 10x + 100}{\sqrt{100 - x^2}}$

$f'(x) = 0 \Rightarrow x = -10$ or 5, but $x > 0 \Rightarrow x = 5$

$$f''(x)\frac{2x^3 - 300x - 1000}{(100 - x)^{3/2}} \Rightarrow f''(5) = \frac{-30}{\sqrt{75}} < 0$$

Maximum ara of trapezium is $75\sqrt{3}$ cm^2 when $x = 5$

35. (d) $(I - A)^3 - 7A = I + A + 3A + 3A - 7A = I$

36. (c) $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

37. (b) Since, distinct elements of *A* have distinct *f*-images in *B*. Hence, *f* is injective and every element of *B* does not have its pre-image in *A*, hence *f* is not surjective.

$\therefore$ *f* is injective andis not surjective.

38. (b) $|A| = 7$, adj $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\therefore$ $14A^{-1} = 14 \times \dfrac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$

39. (b) $y = x^3 - 11x + 5 \Rightarrow \dfrac{dy}{dx} = 3x^2 - 11$

Slope of line $y = x - 11$ is 1 $\Rightarrow$ $3x^2 - 11 = 1$ $\Rightarrow$ $x = \pm 2$

$\therefore$ Pointis (2, –9) as (–2, 19) does not satisfy the equation of the given line

40. (c) $A^2 = 3I$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow 3 - \alpha^2 - \beta\gamma = 0$$

Section-C

41. (c) As Z is maximum at (30, 30) and (0, 40)

$\Rightarrow$ $30a + 30b = 40b$ $\Rightarrow$ $b - 3a = 0$

42. (a) $y = mx + 1$ ...(1)

$y^2 = 4x$ ...(2)

Substituting (1) in (2) : $(mx + 1)^2 = 4x$

$\Rightarrow$ $m^2x^2 + (2m - 4)x + 1 = 0$ ...(3)

As line is tangent to the curve

$\Rightarrow$ Line touches the curve at only one point

$\Rightarrow$ $(2m - 4)^2 - 4m^2 = 0$ $\Rightarrow$ $m = 1$

43. (c) Let $f(x) = [x(x - 1) + 1]^{1/3}$, $0 \le x \le 1$

$f'(x) = \dfrac{2x - 1}{3(x^2 - x + 1)^{2/3}}$ let $f'(x) = 0$ $\Rightarrow$ $x = \dfrac{1}{2} \in [0, 1]$

$f(0) = 1$, $f\left(\dfrac{1}{2}\right) = \left(\dfrac{3}{4}\right)^{\frac{1}{3}}$ and $f(1) = 1$

$\therefore$ Maximum value of $f(x)$ is 1.

44. (b) Feasible region is bounded in the first quadrant

45. (d) $|A| = 2 + 2\sin^2\alpha$

As $-1 \le \sin\alpha \le 1$, $0 \le \alpha \le 2\pi$

$\Rightarrow$ $2 \le 2 + 2\sin^2\alpha \le 4$ $\Rightarrow$ $|A| \in [2, 4]$

46. (d) Fuel cost per hour = k (speed)^2

$\Rightarrow$ $48 = k.\ 16^2$ $\Rightarrow$ $k = \dfrac{3}{16}$

47. (b) Total cost of running train (let C) = $\frac{3}{16}v^2t+1200t$

Distance covered = 500 km $\Rightarrow$ time $=\frac{500}{v}$ hrs

Total cost of running train 500 km = $\frac{3}{16}v^2\left(\frac{500}{V}\right)+1200\left(\frac{500}{v}\right)$

$\Rightarrow \quad C=\frac{375}{4}v+\frac{600000}{v}$

48. (c) $\frac{dC}{dv}=\frac{375}{4}-\frac{600000}{v^2}$

Let $\frac{dC}{dv}=0 \Rightarrow v=80$ km/h

49. (c) Fuel cost for running 500 km $\frac{375}{v}=\frac{375}{4}\times 80=7500$

50. (d) Total cost for running 500 km $=\frac{375}{4}v+\frac{60000}{v}$

$=\frac{375\times 80}{4}+\frac{600000}{80}=15000$