# DEVELOPMENT OF SUPPORT MATERIAL IN MATHEMATICS FOR CLASS XI

## GROUP LEADER

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# REVIEW OF SUPPORT MATERIAL : 2015-16

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COURSE STRUCTURE

CLASS XI

One Paper Three Hours Max. Marks. 100

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100

Unit-I : Sets and Functions

1. Sets : (12) Periods


2. Relations and Functions : (14) Periods

Ordered pairs, Cartesian product of sets. Number of elements in the cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto R × R × R). Definition of relation, pictorial diagrams,
domain, codomain and range of a relation. Function as a special kind of relation from one set to another. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum exponential, logarithmic functions and greatest integer functions, with their graphs. Sum, difference, product and quotients of functions.

3. **Trigonometric Functions** :

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity \( \sin^2 x + \cos^2 x = 1 \), for all \( x \). Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing \( \sin (x \pm y) \) and \( \cos (x \pm y) \) in terms of \( \sin x, \sin y, \cos x \) and \( \cos y \). Deducing the identities like the following:

\[
\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}, \quad \cot(x \pm y) = \frac{\cot x \cot y \pm 1}{\cot y \pm \cot x},
\]

\[
\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2},
\]

\[
\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2},
\]

\[
\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2},
\]

\[
\cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}.
\]

Identities related to \( \sin 2x, \cos 2x, \tan 2x, \sin 3x, \cos 3x \) and \( \tan 3x \). General solution of trigonometric equations of the type \( \sin \theta = \sin \alpha, \cos \theta = \cos \alpha \) and \( \tan \theta = \tan \alpha \). Proof and simple applications of sine and cosine formulae.

**Unit-II : Algebra**

1. **Principle of Mathematical Induction** :

Process of the proof by induction, motivating the applications of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.
2. Complex Numbers and Quadratic Equations : (10) Periods

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve some of the quadratic equations. Algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations in the complex number system. Square root of a complex number.

3. Linear Inequalities : (10) Periods

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Graphical solution of system of linear inequalities in two variables.

4. Permutations and Combinations : (12) Periods

Fundamental principle of counting. Factorial $n$ ($n!$) Permutations and combinations, derivation of formulae and their connections, simple applications.

5. Binomial Theorem : (08) Periods

History, statement and proof of the binomial theorem for positive integral indices. Pascal’s triangle, General and middle term in binomial expansion, simple applications.

6. Sequence and Series : (10) Periods

Sequence and Series. Arithmetic progression (A.P.) arithmetic mean (A.M.) Geometric progression (G.P.), general term of a G.P., sum of $n$ terms of a G.P., Arithmetic and Geometric series, Infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M. Sum to $n$ terms of the special series $\sum_{k=1}^{n} k$, $\sum_{k=1}^{n} k^2$ and $\sum_{k=1}^{n} k^3$.

Unit-III : Coordinate Geometry

1. Straight Lines : (09) Periods

Brief recall of two dimensional geometry from earlier classes. Shifting of origin. Slope of a line and angle between two lines. Various forms of equations of a line : parallel to axes, point-slope form, slope-intercept form, two-point form, intercept form and normal form. General equation of a line. Equation of family of lines passing through the point of intersection of two lines. Distance of a point from a line.
2. Conic Sections: 
Sections of a cone: circles, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

3. Introduction to Three-Dimensional Geometry
Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

Unit-IV : Calculus
1. Limits and Derivatives:
Limit of function introduced as rate of change of distance function and its geometric meaning. Limits of Polynomials and rational function, trigonometric, exponential and logarithmic functions.
\[
\lim_{x \to o} \log_e (1 + x), \quad \lim_{x \to o} \frac{e^x - 1}{x}
\]
Definition of derivative, relate it to slope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

Unit-V : Mathematical Reasoning
1. Mathematical Reasoning:
Mathematically acceptable statements. Connecting words/phrases-consolidating the understanding of “if and only if (necessary and sufficient) condition”, “implies”, “and/or”, “implied by”, “and”, “or”, “there exists” and their use through variety of examples related to real life and Mathematics. Validating the statements involving the connecting words, difference between contradiction, converse and contrapositive.

Unit-VI : Statistics and Probability
1. Statistics
Measure of dispersion, range, mean deviation, variance and standard deviation of ungrouped/grouped data.
Analysis of frequency distributions with equal mean but different variances.
2. Probability
Random experiments, outcomes; Sample spaces (set representation); Event; Occurrence of events, "not", "and" and "or" events. Exhaustive events, mutually exclusive events. Axiomatic (set theoretic) probability connections with the theories of earlier classes. Probability of an event, probability of "not", "and" and "or" events.
CHAPTER - 1

SETS

KEY POINTS

- A set is a well-defined collection of objects.
- There are two methods of representing a set :
  (a) Roster or Tabular form e.g. :-
      natural numbers less than 5 = \{1, 2, 3, 4\}
  (b) Set-builder form or Rule method e.g. : Vowels in English alphabet
      = \{x : x \text{ is a vowel in the English alphabet} \}
- Types of sets :-
  (i) Empty set or Null set or void set
  (ii) Finite set
  (iii) Infinite set
  (iv) Singleton set
- Subset := A set A is said to be a subset of set B if \( a \in A \Rightarrow a \in B, \forall a \in A \). We write it as \( A \subseteq B \)
- Equal sets := Two sets A and B are equal if they have exactly the same elements i.e. \( A = B \) if \( A \subseteq B \) and \( B \subseteq A \)
- Power set : The collection of all subsets of a set A is called power set of A, denoted by \( P(A) \) i.e. \( P(A) = \{ B : B \subseteq A \} \)
- If A is a set with \( n(A) = m \) then \( n[P(A)] = 2^m \).
- Equivalent sets : Two finite sets A and B are equivalent, if their cardinal numbers are same i.e., \( n(A) = n(B) \).
- Proper set and super set : If \( A \subseteq B \) then A is called the proper subset of B and B is called the superset of A.
Types of Intervals

Open Interval \((a, b) = \{ x \in \mathbb{R} : a < x < b \}\)

Closed Interval \([a, b] = \{ x \in \mathbb{R} : a \leq x \leq b \}\)

Semi open or Semi closed Interval,

\((a, b] = \{ x \in \mathbb{R} : a < x \leq b\}\)

\([a, b) = \{ x \in \mathbb{R} : a \leq x < b\}\)

- Union of two sets \(A\) and \(B\) is,
  \[A \cup B = \{ x : x \in A \text{ or } x \in B \}\]

- Intersection of two sets \(A\) and \(B\) is,
  \[A \cap B = \{ x : x \in A \text{ and } x \in B\}\]

- Disjoint sets: Two sets \(A\) and \(B\) are said to be disjoint if \(A \cap B = \emptyset\)
• Difference of sets $A$ and $B$ is,
$$A - B = \{ x : x \in A \text{ and } x \notin B \}$$

• Difference of sets $B$ and $A$ is,
$$B - A = \{ x : x \in B \text{ and } x \notin A \}$$

• Complement of a set $A$, denoted by $A'$ or $A^c$ is
$$A' = A^c = U - A = \{ x : x \in U \text{ and } x \notin A \}$$

• Properties of complement sets:
  1. Complement laws
     (i) $A \cup A' = U$  (ii) $A \cap A' = \emptyset$  (iii) $(A')' = A$
  2. De Morgan's Laws
     (i) $(A \cup B)' = A' \cap B'$  (ii) $(A \cap B)' = A' \cup B'$
Note: This law can be extended to any number of sets.

3. \( \phi = \bigcup \) and \( \bigcup' = \phi \)

4. If \( A \subset B \) then \( B' \subset A' \)

Laws of Algebra of sets.

(i) \( A \cup \phi = A \)

(ii) \( A \cap \phi = \phi \)

\( A - B = A \cap B' = A - (A \cap B) \)

Commutative Laws :-

(i) \( A \cup B = B \cup A \)

(ii) \( A \cap B = B \cap A \)

Associative Laws :-

(i) \( (A \cup B) \cup C = A \cup (B \cup C) \)

(ii) \( (A \cap B) \cap C = A \cap (B \cap C) \)

Distributive Laws :-

(i) \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \)

(ii) \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \)

If \( A \subset B \), then \( A \cap B = A \) and \( A \cup B = B \)

When \( A \) and \( B \) are disjoint \( n(A \cup B) = n(A) + n(B) \)

When \( A \) and \( B \) are not disjoint

\( n(A \cup B) = n(A) + n(B) - n(A \cap B) \)

\( n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \)

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Which of the following are sets? Justify your answer.

1. The collection of all the months of a year beginning with letter M

2. The collection of difficult topics in Mathematics.

Let \( A = \{1,3,5,7,9\} \). Insert the appropriate symbol \( \in \) or \( \notin \) in blank spaces \( :- (\text{Question- 3,4}) \)
3. \(\mathbb{Z} = \{x : x \text{ is an integer, } -1 \leq x < 4\}\)

4. \(\mathbb{Z} = \{x : x \text{ is an integer, } -1 \leq x < 4\}\)

5. Write the set \(A = \{x : x \text{ is an integer, } -1 \leq x < 4\}\) in roster form.

6. List all the elements of the set,

\[
A = \left\{x : x \in \mathbb{Z}, \frac{-1}{2} < x < \frac{11}{2}\right\}
\]

7. Write the set \(B = \{3,9,27,81\}\) in set-builder form.

Which of the following are empty sets? Justify. (Question- 8,9)

8. \(A = \{x : x \in \mathbb{N} \text{ and } 3 < x < 4\}\)

9. \(B = \{x : x \in \mathbb{N} \text{ and } x^2 = x\}\)

Which of the following sets are finite or Infinite? Justify. (Question-10,11)

10. The set of all the points on the circumference of a circle.

11. \(B = \{x : x \in \mathbb{N} \text{ and } x \text{ is an even prime number}\}\)

12. Are sets \(A = \{-2,2\}, B = \{x : x \in \mathbb{Z}, x^2 - 4 = 0\}\) equal? Why?

13. Write \((-5,9]\) in set-builder form.

14. Write \(\{x : x \in \mathbb{R}, -3 \leq x < 7\}\) as interval.

15. If \(A = \{1,3,5\}\), how many elements has \(P(A)\) ?

16. Write all the possible subsets of \(A = \{5,6\}\).

If \(A = \{2,3,4,5\}, B = \{3,5,6,7\}\) find (Question- 17,18)

17. \(A \cup B\)

18. \(A \cap B\)

19. If \(A = \{1,2,3,6\} , B = \{1, 2, 4, 8\}\) find \(B - A\)

20. If \(A = \{p, q\}, B = \{p, q, r\}\), is \(B\) a superset of \(A\)? Why?

21. Are sets \(A = \{1,2,3,4\}, B = \{x : x \in \mathbb{N} \text{ and } 5 \leq x \leq 7\}\) disjoint? Why?
22. If X and Y are two sets such that \( n(X) = 19 \), \( n(Y) = 37 \) and \( n(X \cap Y) = 12 \), find \( n(X \cup Y) \).

23. Consider the following sets
\[ \phi, A = \{2, 5\}, B = \{1, 2, 3, 4\}, C = \{1, 2, 3, 4, 5\} \]
Insert the correct symbol \( \subset \) or \( \subsetneq \) between each pair of sets
(i) \( \phi \subset B \)
(ii) \( A \subset B \)
(iii) \( A \subset C \)
(iv) \( B \subset C \)

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

24. If \( \mathbb{U} = \{1,2,3,4,5,6,7,8,9\} \), \( A = \{2,3,5,7,9\} \), \( B = \{1,2,4,6\} \), verify
(i) \( (A \cup B)' = A' \cap B' \)
(ii) \( B - A = B \cap A' = B - (A \cap B) \)

25. Let A, B be any two sets. Using properties of sets prove that,
(i) \( (A - B) \cup B = A \cup B \)
(ii) \( (A \cup B) - A = B - A \)

[Hint : \( A - B = A \cap B' \) and use distributive law.]

26. In a group of 800 people, 500 can speak Hindi and 320 can speak English. Find
(i) How many can speak both Hindi and English?
(ii) How many can speak Hindi only?

27. A survey shows that 84% of the Indians like grapes, whereas 45% like pineapple. What percentage of Indians like both grapes and pineapple?

28. In a survey of 450 people, it was found that 110 play cricket, 160 play tennis and 70 play both cricket as well as tennis. How many play neither cricket nor tennis?

29. In a group of students, 225 students know French, 100 know Spanish and 45 know both. Each student knows either French or Spanish. How many students are there in the group?

30. If \( A = [-3, 5) \), \( B = (0, 6] \) then find (i) \( A - B \), (ii) \( A \cup B \)
Hots (4 Marks)

31. Show that \( n\{P\{P(P(\emptyset))}\} = 4 \)

LONG ANSWER TYPE QUESTIONS (6 MARKS)

32. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families by newspaper C. 5% families buy A and B, 3%, buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, find the no of families which buy

(1) A only
(2) B only
(3) none of A, B and C
(4) exactly two newspapers
(5) exactly one newspaper
(6) A and C but not B
(7) atleast one of A, B, C.

33. In a group of 84 persons, each plays atleast one game out of three viz. tennis, badminton and cricket. 28 of them play cricket, 40 play tennis and 48 play badminton. If 6 play both cricket and badminton and 4 play tennis and badminton and no one plays all the three games, find the number of persons who play cricket but not tennis.

34. In a class, 18 students took Physics, 23 students took Chemistry and 24 students took Mathematics of these 13 took both Chemistry and Mathematics, 12 took both Physics and Chemistry and 11 took both Physics and Mathematics. If 6 students offered all the three subjects, find :

(1) The total number of students.
(2) How many took Maths but not Chemistry.
(3) How many took exactly one of the three subjects.

35. Using properties of sets and their complements prove that

(1) \( (A / B) \cup (B / A) = (A \cup B) / (A \cap B) \)
(2) \( (A \setminus B) \cup (A \setminus C) = A \setminus (B \cap C) \)
(3) \( A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C) \)
(4) \( A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C) \)
(5) \( (A - B) \cap (A - C) = A - (B \cup C) \) where A, B, C are any three sets.
ANSWERS

1. Set
2. Not a set
3. \(\notin\)
4. \(\in\)
5. \(A = \{-1, 0, 1, 2, 3\}\)
6. \(A = \{0,1,2,3,4,5\}\)
7. \(B = \{x : x = 3^n, n \in N\text{ and }1 \leq n \leq 4\}\)
8. Empty set because no natural number is lying between 3 and 4
9. Non-empty set because \(B = \{1\}\)
10. Infinite set because circle is a collection of infinite points whose distances from the centre is constant called radius.
11. Finite set because \(B = \{2\}\)
12. Yes because \(x^2 - 4 = 0\); \(x = 2, -2\) both are integers
13. \(\{x : x \in R, -5 < x \leq 9\}\)
14. \([-3,7]\)
15. \(2^3 = 8\)
16. \(\phi, \{5\}, \{6\}, \{5,6\}\)
17. \(A \cup B = \{2,3,4,5,6,7\}\)
18. \(A \cap B = \{3, 5\}\)
19. \(B - A = \{4,8\}\)
20. Yes, because \(A\) is a subset of \(B\)
21. Yes, because \(A \cap B = \phi\)
22. \(n(X \cup Y) = 44\)
23. (i) \(\subset\); (ii) \(\not\subset\); (iii) \(\subset\); (iv) \(\subset\)
26. (i) 20 people can speak both Hindi and English
   (ii) 480 people can speak Hindi only
27. 29% of the Indians like both grapes and pineapple.
28. Hint: \(\cup\) - set of people surveyed
   \(A\) - set of people who play cricket
   \(B\) - set of people who play tennis
Number of people who play neither cricket nor tennis

\[ n[(A \cup B)'] = n(U) - n(A \cup B) \]

\[ = 450 - 200 \]

\[ = 250 \]

29. There are 280 students in the group.
30. (i) \([-3, 0]\); (ii) \([-3, 6]\)
32. (i) 3300 (ii) 1400 (iii) 4000 (iv) 800 (v) 4800 (vi) 400 (vii) 5800
33. 6
34. (i) 35 (ii) 11 (iii) 11
CHAPTER – 2
RELATIONS AND FUNCTIONS

KEY POINTS

- Cartesian Product of two non-empty sets A and B is given by,
  \[ A \times B = \{ (a,b) : a \in A, b \in B \} \]
- If \((a,b) = (x, y)\), then \(a = x\) and \(b = y\)
- Relation \(R\) from a non-empty set \(A\) to a non-empty set \(B\) is a subset of \(A \times B\).
- Domain of \(R\) = \(\{a : (a,b) \in R\}\)
- Range of \(R\) = \(\{b : (a,b) \in R\}\)
- Co-domain of \(R\) = Set \(B\)
- Range \(\subseteq\) Co-domain
- If \(n(A) = p, n(B) = q\) then \(n(A \times B) = pq\) and number of relations = \(2^{pq}\)
- Image: If the element \(x\) of \(A\) corresponds to \(y \in B\) under the function \(f\), then we say that \(y\) is image of \(x\) under ‘\(f\)’
  \[ f(x) = y \]
- If \(f(x) = y\), then \(x\) is preimage of \(y\).
- A relation \(f\) from a set \(A\) to a set \(B\) is said to be a function if every element of set \(A\) has one and only one image in set \(B\).
- \(D_f = \{x : f(x) \text{ is defined}\}\)
- \(R_f = \{f(x) : x \in D_f\}\)
- Let \(A\) and \(B\) be two non-empty finite sets such that \(n(A) = p\) and \(n(B) = q\) then number of functions from \(A\) to \(B\) = \(q^p\).
- Identity function, \(f : R \rightarrow R; f(x) = x \ \forall x \in R\) where \(R\) is the set of real numbers.
  \[ D_f = R \quad R_f = R \]
- Constant function, \( f : \mathbb{R} \rightarrow \mathbb{R} \); \( f(x) = c \) \( \forall x \in \mathbb{R} \) where \( c \) is a constant
  
  \[ D_f = \mathbb{R} \quad R_f = \{c\} \]

- Modulus function, \( f : \mathbb{R} \rightarrow \mathbb{R} \); \( f(x) = |x| \) \( \forall x \in \mathbb{R} \)
  
  \[ D_f = \mathbb{R} \quad R_f = \{x : x \in \mathbb{R} : x \geq 0\} \]

- Signum function, \( f : \mathbb{R} \rightarrow \mathbb{R} \); \( f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases} \)
  
  \[ D_f = \mathbb{R} \quad R_f = \{-1,0,1\} \]
• Greatest Integer function, $f : \mathbb{R} \to \mathbb{R}$; $f(x) = \lfloor x \rfloor$, $x \in \mathbb{R}$ assumes the value of the greatest integer, less than or equal to $x$

$D_f = \mathbb{R}$ \hspace{0.5cm} $R_f = \mathbb{Z}$

• $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

$D_f = \mathbb{R}$ \hspace{0.5cm} $R_f = [0, \infty)$

• $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^3$

$D_f = \mathbb{R}$ \hspace{0.5cm} $R_f = \mathbb{R}$
- Exponential function, \( f : \mathbb{R} \to \mathbb{R} ; f(x) = a^x, \ a > 0, \ a \neq 1 \)

\[ D_f = \mathbb{R} \quad R_f = (0, \infty) \]

- Natural exponential function, \( f(x) = e^x \)

\[ e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots, \ 2 < e < 3 \]

- Logarithmic functions, \( f : (0, \infty) \to \mathbb{R} ; f(x) \log_a x, \ a > 0, \ a \neq 1 \)

\[ D_f = (0, \infty) \quad R_f = \mathbb{R} \]

- Natural logarithmic function \( f(x) \log_e x \) or \( \log x \).

- Let \( f : X \to \mathbb{R} \) and \( g : X \to \mathbb{R} \) be any two real functions where \( x \subseteq \mathbb{R} \) then

\[ (f \pm g)(x) = f(x) \pm g(x) \quad \forall \ x \in X \]

\[ (fg)(x) = f(x) \ g(x) \quad \forall \ x \in X \]

\[ \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} \quad \forall \ x \in X \text{ provided } g(x) \neq 0 \]
VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find a and b if \((a - 1, b + 5) = (2, 3)\)

   If \(A = \{1,3,5\}, B = \{2,3\}\) find : (Question-2, 3)

2. \(A \times B\)

3. \(B \times A\)

   Let \(A = \{1,2\}, B = \{2,3,4\}, C = \{4,5\}\), find (Question-4,5)

4. \(A \times (B \cap C)\)

5. \(A \times (B \cup C)\)

6. If \(P = \{1,3\}, Q = \{2,3,5\}\), find the number of relations from \(A\) to \(B\)

7. If \(A = \{1,2,3,5\}\) and \(B = \{4,6,9\}\),
   \[R = \{(x, y) : |x - y| \text{ is odd, } x \in A, y \in B\}\]

   Write \(R\) in roster form

   Which of the following relations are functions. Give reason. (Questions 8 to 10)

8. \(R = \{(1,1), (2,2), (3,3), (4,4), (4,5)\}\)

9. \(R = \{(2,1), (2,2), (2,3), (2,4)\}\)

10. \(R = \{(1,2), (2,5), (3,8), (4,10), (5,12), (6,12)\}\)

   Which of the following arrow diagrams represent a function? Why? (Question-11,12)

11. [Diagram 1]

12. [Diagram 2]
Let \( f \) and \( g \) be two real valued functions, defined by, \( f(x) = x^2 \), \( g(x) = 3x + 2 \), find : (Question 13 to 16)

13. \((f + g)(-2)\)
14. \((f - g)(1)\)
15. \((fg)(-1)\)
16. \(\left(\frac{f}{g}\right)(0)\)

17. If \( f(x) = x^3 \), find the value of, \( \frac{f(5) - f(1)}{5 - 1} \)

18. Find the domain of the real function, \( f(x) = \sqrt{x^2 - 4} \)

19. Find the domain of the function, \( f(x) = \frac{x^2 + 2x + 3}{x^2 - 5x + 6} \)

Find the range of the following functions, (Question- 20,21)

20. \( f(x) = \frac{1}{4 - x^2} \)
21. \( f(x) = x^2 + 2 \)

22. Find the domain of the relation,
\[ R = \{ (x, y) : x, y \in \mathbb{Z}, xy = 4 \} \]

Find the range of the following relations : (Question-23, 24)

23. \( R = \{(a,b) : a, b \in \mathbb{N} \text{ and } 2a + b = 10\} \)
24. \( R = \left\{ \left(x, \frac{1}{x}\right) : x \in \mathbb{Z}, 0 < x < 6 \right\} \)
SHORT ANSWER TYPE QUESTIONS (4 MARKS)

25. Let $A = \{1,2,3,4\}$, $B = \{1,4,9,16,25\}$ and $R$ be a relation defined from $A$ to $B$ as,
   
   \[ R = \{(x, y) : x \in A, y \in B \text{ and } y = x^2\} \]

   (a) Depict this relation using arrow diagram.
   (b) Find domain of $R$.
   (c) Find range of $R$.
   (d) Write co-domain of $R$.

26. Let $R = \{(x, y) : x, y \in \mathbb{N} \text{ and } y = 2x\}$ be a relation on $\mathbb{N}$. Find:
   
   (i) Domain
   (ii) Codomain
   (iii) Range

   Is this relation a function from $\mathbb{N}$ to $\mathbb{N}$?

27. Let $f(x) = \begin{cases} x^2, & \text{when } 0 \leq x \leq 2. \\ 2x, & \text{when } 2 \leq x \leq 5 \end{cases}$
   
   $g(x) = \begin{cases} x^2, & \text{when } 0 \leq x \leq 3. \\ 2x, & \text{when } 3 \leq x \leq 5 \end{cases}$

   Show that $f$ is a function while $g$ is not a function.

28. Find the domain and range of,
   
   $f(x) = |2x - 3| - 3$

29. Draw the graph of the Greatest Integer function

30. Draw the graph of the Constant function, $f : \mathbb{R} \to \mathbb{R}; f(x) = 2 \quad \forall x \in \mathbb{R}$.

   Also find its domain and range.

31. Draw the graph of the function $|x - 2|

   Find the domain and range of the following real functions

   (Question 33 to 38)

32. $f(x) = \sqrt{x^2 + 4}$
33. \( f(x) = \frac{x + 1}{x - 2} \)

34. \( f(x) = \frac{|x + 1|}{x - 1} \)

35. \( f(x) = \frac{x^2 - 9}{x - 3} \)

36. \( f(x) = \frac{4 - x}{x - 4} \)

37. \( f(x) = 1 - |x - 3| \)

**ANSWERS**

1. \( a = 3, \ b = -2 \)

2. \( A \times B = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\} \)

3. \( B \times A = \{(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\} \)

4. \( \{(1,4), (2,4)\} \)

5. \( \{(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)\} \)

6. \( 2^6 = 64 \)

7. \( R = \{(1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6)\} \)

8. Not a function because 4 has two images.

9. Not a function because 2 does not have a unique image.

10. Function because every element in the domain has its unique image.

11. Function because every element in the domain has its unique image.

12. Not a function because 2 is having more than one image 0 and 3.

13. 0

14. -4

15. -1

16. 0
17. 31
18. \((-\infty, -2] \cup [2, \infty)\)

19. \(R - \{2,3\}\)
20. \((-\infty, 0) \cup \left[\frac{1}{4}, \infty\right)\)

21. \([2,\infty)\)
22. \([-4, -2, 1, 2, 4]\)

23. \(\{2,4,6,8\}\)
24. \(\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}\)

25. (a)

(b) \(\{1, 2, 3, 4\}\)
(c) \(\{1, 4, 9, 16\}\)
(d) \(\{1, 4, 9, 16, 25\}\)

26. (i) \(\mathbb{N}\)
(ii) \(\mathbb{N}\)
(iii) Set of even natural numbers

yes, \(R\) is a function from \(\mathbb{N}\) to \(\mathbb{N}\).

28. Domain is \(R\)
Range is \([-3, \infty)\)

30. Domain = \(R\)
Range = \(\{2\}\)

31.
32. Domain = R
   Range   = {2, \infty}
33. Domain = R – {2}
   Range   = R – {1}
34. Domain = R
   Range   = {1, –1}
35. Domain = R – {3}
   Range   = R – {6}
36. Domain = R – 4}
   Range   = {–1}
37. Domain = R
   Range   = \{–x, 1\}
CHAPTER - 3

TRIGONOMETRIC FUNCTIONS

KEY POINTS

- A radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle. We denote 1 radian by $1^c$.

- $\pi$ radian = 180 degree

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian}$$

- If an arc of length $l$ makes an angle $\theta$ radian at the centre of a circle of radius $r$, we have

$$\theta = \frac{l}{r}$$

- Quadrant → I II III IV

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- Function $-x$ $\frac{\pi}{2} - x$ $\frac{\pi}{2} + x$ $\pi - x$ $\pi + x$ $2\pi - x$ $2\pi + x$

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**Some Standard Results**

- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$\cot(x + y) = \frac{\cot x \cdot \cot y - 1}{\cot y + \cot x}$$

- $\sin (x - y) = \sin x \cos y - \cos x \sin y$
- $\cos (x - y) = \cos x \cos y + \sin x \sin y$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$$

$$\cot(x - y) = \frac{\cot x \cdot \cot y + 1}{\cot y - \cot x}$$

- $\tan(x + y + z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y \tan z - \tan z \tan x}$

- $2\sin x \cos y = \sin(x + y) + \sin(x - y)$
- $2\cos x \sin y = \sin(x + y) - \sin(x - y)$
- $2\cos x \cos y = \cos(x + y) + \cos(x - y)$
- $2\sin x \sin y = \cos(x - y) - \cos(x + y)$
\[ \sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2} \]
\[ \sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2} \]
\[ \cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2} \]
\[ \cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2} \]
\[ \sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x} \]
\[ \cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x} \]
\[ \tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \]
\[ \sin 3x = 3 \sin x - 4 \sin^3 x \]
\[ \cos 3x = 4 \cos^3 x - 3 \cos x \]
\[ \tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \]
\[ \sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x \]
\[ \cos(x + y) \cos(x - y) = \cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x \]

- **Principal solutions** – The solutions of a trigonometric equation for which \( 0 \leq x < 2 \pi \) are called its principal solutions.
- **General solution** – A solution of a trigonometric equation, generalised by means of periodicity, is known as the general solution.
General solutions of trigonometric equations:

\[ \sin \theta = 0 \Rightarrow \theta = n \pi, \; n \in \mathbb{Z} \]

\[ \cos \theta = 0 \Rightarrow \theta = (2n + 1)\frac{\pi}{2}, \; n \in \mathbb{Z} \]

\[ \tan \theta = 0 \Rightarrow \theta = n \pi, \; n \in \mathbb{Z} \]

\[ \sin \theta = \sin \alpha \Rightarrow \theta = n \pi + (-1)^n \alpha, \; n \in \mathbb{Z} \]

\[ \cos \theta = \cos \alpha \Rightarrow \theta = 2n \pi \pm \alpha, \; n \in \mathbb{Z} \]

\[ \tan \theta = \tan \alpha \Rightarrow \theta = n \pi + \alpha, \; n \in \mathbb{Z} \]

- Law of sines or sine formula

The lengths of sides of a triangle are proportional to the sines of the angles opposite to them i.e.,

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

- Law of cosines or cosine formula

In any \( \triangle ABC \)

\[ \cos A = \frac{b^2 + c^2 - a^2}{2bc} \]

\[ \cos B = \frac{c^2 + a^2 - b^2}{2ca} \]

\[ \cos C = \frac{a^2 + b^2 - c^2}{2ab} \]

**VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)**

1. Find the radian measure corresponding to
   (i) \( 5^\circ \; 37' \; 30'' \); (ii) \( -37^\circ \; 30' \)

2. Find the degree measure corresponding to (i) \( \left( \frac{11}{16} \right)^c \); (ii) \( -4^c \)
3. (i) Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°.
   (ii) Find each interior angle of a regular decagon in radian.

4. Find the value of $\tan \frac{19\pi}{3}$

5. Find the value of (i) $\sin(-1125^\circ)$; (ii) $\cos(-2070^\circ)$

6. Find the value of (i) $\tan 15^\circ$; (ii) $\sin 75^\circ$

7. If $\sin A = \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$, find $\cos A$

8. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$ then find the value of $A + B$.

9. Express $\sin 12\theta + \sin 4\theta$ as the product of sines and cosines.

10. Express $2 \cos 4x \sin 2x$ as an algebraic sum of sines or cosines.

11. Write the range of $\cos \theta$

12. What is domain of $\sec \theta$?

13. Find the principal solutions of (i) $\cosec x = -1$; (ii) $\cos x = 1$.

14. Write the general solution of $\sin \left( x + \frac{\pi}{12} \right) = 0$

15. If $\sin x = \frac{\sqrt{5}}{3}$ and $0 < x < \frac{\pi}{2}$ find the value of $\cos 2x$

16. If $\cos x = \frac{-1}{3}$ and $x$ lies in quadrant III, find the value of $\sin \frac{x}{2}$

17. Evaluate: $2 \sin 75^\circ \sin 15^\circ$

18. If $\tan \alpha = \frac{1}{7}$, then find $\cos 2\alpha$.

19. Evaluate: $\sin (\pi + x) \sin (\pi - x) \cosec^2 x$

20. What is sign of $\cos \frac{x}{2} - \sin \frac{x}{2}$ when
   (i) $0 < x < \pi/2$   (ii) $0 < x < \pi$
21. What is maximum value of \(3 - 7\cos 5x\) ?

22. Find the range of \(f(x) = \sin \pi x\)

22A. In any \(\Delta ABC\), if \(a = 2\), \(b = 3\) and \(\sin A = \frac{2}{3}\), find \(\angle B\).

22B. Find-the angle between hour hand and minute hand of a clock at quarter to five.

**SHORT ANSWER TYPE QUESTIONS (4 MARKS)**

23. A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces 72° at the centre, find the length of the rope.

24. If the angles of a triangle are in the ratio 3:4:5, find the smallest angle in degrees and the greatest angle in radians.

25. If \(\sin x = \frac{12}{13}\) and \(x\) lies in the second quadrant, show that \(\sec x + \tan x = -\frac{5}{2}\).

26. If \(\sec x = \sqrt{2}\) and \(\frac{3\pi}{2} < x < 2\pi\) find the value of \(\frac{1 + \tan x + \csc x}{1 + \cot x - \csc x}\).

27. Prove that \(\tan 50^\circ = \tan 40^\circ + 2 \tan 10^\circ\)

28. If \(\cot \alpha = \frac{1}{2}\), \(\sec \beta = -\frac{5}{3}\) where \(\pi < \alpha < \frac{3\pi}{2}\) and \(\frac{\pi}{2} < \beta < \pi\), find the value of \(\tan (\alpha + \beta)\)

29. \(\tan 13x = \tan 4x + \tan 9x + \tan 4x \tan 9x \tan 13x\)

Prove the following Identities

30. \(\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4 \cos 2\theta \cos 4\theta\)

31. \(\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2 \tan 2x\)

32. \(\frac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x \sin x + \cos 6x \cos x} = \tan 2x\)

33. \(\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \tan \frac{\theta}{2}\)
34. \( \tan \alpha \cdot \tan(60^\circ - \alpha) \cdot \tan(60^\circ + \alpha) = \tan 3\alpha \)

35. Show that
   
   (i) \( \cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8} \)
   
   (ii) \( \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16} \)

36. Show that \( \sqrt{2 + \sqrt{2 + 2 \cos 4\theta}} = 2 \cos \theta \)

37. Prove that \( \frac{\cos x}{1 - \sin x} = \tan \left( \frac{\pi}{4} + \frac{x}{2} \right) \)

38. \( \cos 10^\circ + \cos 110^\circ + \cos 130^\circ = 0 \)

39. \( \frac{\sin(x + y) - 2 \sin x + \sin(x - y)}{\cos(x + y) - 2 \cos x + \cos(x - y)} = \tan x \)

40. \( \sin x + \sin 2x + \sin 4x + \sin 5x = 4 \cos \frac{x}{2} \cos \frac{3x}{2} \sin(3x) \).

41. \( \frac{\sec 8x - 1}{\sec 4x - 1} = \frac{\tan 8x}{\tan 2x} \).

42. Evaluate (i) \( \cos 36^\circ \) (ii) \( \frac{\pi}{8} \) (iii) \( \tan \frac{13\pi}{12} \)

43. Draw the graph of \( \cos x \) in \([0, 2\pi]\).

Find the general solution of the following equations (Q.No. 44 to Q. No. 49)

44. If \( \frac{\sin(x + y)}{\sin(x - y)} = \frac{a + b}{a - b} \)

   then prove that \( \frac{\tan x}{\tan y} = \frac{a}{b} \)

   [Hint : By applying Companando and Dividendo
   \[ \frac{A}{B} = \frac{C}{D} \Rightarrow \frac{A + B}{A - B} = \frac{C + D}{C - D} \]

45. (i) \( \sin 7x = \sin 3x \) (ii) \( \cos 3x - \sin 2x = 0 \)
46. \(3 \cos x - \sin x = 1\).
47. \(3 \tan x + \cot x = 5 \csc x\).
48. \(\tan x + \tan 2x + \sqrt{3} \tan x \tan 2x = \sqrt{3}\).
49. \(\tan x + \sec x = \sqrt{3}\).
50. In any triangle ABC, prove that
   
   \[a \sin B - \sin C + b \sin C - \sin A + c \sin A - \sin B = 0.\]
51. In any triangle ABC, prove that (Q. 51 to Q. 55)
   
   \[(a - b)^2 \cos^2 \frac{C}{2} + (a + b)^2 \sin^2 \frac{C}{2} = c^2.\]
52. \((b^2 - c^2) \cot A + (c^2 - a^2) \cot B + (a^2 - b^2) \cot C = 0.\)
53. (i) \(a = b \cos C + c \cos B\)
    (ii) \(\frac{c}{a - b} = \frac{\tan A/2 + \tan B/2}{\tan A/2 - \tan B/2}\)
54. (i) \(\frac{a + b}{c} = \frac{\cos \frac{A - B}{2}}{\sin \frac{C}{2}}.\)
    (ii) \(\frac{a^2 + b^2}{a^2 + c^2} = \frac{1 + \tan (A - B) \cos C}{1 + \cos (A - C) \cos B}\)
55. (i) If \(\cos A = \frac{\sin B}{2 \sin C}\) then prove that the triangle is isosceles.
    (ii) If a \(\cos A = b \cos B\) then prove that the triangle is either isosceles or right angled

**LONG ANSWER TYPE QUESTIONS (6 MARKS)**

56. Prove that
   
   \(\cos A \cos 2A \cos 4A \cos 8A = \frac{\sin 16A}{16 \sin A}\)
57. Prove that \(\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}\)
58. Find the general solution of
    (i) \(\sin 2x + \sin 4x + \sin 6x = 0\)
    (ii) \(4 \sin x \sin 2x \sin 4x = \sin 3x\)
59. Find the general solution of
   
   \(\cos \theta \cos 2\theta \cos 3\theta = \frac{1}{4}\)
60. Draw the graph of \( \tan x \) in \( \left[ -\frac{3\pi}{2}, \frac{3\pi}{2} \right] \)

61. In any triangle \( \triangle ABC \), prove that
\[
\frac{b^2 - c^2}{a^2} \sin 2A + \frac{c^2 - a^2}{b^2} \sin 2B + \frac{(a^2 - b^2)}{c^2} \sin 2C = 0
\]

62. Prove that
\[
4 \sin \alpha \sin \left( \alpha + \frac{\pi}{3} \right) \sin \left( \alpha + \frac{2\pi}{3} \right) = \sin 3\alpha
\]

63*. Prove that
\[
\sin^3 x + \sin^3 \left( \frac{2\pi}{3} + x \right) + \sin^3 \left( \frac{4\pi}{3} + x \right) = -\frac{3}{4} \sin 3x
\]

64. In any \( \triangle ABC \) prove that
\[
(c^2 + b^2 - a^2) \tan A = (a^2 + c^2 - b^2) \tan B = (a^2 + b^2 - c^2) \tan C
\]

65. In any \( \triangle ABC \) prove that
\[
\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c} \quad \text{if} \quad \angle C = 60^\circ
\]

66. If \( \alpha \) and \( \beta \) are distinct roots of
\[
\alpha \cos \theta + b \sin \theta = c \quad \text{prove that} \quad \sin (\alpha + \beta) = \frac{2ab}{a^2 + b^2}
\]

ANSWERS

1. (i) \( \left( \frac{\pi}{32} \right)^c \); (ii) \( \left( \frac{5\pi}{24} \right)^c \)
2. (i) \( 39^\circ 22'30'' \); (ii) \( -229^\circ 5'27'' \)

3. (i) \( \frac{5\pi}{12} \text{ cm} \); (ii) \( \frac{4\pi}{5} \)
4. \( \sqrt{3} \)

5. (i) \( \frac{-1}{\sqrt{2}} \); (ii) \( 0 \)
6. (i) \( 2 - \sqrt{3} \); (ii) \( \frac{\sqrt{6} - \sqrt{2}}{4} \)

7. \( \frac{-4}{5} \)
8. \( 45^\circ \)
9. $2 \sin 8\theta \cos 4\theta$

10. $\sin 6x - \sin 2x$

11. $[-1, 1]$

12. $R - \left\{ (2n + 1) \frac{\pi}{2}, n \in \mathbb{Z} \right\}$

13. (i) $\frac{3\pi}{2}$; (ii) $0, \pi$

14. $n\pi - \frac{\pi}{12}$

15. $-\frac{1}{9}$

16. $\frac{\sqrt{6}}{3}$

17. $\frac{1}{2}$

18. $\frac{24}{25}$

19. $-1$

20. (i) Positive; (ii) Negative

21. $10$

22. $[-1, 1]$

22A. $\frac{\pi}{2}$

22B. $127.5^\circ$

23. $70 \text{ m}$

24. $45^\circ, \frac{5\pi}{12} \text{ radians}$

26. $-1$

28. $\frac{2}{11}$

42. (i) $\frac{1+\sqrt{5}}{4}$ (ii) $\sqrt{2} - 1$ (iii) $2 - \sqrt{3}$

45. (i) $(2n + 1) \frac{\pi}{10}, \frac{n\pi}{2}, n \in \mathbb{Z}$ (ii) $\frac{1}{5} \left( \frac{2n\pi + \frac{\pi}{2}}{2} \right), 2n\pi - \frac{\pi}{2}, n \in \mathbb{Z}$

46. $2n\pi \pm \frac{\pi}{3} - \frac{\pi}{6}, n \in \mathbb{Z}$

47. $2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$

48. $\frac{n\pi}{3} + \frac{\pi}{9}$

49. $2n\pi + \frac{\pi}{6}$

58. (i) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$ (ii) $n\pi, m\pi \pm \frac{\pi}{3}, m, n \in \mathbb{Z}$

59. $(2n + 1) \frac{\pi}{8}, n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$
CHAPTER - 4

PRINCIPLE OF MATHEMATICAL INDUCTION

KEY POINTS

- Induction and deduction are two basic processes of reasoning.
- Deduction is the application of a general case to a particular case. In contrast to deduction, induction is process of reasoning from particular to general.
- Principle of Mathematical Induction:
  Let P(n) be any statement involving natural number n such that
  (i) P(1) is true, and
  (ii) If P(k) is true implies that P(k +1) is also true for some natural number k
  then P(n) is true ∀ n ∈ N

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Using the principle of mathematical induction prove the following for all n ∈ N:

1. If $P(n) : 1 + 4 + 7 + .... + (3n - 2) = \frac{1}{2}n(3n-1)$
   Verify $P(n)$ for $n = 1, 2, 10$

2. Given $P(n) : 1 + 2 + 3 + ...... + n < \frac{1}{8}(2n+1)^2$
   Verify $P(n)$ for $n = 1, 2$

3. $P(n) = 3^{2n} + 2 - 8n - 9$ is a multiple of 64
   Verify $P(n)$ for $n = 1$ and 2.

4. $3.6 + 6.9 + 9.12 + ......... + 3n (3n + 3) = 3n(n + 1)(n + 2)$

5. $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) - \cdots \left(1 - \frac{1}{n+1}\right) = \frac{1}{n+1}$
6. $n^2 + n$ is an even natural number.
7. $2^{3n} - 1$ is divisible by 7.
8. $3^{2n}$ when divided by 8 leaves the remainder 1.
9. $4^n + 15n - 1$ is divisible by 9.
10. $n^3 + (n + 1)^3 + (n + 2)^3$ is a multiple of 9.
11. $x^{2n - 1} - 1$ is divisible by $x - 1$, $x \neq 1$.
12. $3^n > n$
13. If $x$ and $y$ are any two distinct integers then $x^n - y^n$ is divisible by $(x - y)$.
14. $n < 2^n$
15. $a + (a + d) + (a + 2d) + \ldots + [a + (n - 1)d] = \frac{n}{2}[2a + (n - 1)d]$.
16. $3x + 6x + 9x + \ldots$ to $n$ terms $= \frac{3}{2}n(n + 1)x$.
17. $11^{2n+2} + 12^{2n+1}$ is divisible by 133.
18. Using induction prove that
   $$\sin x + \sin 2x + \sin 3x + \ldots + \sin nx = \frac{\sin \left( \frac{n+1}{2} \right)x \sin \frac{nx}{2}}{\sin \frac{x}{2}}$$
19. Using PMI, prove
   $$7 + 77 + 777 + \ldots + \text{to } n \text{ terms} = \frac{7}{81}(10^{n+1} - 9n - 10) \forall n \in N.$$
20. Using PMI prove that
   $$\sin n + \sin 3x + \ldots + \sin (2n - 1)x = \frac{\sin^2 nx}{\sin x}, \forall n \in N.$$
21. Using PMI, prove that
   $$\cos \alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \ldots \cos(2^{n-1}\alpha) = \frac{\sin^2 n\alpha}{2^n \sin \alpha}, \forall n \in N$$
22. $\forall n \geq 2$, $n \in N$ prove that
   $$1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} < 2 - \frac{1}{n}$$
CHAPTER - 5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

KEY POINTS

- The imaginary number $\sqrt{-1} = i$, is called iota
- For any integer $k$, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ if both $a$ and $b$ are negative real numbers
- A number of the form $z = a + ib$, where $a, b \in \mathbb{R}$ is called a complex number.
  
  $a$ is called the real part of $z$, denoted by $\text{Re}(z)$ and $b$ is called the imaginary part of $z$, denoted by $\text{Im}(z)$
- $a + ib = c + id$ if $a = c$, and $b = d$
- $z_1 = a + ib$, $z_2 = c + id$.
  
  In general, we cannot compare and say that $z_1 > z_2$ or $z_1 < z_2$
  
  but if $b, d = 0$ and $a > c$ then $z_1 > z_2$
  
  i.e. we can compare two complex numbers only if they are purely real.
- $0 + i0$ is additive identity of a complex number.
- $-z = -a + i(-b)$ is called the Additive Inverse or negative of $z = a + ib$
- $1 + i0$ is multiplicative identity of complex number.
- $\bar{z} = a - ib$ is called the conjugate of $z = a + ib$
\[ z^{-1} = \frac{1}{z} = \frac{a - ib}{a^2 + b^2} = \frac{\overline{z}}{|z|^2} \]  

is called the multiplicative inverse of \( z = a + ib \) (\( a \neq 0, b \neq 0 \)).

- The coordinate plane that represents the complex numbers is called the complex plane or the Argand plane.
- Polar form of \( z = a + ib \) is, 
  \[ z = r (\cos \theta + i \sin \theta) \]  
  where \( r = \sqrt{a^2 + b^2} = |z| \) is called the modulus of \( z \), 
  \( \theta \) is called the argument or amplitude of \( z \).
- The value of \( \theta \) such that, \(-\pi < \theta \leq \pi\) is called the principle argument of \( z \).
- \(|z_1 + z_2| \leq |z_1| + |z_2|\)
- \(|z_1z_2| = |z_1| \cdot |z_2|\)
- \( \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}, \quad |z^n| = |z|^n, \quad |z| = |\overline{z}| = |-z| = |\overline{\overline{z}}|, \quad z \overline{z} = |z|^2\)
- \(|z_1 + z_2| \leq |z_1| + |z_2|\)
- \(|z_1 - z_2| \geq ||z_1| - |z_2||\)
- If \( z_1 = r_1 (\cos \theta_1 + i \sin \theta_1) \)  
  \( z_2 = r_2 (\cos \theta_2 + i \sin \theta_2) \)  
  then \( z_1z_2 = r_1 r_2 [\cos (\theta_1 + \theta_2) + i \sin (\theta_1 + \theta_2)] \)
  
  \[ \frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)] \]
- For the quadratic equation \( ax^2 + bx + c = 0 \), \( a, b, c \in \mathbb{R}, a \neq 0 \),  
  if \( b^2 - 4ac < 0 \) then it will have complex roots given by,  
  \[ x = \frac{-b \pm i\sqrt{4ac - b^2}}{2a} \]
\[ \sqrt{a + ib} \] is called square root of \( z = a + ib \)

\[ \sqrt{a + ib} = x + iy \]

Squaring both sides we get

\[ a + ib = x^2 - y^2 + 2i(xy) \]

\[ x^2 - y^2 = a, \quad 2xy = b. \] Solving these we get \( x \) and \( y \).

**VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)**

1. Evaluate:
   (i) \( \sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625} \)
   (ii) \( i\sqrt{-16} + i\sqrt{-25} + \sqrt{49} - i\sqrt{49} + 14 \)
   (iii) \( (\sqrt{-1})^{31} \)
   (iv) \( i^{35} + \frac{1}{i^{35}} \)
   (v) \( i^5 + i^{10} + i^{15} \)
   (vi) \( (\sqrt{3} + \sqrt{2}) (2\sqrt{3} - i) \)
   (vii) \( \frac{(3 + \sqrt{5}i) (3 - \sqrt{5}i)}{(-3 + \sqrt{2}i) - (\sqrt{3} - \sqrt{2}i)} \)

2. Find \( x \) and \( y \) if \( 3x + (2y - 3) i \) is equal to \( 2 + 4i \)

3. Find additive inverse of \( 6i - i\sqrt{49} \)

4. Find multiplicative inverse of \( 2 + i \)

5. Find modulus of \( \sqrt{-25} + 7 \)

6. If \( z_1 = 2 + 4i, z_2 = 3 - 5i \) then find
   (i) \( R_e (z_1z_2) \)
   (ii) \( I_m (z_1z_2) \)
   (iii) \( R_e (z_1 - z_2) \)
   (iv) \( I_m (z_1 + z_2) \)

7. Express \( \frac{1}{1+i} \) in the form of \( a + ib \).

8. If modulus of \( z \) is 2 and argument of \( z \) is \( \frac{5\pi}{6} \) then write \( z \) in form of \( a + ib \).
9. If \( z_1 = \sqrt{2}(\cos 30° + i \sin 30°) \), \( z_2 = \sqrt{3}(\cos 60° + i \sin 30°) \)
   Find \( R_e (z_1z_2) \)

10. Find the value of \((-\sqrt{-1})^{4n}\) \( n \) is any positive integer.

11. Find conjugate of \( i^7 \)

12. Find the solution of equation \( x^2 + 3 = 0 \) in complex numbers.

13. Represent \( \sin \frac{\pi}{3} + i \cos \frac{\pi}{3} \) in polar form

14. Represent in \((a + ib) 3(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})\)

15. If \( z_1 = \cos 30° + i \sin 30° \)
   \( z_2 = \cos 60° + i \sin 60° \)
   then find (i) \( \left| \frac{z_1}{z_2} \right| \); (ii) \( |z_1z_2| \)

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

16. For Complex numbers \( z_1 = -1 + i \), \( z_2 = 3 - 2i \)
   show that,
   \[ \text{Im} (z_1z_2) = \text{Re} (z_1) \text{Im}(z_2) + \text{Im} (z_1) \text{Re} (z_2) \]

17. If \( x + iy = \frac{1+i}{1-i} \), prove that \( x^2 + y^2 = 1 \)

18. Find real value of \( \theta \) such that,
   \[ \frac{1 + i \cos \theta}{1 - 2i \cos \theta} \] is a real number

19. If \( \left| \frac{z - 5i}{z + 5i} \right| = 1 \), show that \( z \) is a real number.

20. Find the value of \( x \) and \( y \) if
   (i) \( x^2 - 7x + 9yi = y^2 i + 20i - 12 \)
   (ii) \( \frac{x - 1}{3 + i} + \frac{y - 1}{3 - i} = i \)
   (iii) \( (x^4 + 2xi) - (3x^2 + iy) = (3 - 5i) + (1 + 2i) \)
21. Express in the form of \( a + ib \)
\[
\frac{(1+i)^3}{4+3i}
\]
22. Convert the following in polar form:

(i) \(-3\sqrt{2} + 3\sqrt{2}i\)  
(ii) \(\frac{(\sqrt{5}-1) - (\sqrt{5}+1)i}{2\sqrt{2}}\)

(iii) \(i(1+i)\)  
(iv) \(\frac{5-i}{2-3i}\)

23. If \((x + iy)^3 = a + ib\), prove that, 
\[
\left(\frac{x}{a} + \frac{y}{b}\right) = 4(a^2 - b^2)
\]
24. For complex numbers \(z_1 = 6 + 3i\), \(z_2 = 3 - i\) find \(\frac{z_1}{z_2}\)
25. If \(\left(\frac{2 + 2i}{2 - 2i}\right)^n = 1\), find the least positive integral value of \(n\).
26. Solve

(i) \(ix^2 + 4x - 4i = 0\)  
(ii) \(x^2 - (2 + i)x - (1 - 7i) = 0\)

(iii) \(2x^2 + 3ix + 2 = 0\)  
(iv) \(x^2 - 2x + \frac{3}{z} = 0\)

27. Find square root of the following complex number

(i) \(-7 + 24i\)  
(ii) \(-3 - 4i\)

(iii) \(-15 - 8i\)  
(iv) \(7 - 30\sqrt{2}\)

28. If \(z_1, z_2\) are complex numbers such that \(\frac{2z_1}{3z_2}\) is purely imaginary number

then find \(\left|\frac{z_1 - z_2}{z_1 + z_2}\right|\).

29. Prove that \(x^4 + 4 = (x + 1 + i)(x + 1 - i)(x - 1 + i)(x - 1 - i)\).

30. If \(z = x + iy\) and \(w = \frac{1 - 2i}{z - i}\) show that \(|w| = 1 \Rightarrow z\) is purely real.
31. If \( z = 2 - 3i \) show that \( z^2 - 4z + 13 = 0 \), hence find the value of 
\( 4z^3 - 3z^2 + 169 \).

32. Find all non-zero complex number \( z \) satisfying \( \overline{z} = iz^2 \).

33. If \( iz^3 + z^2 - z + i = 0 \) then show that \( |z| = 1 \).

**LONG ANSWER TYPE QUESTIONS (6 MARKS)**

34. If \( z_1, z_2 \) are complex numbers such that, 
\[
\left| \frac{z_1 - 3z_2}{3 - z_1z_2} \right| = 1 \text{ and } |z_2| \neq 1
\]
then find \( |z_1| \)

35. If \( x = -1 + i \) then find the value of \( x^4 + 4x^3 + 4x^2 + 2 \)

*36. If \( z = x + iy \) and \( w = \frac{1 - iz}{z - i} \) show that if \( |w| = 1 \) then \( z \) is purely real.

*37. Prove by Principle of Mathematical Induction
\[
(cos \ \theta + i \ sin \ \theta)^n = cos \ n\theta + i \ sin \ n\theta
\]
if \( n \) is any natural number.

38. Solve \( x^2 - (7 - i) x + 18 - i = 0 \)

39. Solve \( 2x^2 - (3 + 7i) x - (3 - 9i) = 0 \)

**ANSWERS**

1. (i) 0; (ii) 19; (iii) 1; (iv) 0; (v) -1;

   (vi) -10 - 198i  (viii) \( \frac{7 \ i}{\sqrt{2}} \)

2. \( x = \frac{2}{3}, y = \frac{7}{2} \)

3. -7 - 6i

4. \( \frac{2}{3} - \frac{i}{3} \)

5. \( \sqrt{74} \)

6. (i) 26; (ii) 2; (iii) -1; (iv) -1
7. \( \frac{1}{2} \quad \frac{i}{2} \)  
8. \(-\sqrt{3} + 1i\)  
9. 0  
10. 1  
11. \(i\)  
12. \(\pm \sqrt{3}i\)  
13. \(2\left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right)\)  
14. \(-3i\)  
15. (i) 1; (ii) 1;  
18. \(\theta = (2n + 1)\frac{\pi}{2}\)  
20. (i) \(x = 4, 3; \quad y = 5, 4\)  
(ii) \(x = -4, y = 6\)  
(iii) \(x = 2\) and \(y = 3\) or \(x = -2\) and \(y = 1/3\)  
21. \(-2\quad \frac{14}{25}i\)  
22. (i) \(6\left(\cos\frac{3\pi}{4} + i \sin\frac{3\pi}{4}\right)\); (ii) \(1\left[\cos\left(-\frac{5\pi}{12}\right) + i \sin\left(-\frac{5\pi}{12}\right)\right]\);  
(iii) \(\sqrt{2}\left[\cos\frac{3\pi}{4} + i \sin\frac{3\pi}{4}\right]\); (iv) \(\sqrt{2}\left[\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right]\)  
24. \(\frac{z_{1}}{z_{2}} = \frac{3(1+i)}{2}\)  
25. \(n = 4\)  
26. (i) \(2i, 2i\)  
(ii) \(3 - i, -1 + 2i\)  
(iii) \(-\frac{1}{2}i, -2i\)  
(iv) \(\frac{2 \pm \sqrt{2}i}{2}\)  
27. (i) \(\pm (3 + 4i)\)  
(ii) \(\pm(1 - 2i)\)  
(iii) \(\pm (1 - 4i)\)  
(iv) \(\pm (5 - 3\sqrt{2}i)\)  
28. 1  
31. 0  
32. \(z = 0, i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i\)  
34. \(|z_{1}| = 3\)  
35. 6  
38. \(4 - 3i\) and \(3 + 2i\)  
39. \(\frac{3}{2} + \frac{1}{2}i\) and \(3i\)
CHAPTER - 6

LINEAR INEQUALITIES

KEY POINTS

- The inequality containing < or > is called strict inequality.
- The inequality containing ≤ or ≥ is called slack inequality.
- Two real numbers or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' form an inequality.
- The inequalities of the form $ax + b > 0$, $ax + b < 0$, $ax + b ≥ 0$, $ax + b ≤ 0$; $a ≠ 0$ are called linear inequalities in one variable $x$.
- The inequalities of the form $ax + by + c > 0$, $ax + by + c < 0$, $ax + by + c ≥ 0$, $ax + by + c ≤ 0$, $a ≠ 0$, $b ≠ 0$ are called linear inequalities in two variables $x$ and $y$.
- Rules for solving inequalities:
  (i) $a ≥ b$ then $a ± k ≥ b ± k$
    where $k$ is any real number.
  (ii) but if $a ≥ b$ then $ka$ is not always $≥ kb$.
    If $k > 0$ (i.e. positive) then $a ≥ b ⇒ ka ≥ kb$
    If $k < 0$ (i.e. negative) then $a ≥ b ⇒ ka ≤ kb$
- Solution Set: A solution of an inequality is a number which when substituted for the variable, makes the inequality true. The set of all solutions of an inequality is called the solution set of the inequality.
- The graph of the inequality $ax + by > c$ is one of the half planes and is called the solution region.
- When the inequality involves the sign $≤$ or $≥$ then the points on the line are included in the solution region but if it has the sign $<$ or $>$ then the points on the line are not included in the solution region and it has to be drawn as a dotted line.
VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Solve $5x < 24$ when $x \in \mathbb{N}$
2. Solve $3x < 11$ when $x \in \mathbb{Z}$
3. Solve $3 - 2x < 9$ when $x \in \mathbb{R}$
4. Show the graph of the solution of $2x - 3 > x - 5$ on number line.
5. Solve $5x - 8 \geq 8$ graphically
6. Solve $\frac{1}{x - 2} \leq 0$
7. Solve $0 < \frac{-x}{3} < 1$

Write the solution in the form of intervals for $x \in \mathbb{R}$ for Questions 8 to 10

8. $\frac{2}{x - 3} < 0$
9. $-3 \leq -3x + 2 < 4$
10. $3 + 2x > -4 - 3x$
11. Draw the graph of the solution set of $x + y \geq 4$.
12. Draw the graph of the solution set of $x \leq y$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Solve the inequalities for real $x$

13. $\frac{2x - 3}{4} + 9 \geq 3 + \frac{4x}{3}$
14. $\frac{2x + 3}{4} - 3 < \frac{x - 4}{3} - 2$
15. $-5 \leq \frac{2 - 3x}{4} \leq 9$
16. $|3x - 2| \leq \frac{1}{2}$
17. $|4 - x| + 1 < 3$
18. $\frac{3}{x - 2} < 1$
19. $\frac{x}{x - 5} > \frac{1}{2}$
20. $\frac{2x + 4}{x - 1} \geq 5$
21. Solve $\frac{|x| - 1}{|x| - 2} \geq 0$, $x \in \mathbb{R}$, $x \neq \pm 2$
22. $3x - 7 > 2(x - 6)$, $6 - x > 11 - 2x$
23. The water acidity in a pool is considered normal when the average PH reading of three daily measurements is between 7.2 and 7.8. If the first two PH readings are 7.48 and 7.85, find the range of PH value for the third reading that will result in the acidity level being normal.
24. While drilling a hole in the earth, it was found that the temperature (T °C) at x km below the surface of the earth was given by $T = 30 + 25(x - 3)$, when $3 \leq x \leq 15$.
   Between which depths will the temperature be between 200°C and 300°C?
25. Solve for real x, $|x + 1| + |x| > 3$
   Solve the following systems of inequalities graphically: (Questions 26, 27)
26. $x + y > 6$, $2x - y > 0$
27. $3x + 4y \leq 60$, $x + 3y \leq 30$, $x \geq 0$, $y \geq 0$

**LONG ANSWER TYPE QUESTIONS (6 MARKS)**

Solve the system of inequalities for real x

28. $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$ and
\[
\frac{2x - 1}{12} - \frac{x - 1}{3} < \frac{3x + 1}{4}
\]

Solve the following system of inequalities graphically (Questions 28 to 30)

29. \[3x + 2y \leq 24, \ x + 2y \leq 16, \ x + y \leq 10, \ x \geq 0, \ y \geq 0\]
30. \[2x + y \geq 4, \ x + y \leq 3, \ 2x - 3y \leq 6\]
31. \[x + 2y \leq 2000, \ x + y \leq 1500, \ y \leq 600, \ x \geq 0, \ y \geq 0\]
32. \[3y - 2x < 4, \ x + 3y > 3 \text{ and } x + y \leq 5\]

ANSWERS

1. \(\{1,2,3,4\}\) 2. \(\{\ldots, -2, -1, 0, 1, 2, 3\}\)
3. \(x > -3\) 6. \(x < 2\)
7. \(-3 < x < 0\) 8. \((-\infty, 3)\)
9. \(\left[\frac{-2}{3}, \frac{5}{3}\right]\) 10. \(\left(-\frac{7}{5}, \infty\right)\)

11. \(X, Y, Y', O, X', (0, 4), (4, 0)\) 12. \(X, Y, Y', O, X', (1, 1), (2, 2)\)
13. \(\left(-\infty, \frac{63}{10}\right]\) 14. \(\left(-\infty, -\frac{13}{2}\right]\)
15. \(\left[-\frac{34}{3}, \frac{22}{3}\right]\) 16. \(\left[\frac{1}{2}, \frac{5}{6}\right]\)
17. \(\left(2, 6\right)\) 18. \(\left(-\infty, 2\right) \cup (5, \infty)\)
19. \(\left(-\infty, -5\right) \cup (5, \infty)\) 20. \(\left(-\infty, 2\right) \cup (7, \infty)\)
21. \([-1, 1] \cup (-\infty, -2) \cup (2, \infty)\]
22. \((5, \infty)\)
23. Between 6.27 and 8.07
24. Between 9.8 m and 13.8 m
25. \((-\infty, -2) \cup (1, \infty)\)
28. \((3, \infty)\)
CHAPTER - 7

PERMUTATIONS AND COMBINATIONS

KEY POINTS

• Multiplication Principle (Fundamental Principle of Counting): If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of different ways of occurrence of the two events in order is \( m \times n \).

• Fundamental Principle of Addition: If there are two events such that they can occur independently in m and n ways respectively, then either of the two events can occur in \( m + n \) ways.

• Factorial: Factorial of a natural number n, denoted by n! or \( n! \) is the continued product of first n natural numbers.

\[ n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1 \]

• Permutation: A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

• The number of permutation of n different objects taken r at a time where \( 0 \leq r \leq n \) and the objects do not repeat is denoted by \( ^nP_r \) or \( P(n, r) \)

\[ ^nP_r = \frac{n!}{(n-r)!} \]

• The number of permutations of n objects, taken r at a time, when repetition of objects is allowed is \( n^r \).

• The number of permutations of n objects of which \( p_1 \) are of one kind, \( p_2 \) are of second kind, .. \( p_k \) are of \( k^{th} \) kind and the rest if any, are of different kind, is

\[ \frac{n!}{p_1!p_2!\ldots p_k!} \]

• Combination: Each of the different selections made by choosing some
or all of a number of objects, without considering their order is called a combination. The number of combination of n object taken r at a time where \(0 \leq r \leq n\), is denoted by \(\binom{n}{r}\) or \(C(n, r)\) or \(^nC_r\) where \(\binom{n}{r} = \frac{n!}{r!(n-r)!}\).

Some important results:

- \(0! = 1\)
- \(\binom{n}{0} = \binom{n}{n} = 1\)
- \(\binom{n}{r} = \binom{n}{n-r}\) where \(0 \leq r \leq n\), and \(r\) are positive integers
- \(\binom{n}{r} = \frac{n!}{r!(n-r)!}\) where \(0 \leq r \leq n\), \(n\) and \(r\) are positive integers.
- \(\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}\) where \(r \leq n\), \(n\), \(r\) \(\in\) \(N\)
- \(\binom{n}{a} = \binom{n}{b}\) if either \(a + b = n\) or \(a = b\)

**VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)**

1. Using the digits 1, 2, 3, 4, 5 how many 3 digit numbers (without repeating the digits) can be made?
2. In how many ways 7 pictures can be hanged on 9 pegs?
3. Ten buses are plying between two places A and B. In how many ways a person can travel from A to B and come back?
4. There are 10 points on a circle. By joining them how many chords can be drawn?
5. There are 10 non collinear points in a plane. By joining them how many triangles can be made?
6. If \(\frac{1}{6!} + \frac{1}{8!} = \frac{x}{9!}\) find \(x\)
7. If \(\binom{n}{4} : \binom{n}{2} = 12\), find \(n\).
8. How many different words (with or without meaning) can be made using all the vowels at a time?
9. Using 1, 2, 3, 4, 5 how many numbers greater than 10000 can be made? (Repetition not allowed)

10. If \(^nC_{12} = ^nC_{13}\) then find the value of \(^{25}C_n\).

11. In how many ways 4 boys can be choosen from 7 boys to make a committee?

12. How many different words can be formed by using all the letters of word SCHOOL?

13. In how many ways can the letters of the word PENCIL be arranged so that I is always next to L.

14. In an examination there are there multiple choice questions and each question has 4 choices. Find the number of ways in which a student can fail to get all answer correct.

15. A gentleman has 6 friends to invite. In how many ways can he send invitation cards to them if he has three servants to carry the cards?

16. If there are 12 persons in a party, and if each two of them Shake hands with each other, how many handshakes happen in the party?

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

17. In how many ways 12 boys can be seated on 10 chairs in a row so that two particular boys always take seat?

18. In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together?

19. From a group of 7 boys and 5 girls, a team consisting of 4 boys and 2 girls is to be made. In how many different ways it can be done?

20. In how many ways can one select a cricket team of eleven players from 17 players in which only 6 players can bowl and exactly 5 bowlers are to be included in the team?

21. A student has to answer 10 questions, choosing atleast 4 from each of part A and B. If there are 6 questions in part A and 7 in part B. In how many ways can the student choose 10 questions?
22. Using the digits 0, 1, 2, 2, 3 how many numbers greater than 20000 can be made?

23. If the letters of the word ‘PRANAV’ are arranged as in dictionary in all possible ways, then what will be 182nd word.

24. From a class of 15 students, 10 are to choosen for a picnic. There are two students who decide that either both will join or none of them will join. In how many ways can the picnic be organized?

25. Using the letters of the word, ‘ARRANGEMENT’ how many different words (using all letters at a time) can be made such that both A, both E, both R and both N occur together.

26. A polygon has 35 diagonal. Find the number of its sides.
   [Hint : Number of diagonal of n sided polygon is given by \( ^nC_2 - n \)]

27. How many different products can be obtained by multiplying two or more of the numbers 2, 5, 6, 7, 9?

28. Determine the number of 5 cards combinations out of a pack of 52 cards if atleast 3 out of 5 cards are ace cards?

29. How many words can be formed from the letters of the word ‘ORDINATE’ so that vowels occupy odd places?

30. Find the number of all possible arrangements of the letters of the word “MATHEMATICS” taken four at a time.

31. Prove that 33! in divisible by \( 2^{15} \). what is the largest integer n such that 33! is divisible by \( 2^n \)?

32. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if a team has
   (i) no girl          (ii) at least 3 girls
   (iii) at least one girl and one boy?

33. Find n if
   \[ 16 \binom{n+2}{8} = 57 \binom{n-2}{4} \]

34. A boy has 3 library tickets and 8 books of his interest in the library of these 8, he does not want to borrow Mathematics Part II. Unless
Mathematics Part I is also borrowed. In how many ways can he choose the these books to be borrowed?

35. Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

**LONG ANSWER TYPE QUESTIONS (6 MARKS)**

36. Using the digits 0, 1, 2, 3, 4, 5, 6 how many 4 digit even numbers can be made, no digit being repeated?

37. There are 15 points in a plane out of which only 6 are in a straight line, then

(a) How many different straight lines can be made?

(b) How many triangles can be made?

38. If there are 7 boys and 5 girls in a class, then in how many ways they can be seated in a row such that

(i) No two girls sit together?

(ii) All the girls never sit together?

39. Using the letters of the word 'EDUCATION' how many words using 6 letters can be made so that every word contains atleast 4 vowels?

40. What is the number of ways of choosing 4 cards from a deck of 52 cards? In how many of these,

(a) 3 are red and 1 is black.

(b) All 4 cards are from different suits.

(c) Atleast 3 are face cards.

(d) All 4 cards are of the same colour.

41. How many 3 letter words can be formed using the letters of the word INEFFECTIVE?
42. How many 5 letter words containing 3 vowels and 2 consonants can be formed using the letters of the word EQUATION so that 3 vowels always occur together?

43. If all letters of word ‘MOTHER’ are written in all possible orders and the word so formed are arranged in a dictionary order, then find the rank of word ‘MOTHER’?

ANSWERS

1. 60
2. \( \frac{9!}{2!} \)
3. 100
4. 45
5. 120
6. 513
7. \( n = 6 \)
8. 120
9. 120
10. 1
11. 35
12. 360
13. 120
14. 63
15. \( 3^6 = 729 \)
16. 66
17. \( 90 \times ^{10}P_8 \)
18. 56, 19,350
20. 2772
21. 266
22. 36
23. PAANVR
24. \( ^{13}C_{10} + ^{13}C_8 \)
25. 5040
26. 10
27. 26
28. 4560
29. 576
30. 2454
31. 31
32. (i) 21; (ii) 91; (iii) 441
33. 19
34. 41
35. 48, 144
36. 420
37. (a) 91 (b) 435
38. (i) $7! \times \begin{pmatrix} 8 \\ 5 \end{pmatrix}$ (ii) $12! - 8! \times 5!$
39. 24480
40. $\binom{52}{4}$
   (a) $\binom{26}{1} \times \binom{26}{3}$ (b) $(13)^4$
   (c) 9295 (Hint: Face cards: 4J + 4K + 4Q)
   (d) $2 \times \binom{26}{4}$
41. 265 (Hint: make 3 cases i.e.
   (i) All 3 letters are different (ii) 2 are identical 1 different
   (iii) All are identical, then form the words.)
42. 1080
43. 309
CHAPTER - 8

BINOMIAL THEOREM

KEY POINTS

1. \((a + b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r, n \in \mathbb{N}\)

2. \(T_{r+1} = \text{General term} = \binom{n}{r} a^{n-r} b^r, 0 \leq r \leq n\)

3. Total number of terms in \((a + b)^n\) is \((n + 1)\)

4. If \(n\) is even, then in the expansion of \((a + b)^n\), middle term is \(\left(\frac{n}{2} + 1\right)^{th}\) term i.e. \(\left(\frac{n + 2}{2}\right)^{th}\) term.

5. If \(n\) is odd, then in the expansion of \((a + b)^n\), middle terms are \(\left(\frac{n + 1}{2}\right)^{th}\) and \(\left(\frac{n + 3}{2}\right)^{th}\) terms

6. In \((a + b)^n\), \(r^{th}\) term from the end is same as \((n - r + 2)^{th}\) term from the beginning.

7. \(r^{th}\) term from the end in \((a + b)^n\)

   \[= r^{th}\] term from the beginning in \((b + a)^n\)

8. In \((1 + x)^n\), coefficient of \(x^r\) is \(\binom{n}{r}\)

9. Some particular cases:
\[(1 + x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r\]

\[(1 - x) = \sum_{r=0}^{n} (-1)^r \binom{n}{r} x^r\]

- Some properties of Binomial coefficients:
  \[\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n} = 2^n\]
  \[\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \ldots + (-1)^n \binom{n}{n} = 0\]
  \[\binom{n}{0} - \binom{n}{2} + \binom{n}{4} + \ldots = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \ldots = 2^{n-1}\]

**VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)**

1. Compute \((98)^2\), using binomial theorem.
2. Expand \(\left(x - \frac{1}{x}\right)^3\) using binomial theorem.
3. Find number of terms in the expansion of the following:
   (i) \(\left(3x - \frac{7}{y^2}\right)^8\)
   (ii) \((1 + 2x + x^2)^7\)
4. Write value of
   \[2n-1 \binom{C_5}{6} + 2n-1 \binom{C_6}{7} + 2n \binom{C_7}{7}\]
   [Hint : Use \(\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}\)]
5. In the expansion, \((1 + x)^{14}\), write the coefficient of \(x^{12}\)
6. Find the sum of the coefficients in \((x + y)^8\)
   [Hint : Put \(x = 1, y = 1\)]
7. If \(\binom{n}{n-3} = 720\), find \(n\).
   [Hint : Express 720 as the product of 3 consecutive positive integers]
8. Find middle term in expansion of \((1 + x)^{2n}\).
9. Find

(i) 3rd term in expansion of \( \left( 3x^2 - \frac{2}{x^3} \right)^8 \)

(ii) 4th term in expansion of \( (x - 2y)^{12} \)

(iii) 4th term from end in the expansion of \( \left( \frac{x^3}{2} - \frac{2}{x^2} \right)^9 \)

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

10. If the first three terms in the expansion of \((a + b)^n\) are 27, 54 and 36 respectively, then find \(a, b\) and \(n\).

11. In \( (3x^2 - \frac{1}{x})^{18} \), which term contains \(x^{12}\)?

12. In \( \left( \frac{\sqrt{x} + \sqrt{3}}{2x^2} \right)^{10} \), find the term independent of \(x\).

13. Evaluate : \( \left( \sqrt{2} + 1 \right)^5 - \left( \sqrt{2} - 1 \right)^5 \) using binomial theorem.

14. Evaluate \( (0.9)^4 \) using binomial theorem.

15. In the expansion of \((1 + x^2)^8\), find the difference between the coefficients of \(x^6\) and \(x^4\).

16. In \( (2x - \frac{3}{x})^8 \), find 7th term from end.

17. In \( \left( 2x^3 - \frac{1}{x^2} \right)^{12} \), find the coefficient of \(x^{11}\).

18. Find the coefficient of \(x^4\) in \((1 - x)^2 (2 + x)^5\) using binomial theorem.
19. Using binomial theorem, show that
   \[3^{2n} + 2 - 8n - 9\] is divisible by 8.

   [Hint : \(3^{2n} + 2 = 9 \left(3^2\right)^n = 9(1 + 8)^n\), Now use binomial theorem.]

20. Prove that,
   \[\sum_{r=0}^{20} C_{20-r} (2 - t)^{20-r} (t - 1)^r = 1\]

21. Find the middle term(s) in \(x - \frac{1}{x}\)^8

22. If the coefficients of three consecutive terms in the expansion of \((1 + x)^n\)
    are in the ratio 1:3:5, then show that \(n = 7\).

23. Show that the coefficient of middle term in the expansion of \((1 + x)^{20}\) is equal to the sum of the coefficients of two middle terms in the expansion of \((1 + x)^{19}\)

   **LONG ANSWER TYPE QUESTIONS (6 MARKS)**

24. Show that the coefficient of \(x^5\) in the expansion of product \((1 + 2x)^6 (1 - x)^7\) is 171.

25. If the 3\(^{rd}\), 4\(^{th}\) and 5\(^{th}\) terms in the expansion of \((x + a)^n\) are 84, 280 and 560 respectively then find the values of \(a\), \(x\) and \(n\)

26. In the expansion of \((1 - x)^{2n-1}\), find the sum of coefficients of \(x^{r-1}\) and \(x^{2n-r}\)

27. If the coefficients of \(x^7\) in \(\left(ax^2 + \frac{1}{bx}\right)^{11}\) and \(x^{-7}\) in \(\left(ax - \frac{1}{bx^2}\right)^{11}\) are equal, then show that \(ab = 1\).

28. If three successive coefficients in expansion of \((1 + x)^n\) are 220, 495 and 792 then find \(n\).

29. In the expansion of \(\left(\frac{3\sqrt{2} + \frac{1}{3\sqrt{3}}}{\sqrt[3]{2}}\right)^n\) the ratio of 7th term from the beginning to the 7th term from the end is 1 : 6 find \(n\).
30. If \(a_1, a_2, a_3\) and \(a_4\) are the coefficients of any four consecutive terms in the expansion of \((1 + x)^n\) prove that \(\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}\).

31. Find \(n\) if the coefficients of 5th, 6th and 7th terms in the expansion of \((1 + x)^n\) are in A.P.

32. Find the ratio of the coefficients of \(x^{15}\) to the term independent of \(x\) in the expansion \(\left(x^2 + \frac{2}{x}\right)^{15}\).

**ANSWERS**

1. 9604
2. \(x^3 - \frac{1}{x^3} - 3x + \frac{3}{x}\)
3. (i) 9; (ii) 15;
4. \(2^{n+1}C_7\)
5. 91
6. 256
7. \(n = 10\)
8. \(2^nC_{n-1}x^n\)
9. (i) \(112 \times 3^6 \times x^{10}\); (ii) \(-1760x^6y^2\); (iii) \(\frac{672}{x^3}\)
10. \(a = 3, b = 2, n = 3\)
11. 9th term
12. \(T_3 = \frac{5}{12}\)
13. 82
14. 0.6561
15. 28
16. 16128 \(x^4\)
17. \(-101376\)
18. 10
19. 21
20. 70
25. \(a = 2, x = 1, n = 7\)
26. 0
28. 12
29. 9
31. 7 or 14
32. \(\frac{1}{32}\) or 1 : 32
CHAPTER - 9

SEQUENCES AND SERIES

KEY POINTS

• A sequence is a function whose domain is the set N of natural numbers or some subset of it.

• A sequence whose range is a subset of R is called a real sequence.

• A sequence is said to be a progression if the term of the sequence can be expressed by some formula

• A sequence is called an arithmetic progression if the difference of a term and previous term is always same, i.e., \( a_{n+1} - a_n = \text{constant} (=d) \) for all \( n \in N \).

• The term ‘series’ is associated with the sequence in following way:
  
  Let \( a_1, a_2, a_3 \ldots \) be a sequence. Then, the expression \( a_1 + a_2 + a_3 + \ldots \ldots \) is called series associated with given sequence.

• A series is finite or infinite according as the given sequence is finite or infinite.

• General A.P. is,
  
  \( a, a + d, a + 2d, \ldots \ldots \)

• \( a_n = a + (n - 1)d = \text{n}^{\text{th}} \text{ term of A.P.} = l \)

• \( S_n = \text{Sum of first} \ n \text{ terms of A.P.} \)

  \[ S_n = \frac{n}{2} [a + l] \text{ where } l = \text{last term.} \]

  \[ S_n = \frac{n}{2} [2a + (n - 1)d] \]

• If \( a, b, c \) are in A.P. then \( a \pm k, b \pm k, c \pm k \) are in A.P.,
ak, bk, ck are also in A.P., k \neq 0, \frac{a}{k}, \frac{b}{k}, \frac{c}{k} are also in A.P. where k \neq 0.

- Three numbers in A.P. 
  \[ a - d, a, a + d \]

- Arithmetic mean between a and b is \( \frac{a + b}{2} \).

- If \( A_1, A_2, A_3, \ldots A_n \) are n numbers inserted between a and b, such that the resulting sequence is A.P. then,
  \[ A_n = a + n \left( \frac{b - a}{n + 1} \right) \]

- \( S_k - S_{k-1} = a_k \)

- \( a_m = n, a_n = m \Rightarrow a_r = m + n - r \)

- \( S_m = S_n \Rightarrow S_{m+n} = 0 \)

- \( S_p = q \) and \( S_q = p \Rightarrow S_{p+q} = -p - q \)

- In an A.P., the sum of the terms equidistant from the beginning and from the end is always same, and equal to the sum of the first and the last term

- If three terms of A.P. are to be taken then we choose then as \( a - d, a, a + d \).

- If four terms of A.P. are to be taken then we choose then as \( a - 3d, a - d, a + d, a + 3d \).

- G.P. (Geometrical Progression)

  \[ a, ar, ar^2, \ldots \ldots \ldots \text{(General G.P.)} \]
  \[ a_n = ar^{n-1} \]
  \[ S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1 \]

- Geometric mean between a and b is \( \sqrt{ab} \)
• Reciprocals of terms in GP always form a G.P.
• If $G_1, G_2, G_3, \ldots, G_n$ are $n$ numbers inserted between $a$ and $b$ so that the resulting sequence is G.P., then
  \[ G_k = a \left( \frac{b}{a} \right)^{\frac{k}{n+1}}, 1 \leq k \leq n \]
• If three terms of G.P. are to be taken then those are \(a, ar, ar^2\).
• If four terms of G.P. are to be taken then we choose those as \(\frac{a}{r^3}, \frac{a}{r}, ar, ar^3\).
• If $a, b, c$ are in G.P. then $ak, bk, ck$ are also in G.P. where $k \neq 0$, \(\frac{a}{k}, \frac{b}{k}, \frac{c}{k}\) are also in G.P. where $k \neq 0$.
• In a G.P., the product of the terms equidistant from the beginning and from the end is always same and equal to the product of the first and the last term.
• If each term of a G.P. be raised to some power then the resulting terms are also in G.P.
• Sum of infinite G.P. is possible if $|r| < 1$ and sum is given by \(\frac{a}{1-r}\)
  \[ \sum_{r=1}^{n} r = \frac{n(n + 1)}{2} \]
  \[ \sum_{r=1}^{n} r^2 = \frac{n(n + 1)(2n + 1)}{6} \]
  \[ \sum_{r=1}^{n} r^3 = \left[ \frac{n(n + 1)}{2} \right]^2 \]

**VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)**

1. If $n^{th}$ term of an A.P. is $6n - 7$ then write its $50^{th}$ term.
2. If $S_n = 3n^2 + 2n$, then write $a_2$.
3. Which term of the sequence, 3, 10, 17, .......... is 136?

4. If in an A.P. 7th term is 9 and 9th term is 7, then find 16th term.

5. If sum of first n terms of an A.P is \(2n^2 + 7n\), write its \(n^{th}\) term.

6. Which term of the G.P.,
   \[2, 1, \frac{1}{2}, \frac{1}{4}, \ldots \ldots \ldots \text{is } \frac{1}{1024}\ ?\]

7. If in a G.P., \(a_3 + a_5 = 90\) and if \(r = 2\) find the first term of the G.P.

8. In G.P. \(2, 2\sqrt{2}, 4, \ldots \), 128\(\sqrt{2}\), find the 4th term from the end.

9. If the product of 3 consecutive terms of G.P. is 27, find the middle term

10. Find the sum of first 8 terms of the G.P. 10, 5, \(\frac{5}{2}\), .......

11. Find the value of \(5^{1/2} \times 5^{1/4} \times 5^{1/8} \ldots \ldots \text{upto infinity.}\)

12. Write the value of \(0.\overline{3}\)

13. The first term of a G.P. is 2 and sum to infinity is 6, find common ratio.

14. Write the \(n^{th}\) term of the series, \(\frac{3}{7.11^2} + \frac{5}{8.12^2} + \frac{7}{9.13^2} + \ldots \ldots \)

15. Find the number of terms in the A.P. 7, 10, 13, ......., 31.

16. In an A.P.,
   \(8, 11, 14, \ldots \ldots \text{find } S_n - S_{n-1}\)

17. Find the number of squares that can be formed on chess board?

**SHORT ANSWER TYPE QUESTIONS (4 MARKS)**

18. Find the least value of \(n\) for which
   \(1 + 3 + 3^2 + \ldots + 3^{n-1} > 1000\)

19. Find the sum of the series
   \[(1 + x) + (1 + x + x^2) + (1 + x + x^2 + x^3) + \ldots \]
20. Write the first negative term of the sequence \(20, 19 \frac{1}{4}, 18 \frac{1}{2}, 17 \frac{3}{4}, \ldots\).

21. Determine the number of terms in A.P. 3, 7, 11, \ldots\ 407. Also, find its 11th term from the end.

22. How many numbers are there between 200 and 500, which leave remainder 7 when divided by 9.

23. Find the sum of all the natural numbers between 1 and 200 which are neither divisible by 2 nor by 5.

24. Find the sum of the sequence,

\[-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, \ldots, -\frac{10}{3}\]

25. If in an A.P. \(\frac{a_{7}}{a_{10}} = \frac{5}{7}\) find \(\frac{a_{4}}{a_{7}}\)

26. In an A.P. sum of first 4 terms is 56 and the sum of last 4 terms is 112. If the first term is 11 then find the number of terms.

27. Solve : \(1 + 6 + 11 + 16 + \ldots\ldots + x = 148\)

28. The ratio of the sum of \(n\) terms of two A.P.'s is \((7n - 1) : (3n + 11)\), find the ratio of their 10th terms.

29. If the 1st, 2nd and last terms of an A.P are \(a, b\) and \(c\) respectively, then find the sum of all terms of the A.P.

30. If \(\frac{b + c - 2a}{a}, \frac{c + a - 2b}{b}, \frac{a + b - 2c}{c}\) are in A.P. then show that \(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\) are also in A.P. [Hint. : Add 3 to each term]

31. The product of first three terms of a G.P. is 1000. If 6 is added to its second term and 7 is added to its third term, the terms become in A.P. Find the G.P.

32. If the continued product of three numbers in G.P. is 216 and the sum of their products in pairs is 156, find the numbers.

33. Find the sum of first \(n\) terms of the series :

\(3 + 7 + 13 + 21 + 31 + \ldots\ldots\)
34. If \( A = 1 + r^a + r^{2a} + \ldots \) up to infinity, then express \( r \) in terms of \('a' \& 'A'\).

35. Find the sum of first terms of the series 0.7 + 0.7 + 0.777 + ...

36. If \( x = a + \frac{a}{r} + \frac{a}{r^2} + \ldots \infty \), \( y = b - \frac{b}{r} + \frac{b}{r^2} - \ldots \infty \) and \( z = c + \frac{c}{r^2} + \frac{c}{r^4} + \ldots \infty \)
prove that \( \frac{xy}{z} = \frac{ab}{c} \).

37. The sum of first three terms of a G.P. is 15 and sum of next three terms is 120. Find the sum of first \( n \) terms.

38. Prove that, \( 0.031 = \frac{7}{225} \)

[Hint : \( 0.031 = 0.03 + 0.001 + 0.0001 + \ldots \) Now use infinite G.P.]

39. If \( \log 2, \log (2^n - 1) \) and \( \log (2^n + 3) \) are in A.P. Show that \( n = \frac{\log 5}{\log 2} \)

**LONG ANSWER TYPE QUESTIONS (6 MARKS)**

40. Prove that the sum of \( n \) numbers between \( a \) and \( b \) such that the resulting series becomes A.P. is \( \frac{n(a + b)}{2} \).

41. A square is drawn by joining the mid points of the sides of a square. A third square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of the first square is 15 cm, then find the sum of the areas of all the squares so formed.

42. If \( a, b, c \) are in G.P., then prove that
\[
\frac{1}{a^2 - b^2} = \frac{1}{b^2 - c^2} - \frac{1}{b^2}
\]

[Hint : Put \( b = ar \), \( c = ar^2 \)]

43. Find two positive numbers whose difference is 12 and whose arithmetic...
mean exceeds the geometric mean by 2.

44. If a is A.M. of b and c and c, G₁, G₂, b are in G.P. then prove that
   \[ G_1^3 + G_2^3 = 2abc \]

45. Find the sum of the series,
   \[ 1.3.4 + 5.7.8 + 9.11.12 + \ldots \ldots \ldots \text{upto}\ n\ \text{terms}. \]

46. Evaluate \[ \sum_{r=1}^{10} (2r - 1)^2 \]

47. The sum of an infinite G.P. is 57 and the sum of the cubes of its term is 9747, find the G.P.

*48. Show that \[ \frac{1 \times 2^2 + 2 \times 3^2 + \ldots + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + \ldots + n^2 \times (n+1)} = \frac{3n+5}{3n+1}. \]

ANSWERS

1. 293
2. 11
3. 20
4. 0
5. 4n + 5
6. 12
7. \( \frac{9}{2} \)
8. 64
9. 3
10. \( 20 \left( 1 - \frac{1}{2^6} \right) \)
11. 5
12. \( \frac{1}{3} \)
13. \( \frac{2}{3} \)
14. \( \frac{2n + 1}{(n + 6)(n + 10)^2} \)
15. 9
16. 3n + 5
17. 204
18. 7
19. \( \frac{n}{1-x} \frac{x^2 (1-x^n)}{(1-x)^2} \)
20. \(-\frac{1}{4}\)
21. 102, 367
22. 33
23. 7999
24. \(\frac{63}{2}\)
25. \(\frac{3}{5}\)
26. 11
27. 36
28. 33 : 17
29. \(\frac{(b+c-2a)(a+c)}{2(b-a)}\)
30. 5, 10, 20, ......; or 20, 10, 5, ......
31. 5, 10, 20, ......; or 20, 10, 5,
32. 18, 6, 2; or 2, 6, 18
33. \(\frac{n}{3}(n^2 + 3n + 5)\)
34. \(\left(\frac{A-1}{A}\right)^{1/a}\)
35. \(\frac{7}{81}[9n-1 + 10^{-n}]\)
36. \(\frac{15}{7}(2^n - 1)\)
37. 41. 450 cm²
38. 16, 4
39. \(\frac{n(n+1)}{3}(48n^2 - 16n - 14)\)
40. 1330
41. 19, \(\frac{38}{3}, \frac{76}{9}\), ......
CHAPTER - 10

STRAIGHT LINES

- Distance between two points \( A(x_1, y_1) \) and \( B(x_2, y_2) \) is given by
  \[
  AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
  \]

- Let the vertices of a triangle \( ABC \) are \( A(x_1, y_1) \) \( B(x_2, y_2) \) and \( C(x_3, y_3) \).
  Then area of triangle \( ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|\)

- Straight line is a curve such that every point on the line segment joining any two points on it lies on it.

- A line is also defined as the locus of a point satisfying the condition \( ax + by + c = 0 \) where \( a, b, c \) are constants.

- Slope or gradient of a line is defined as \( m = \tan \theta \), \((\theta \neq 90^\circ)\), where \( \theta \) is angle which the line makes with positive direction of x-axis measured in anticlockwise direction, \( 0 \leq \theta < 180^\circ \)

- Slope of x-axis is zero and slope of y-axis is not defined.

- Three points \( A, B \) and \( C \) lying in a plane are collinear, if slope of \( AB = \) Slope of \( BC \).

- Slope of a line through given points \( (x_1, y_1) \) and \( (x_2,y_2) \) is given by
  \[
  \frac{y_2 - y_1}{x_2 - x_1}
  \]

- Two lines are parallel to each other if and only if their slopes are equal.

- Two lines are perpendicular to each other if and only if their slopes are negative reciprocal of each other.

- Acute angle \( \alpha \) between two lines, whose slopes are \( m_1 \) and \( m_2 \) is given by
  \[
  \tan \alpha = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|, 1 + m_1 m_2 \neq 0
  \]

- \( x = a \) is a line parallel to y-axis at a distance of a units from y-axis.
  \( x = a \) lies on right or left of y-axis according as a is positive or negative.
y = b is a line parallel to x-axis at a distance of ‘b’ units from x-axis. y=b lies above or below x-axis, according as b is positive or negative.

- Equation of a line passing through given point \((x_1, y_1)\) and having slope m is given by
  \[ y - y_1 = m(x - x_1) \]

- Equation of a line passing through given points \((x_1, y_1)\) and \((x_2, y_2)\) is given by
  \[ y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \]

- Equation of a line having slope m and y-intercept c is given by
  \[ y = mx + c \]

- Every first degree equation in \(x, y\) represents a straight line.

- Equation of line having intercepts a and b on x-axis and y-axis respectively is given by
  \[ \frac{x}{a} + \frac{y}{b} = 1 \]

- Equation of line in normal form is given by \(x \cos \alpha + y \sin \alpha = p\),
  \[ p = \text{Length of perpendicular segment from origin to the line} \]
  \[ \alpha = \text{Angle which the perpendicular segment makes with positive direction of x-axis} \]

- Equation of line in general form is given by \(Ax + By + C = 0\), A, B and C are real numbers and at least one of A or B is non zero.

- Distance of a point \((x_1, y_1)\) from line \(Ax + By + C = 0\) is given by
  \[ d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \]

- Distance between two parallel lines \(Ax + By + C_1 = 0\) and \(Ax + By + C_2 = 0\) is given by
  \[ d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} \]
Shifting of origin to a new point without changing the direction of the axes is known as translation of axes.

Let OX, OY be the original axes and O' be the new origin. Let coordinates of O' referred to original axes be (h, k). Let P(x, y) be point in plane

Let O'X' and O'Y' be drawn parallel to and in same direction as OX and OY respectively. Let coordinates of P referred to new axes O'X' and O'Y' be (x', y') then \( x = x' + h, \ y = y' + k \)

or \( x' = x - h, \ y' = y - k \)

Thus

(i) The point whose coordinates were (x, y) has new coordinates \((x - h, y - k)\) when origin is shifted to \((h, k)\).

(ii) Coordinates of old origin referred to new axes are \((-h, -k)\).

- Equation of family of lines parallel to \(Ax + By + C = 0\) is given by \(Ax + By + k = 0\), for different real values of \(k\)
- Equation of family of lines perpendicular to \(Ax + By + C = 0\) is given by \(Bx - Ay + k = 0\), for different real values of \(k\).
- Equation of family of lines through the intersection of lines \(A_1x + B_1y + C_1 = 0\) and \(A_2x + B_2y + C_2 = 0\) is given by \((A_1x + B_1y + C_1) + k(A_2x + B_2y + C_2) = 0\), for different real values of \(k\).
VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Three consecutive vertices of a parallelogram are (–2, –1), (1, 0) and (4, 3), find the fourth vertex.

2. For what value of k are the points (8, 1), (k, –4) and (2, –5) collinear?

3. The mid point of the segment joining (a, b) and (–3, 4b) is (2, 3a + 4). Find a and b.

4. Coordinates of centroid of \( \triangle ABC \) are (1, –1). Vertices of \( \triangle ABC \) are \( A(-5, 3) \), \( B(p, -1) \) and \( C(6, q) \). Find p and q.

5. In what ratio y-axis divides the line segment joining the points (3,4) and (–2, 1) ?

6. What are the possible slopes of a line which makes equal angle with both axes?

7. Determine \( x \) so that slope of line through points (2, 7) and (x, 5) is 2.

8. Show that the points (a, 0), (0, b) and (3a – 2b) are collinear.

9. Find the equation of straight line cutting off an intercept –1 from \( y \) axis and being equally inclined to the axes.

10. Write the equation of a line which cuts off equal intercepts on coordinate axes and passes through (2, 5).

11. Find k so that the line \( 2x + ky - 9 = 0 \) may be perpendicular to \( 2x + 3y - 1 = 0 \)

12. Find the acute angle between lines \( x + y = 0 \) and \( y = 0 \)

13. Find the angle which \( \sqrt{3}x + y + 5 = 0 \) makes with positive direction of x-axis.

14. If origin is shifted to (2, 3), then what will be the new coordinates of (–1, 2)?

15. On shifting the origin to \( (p, q) \), the coordinates of point (2, –1) changes to (5, 2). Find p and q.

16. Determine the equation of line through a point (–4, –3) and parallel to x-axis.
SHORT ANSWER TYPE QUESTIONS (4 MARKS)

17. If the image of the point (3, 8) in the line \( px + 3y - 7 = 0 \) is the point \((-1, -4)\), then find the value of \( p \).

18. Find the distance of the point (3,2) from the straight line whose slope is 5 and is passing through the point of intersection of lines \( x + 2y = 5 \) and \( x - 3y + 5 = 0 \).

19. The line \( 2x - 3y = 4 \) is the perpendicular bisector of the line segment \( AB \). If coordinates of \( A \) are \((-3, 1)\) find coordinates of \( B \).

20. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line \( y = 2x + c \). Find \( c \) and remaining two vertices.

21. If two sides of a square are along \( 5x - 12y + 26 = 0 \) and \( 5x - 12y - 65 = 0 \) then find its area.

22. Find the equation of a line with slope \(-1\) and whose perpendicular distance from the origin is equal to 5.

23. If a vertex of a square is at \((1, -1)\) and one of its side lie along the line \( 3x - 4y - 17 = 0 \) then find the area of the square.

24. What is the value of \( y \) so that line through \((3, y)\) and \((2, 7)\) is parallel to the line through \((-1, 4)\) and \((0, 6)\) ?

25. In what ratio, the line joining \((-1, 1)\) and \((5, 7)\) is divided by the line \( x + y = 4 \)?

26. Find the equation of the lines which cut-off intercepts on the axes whose sum and product are 1 and \(-6\) respectively.

27. Find the area of the triangle formed by the lines \( y = x \), \( y = 2x \), \( y = 3x + 4 \).

28. Find the coordinates of the orthocentre of a triangle whose vertices are \((-1, 3)\) \((2, -1)\) and \((0, 0)\). [Orthocentre is the point of concurrency of three altitudes].

29. Find the equation of a straight line which passes through the point of intersection of \( 3x + 4y - 1 = 0 \) and \( 2x - 5y + 7 = 0 \) and which is perpendicular to \( 4x - 2y + 7 = 0 \).
30. If the image of the point (2, 1) in a line is (4, 3) then find the equation of line.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

31. Find points on the line $x + y + 3 = 0$ that are at a distance of $\sqrt{5}$ units from the line $x + 2y + 2 = 0$.

32. Find the equation of a straight line which makes acute angle with positive direction of x-axis, passes through point(−5, 0) and is at a perpendicular distance of 3 units from origin.

33. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (−3, 1) and (1,1). Find the equation of other three sides.

34. If (1,2) and (3, 8) are a pair of opposite vertices of a square, find the equation of the sides and diagonals of the square.

35. Find the equations of the straight lines which cut off intercepts on x-axis twice that on y-axis and are at a unit distance from origin.

36. Two adjacent sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals is $11x + 7y = 4$, find the equation of the other diagonal.

37. A line is such that its segment between the lines $5x − y + 4 = 0$ and $3x + 4y − 4 = 0$ is bisected at the point (1, 5). Obtain its equation.

38. If one diagonal of a square is along the line $8x − 15y = 0$ and one of its vertex is at (1, 2), then find the equation of sides of the square passing through this vertex.

39. If the slope of a line passing through the point A(3, 2) is 3/4 then find points on the line which are 5 units away from the point A.

40. Find the equation of straight line which passes through the intersection of the straight line $3x + 2y + 4 = 0$ and $x − y − 2 = 0$ and forms a triangle with the axis whose area is 8 sq. unit.

ANSWERS

1. (1, 2) 
2. $k = 3$
3. $a = 7, b = 10$ 
4. $p = 2, q = −5$
5. $3 : 2$ (internally)
6. $\pm 1$
7. $1$
8. $y = x - 1$ and $y = -x - 1$.
9. $x + y = 7$
10. $\frac{\pi}{4}$
11. $\frac{2\pi}{3}$
12. $(-3, -1)$
13. $p = -3, q = -3$
14. $y + 3 = 0$
15. $1$
16. $\frac{10}{\sqrt{26}}$
17. $(1, -5)$
18. $c = -4, (2,0), (4, 4)$
19. $49$ square units
20. $2x - 3y - 6 = 0$ and $-3x + 2y - 6 = 0$
21. $4$ square units
22. $x + y + 5\sqrt{2} = 0, x + y - 5\sqrt{2} = 0$
23. $y = 9$
24. $1 : 2$
25. $2x - 3y - 6 = 0$ and $-3x + 2y - 6 = 0$
26. $4$ square units
27. $(-4, -3)$
28. $x + 2y = 1$
29. $x + y - 5 = 0$
30. $(1, -4), (-9, 6)$
31. $3x - 4y + 15 = 0$
32. $4x + 7y - 25 = 0$
33. $7x - 4y = 0$
34. $x - 2y - 3 = 0, 2x + y - 14 = 0,$
35. $3x - y - 1 = 0, x + 3y - 17 = 0$
36. $x = y$
37. $107x - 3y - 92 = 0$
38. $23x - 7y - 9 = 0$ and $7x + 23y - 53 = 0$
39. $(-1, -1)$ or $(7, 5)$
40. $x - 4y - 8 = 0$ or $x + 4y + 8 = 0$
KEY POINTS

- Circle, ellipse, parabola and hyperbola are curves which are obtained by intersection of a plane and cone in different positions.

- Circle: It is the set of all points in a plane that are equidistant from a fixed point in that plane.
  
  Equations of circle: 
  
  
  \[(x - h)^2 + (y - k)^2 = r^2\]
  Centre \((h, k)\), radius \(r\)

- Parabola: It is the set of all points in a plane which are equidistant from a fixed point (focus) and a fixed line (directrix) in the plane. Fixed point does not lie on the line.

\[
\begin{align*}
  &\text{Equation: } y^2 = 4ax \\
  &\text{Focus: } F(a,0) \\
  &\text{Directrix: } x = -a \\
  &\text{Direction: } X'Y' \\
  &\text{Equation: } y^2 = -4ax \\
  &\text{Focus: } F(-a,0) \\
  &\text{Directrix: } x = a \\
  &\text{Direction: } X'Y' \\
  &\text{Equation: } x^2 = 4ay \\
  &\text{Focus: } F(0,a) \\
  &\text{Directrix: } y = -a \\
  &\text{Direction: } X'Y' \\
  &\text{Equation: } x^2 = -4ay \\
  &\text{Focus: } F(0,-a) \\
  &\text{Directrix: } y = a \\
  &\text{Direction: } X'Y'
\end{align*}
\]
Main facts about the Parabola

<table>
<thead>
<tr>
<th>Equation</th>
<th>$y^2 = 4ax$</th>
<th>$y^2 = -4ax$</th>
<th>$x^2 = 4ay$</th>
<th>$x^2 = -4ay$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Axis</strong></td>
<td>$y = 0$</td>
<td>$y = 0$</td>
<td>$x = 0$</td>
<td>$x = 0$</td>
</tr>
<tr>
<td><strong>Directrix</strong></td>
<td>$x + a = 0$</td>
<td>$x - a = 0$</td>
<td>$y + a = 0$</td>
<td>$y - a = 0$</td>
</tr>
<tr>
<td><strong>Focus</strong></td>
<td>$(a, 0)$</td>
<td>$(-a, 0)$</td>
<td>$(0, a)$</td>
<td>$(0, -a)$</td>
</tr>
<tr>
<td><strong>Length of latus-rectum</strong></td>
<td>$4a$</td>
<td>$4a$</td>
<td>$4a$</td>
<td>$4a$</td>
</tr>
<tr>
<td><strong>Equation of latus-rectum</strong></td>
<td>$x - a = 0$</td>
<td>$x + a = 0$</td>
<td>$y - a = 0$</td>
<td>$y + a = 0$</td>
</tr>
</tbody>
</table>

- **Latus Rectum**: A chord through focus perpendicular to axis of parabola is called its latus rectum.

- **Ellipse**: It is the set of points in a plane the sum of whose distances from two fixed points in the plane is a constant and is always greater than the distances between the fixed points.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a > b > 0$, $b > a > 0$

$$c = \sqrt{a^2 - b^2}$$

Main facts about the ellipse

<table>
<thead>
<tr>
<th>Equation</th>
<th>$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a &gt; 0$, $b &gt; 0$</th>
<th>$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, $a &gt; 0$, $b &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Centre</strong></td>
<td>$(0,0)$</td>
<td>$(0,0)$</td>
</tr>
<tr>
<td><strong>Major axis lies along</strong></td>
<td>$x$–axis</td>
<td>$y$–axis</td>
</tr>
<tr>
<td><strong>Length of major axis</strong></td>
<td>$2a$</td>
<td>$2a$</td>
</tr>
<tr>
<td><strong>Length of minor axis</strong></td>
<td>$2b$</td>
<td>$2b$</td>
</tr>
<tr>
<td>Foci</td>
<td>$(-c, 0), (c, 0)$</td>
<td>$(0, -c), (0, c)$</td>
</tr>
<tr>
<td>Vertices</td>
<td>$(-a, 0), (a, 0)$</td>
<td>$(0, -a), (0, a)$</td>
</tr>
<tr>
<td>Eccentricity $e$</td>
<td>$\frac{c}{a}$</td>
<td>$\frac{c}{a}$</td>
</tr>
<tr>
<td>Length of latus–rectum</td>
<td>$\frac{2b^2}{a}$</td>
<td>$\frac{2b^2}{a}$</td>
</tr>
</tbody>
</table>

- **Latus rectum**: Chord through foci perpendicular to major axis called latus rectum.
- **Hyperbola**: It is the set of all points in a plane, the differences of whose distance from two fixed points in the plane is a constant.

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1
\]

\[
y^2 - \frac{x^2}{b^2} = 1
\]

\[
c = \sqrt{a^2 + b^2}
\]

**Main facts about the Hyperbola**

| Equation          | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \quad a > 0, \ b > 0$ | $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1, \quad a > 0, \ b > 0$ |
| Centre            | $(0, 0)$                                               | $(0, 0)$                                               |
| Transverse axis   | $x$–axis                                              | $y$–axis                                              |
| Length of transverse axis | $2a$             | $2a$             |
| Length of conjugate axis | $2b$             | $2b$             |
| Foci              | $(-c, 0), (c, 0)$                                    | $(0, -c), (0, c)$                                    |
| Vertices          | $(-a, 0), (a, 0)$                                    | $(0, -a), (0, a)$                                    |
| Eccentricity $e$  | $\frac{c}{a}$                                         | $\frac{c}{a}$                                         |
| Length of latus–rectum | $\frac{2b^2}{a}$ | $\frac{2b^2}{a}$ |
Latus Rectum: Chord through foci perpendicular to transverse axis is called latus rectum.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find the centre and radius of the circle
   \[3x^2 + 3y^2 + 6x - 4y - 1 = 0\]

2. Does \(2x^2 + 2y^2 + 3y + 10 = 0\) represent the equation of a circle? Justify.

3. Find equation of circle whose end points of one of its diameter are \((-2, 3)\) and \((0, -1)\).

4. Find the value(s) of \(p\) so that the equation \(x^2 + y^2 - 2px + 4y - 12 = 0\) may represent a circle of radius 5 units.

5. If parabola \(y^2 = px\) passes through point \((2, -3)\), find the length of latus rectum.

6. Find the coordinates of focus, and length of latus rectum of parabola \(3y^2 = 8x\).

7. Find the eccentricity of the ellipse
   \[\frac{x^2}{25} + \frac{y^2}{9} = 1\]

8. Find the centre and radius of the circle \(x^2 + y^2 - 6x + 4y - 12 = 0\)

9. Find the length of major and minor axis of the following ellipse,
   \[16x^2 + 25y^2 = 400\]

10. Find the eqn. of hyperbola satisfying given conditions foci \((\pm 5, 0)\) and transverse axis is of length 8.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

11. If one end of a diameter of the circle \(x^2 + y^2 - 4x - 6y + 11 = 0\) is \((3, 4)\), then find the coordinates of the other end of diameter.

12. Find the equation of the ellipse with foci \((\pm 5, 0)\) and \(x = \frac{36}{5}\) as one of the directrices.
13. Find equation of an ellipse having vertices (0, ± 5) and foci (0, ± 4).

14. If the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then obtain the equation of a hyperbola.

15. Find the equation for the ellipse that satisfies the given condition:
   Major axis on the x-axis and passes through the points (4, 3) and (6, 2).

16. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, find the equation of the hyperbola if its eccentricity is 2.

17. Find the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which passes through the points (3, 0) and $(3\sqrt{2}, 2)$.

18. If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentricity.

19. Find equation of circle concentric with circle $4x^2 + 4y^2 - 12x - 16y - 21 = 0$ and of half its area.

20. Find the equation of a circle whose centre is at (4, -2) and $3x - 4y + 5 = 0$ is tangent to circle.

**LONG ANSWER TYPE QUESTIONS (6 MARKS)**

21. Prove that the points (1, 2), (3, – 4), (5, – 6) and (11, – 8) are concyclic.

22. A circle has radius 3 units and its centre lies on the line $y = x – 1$. If it is passes through the point (7, 3) then find the equations of the circle.

23. Find the equation of the circle which passes through the points (20, 3), (19, 8) and (2, – 9). Find its centre and radius.

24. Find the equation of circle having centre (1, – 2) and passing through the point of intersection of the lines $3x + y = 14$ and $2x + 5y = 18$.

**ANSWERS**

1. $\left(-1, \frac{2}{3}\right)$, $\frac{4}{3}$

2. No
3. \(x^2 + y^2 + 2x - 2y - 3 = 0\) or \((x + 1)^2 + (y - 1)^2 = 5\)

4. \(-3, +3\)

5. \(\frac{9}{2}\)

6. \(\left(\frac{2}{3}, 0\right), \frac{8}{3}\)

7. \(\frac{4}{5}\)

8. \((3, -2), 5\)

9. \(10, 8\)

10. \(\frac{x^2}{16} - \frac{y^2}{9} = 1\)

11. \((1, 2)\)

12. \(\frac{x^2}{36} + \frac{y^2}{11} = 1\)

13. \(\frac{x^2}{9} + \frac{y^2}{25} = 1\)

14. \(x^2 - y^2 = 32\)

15. \(\frac{x^2}{52} + \frac{y^2}{13} = 1\)

16. \(\frac{x^2}{4} - \frac{y^2}{12} = 1\)

17. \(\sqrt{13}/3\)

18. \(\frac{\sqrt{3}}{2}\)

19. \(2x^2 + 2y^2 - 6x + 8y + 1 = 0\)

20. \(x^2 + y^2 - 8x + 4y - 5 = 0\)

22. \[\begin{cases} x^2 + y^2 - 8x - 6y + 16 = 0 \\ x^2 + y^2 - 14x - 12y + 76 = 0 \end{cases}\]

23. \(x^2 + y^2 - 14x - 6y - 111 = 0\)

centre \((7, 3)\) radius = 13

24. \(x^2 + y^2 - 2x + 4y - 20 = 0\)
Three mutually perpendicular lines in space define three mutually perpendicular planes, called Coordinate planes, which in turn divide the space into eight parts known as octants and the lines are known as Coordinate axes.

- **Coordinate axes**: XOX', YOY', ZOZ'
- **Coordinate planes**: XOY, YOZ, ZOX or XY, YX, ZX planes
- **Octants**: OXYZ, OX'YZ, OXY'Z, OXYZ'
  
  OX' Y'Z, OXY'Z', OX'YZ', OXY'Z'

- Coordinates of a point P are the perpendicular distances of P from three coordinate planes YZ, ZX and XY respectively.

- The distance between the point A(x₁, y₁, z₁) and B(x₂, y₂, z₂) is given by

\[
AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}
\]

- Let P(x₁, y₁, z₁) and Q(x₂, y₂, z₂) be two points in space and let R be a point on line segment PQ such that it divides PQ in the ratio m₁ : m₂

(i) Internally, then the coordinates of R are
\[
\left( \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2} \right)
\]

(ii) externally, then coordinates of R are
\[
\left( \frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2} \right)
\]

- Coordinates of centroid of a triangle whose vertices are \((x_1, y_1, z_1)\), \((x_2, y_2, z_2)\) and \((x_3, y_3, z_3)\) are
\[
\left( \frac{x_1 + y_1 + z_1}{3}, \frac{x_2 + y_2 + z_2}{3}, \frac{x_3 + y_3 + z_3}{3} \right)
\]

**VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)**

1. Find the image of \((-5, 4, -3)\) in \(xz\) plane.
2. Name the octant in which \((-5, 4, -3)\) lies.
3. What is the perpendicular distance of the point \(P(6, 7, 8)\) from \(xy\) plane.
4. Find the distance of point \(P(3, -2, 1)\) from \(z\)-axis.
5. Write coordinates of foot of perpendicular from \((3, 7, 9)\) on \(x\) axis.
6. If the distance between the points \((a, 2, 1)\) and \((1, -1, 1)\) is 5, then find the value(s) of \(a\).
7. Find the coordinates of the foot of perpendicular drawn from the point \((2, 4, 5)\) on \(y\) axis.
8. Let \(A, B, C\) be the feet of perpendiculars from point \(P\) on the \(xy\), \(yz\) and \(xZ\) planes respectively. Find the coordinates of \(A, B, C\) where the point \(P\) is \((4, -3, -5)\).

**SHORT ANSWER TYPE QUESTIONS (4 MARKS)**

9. Show that points \((4, -3, -1), (5, -7, 6)\) and \((3, 1, -8)\) are collinear.
10. Find the point on \(y\)-axis which is equidistant from the point \((3, 1, 2)\) and \((5, 5, 2)\).
11. Determine the point in \(yz\) plane which is equidistant from three points
A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

12. The centroid of \( \triangle ABC \) is at (1,1,1). If coordinates of A and B are (3,–5,7) and (–1, 7, –6) respectively, find coordinates of point C.

13. Find the length of the medians of the triangle with vertices \( A(0, 0, 6) \), \( B(0, 4, 0) \) and \( C(6, 0, 0) \).

14. Find the ratio in which the line joining the points A(2, 1, 5) and B(3, 4, 3) is divided by the plane \( 2x + 2y - 2z = 1 \). Also, find the coordinates of the point of division.

15. If the extremities (end points) of a diagonal of a square are (1,–2,3) and (2,–3,5) then find the length of the side of square.

16. Three consecutive vertices of a parallelogram ABCD are A(6, –2, 4) B(2, 4,–8), C(–2, 2, 4). Find the coordinates of the fourth vertex.

17. If the points A(1, 0, –6), B(–5, 9, 6) and C(–3, p, q) are collinear, find the value of p and q.

18. Show that the point A(1, 3, 0), B(–5, 5, 2), C(–9, –1, 2) and D(–3, –3, 0) are the vertices of a parallelogram ABCD, but it is not a rectangle.

19. The mid points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, –1) Find its vertices and hence find centroid.

20. Find the coordinate of the point P which is five-sixth of way from A(–2, 0, 6) to B(10, –6, –12).

21. Prove that the points (0, –1, –7), (2, 1, –9) and (6, 5, –13) are collinear. Find the ratio in which first point divides the join of the other two.

22. Let A(3, 2, 0), B(5, 3, 2) C(–9, 6, –3) be three points forming a triangle. AD, the bisector of \( \angle ABC \), meets BC in D. Find the coordinates of the point D.

ANSWERS

1. (–5, –4, –3)                2. OX’ YZ’
3. 8                           4. \( \sqrt{13} \) units
5. (3,0,0)                    6. 5, –3
7. (0, 4, 0)  8. (4, –3, 0), (0, –3, –5), (4, 0, –5)

10. (0, 5, 0)  11. (0, 1, 3)

12. (1, 1, 2)  13. 7, \sqrt{34}, 7

14. 5 : 7, \left(\frac{29}{12}, \frac{9}{4}, \frac{25}{6}\right)  15. \sqrt{3} \text{ units}

16. (2, –4, 16)  17. p = 6, q = 2

19. Vertices (–3, 4, –7), (7, 2, 5), (3, 21, 17) centroid \left(\frac{7}{3}, 6, 5\right)

21. 1 : 3 externally  22. \left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)
CHAPTER - 13

LIMITS AND DERIVATIVES

KEY POINTS

- \( \lim_{x \to c} f(x) = l \) if and only if
  \( \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) \)

- \( \lim_{x \to c} \alpha = \alpha \), where \( \alpha \) is a fixed real number.

- \( \lim_{x \to c} x^n = c^n \), for all \( n \in \mathbb{N} \)

- \( \lim_{x \to c} f(x) = f(c) \), where \( f(x) \) is a real polynomial in \( x \).

Algebra of limits

Let \( f, g \) be two functions such that \( \lim_{x \to c} f(x) = l \) and \( \lim_{x \to c} g(x) = m \), then

- \( \lim_{x \to c} [\alpha f(x)] = \alpha \lim_{x \to c} f(x) = \alpha l \) for all \( \alpha \in \mathbb{R} \)

- \( \lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = l \pm m \)

- \( \lim_{x \to c} [f(x)g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x) = lm \)

- \( \lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{l}{m}, \) \( m \neq 0, g(x) \neq 0 \)
\[ \lim_{x \to c} \frac{1}{f(x)} = \frac{1}{\lim_{x \to c} f(x)} = \frac{1}{l} \text{ provided } l \neq 0 \text{ and } f(x) \neq 0 \]

\[ \lim_{x \to c} [(f(x))^n] = \left( \lim_{x \to c} f(x) \right)^n = l^n, \text{ for all } n \in \mathbb{N} \]

Some important theorems on limits

- \[ \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(-x) \]
- \[ \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1} \]
- \[ \lim_{x \to 0} \frac{\sin x}{x} = 1 \text{ where } x \text{ is measured in radians.} \]
- \[ \lim_{x \to 0} x \sin \left( \frac{1}{x} \right) = 0 \]
- \[ \lim_{x \to 0} \tan x = \lim_{x \to 0} x \sin \left( \frac{1}{x} \right) = 1 \left[ \text{Note that } \lim_{x \to 0} \frac{\cos x}{x} \neq 1 \right] \]
- \[ \lim_{x \to 0} \frac{1 - \cos x}{x} = 0 \]
- \[ \lim_{x \to 0} \frac{e^x - 1}{x} = 1 \]
- \[ \lim_{x \to 0} \frac{a^x - 1}{x} = \log_a e \]
- \[ \lim_{x \to 0} \frac{\log(1 + x)}{x} = 1 \]
- \[ \lim_{x \to 0} (1 + x)^{1/x} = e \]
Derivative of a function at any point

- A function $f$ is said to have a derivative at any point $x$ if it is defined in some neighbourhood of the point $x$ and $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$ exists.

The value of this limit is called the derivative of $f$ at any point $x$ and is denoted by $f'(x)$ i.e.

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Algebra of derivatives:

- $\frac{d}{dx}(cf(x)) = c \cdot \frac{d}{dx}(f(x))$ where $c$ is a constant
- $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$
- $\frac{d}{dx} (f(x) \cdot g(x)) = f(x) \cdot \frac{d}{dx} (g(x)) + g(x) \cdot \frac{d}{dx} (f(x))$
- $\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{g(x) \cdot \frac{d}{dx} (f(x)) - f(x) \cdot \frac{d}{dx} (g(x))}{(g(x))^2}$
- If $y = f(x)$ is a given curve then slope of the tangent to the curve at the point $(h, k)$ is given by $\frac{dy}{dx} \bigg|_{(h,k)}$ and is denoted by ‘$m$’.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Evaluate the following Limits:

1. $\lim_{x \to 3} \frac{\sqrt{2x + 3}}{x + 3}$
2. $\lim_{x \to 0} \frac{\sin 3x}{x}$
3. \[ \lim_{x \to 1} \frac{\sqrt{1 + x} - \sqrt{1 - x}}{1 + x} \]

4. \[ \lim_{n \to \infty} \frac{1 + 2 + 3 + \ldots + n}{n^2} \]

5. Find \( n \), if \( \lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80 \), \( n \in \mathbb{N} \).

6. \[ \lim_{x \to 0} \frac{\tan^2 3x}{x^2} \]

7. \[ \lim_{x \to 2} \left( x^2 - 5x + 1 \right) \]

Differentiate the following functions with respect to \( x \):

8. \[ \frac{x}{2} + \frac{2}{x} \]

9. \( x^2 \tan x \)

10. \[ \frac{x}{\sin x} \]

11. \( \log x \)

12. \( 2^x \)

13. If \( f(x) = x^2 - 5x + 7 \), find \( f'(3) \)

14. If \( y = \sin x + \tan x \), find \( \frac{dy}{dx} \) at \( x = \frac{\pi}{3} \)

15. \( 3^x + x^2 + 3^3 \)

16. \( (x^2 - 3x + 2) (x + 2) \)

17. \( e^{3\log x} \) (Hint: \( e^{\log k} = k \))
SHORT ANSWER TYPE QUESTIONS (4 MARKS)

18. If \( f(x) = \begin{cases} 
5x - 4, & 0 < x \leq 1, \\
4x^3 - 3x, & 1 < x < 2 
\end{cases} \), show that \( \lim_{x \to 1} f(x) \) exists.

19. If \( f(x) = \begin{cases} 
\frac{x - |x|}{x}, & x \neq 0, \\
2, & x = 0 
\end{cases} \), show that \( \lim_{x \to 0} f(x) \) does not exist.

20. Let \( f(x) = \begin{cases} 
\cos x, & x \geq k \\
x + k, & x < 0 
\end{cases} \) find the value of constant \( k \), given that \( \lim_{x \to 0} f(x) \) exist.

Evaluate the following Limits :

21. If \( \lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2} \), then find the value of \( k \).

22. Evaluate \( \lim_{x \to 0} \left[ \frac{\sin x^0}{x} \right] \)

23. \( \lim_{x \to 0} \frac{x}{\sqrt{1 + x} - \sqrt{1 - x}} \)

24. Evaluate \( \lim_{x \to 9} \left[ \frac{x^{3/2} - 27}{x^2 - 81} \right] \)

25. \( \lim_{x \to a} \frac{(x + 2)^5 - (a + 2)^5}{x - a} \)

26. Evaluate \( \lim_{x \to 0} \frac{\cos ax - \cos bx}{\cos x - 1} \)

27. \( \lim_{x \to 0} \frac{\tan x - \sin x}{x^3} \)
28. \[ \lim_{x \to 0} \frac{x \tan x}{1 - \cos x} \]

29. \[ \lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3} \]

30. \[ \lim_{x \to a} \frac{\cos x - \cos a}{\cot x - \cot a} \]

31. \[ \lim_{x \to \pi} \frac{1 + \sec^3 x}{\tan^2 x} \]

32. Evaluate \[ \lim_{x \to 0} \frac{e^x - e^{\sin x}}{\sin x - x} \]

33. \[ \lim_{x \to 1} \frac{x - 1}{\log_e x} \]

34. \[ \lim_{x \to e} \frac{\log x - 1}{x - e} \]

35. Evaluate \[ \lim_{x \to 2} \left[ \frac{4}{x^3 - 2x^2} + \frac{1}{2 - x} \right] \]

36. \[ \lim_{x \to a} \frac{\sqrt{a + 2x} - \sqrt{3x}}{\sqrt{3a + x} - 2\sqrt{x}} \]

37. \[ \lim_{x \to 0} \frac{\sin(2 + x) - \sin(2 - x)}{x} \]

Differentiate the following functions with respect to \( x \) from first principle:

38. \[ \sqrt{2x + 3} \]

39. \[ \frac{x^2 + 1}{x} \]

40. \[ e^x \]

41. \[ \log x \]
42. Using definition, find derivative of \( f(x) = \sin^2 x \).

43. \( \cot x \)  

44. \( a^x \)

Differentiate the following functions with respect to \( x \):

45. \( \frac{(3x + 1)(2\sqrt{x} - 1)}{\sqrt{x}} \)

46. If for \( f(x) = \lambda x^2 + \mu x + 12 \), \( f'(4) = 15 \) and \( f'(2) = 11 \) then find \( \lambda \) and \( \mu \).

47. \( (x - \frac{1}{x})\left(x^2 - \frac{1}{x^2}\right) \)

48. \( \frac{\sin x - x \cos x}{x \sin x + \cos x} \)

49. \( x^3 e^x \sin x \)

50. \( x^5 e^x + x^6 \log x \)

51. Differentiate \( \frac{2^x \cot x}{\sqrt{x}} \) w.r.t. \( x \).

52. \( x^n \log_a x \ e^x \)

53. \( \frac{e^x + \log x}{\sin x} \)

54. \( \frac{1 + \log x}{1 - \log x} \)

55. \( e^x \sin x + x^n \cos x \)

56. If \( y = \sqrt{x} + \frac{1}{\sqrt{x}} \), prove that \( 2x \frac{dy}{dx} + y = 2\sqrt{x} \)

57. If \( y = \sqrt{1 - \cos 2x} \) \( \frac{1 + \cos 2x}{1 + \cos 2x} \), find \( \frac{dy}{dx} \)
58. If \( y = \sqrt[3]{x} + \sqrt[3]{a} \), prove that

\[
(2xy) \frac{dy}{dx} = \frac{x}{a} - \frac{a}{x}
\]

59. For the curve \( f(x) = (x^2 + 6x - 5)(1-x) \), find the slope of the tangent at \( x = 3 \).

**LONG ANSWER TYPE QUESTIONS (6 MARKS)**

Differentiate the following functions with respect to \( x \) from first principle:

60. Using definition, find derivative of \( f(x) = \sqrt{\cos x} \)

61. Find the derivative of \( f(x) = \tan(ax + b) \) by first principle.

Evaluate the following limits:

62. Evaluate \( \lim_{x \to 0} \left[ \frac{a^x - b^x}{x} \right] \)

63. Evaluate \( \lim_{x \to \frac{\pi}{2}} \left[ \frac{\cos^2 x}{\sqrt{2} - \sqrt{1 + \sin x}} \right] \)

**ANSWERS**

1. \( \frac{1}{2} \)

2. 3

3. \( \frac{1}{\sqrt{2}} \)

4. \( \frac{1}{2} \)

5. 5

6. 9

7. \(-5\)

8. \( \frac{1}{2} - \frac{2}{x^2} \)

9. \( 2 \tan x + x^2 \sec^2 x \)

10. \( \csc x - x \cot x \csc x \)
11. 0
12. $2^x \log_e 2$
13. 1
14. $k = \frac{9}{2}$
15. $3^x \log_e^3 + 3x^2$
16. $3x^2 - 2x - 4$
17. $3x^2$
18. $k = 1$
19. $k = \frac{8}{3}$
20. $\frac{\pi}{180}$
21. 1
22. $\frac{1}{4}$
23. $\frac{5}{2} (a + 2)^{\frac{3}{2}}$
24. $\frac{a^2 - b^2}{c}$
25. $\frac{1}{2}$
26. 2
27. $\frac{1}{16}$
28. $\sin^3 a$
29. $-\frac{3}{2}$
30. $-1$
31. 1
32. $\frac{1}{e}$
33. $-1$
34. $\frac{2}{3\sqrt{3}}$
35. $2 \cos 2$
36. $\frac{1}{\sqrt{2x + 3}}$
37. $\frac{x^2 - 1}{x^2}$
38. $e^x$
39. $\frac{1}{x}$
40. $\sin 2x$
41. $-\csc^2 x$
42. $a^x \log_e a$
45. \[ 6 - \frac{3}{2} x^{-\frac{1}{2}} + \frac{1}{2} x^{\frac{3}{2}} \]

46. \[ \lambda = 1 ; \mu = 7 \]

47. \[ 3x^2 + \frac{1}{x^2} - 1 - \frac{3}{x^4} \]

48. \[ \frac{x^2}{(x \sin x + \cos x)^2} \]

49. \[ x^2e^x (3 \sin x + x \sin x + x \cos x) \]

50. \[ x^4 (5e^x + xe^x + x + 6x \log x) \]

51. \[ \frac{2^n}{\sqrt{x}} \left( \log 2 \cot x - \cos e^2x - \frac{\cot x}{2^n} \right) \]

52. \[ e^x n^{-1} \left( n \log a x + \log a + x \log a x \right) \]

53. \[ \frac{\left( e^x + \frac{1}{x} \right) \sin x - \left( e^x + \log x \right) \cos x}{\sin^2 x} \]

54. \[ \frac{2}{x (1 - \log x)^2} \]

55. \[ e^x \left( 1 + \frac{1}{x} + x + \log x \right) \]

56. \[ \sec^2 x \]

57. \[ -46 \]

58. \[ \frac{1}{2} \sqrt{\tan x \sin x} \]

59. \[ \log \left( \frac{a}{b} \right) \]

60. \[ 4\sqrt{2} \]
A sentence is called a statement if it is either true or false but not both.

The denial of a statement p is called its negative and is written as ~p and read as not p.

Compound statement is made up of two or more simple statements. These simple statements are called component statements.

‘And’, ‘or’, ‘if–then’, ‘only if’ ‘If and only if’ etc. are connecting words, which are used to form a compound statement.

Compound statement with ‘And’ is

(a) true if all its component statements are true

(b) false if any of its component statement is false

Compound statement with ‘Or’ is

(a) true when at least one component statement is true

(b) false when both the component statements are false

A statement with “if p then q” can be rewritten as

(a) p implies q

(b) p is sufficient condition for q

(c) q is necessary condition for p

(d) p only if q

(e) (~q) implies (~p)
• If, in a compound statement containing the connective “or” all the alternatives cannot occur simultaneously, then the connecting word “or” is called as exclusive “or”.

• If, in a compound statement containing the connective “or”, all the alternative can occur simultaneously, then the connecting word “or” is called as inclusive “or”.

• Contrapositive of the statement \( p \Rightarrow q \) is the statement \( \sim q \Rightarrow \sim p \)

• Converse of the statement \( p \Rightarrow q \) is the statement \( q \Rightarrow p \)

• “For all”, “For every” are called universal quantifiers

• A statement is called valid or invalid according as it is true or false.

**VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)**

Identify which of the following are statements (Q. No 1 to 7)

1. Prime factors of 6 are 2 and 3.
2. \( x^2 + 6x + 3 = 0 \)
3. The earth is a planet.
4. There is no rain without clouds.
5. All complex numbers are real numbers.
6. Tomorrow is a holiday.
7. Answer this question.

Write negation of the following statements (Q. No 8 to 12)

8. All men are mortal.
9. \( \pi \) is not a rational number.
10. Every one in Spain speaks Spanish.
11. Zero is a positive number.

Write the component statements of the following compound statements
12. 7 is both odd and prime number.
13. All integers are positive or negative.
14. 36 is a multiple of 4, 6 and 12.
15. Jack and Jill went up the hill.

Identify the type ‘Or’ (Inclusive or Exclusive) used in the following statements (Q. No. 16 to 19)

16. Students can take French or Spanish as their third language.
17. To enter in a country you need a visa or citizenship card.
18. $\sqrt{2}$ is a rational number or an irrational number.
19. 125 is a multiple of 5 or 8.

Which of the following statements are true or false. Give Reason. (Question No. 20 to 23)

20. 48 is a multiple of 6, 7 and 8
21. $\pi > 2$ and $\pi < 3$.
22. Earth is flat or it revolves around the moon.
23. $\sqrt{2}$ is a rational number or an irrational number.

Identify the quantifiers in the following statements (Q. No. 24 to 26)

24. For every integer $p$, $\sqrt{p}$ is a real number.
25. There exists a capital for every country in the world.
26. There exists a number which is equal to its square.

Write the converse of the following statements (Q. No. 27 to 30)

27. If a number $x$ is even then $x^2$ is also even.
28. If $3 \times 7 = 21$ then $3 + 7 = 10$
29. If $n$ is a prime number then $n$ is odd.
30. Some thing is cold implies that it has low temperature.

Write contrapositive of the following statements (Q. No. 31 to 36)

31. If 5 > 7 then 6 > 7.

32. \( x \) is even number implies that \( x^2 \) is divisible by 4.

33. If a triangle is equilateral, it is isosceles.

34. Only if he does not tire he will win.

35. If a number is divisible by 9, then it is divisible by 3.

36. Something is cold implies that it has low temperature.

37. Check the validity of the statement ‘An integer \( x \) is even if and only if \( x^2 \) is even.’

**ANSWERS**

1. Statement
2. Not a statement
3. Statement
4. Statement
5. Statement
6. Not a Statement
7. Not a statement
8. All men are not mortal
9. \( \pi \) is a rational number.
10. Everyone in Spain doesn’t speak Spanish.
11. Zero is not a positive number.
12. 7 is an odd number. 7 is a prime number.
13. All integer are positive. All integers are negative.
14. 36 is a multiple of 4.
   
   36 is a multiple of 6.
   
   36 is a multiple of 12.
15. Jack went up the hill.
   
   Jill went up the hill.
16. Exclusive
17. Inclusive
18. Exclusive
19. Exclusive
20. False, 48 is not a multiple of 7
21. False, \(\pi\) lies between 3 and 4
22. False
23. True
24. For every
25. For every, there exists
26. There exists
27. If \(x^2\) is even then \(x\) is even
28. If \(3 + 7 = 10\) then \(3 \times 7 = 21\)
29. If \(n\) is odd then \(n\) a prime number.
30. If some thing has low temperature then it is cold.
31. If \(6 \leq 7\) then \(5 \leq 7\)
32. If \(x^2\) is not divisible by 4 then \(x\) is not even.
33. If a trignale is not iosceles, then it is not equilateral.
34. If he tires, then he will not win.
35. If a number is not divisible by 3, then it is not divisible by 9.
36. If something does not have low temperature, then it is not cold.
37. Valid.
CHAPTER - 15

STATISTICS

- Range = Largest observation – smallest observation.
- Mean deviation for ungrouped data or raw data

\[
M.D.(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{n}
\]

\[
M.D.(M) = \frac{\sum |x_i - M|}{n}, \quad M = \text{Median}
\]

- Mean deviation for grouped data (Discrete frequency distribution and Continuous frequency distribution).

\[
M.D.(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{N}
\]

\[
M.D.(M) = \frac{\sum f_i |x_i - M|}{N}
\]

where \( N = \sum f_i \)

- Standard deviation ‘\( \sigma \)’ is positive square root of variance.

\[
\sigma = \sqrt{\text{Variance}}
\]

- Variance \( \sigma^2 \) and standard deviation (SD) \( \sigma \) for ungrouped data

\[
\sigma^2 = \frac{1}{n} \sum (x_i - \bar{x})^2
\]

\[
\text{SD} = \sigma = \sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2}
\]
• Standard deviation of a discrete frequency distribution

\[ \sigma = \frac{1}{N} \sqrt{\sum f_i (x_i - \bar{x})^2} = \frac{1}{N} \sqrt{\sum f_i x_i^2 - (\sum f_i x_i)^2} \]

• Standard deviation of a continuous frequency distribution

\[ \sigma = \frac{1}{N} \sqrt{\sum f_i (x_i - \bar{x})^2} = \frac{1}{N} \sqrt{\sum f_i x_i^2 - (\sum f_i x_i)^2} \]

where \( x_i \) are the midpoints of the classes.

• Short cut method to find variance and standard deviation

\[ \sigma^2 = \frac{h^2}{N^2} \left[ N \sum f_i y_i^2 - (\sum f_i y_i)^2 \right] \]

\[ \sigma = \frac{h}{N} \sqrt{N \sum f_i y_i^2 - (\sum f_i y_i)^2} \]

where \( y_i = \frac{x_i - A}{h} \)

• Coefficient of variation (C.V) = \( \frac{\sigma}{\bar{x}} \times 100 \), \( \bar{x} \neq 0 \)

• If each observation is multiplied by a positive constant \( k \) then variance of the resulting observations becomes \( k^2 \) times of the original value and standard deviation becomes \( k \) times of the original value.

• If each observation is increased by \( k \), where \( k \) is positive or negative, the variance and standard deviation remains same.

• Standard deviation is independent of choice of origin but depends on the scale of measurement.

• The series having higher coefficient of variation is called more variable than the other. While the series having lesser coefficient of variation is called more consistent or more stable. For series with equal means the series with lesser standard deviation is more stable.
VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find range of the following data:
   \[
   \begin{array}{ccccccc}
   x_i & 3 & 4 & 7 & 9 & 10 & 15 \\
   f_i & 2 & 12 & 5 & 8 & 9 & 3 \\
   \end{array}
   \]

2. Find range of the following data:
   \[
   \begin{array}{ccccccc}
   \text{Class} & 10-19 & 20-29 & 30-39 & 40-49 & 50-59 \\
   \text{Frequency} & 5 & 4 & 5 & 3 & 2 \\
   \end{array}
   \]

3. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what are the new mean and the new variance?

4. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what are the new mean and the new standard deviation?

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

5. The frequency distribution
   \[
   \begin{array}{ccccccc}
   x & A & 2A & 3A & 4A & 5A & 6A \\
   f & 2 & 1 & 1 & 1 & 1 & 1 \\
   \end{array}
   \]
   Where A is a positive integer, has a variance of 160. Determine the value of A.

   Calculate the mean deviation about mean for the following data

6. 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

   Calculate the mean deviation about median for the following data

7. 40, 42, 44, 46, 48

8. 22, 24, 30, 27, 29, 35, 25, 28, 41, 42
Calculate the mean, variance and standard deviation of the following data

9. 6, 7, 10, 12, 13, 4, 8 12
10. 15, 22, 27, 11, 9, 21, 14, 9
11. Coefficients of variation of two distribution are 60 and 80 and their standard deviations are 21 and 36. What are their means?
12. Life of bulbs produced by two factors A and B given below

<table>
<thead>
<tr>
<th>Length of Life in (Hours)</th>
<th>Factory A (Number of bulbs)</th>
<th>Factory B (Number of bulbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>550-650</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>650-750</td>
<td>22</td>
<td>60</td>
</tr>
<tr>
<td>750-850</td>
<td>52</td>
<td>24</td>
</tr>
<tr>
<td>850-950</td>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>950-1050</td>
<td>16</td>
<td>12</td>
</tr>
</tbody>
</table>

The bulb of which factory are more consistent from point of view of length.

13. The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observations are 1, 2 and 6. Find the other two observations.
14. Calculate the possible values of x if standard deviation of the numbers 2, 3, 2x and 11 is 3.5.
15. Mean and standard deviation of the data having 18 observations were found to be 7 and 4 respectively. Later it was found that 12 was miscopied as 21 in calculation. Find the correct mean and the correct standard deviation.

**LONG ANSWER TYPE QUESTIONS (6 MARKS)**

Calculate the mean deviation about mean for the following data.

<table>
<thead>
<tr>
<th>Size</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marks</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>30</td>
<td>3</td>
</tr>
<tr>
<td>50</td>
<td>5</td>
</tr>
<tr>
<td>70</td>
<td>1</td>
</tr>
<tr>
<td>90</td>
<td>1</td>
</tr>
</tbody>
</table>
Calculate the mean deviation about median for the following data

<table>
<thead>
<tr>
<th>Marks</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
</tr>
</tbody>
</table>

19. Find the mean and variance of the frequency distribution given below

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>$f_i$</th>
<th>$x$</th>
<th>$x^2$</th>
<th>$(x+1)^2$</th>
<th>$2x$</th>
<th>$2x+1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>x–2</td>
<td>x</td>
<td>$x^2$</td>
<td>$2x$</td>
<td>$2x+1$</td>
</tr>
</tbody>
</table>

where $x$ is a positive integer. Determine the mean and standard deviation of the marks.

20. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test:

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>$f_i$</th>
<th>$x–2$</th>
<th>$x$</th>
<th>$x^2$</th>
<th>$(x+1)^2$</th>
<th>$2x$</th>
<th>$2x+1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>x–2</td>
<td>x</td>
<td>$x^2$</td>
<td>$(x+1)^2$</td>
<td>$2x$</td>
<td>$2x+1$</td>
</tr>
</tbody>
</table>

21. Find the mean and variance of the frequency distribution given below

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ≤ $x$ &lt; 3</td>
<td>6</td>
</tr>
<tr>
<td>3 ≤ $x$ &lt; 5</td>
<td>4</td>
</tr>
<tr>
<td>5 ≤ $x$ &lt; 7</td>
<td>5</td>
</tr>
<tr>
<td>7 ≤ $x$ &lt; 10</td>
<td>1</td>
</tr>
</tbody>
</table>

22. Calculate the mean deviation about mean (Q. No. 22 & 23)

<table>
<thead>
<tr>
<th>Classes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-20</td>
<td>2</td>
</tr>
<tr>
<td>20-30</td>
<td>3</td>
</tr>
<tr>
<td>30-40</td>
<td>8</td>
</tr>
<tr>
<td>40-50</td>
<td>14</td>
</tr>
<tr>
<td>50-60</td>
<td>8</td>
</tr>
<tr>
<td>60-70</td>
<td>3</td>
</tr>
<tr>
<td>70-80</td>
<td>2</td>
</tr>
</tbody>
</table>

23. Marks

<table>
<thead>
<tr>
<th>$x$</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>5</td>
</tr>
<tr>
<td>10-20</td>
<td>8</td>
</tr>
<tr>
<td>20-30</td>
<td>15</td>
</tr>
<tr>
<td>30-40</td>
<td>16</td>
</tr>
<tr>
<td>40-50</td>
<td>6</td>
</tr>
</tbody>
</table>

24. Find the mean deviation about the median

<table>
<thead>
<tr>
<th>Weight (in kg.)</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-40</td>
<td>5</td>
</tr>
<tr>
<td>40-50</td>
<td>8</td>
</tr>
<tr>
<td>50-60</td>
<td>15</td>
</tr>
<tr>
<td>60-70</td>
<td>16</td>
</tr>
<tr>
<td>70-80</td>
<td>6</td>
</tr>
<tr>
<td>80-90</td>
<td>5</td>
</tr>
</tbody>
</table>
25. Calculate the mean deviation about median for the following distribution

<table>
<thead>
<tr>
<th>Classes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>5</td>
</tr>
<tr>
<td>10-20</td>
<td>10</td>
</tr>
<tr>
<td>20-30</td>
<td>20</td>
</tr>
<tr>
<td>30-40</td>
<td>5</td>
</tr>
<tr>
<td>40-50</td>
<td>10</td>
</tr>
</tbody>
</table>

26. Find the mean and standard deviation for the following

<table>
<thead>
<tr>
<th>C.I.</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>25-35</td>
<td>21</td>
</tr>
<tr>
<td>35-45</td>
<td>12</td>
</tr>
<tr>
<td>45-55</td>
<td>30</td>
</tr>
<tr>
<td>55-65</td>
<td>45</td>
</tr>
<tr>
<td>65-75</td>
<td>50</td>
</tr>
<tr>
<td>75-85</td>
<td>37</td>
</tr>
<tr>
<td>85-95</td>
<td>5</td>
</tr>
</tbody>
</table>

27. The mean and standard deviation of some data taken for the time to complete a test are calculated with following results

Number of observations = 25, mean = 18.2
Standard deviation = 3.25 seconds

Further another set of 15 observations $x_1, x_2, ..., x_{15}$ also in $\sum_{i=1}^{15} x_i^2 = 5524$.

Calculate the standard deviation based on all 40 observations.

28. Find the coefficient of variation of the following data

<table>
<thead>
<tr>
<th>Classes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-29</td>
<td>5</td>
</tr>
<tr>
<td>30-39</td>
<td>12</td>
</tr>
<tr>
<td>40-49</td>
<td>15</td>
</tr>
<tr>
<td>50-59</td>
<td>20</td>
</tr>
<tr>
<td>60-69</td>
<td>18</td>
</tr>
<tr>
<td>70-79</td>
<td>10</td>
</tr>
<tr>
<td>80-89</td>
<td>6</td>
</tr>
<tr>
<td>90-99</td>
<td>4</td>
</tr>
</tbody>
</table>

29. Which group of students is more stable- Group A or Group B?

<table>
<thead>
<tr>
<th>Classes</th>
<th>Number in Group A</th>
<th>Number in Group B</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-15</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>15-25</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>25-35</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>35-45</td>
<td>30</td>
<td>33</td>
</tr>
<tr>
<td>45-55</td>
<td>23</td>
<td>15</td>
</tr>
<tr>
<td>55-65</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>65-75</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

30. Mean and standard deviation of 100 observations were found to be 40 and 10 respectively. If at the time of calculation two observation were wrongly taken as 30 and 70 in place of 3 and 27 respectively. Find correct standard deviation.
31. For a distribution $\Sigma (x_i - 5) = 3$, $\Sigma (x_i - 5)^2 = 43$ and total number of items is 18. Find the mean and standard deviation.

ANSWERS

1. 12
2. 50
3. 48, 256
4. 35, 4
5. $A = 7$
6. 2.33
7. 2.4
8. 4.7
9. 9, 9.25, 3.04
10. 16, 38.68, 6.22
11. 35, 45
12. Factory A
13. 4, 9
14. 3, 7/3
15. 6.5, 2.5
16. 2.8
17. 16
18. 0.8
19. 10.1
20. Mean 2.8, Standard deviation 1.12
21. Mean 5.5, Variance 4.26
22. 10
23. 9.44
24. 11.44
25. 9
26. 61.1, 15.93
27. 3.87
28. 31.24
29. Group A
30. 10.24
31. Mean 5.17, Standard deviation 1.53
Random Experiment: If an experiment has more than one possible outcome and it is not possible to predict the outcome in advance then experiment is called random experiment.

Sample Space: The collection of all possible outcomes of a random experiment is called sample space associated with it. Each element of the sample space(set) is called a sample point.

Some examples of random experiments and their sample spaces

(i) A coin is tossed
   \[ S = \{H, T\}, \ n(S) = 2 \]
   Where \( n(S) \) is the number of elements in the sample space \( S \).

(ii) A die is thrown
    \[ S = \{1, 2, 3, 4, 5, 6\}, \ n(S) = 6 \]

(iii) A card is drawn from a pack of 52 cards
     \[ n(S) = 52. \]

(iv) Two coins are tossed
     \[ S = \{HH, HT, TH, TT\}, \ n(S) = 4. \]

(v) Two dice are thrown
    \[ S = \left\{ \begin{array}{c} 11, 12, 13, 14, 15, 16, \\ 21, 22, \cdots, 26, \\ \vdots \\ 61, 62, \cdots, 66 \end{array} \right\} \]
    \[ n(S) = 36 \]

(vi) Two cards are drawn from a well shuffled pack of 52 cards
(a) with replacement \( n(S) = 52 \times 52 \)
(b) without replacement \( n(S) = \binom{52}{2} \)

- **Event**: A subset of the sample space associated with a random experiment is called an event.
- **Simple Event**: Simple event is a single possible outcome of an experiment.
- **Compound Event**: Compound event is the joint occurrence of two or more simple events.
- **Sure Event**: If event is same as the sample space of the experiment, then event is called sure event.
- **Impossible Event**: Let \( S \) be the sample space of the experiment, \( \emptyset \subseteq S \), \( \emptyset \) is an event called impossible event.
- **Exhaustive and Mutually Exclusive Events**: Events \( E_1, E_2, E_3 \cdots \cdots - E_n \) are mutually exclusive and exhaustive if \( E_1 \cup E_2 \cup E_3 \cup \cdots \cdots \cup E_n = S \) and \( E_i \cap E_j = \emptyset \) for all \( i \neq j \)
- **Probability of an Event**: For a finite sample space \( S \) with equally likely outcomes, probability of an event \( A \) is \( P(A) = \frac{n(A)}{n(S)} \), where \( n(A) \) is number of elements in \( A \) and \( n(S) \) is number of elements in set \( S \) and \( 0 \leq P(A) \leq 1 \).
  - (a) If \( A \) and \( B \) are any two events then
    \[
P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)
    \]
    \[
    = P(A) + P(B) - P(A \text{ and } B)
    \]
  - (b) If \( A \) and \( B \) are mutually exclusive events then
    \[
P(A \cup B) = P(A) + P(B)
    \]
  - (c) \( P(A) + P(\overline{A}) = 1 \)
    or \( P(A) + P(\text{not } A) = 1 \)
  - (d) \( P(\text{Sure event}) = 1 \)
(e) \( P(\text{impossible event}) = 0 \)

- \( P(A - B) = P(A) - P(A \cap B) = P(A \cap \overline{B}) \)
- If \( S = \{w_1, w_2, \ldots, w_n\} \) then
  (i) \( 0 \leq P(w_i) \leq 1 \) for each \( w_i \in S \)
  (ii) \( P(w_1) + P(w_2) + \ldots + P(w_n) = 1 \)
  (iii) \( P(A) = \sum P(w_i) \) for any event \( A \) containing elementary events \( w_i \).

- \( P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B) \)
- Addition theorem for three events

  Let \( E, F \) and \( G \) be any three events associated with a random experiment, then

  \[
P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(E \cap G) + P(E \cap F \cap G)
\]

- Let \( E \) and \( F \) be two events associated with a random experiment then
  (i) \( P(E \cap \overline{F}) = P(E) - P(E \cap F) \)
  (ii) \( P(\overline{E} \cap F) = P(F) - P(E \cap F) \)
  (iii) \( P(\overline{E} \cap \overline{F}) = P(\overline{E} \cup \overline{F}) = 1 - P(E \cup F) \)

**VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)**

Describe the Sample Space for the following experiments (Q. No. 1 to 4)

1. A coin is tossed twice and number of heads is recorded.
2. A card is drawn from a deck of playing cards and its colour is noted.
3. A coin is tossed repeatedly until a tail comes up for the first time.
4. A coin is tossed. If it shows head we draw a ball from a bag consisting of 2 red and 3 black balls. If it shows tail, coin is tossed again.

5. What is the probability of 53 Sundays and 53 Mondays in a leap year.

6. What is the probability of 53 Sundays or 53 Mondays in a leap year.

7. Three coins are tossed. Write three events which are mutually exclusive and exhaustive.

8. A coin is tossed n times. What is the number of elements in its sample space?

   If E, F and G are the subsets representing the events of a sample space S. What are the sets representing the following events? (Q No 9 to 12).

9. Out of three events atleast two events occur.

10. Out of three events only one occurs.

11. Out of three events only E occurs.

12. Out of three events exactly two events occur.

13. If probability of event A is 1 then what is the type of event ‘not A’?

14. One number is chosen at random from the numbers 1 to 21. What is the probability that it is prime?

15. What is the probability that a given two digit number is divisible by 15?

16. If \( P(A \cup B) = P(A) + P(B) \), then what can be said about the events A and B?

17. If A and B are mutually exclusive events then what is the probability of \( A \cap B \) ?

18. If A and B are mutually exclusive and exhaustive events then what is the probability of \( A \cup B \) ?

19. A box contain 1 red and 3 identical white balls. Two balls are drawn at random in succession with replacement. Write sample space for this
20. A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.

21. A card is drawn from a pack of 52 cards. Find the probability of getting:
   (i) a jack or a queen
   (ii) a king or a diamond
   (iii) a heart or a club
   (iv) either a red or a face card.
   (v) neither a heart nor a king
   (vi) neither an ace nor a jack

**SHORT ANSWER TYPE QUESTIONS (4 MARKS)**

22. Find the probability that in a random arrangement of the letters of the word UNIVERSITY two I's come together.

23. An urn contains 5 blue and an unknown number $x$ of red balls. Two balls are drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, find $x$.

24. Out of 8 points in a plane 5 are collinear. Find the probability that 3 points selected at random form a triangle.

25. Find the probability of at most two tails or at least two heads in a toss of three coins.

26. A, B and C are events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.8$, $P(A \cap B) = 0.08$, $P(A \cap C) = 0.28$ and $P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \geq 0.75$ then prove that $P(B \cap C)$ lies in the interval $[0.23, 0.48]$

   \[Hint : P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)\]}
27. For a post three persons A, B and C appear in the interview. The probability of A being selected is twice that of B and the probability of B being selected is twice that of C. The post is filled. What are the probabilities of A, B and C being selected?

28. A and B are two candidates seeking admission in college. The probability that A is selected is 0.5 and the probability that both A and B are selected is utmost 0.3. Show that the probability of B being selected is utmost 0.8.

29. \( S = \{1, 2, 3, \ldots, 30\}, \ A = \{x : x \text{ is multiple of 7}\} \ B = \{x : x \text{ is multiple of 5}\} \ C = \{x : x \text{ is a multiple of 3}\} \). If \( x \) is a member of \( S \) chosen at random find the probability that

(i) \( x \in A \cup B \)

(ii) \( x \in B \cap C \)

(iii) \( x \in A \cap C' \)

30. One number is chosen at random from the number 1 to 100. Find the probability that it is divisible by 4 or 10.

31. A bag contains 5 red, 4 blue and an unknown number of \( m \) green balls. Two balls are drawn. If probability of both being green is \( \frac{1}{7} \) find \( m \).

32. The number lock of a suitcase has 4 wheels with 10 digits, i.e. from 0 to 9. The lock open with a sequence of 4 digits with repeats allowed. What is the probability of a person getting the right sequence to open the suitcase?

33. If \( A \) and \( B \) are any two events having \( P(A \cup B) = \frac{1}{2} \) and \( P(A) = \frac{2}{3} \) then find the \( P(\overline{A} \cap B) \).

34. Three of the six vertices of a regular hexagon are chosen at random. What is probability that the triangle with these vertices is equilateral?

35. A typical PIN (Personal identification number) is a sequence of any four symbol chosen from the 26 letters in the alphabet and ten digits. If all PINs are equally likely, what is the probability that a randomly chosen PIN contains a repeated symbol?
36. If A, B, C are three mutually exclusive and exhaustive events of an experiment such that \(3 \ P(A) = 2 \ P(B) = P(C)\), then find the value of \(P(A)\).

37. An urn contains 9 red, 7 white and 4 black balls. If two balls are drawn at random. Find the probability that the balls are of same colour.

**ANSWERS**

1. \(\{0, 1, 2\}\)  
2. \(\{\text{Red, Black}\}\)
3. \(\{T, HT, HHT, HHHT........\}\)
4. \(\{HR_1, HR_2, HB_1, HB_2, HB_3, TH, TT\}\)
5. \(\frac{1}{7}\)  
6. \(\frac{3}{7}\)
7. \(A = \{HHH, HHT, HTH, THH\}\)  
   \(B = \{HTT, THT, HTT\}\)
   \(C = \{TTT\}\)
8. \(2^n\)
9. \((E \cap F \cap G) \cup (E' \cap F \cap G) \cup (E \cap F' \cap G) \cap (E \cap F \cap G')\)
10. \((E \cap F' \cap G) \cup (E' \cap F \cap G') \cup (E' \cap F' \cap G)\)
11. \((E \cap F' \cap G')\)
12. \((E \cap F \cap G') \cup (E \cap F' \cap G) \cup (E' \cap F \cap G)\)
13. Impossible event
14. \(\frac{8}{21}\)
15. \(\frac{1}{15}\)
16. Mutually exclusive events.
17. 0
18. 1
19. \(S = \{RR, RW, WR, WW\}\)
20. \(S = \{RW, WR, WW\}\)
21. (i) \(\frac{2}{13}\);  
   (ii) \(\frac{4}{13}\);
(iii) \( \frac{1}{2} \);  \hspace{1cm}  (iv) \( \frac{8}{13} \);

(v) \( \frac{9}{13} \);  \hspace{1cm}  (vi) \( \frac{11}{13} \)

22. \( \frac{1}{5} \)  \hspace{1cm}  23. 3

24. \( \frac{23}{28} \)  \hspace{1cm}  25. \( \frac{7}{8} \)

26. 0.23 \( \leq \) P(B) \( \leq \) 0.48  \hspace{1cm}  27. \( \frac{4}{7}, \frac{2}{7}, \frac{1}{7} \)

28. (i) \( \frac{1}{3} \), (ii) \( \frac{1}{15} \), (iii) \( \frac{1}{10} \)  \hspace{1cm}  29. \( \frac{1}{7} \)

30. \( \frac{3}{10} \)  \hspace{1cm}  31. \( \frac{3}{10} \).

32. \( \frac{1}{10000} \)  \hspace{1cm}  33. \( \frac{1}{6} \)

34. \( \frac{1}{10} \)  \hspace{1cm}  35. \( \frac{2,65,896}{1,67,9,616} \)

36. \( \frac{2}{11} \)  \hspace{1cm}  037. \( \frac{63}{190} \)
MODEL TEST PAPER-I

Time : 3 hours
Maximum Marks : 100

General Instructions:
(i) All questions are compulsory.
(ii) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, section B comprises of 13 questions of 4 marks each and section C comprises of 7 questions of six marks each.
(iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.
(iv) There is no overall choice. However internal choice have been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculator is not permitted.

SECTION A

1. Find the value of \( \sin \left( -\frac{11\pi}{3} \right) \).

2. Evaluate \( \lim_{x \to 3} \frac{e^x - e^3}{x - 3} \).

3. Find the distance between the parallel lines \( 15x + 8y - 34 = 0 \) and \( 15x + 8y + 31 = 0 \).

4. Find the derivative of \( x^5(3 - 6x^{-9}) \) with respect to \( x \).

5. If \( P(A) = \frac{3}{5} \) and \( P(B) = \frac{1}{5} \), find \( P(A \text{ or } B) \), if \( A \) and \( B \) are mutually exclusive events.
6. Find the domain of the function \( f(x) = \frac{x^2 + 3x + 5}{x^2 - 5x + 4} \).

SECTION B

7. Convert the complex number \( \frac{-16}{1 + i\sqrt{3}} \) into polar form.

8. Find domain and range of the real function \( f(x) = \frac{1}{1 - x^2} \).

9. If the sum of \( n \) terms of an A.P. is \( (3n^2 + 5n) \) and its \( m \)th term is 164, find the value of \( m \).

   OR

   Find the sum to \( n \) terms of the series:

   \[
   \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \ldots
   \]

10. Find the co-ordinates of the foot of the perpendicular drawn from the point \((-1, 3)\) to the line \(3x - 4y - 16 = 0\).

11. Write the contrapositive of the following statements:
   (i) If you are born in India, then you are a citizen of India.
   (ii) If a triangle is equilateral, it is isosceles.

   Write the converse of the following statements:
   (iii) If a number \( n \) is even, then \( n^2 \) is even.
   (iv) If \( x \) is a prime number, then \( x \) is odd.

12. Evaluate: \( \lim_{x \to \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2} \).

   OR

   Find the derivative of \( f \) from the first principle, where \( f \) is given by:

   \[ f(x) = \frac{2x + 3}{x - 2} \]
13. Find the term independent of x in the expansion of \( \left( \frac{3}{2} x^2 - \frac{1}{3x} \right)^9 \).

14. How many numbers greater than 10,00,000 can be formed by using the digits 1, 2, 0, 2, 4, 2, 4?

15. Find the equation of the circle passing through the points (2, 3) and (-1, 1) and whose centre is on the line \( x - 3y - 11 = 0 \).

   OR

   Find the equation of the ellipse, with major axis along the x-axis and passing through the points (4, 3) and (-1, 4).

16. Let \( A = \{1, 2, 3, \ldots, 20\} \). Define a relation \( R \) from \( A \) to \( A \) by

\[
R = \{(a, b) : a - 2b = 0, \ a, b \in A\}.
\]

Depict the relation using roaster form. Write domain and range of the relation.

17. Four cards from a pack of 52 cards are chosen at random. Find the number of ways when :

(i) four cards are of same suit.

(ii) four cards belong to four different suits.

(iii) four cards are face cards.

18. Find the co-ordinates of a point on y-axis which are at a distance of \( 5\sqrt{2} \) from the point \( P(3, -2, 5) \).

19. Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that

(i) both Anil and Ashima will not qualify the examination.

(ii) atleast one of them will not qualify the examination and

(iii) only one of them will qualify the examination.

   OR

In a relay race there are 5 teams A, B, C, D and E.
(i) What is the probability that A, B and C finish first, second and third respectively?

(ii) What is the probability that A, B and C are first three to finish (in any order). (Assume that all finishing orders are equally likely)?

SECTION C

20. Prove the following by using the principle of Mathematical induction for all \( n \in \mathbb{N} \):

\[
\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \cdots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}.
\]

21. Find the value of \( \tan \left( \frac{13\pi}{12} \right) \).

OR

Prove that \( \cos 2x \cos \frac{x}{2} - \cos 3x \cos \frac{9x}{2} = \sin 5x \sin \frac{5x}{2} \).

22. Calculate mean, variance and standard deviation for the following distribution:

<table>
<thead>
<tr>
<th>Classes</th>
<th>30–40</th>
<th>40–50</th>
<th>50–60</th>
<th>60–70</th>
<th>70–80</th>
<th>80–90</th>
<th>90–100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>3</td>
<td>7</td>
<td>12</td>
<td>15</td>
<td>8</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

OR

The mean and standard deviation of a group of 100 observations were found to be 20 and 3 respectively. Later on, it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

23. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read H and T, 8 read both T and I and 3 read all the three newspapers. Find.

(a) the number of people who read at least one of the newspapers.

(b) the number of people who read exactly one newspaper. Write the names of any 3 newspapers. What is the importance of newspaper in our life? Explain briefly.
24. Find the value of n so that \( \frac{a^{n+1} + b^{n+1}}{a^n + b^n} \) may be the geometric mean between \( a \) and \( b \).

25. Find general solution of equation \( \sin x + \sin 2x + \sin 3x + \sin 4x = 0 \).

26. The longest side of a triangular national park is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangular national park is at least 61 cm, find the minimum length of the shortest side. Name any one national park of India. What are the needs of national parks in India?

ANSWERS

SECTION A

1. \( \frac{\sqrt{3}}{2} \)

2. \( e^3 \)

3. \( \frac{65}{17} \) units

4. \( 15x^4 + \frac{24}{x^5} \)

5. \( \frac{4}{5} \)

6. Domain = \( \mathbb{R} - \{1, 4\} \)

SECTION B

7. \( \frac{-16}{1 + i\sqrt{3}} = -4 + 4\sqrt{3} \)

\[ \mu = 8 ; \theta = \frac{2\pi}{3} \]

So polar form is \( 8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) \)

8. \( f(x) \) is real if \( 1 - x^2 \neq 0 \)

if \( x^2 \neq 1 \)

if \( x \neq 1 \)

\[ \therefore \text{ Domain } = \mathbb{R} - \{-1, 1\} \]
Consider \( y \in \text{domain} \)

Let \( y = f(x) \quad \Rightarrow \quad y = \frac{1}{1-x^2} \)

\[ \Rightarrow \quad \frac{1}{y} = 1 - x^2 \quad \Rightarrow \quad x^2 = 1 - \frac{1}{y} \]

\[ \Rightarrow \quad x = \pm \sqrt{y - 1} \]

\( x \) is real of \( \frac{y - 1}{y} \geq 0 \) and \( y \neq 0 \)

\[
\begin{array}{cccccc}
+ & - & + & - & + & \\
0 & 1 & & & & \\
\end{array}
\]

If \( y < 0 \) or \( y \leq 1 \)

\[ \therefore \text{Range } (-\infty, 0) \cup [1, \infty] \]

9. Given, \( S_n = 3n^2 + 5n \) \quad ...(i)

So, \( S_{n-1} = 3(n-1)^2 + 5(n-1) \) \quad ...(ii)

Eq. (i) – (ii), \( S_n - S_{n-1} = 6n + 2 \)

\[ T_n = 6n + 2 \]

Hence, \( T_m = 6m + 2 = 164 \)

Or

\[ m = 27 \]

OR

Let \( T_n \) denotes the \( n^{th} \) term, then

\[ T_n = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \]

Put \( n = 1, 2, 3, \ldots \), we get

\[ T_1 = \frac{1}{1} - \frac{1}{2} \]
\[ T_2 = \frac{1}{2} - \frac{1}{3} \]
\[ T_3 = \frac{1}{3} - \frac{1}{4} \]
\[ \vdots \]
\[ T_n = \frac{1}{n} - \frac{1}{n+1} \]

Adding these, we get
\[ S_n = 1 - \frac{1}{n+1} = \frac{n}{n+1} \]

10. Slope of the line \(3x - 4y - 16 = 0\) ...(i) is \(\frac{3}{4}\).

Slope of any line \(\perp\) to it = \(-\frac{4}{3}\)

Equation of the line passing through \((-1, 3)\) and having the slope \(-\frac{4}{3}\) is given by
\[ y - 3 = -\frac{4}{3}(x + 1) \]
\[ 4x + 3y - 5 = 0 \] ...(ii)

Solving (i) and (ii), we get
\[ x = \frac{68}{25} \text{ and } y = -\frac{49}{25} \]

So the required foot of \(\perp\) is \(\left(\frac{68}{25}, -\frac{49}{25}\right)\).
11. (i) If you are not a citizen of India, then you were not born in India.

(ii) If a triangle is not isosceles, then it is not equilateral.

(iii) If a number $n^2$ is even, then $n$ is even.

(iv) If a number $x$ is odd, then $x$ is a prime number.

12. \[
\lim_{x \to \frac{n}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2}
\]

Put $x = \frac{\pi}{2} + h$

\[
= \lim_{h \to 0} \frac{1 + \cos 2\left(\frac{\pi}{2} + h\right)}{\left[\pi - 2\left(\frac{\pi}{2} + h\right)\right]^2}
\]

\[
= \lim_{h \to 0} \frac{1 + \cos(\pi + 2h)}{(\pi - \pi - 2h)^2}
\]

\[
= \lim_{h \to 0} \frac{1 - \cos 2h}{4h^2}
\]

\[
= \lim_{h \to 0} \frac{2 \sin^2 h}{4h^2}
\]

\[
= \frac{1}{2}
\]

OR

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{2(x + h) + 3}{(x + h - 2)} - \frac{2x + 3}{x - 2}
\]

\[
= \lim_{h \to 0} \frac{2x + 3}{h}
\]
\[
\lim_{h \to 0} \frac{(2x + 2h + 3)(x - 2) - (2x + 3)(x + h - 2)}{h(x + h - 2)(x - 2)} = \\
\lim_{h \to 0} \frac{(2x + 3)(x - 2) + 2h(x - 2) - (2x + 3)(x - 2) - h(2x + 3)}{h(x - 2)(x + h - 2)}
\]

\[f'(x) = \frac{7}{(x - 2)^2}\]

13. 
\[T_{r + 1} = \binom{9}{r} \left(\frac{3}{2}x^2\right)^{9-r} \left(-\frac{1}{3x}\right)^r\]

\[T_{r + 1} = \binom{9}{r} \frac{3^{9-2r} \cdot 8^{18-3r} \cdot (-1)^r}{2^{9-r}}\]

From term independent of \(x\), \(r = 6\)

\[\therefore \quad T_7 = \binom{9}{6} \cdot \frac{3^{-3} \cdot (-1)^6}{2^{3}}\]

\[T_7 = \frac{7}{8}\]

14. Since 10,000,000 is a 7-digit number and the number of digits to be used is also 7, therefore, the numbers to be counted will be 7-digit only. Also, the numbers have to be greater than 10,000,000; so they can begin either with 1, 2 or 4.

The number of numbers beginning with 1 = \(\frac{6!}{3!2!} = 60\)

Total numbers beginning with 2 = \(\frac{6!}{2!2!} = 180\)

and the total no. beginning with 4 = \(\frac{6!}{3!} = 120\)

\[\therefore \quad \text{The required no. of numbers} = 60 + 180 + 120 = 360\]
15. Let centre of reqd. circle be (h, k)

\[ \sqrt{(h - 2)^2 + (k - 3)^2} = \sqrt{(h + 1)^2 + (k - 1)^2} \]

\[ \Rightarrow \quad 6h + 4k = 11 \quad \text{...(i)} \]

Since centre lie on line \( x - 3y = 11 \)

\[ h - 3k = 11 \quad \text{...(ii)} \]

Solving equation (i) and (ii) to get \( h = \frac{7}{2}, \quad k = -\frac{5}{2} \)

Also \( \quad \text{Radius} = \sqrt{\left(\frac{7}{2} - 2\right)^2 + \left(\frac{5}{2} - 3\right)^2} = \frac{130}{4} \)

Equation \( x^2 + y^2 - 7x + 5y - 14 = 0 \)

OR

The standard equation of the ellipse is \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \).

Since the point \((4, 3)\) and \((-1, 4)\) lie on the ellipse,

\[ \Rightarrow \quad \frac{16}{a^2} + \frac{9}{b^2} = 1 \]

and \[ \quad \frac{1}{a^2} + \frac{16}{b^2} = 1 \]

Solving, \( a^2 = \frac{247}{7} \), \( b^2 = d \)

Hence, the required equation of the ellipse is :

\[ \frac{x^2}{\frac{247}{7}} + \frac{y^2}{\frac{247}{15}} = 1 \quad \Rightarrow \quad 7x^2 + 15y^2 = 247 \]

16. \( R = \{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7), (16, 8), (18, 9), (20, 10)\} \)

Domain : \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}

Range : \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}
17. (i) No. of ways when four cards of same suit
\[ = 4 \times \binom{13}{4} \]
\[ = 2860 \]

(ii) No. of ways when four cards are one from each suit
\[ = \binom{13}{1} \times \binom{13}{1} \times \binom{13}{1} \times \binom{13}{1} \]
\[ = (13)^4 \]

(iii) No. of ways when four cards are face cards
\[ = \binom{12}{4} \]
\[ = 495 \]

18. Let a point on y-axis be Q(0, b, 0). This is at a distance \(5\sqrt{2}\) from P(3, −2, 5).

\[ |PQ| = 5\sqrt{2} \]

\[ \sqrt{(3-0)^2 + (-2-b)^2 + (5-0)^2} = 5\sqrt{2} \]

\[ b = -6, 2 \]

So, the pts. are (0, −6, 0) and (0, 2, 0).

19. \(P(E) = 0.05, P(E) = 0.10, P(E \cap F) = 0.02\)

(i) \(P(E' \cap F') = P(E \cap F)'\)
\[ = 1 - P(E \cup F) \]
\[ = 1 - [P(E) + P(F) - P(E \cap F)] \]
\[ = 1 - 0.13 = 0.87 \]

(ii) \(P(\text{at least one of them will not qualify})\)
\[ = 1 - P(\text{both of them will qualify}) \]
\[ = 1 - 0.02 = 0.98 \]

(iii) \(P(\text{only one of them will qualify})\)
\[ = P(E \cap F' \text{ or } E' \cap F) \]
\[ P(E \cap F') + P(E' \cap F) = P(E) - P(E \cap F) + P(F) - P(E \cap F) \]
\[ = 0.05 - 0.02 + 0.10 - 0.02 = 0.11 \]

OR

If we consider the sample space consisting of all finishing orders in the first 3 places, we will have \( 5^3 = 60 \). Sample points, each with a probability of \( \frac{1}{60} \).

(i) A, B, and C finish first, second and third respectively. There is only one finishing order for this \( i.e., \) ABC.

\[ \therefore P(A, B \text{ and } C \text{ finish 1st, 2nd and 3rd resp.}) = \frac{1}{60}. \]

(ii) A, B C are the first 3 finishers. There will be 3! arrangements for A, B and C. Therefore, the sample points corresponding to this event will be 3! in number.

So, \( P(A, B \text{ and } C \text{ are first 3 to finish}) = \frac{3!}{60} = \frac{6}{60} = \frac{1}{10} \)

20. Let \( P(n) : \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2n - 1)(2n + 3)} \)

Now \( P(1) \) is \( \frac{1}{3.5} = \frac{1}{3(5)} \) which is true, so \( P(1) \) is true.

Let \( P(m) \) be true for \( m \in \mathbb{N} \).

\[ \Rightarrow \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2m + 1)(2m + 3)} = \frac{m}{3(2m + 3)} \quad \text{...(i)} \]

We shall prove that \( P(m + 1) \) is true \( i.e., \)

\[ \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2m + 1)(2m + 3)} + \frac{1}{[2(m + 1) + 1][2(m + 1) + 3]} = \frac{m + 1}{3[2(m + 1) + 3]} \]
Adding \( \frac{1}{(2m + 3) (2m + 5)} \) to both sides of (i), we get

\[
\frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2m + 1) (2m + 3)} + \frac{1}{(2m + 3) (2m + 5)}
\]

\[
= \frac{m}{3(2m + 3)} + \frac{1}{(2m + 3) (2m + 5)}
\]

\[
= \frac{m(2m + 5) + 3}{3(2m + 3) (2m + 5)} = \frac{2m^2 + 5m + 3}{3(2m + 3) (2m + 5)}
\]

\[
= \frac{m + 1}{3(2m + 5)}
\]

\( \Rightarrow \) P(m + 1) is true.

Thus, P(1) is true and P(m) \( \Rightarrow \) P(m + 1) is true. Hence, by induction P(n) is true for all n.

21. \( \tan \left( \frac{13\pi}{12} \right) = \tan \left( \pi + \frac{\pi}{12} \right) = \tan \frac{\pi}{12} = \tan \left( \frac{\pi}{4} - \frac{\pi}{6} \right) \)

\[
= \frac{\tan \frac{\pi}{4} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{4} \tan \frac{\pi}{6}}
\]

\[
= \frac{1 - \frac{1}{\sqrt{3}}}{\sqrt{3} + 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}
\]

OR

\[
\text{L.H.S.} = \frac{1}{2} \left[ 2\cos 2x \cos \frac{x}{2} - 2\cos \frac{9x}{2} \cos 3x \right]
\]

\[
= \frac{1}{2} \left[ \cos \left( 2x + \frac{x}{2} \right) + \cos \left( 2x - \frac{x}{2} \right) - \cos \left( \frac{9x}{2} + 3x \right) \right]
\]

\[
- \cos \left( \frac{9x}{2} - 3x \right)
\]

\[
= \frac{1}{2} \left[ \cos \left( \frac{5x}{2} \right) + \cos \left( \frac{3x}{2} \right) - \cos \left( \frac{15x}{2} \right) \right]
\]

\[
- \cos \left( \frac{3x}{2} \right)
\]
\[
\frac{1}{2} \left[ \cos \frac{5x}{2} - \cos \frac{15x}{2} \right] \\
= \frac{1}{2} \left[ \frac{5x}{2} + \frac{15x}{2} \sin \frac{5x}{2} - \frac{15x}{2} \sin \frac{5x}{2} \right] \\
= -\sin 5x \sin \left( \frac{-5x}{2} \right) \\
= \sin 5x \sin \frac{5x}{2}
\]

22. Let the assumed mean \( A = 65 \), Here \( h = 10 \).

<table>
<thead>
<tr>
<th>Class</th>
<th>Frequency</th>
<th>Mid-point</th>
<th>( y = x - 65 )</th>
<th>( y_i^2 )</th>
<th>( f_i y_i )</th>
<th>( f_i y_i^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 – 40</td>
<td>3</td>
<td>35</td>
<td>-3</td>
<td>9</td>
<td>-9</td>
<td>27</td>
</tr>
<tr>
<td>40 – 50</td>
<td>7</td>
<td>45</td>
<td>-2</td>
<td>4</td>
<td>-14</td>
<td>28</td>
</tr>
<tr>
<td>50 – 60</td>
<td>12</td>
<td>55</td>
<td>-1</td>
<td>1</td>
<td>-12</td>
<td>12</td>
</tr>
<tr>
<td>60 – 70</td>
<td>15</td>
<td>65</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>70 – 80</td>
<td>8</td>
<td>75</td>
<td>1</td>
<td>1</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>80 – 90</td>
<td>3</td>
<td>85</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>12</td>
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<tr>
<td>90 – 100</td>
<td>2</td>
<td>95</td>
<td>3</td>
<td>9</td>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-15</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>N = 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\bar{x} = A + \frac{\sum f_i y_i}{N} \times h = 65 - \frac{15}{50} \times 10 = 62
\]

\[
\sigma^2 = \frac{\frac{h^2}{N} \left[ \sum y_i^2 - (\sum f_i y_i)^2 \right]}{N^2}
\]

\[
= \frac{(10)^2}{(50)^2} \left[ 50 \times 105 - (-15)^2 \right]
\]

\[
= \frac{1}{25} [5250 - 225] = 201
\]
\[ \sigma = \sqrt{201} = 14.18 \]

OR

Observed mean = 20 and observed S.D. = 3

\[ \therefore \text{Observed sum of observations} = 20 \times 100 = 2000 \]

Also,

\[
\begin{align*}
\text{S.D.} = 3 & \Rightarrow \text{Variance} = 9 \\
\frac{\text{Observed sum of squares}}{100} - (\text{mean})^2 & = 9 \\
\Rightarrow \text{Observed sum of squares} & = 100[9 + (20)^2] = 40900 \\
\text{Sum of remaining 97 observations} & = 2000 - [21 + 21 + 18] = 1940 \\
\text{Sum of squares of remaining 97 observations} & = 40900 - (21^2 + 21^2 + 18^2) = 39694 \\
\therefore \text{Mean of remaining 97 items} & = \frac{1940}{97} = 20 \\
\text{and S.D. of remaining 97 items} & = \sqrt{\frac{39694}{97} - \left(\frac{1940}{97}\right)^2} \\
& = 3.036
\end{align*}
\]

23. If \( H \) = Set of people who read newspaper H

\( T \) = Set of people who read newspaper T

\( I \) = Set of people who read newspaper I

\[
\begin{align*}
n(\cup) & = 60, \ n(H) = 25, \ n(I) = 26, \ n(l) = 26 \\
n(H \cap I) & = 9, \ n(H \cap T) = 11, \ n(T \cap I) = 8, \ n(H \cap T \cap I) = 3
\end{align*}
\]
(a) No. of people who read at least one of the newspaper

\[ n(H \cup T \cup I) = m(H) + n(T) + n(I) - n(H \cap T) - n(T \cap I) - (H \cap I) + n(H \cap T \cap I) \]

\[ = 52 \]

(b) No. of people who read exactly one newspaper

\[ \begin{align*}
&= [n(H) - n(H \cap T) - n(H \cap I) + n(H \cap T \cap I)] \\
&+ [n(T) - n(H \cap T) - n(I \cap T) + n(H \cap T \cap I)] \\
&+ [n(I) - n(H \cap I) - n(T \cap I) + n(H \cap T \cap I)] \\
&= [25 - 11 + 3] + [26 - 11 - 8 + 3] + [26 - 9 - 8 + 3] \\
&= 8 + 10 + 12 \\
&= 30
\end{align*} \]

\[ \text{24.} \]

\[ \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \text{G.M. between } a \text{ and } b \]

\[ \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab} \]

\[ a^{n+1} + b^{n+1} = \sqrt{ab}(a^n + b^n) \]

\[ a^{n + \frac{1}{2}}[a^{1/2} - b^{1/2}] = b^{n + \frac{1}{2}}[a^{1/2} - b^{1/2}] \]

\[ \frac{a^{n + \frac{1}{2}}}{b^{n + \frac{1}{2}}} = 1 \]

\[ n + \frac{1}{2} = 0 \Rightarrow n = -\frac{1}{2} \]
25. \( \sin x + \sin 2x + \sin 3x + \sin 4x = 0 \)

\[
2 \sin \frac{5x}{2} \cos \frac{3x}{2} + 2 \sin \frac{5x}{2} \cos \frac{x}{2} = 0
\]

\[
2 \sin \frac{5x}{2} \left[ \cos \frac{3x}{2} + \cos \frac{x}{2} \right] = 0
\]

\[
2 \sin \frac{5x}{2} \cos x \cdot \cos \frac{x}{2} = 0
\]

\( x = \frac{2n\pi}{5}, \quad x = (2n + 1)\frac{\pi}{2} \) or \( x = (2n + 1)\pi, \quad n \in \mathbb{Z} \)

26. Let the shortest side be \( x \) cm, then the longest side is \( 3x \) cm and the third side \( (3x - 2) \) cm.

\[
\therefore \quad \text{Perimeter} = x + 3x + 3x - 2 = 7x - 2
\]

\( 7x - 2 \geq 61 \)

\( x \geq 9 \)

So, minimum length of the shortest side is 9 cm.
MODEL TEST PAPER – II

Time : 3 hours
Maximum Marks : 100

General Instructions :
(i) All questions are compulsory.
(ii) The question paper consists of 26 questions divided into three Sections A, B and C. Section A comprises of 6 questions of one mark each. Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
(iii) There is no overall choice. However, an internal choice has been provided in 4 questions.

SECTION A

1. Find domain and range of the function \( f(n) = e^n \).

2. If there are 12 persons in a party, and if each two of them shake hands with each other, how many hand-shakes happen in the party?

3. Find the equation of a vertical line passing through \((-2, 6)\).

4. Find perpendicular distance of the point \((6, 7, 8)\) from xy-plane.

5. Find the point on y-axis which is at a distance \(\sqrt{10}\) from the point \((1, 2, 3)\).

6. Write the contrapositive of the following statement :
   
   \( x \) is an even number implies that \( x \) is divisible by 4.

SECTION B

7. If \( \mathbb{U} = \{1, 2, 3, 4, 5, \ldots, 15\}, A \) is the set of odd natural no. < 15.
   
   Find (i) \( A - B \) (ii) \( B' \)
Use the properly of sets to prove that for all the sets A and B, 
\[ A - (A \cap B) = A - B. \]

8. Find the domain of the function \( f \) given by

\[ f(x) = \frac{1}{\sqrt{\lfloor x \rfloor^2 - \lfloor x \rfloor - 6}} \]
where \( \lfloor x \rfloor \) is greatest integer function.

9. If \( R = \{(x, y) : x, y \in \mathbb{N}, x + 2y = 13\} \). Write \( R \) as an ordered pair and also find the domain and Range.

10. Solve the equation \( \sin \theta + \sin 3\theta + \sin 5\theta = 0. \)

11. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I’s not come together?

12. There are 20 points in a plane, no three of which are in the same straight line, excepting 4 points which are collinear. Find the (i) number of straight lines obtained from the pairs of these points (ii) number of triangles that can be formed with the vertices as these points.

13. If the coefficients of \( x \) and \( x^2 \) in the expansion of \( (1 + x)^m (1 - x)^n \) are 3 and -6 respectively. Find the values of \( m \) and \( n. \)

OR

Find the term independent of \( x \) in the expansion of \( \left( \frac{\sqrt{x}}{\sqrt[3]{3}} + \frac{\sqrt[3]{3}}{2x^2} \right)^{10}. \)

14. Find the equation of straight line passing through the intersection of \( x - y + 1 = 0 \) and \( 2x - 3y + 5 = 0 \) and which is at a distance of \( \frac{7}{5} \) from the point \( (3, 2). \)

OR

Find the equation of lines through the point \( (3, 2) \) which makes an angle of 45° with the line \( x - 2y = 3. \)

15. Find the equation of a circle concentric with the circle \( 2x^2 + 2y^2 - 6x + 8y + 1 = 0 \) and of double its area.

16. Find the equation of the ellipse which passes through the point \( (-3, 1) \) and has eccentricity \( \frac{\sqrt{2}}{5} \), with x-axis as its major axis and centre at the origin.
17. Evaluate \[ \lim_{x \to \frac{\pi}{6}} \frac{2\sin^2 x + \sin n - 1}{2\sin^2 x - 3\sin x + 1} \]

OR

Find the derivative of \( f(x) = \tan (2x + 5) \) by using first principle of derivative.

18. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

19. A bag contains 50 tickets numbered 1, 2, 3, ... 50 out of which five are drawn at random and arranged in ascending order of magnitude \( (x_1 < x_2 < x_3 < x_4 < x_5) \). Find the probability that \( x_3 = 30 \).

SECTION C

20. From 50 students taking examination is Mathematics, Physics and Chemistry, each of the student has passed in at least one of the subject, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 Mathematics and Chemistry and at most 20 Physics and Chemistry. What is the largest possible number that could have passed all three examination?

21. Find the value of \( \tan 7.5^\circ - \tan 22.5^\circ + \tan 82.5^\circ - \tan 67.5^\circ \)

OR

If \( x \cos \theta = y \cos \left( \theta + \frac{2\pi}{3} \right) = z \cos \left( \theta + \frac{4\pi}{3} \right) \) then find the value of \( xy + yz + zx \).

22. Prove the following by principle of mathematical induction for all \( n \in \mathbb{N} \).

\[ \frac{1}{1.2.3} + \frac{1}{2.3.5} + \ldots + \frac{1}{n(n + 1)(n + 2)} = \frac{n(n + 3)}{4(n + 1)(n + 2)} \]

23. Solve \( 2x^2 - (3 + 7i)x - (3 - 9i) = 0 \).
24. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest piece if the third piece is to be at least 5 cm longer than the second? These three pieces are to be used as display boards for ‘SAY NO TO JUNK FOOD’. Would you like to participate in this campaign and why?

25. Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2 R^n = S^n$.

26. From a frequency distribution consisting of 18 observations, the mean and the standard deviation were found to be 7 and 4 respectively. But on comparison with the original data, it was found that a figure 12 was miscopied as 21 in calculation. Calculate the correct mean and standard deviation.

OR

Find mean and standard deviation for the following distribution:

<table>
<thead>
<tr>
<th>Classes</th>
<th>0-30</th>
<th>30-60</th>
<th>60-90</th>
<th>90-120</th>
<th>120-150</th>
<th>150-180</th>
<th>180-210</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>10</td>
<td>3</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

ANSWERS

SECTION A

1. Domain = R, Range = $(0, \infty)$  
2. 66  
3. $x + 2 = 0$  
4. 8 units  
5. $(0, 2, 0)$  
6. If x is not divisible by 4 then x is not an even number

SECTION B

7. (i) $A - B = \{1, 9\}$  
(ii) $B^c = \{1, 4, 6, 8, 9, 10, 12, 14, 15\}$
8. Domain = \((-\infty, -2) \cup [4, \infty)\)

9. \(R = \{(1, 6), (3, 5), (5, 4), (7, 3), (9, 2), (11, 1)\}\)
   Domain = \{1, 3, 5, 7, 9, 11\}
   Range = \{1, 2, 3, 4, 5, 6\}

10. \(\theta = \frac{n\pi}{3}, \quad n \in \mathbb{Z}\)

11. 33810

12. (i) 185 \quad (ii) 1136

13. \(m = 12, \quad n = 9 \quad \text{or} \quad \frac{5}{12}\)

14. \(3x - 4y + 6 = 0, \quad 4x - 3y + 1 = 0\)
   OR
   \(3x - y = 7 \quad \text{or} \quad x + 3y = 9\)

15. \(4x^2 + 4y^2 - 12x + 16y - 21 = 0\)

16. \(3x^2 + 5y^2 = 32\)

17. \(2 \sec^2 (2x + 5)\)

18. 0.55

19. \(\frac{551}{15134}\)

20. 14

21. \(2\sqrt{6} \quad \text{or} \quad 0\)

22. \(x = \frac{3}{2} + \frac{1}{2}i \quad \text{or} \quad x = 3i\)

24. \(8 \leq x \leq 22\)

26. 6.5, 2.5
MODEL TEST PAPER – III

Time : 3 hours
Maximum Marks : 100

General Instructions :
(i) All questions are compulsory.
(ii) The question paper consists of 26 questions divided into three Sections A, B and C. Section A comprises of 6 questions of one mark each. Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
(iii) There is no overall choice. However, an internal choice has been provided in 4 questions.

SECTION A

1. Write the following intervals in set builder form :
   \((-\infty, -1]\)

2. Find the value of \(n\), if \(\frac{1}{5!} + \frac{1}{6!} = \frac{n}{6!}\)

3. Find the coordinates of the end points of the latus rectum of the parabola \(y^2 = 8x\).

4. Find the components statements of the following :
   A square is a quadrilateral and its four sides equal.

5. Write the contrapositive of the following statement :
   “If 7 is greater than 5, then 8 is greater than 6”

6. Write the negation of the following statements :
   For every positive real number \(x\), the number \(x - 1\) is also positive.
SECTION B

7. Find the domain and the range of the function \( f(x) = 3x^2 - 5 \). Also find \( f(-3) \) and the numbers which are associated with the number 43 in its range.

8. If \( A = \{2, 4, 6, 9\} \), \( B = \{4, 6, 18, 27, 54\} \) and a relation \( R \) from \( A \) to \( B \) is defined by \( R = \{(a, b) : a \in A, b \in B, a \text{ is a factor of } b \text{ and } a < b\} \), then find \( R \) in roster form. Also find its domain and range.

9. If \( x \) lies in the first quadrant and \( \cos x = \frac{8}{17} \), then find the value of \( \cos \left( \frac{\pi}{6} + x \right) + \cos \left( \frac{\pi}{4} - x \right) + \cos \left( \frac{2\pi}{3} - x \right) \).

10. If \( \sec x \cos 5x + 1 = 0 \) where \( 0 < x < \pi/2 \), then find the value of \( x \).

11. If \( z = x + iy \) and \( z^2 = z^3 - 2 \), show that \( 3(x^2 + y^2) - 20x + 32 = 0 \).

12. By using PMI, prove that \( 7^n - 2^n \) is divisibly by 5 for all \( n \in \mathbb{A} \).

13. By using PMI prove that \( (1 + x)^n \geq 1 + nx \) forall \( n \in \mathbb{N} \) and \( n > -1 \).

14. How many four letter words can be formed by using the letters of the word 'INEFFECTIVE'?

15. If the coefficient of \( 5^{th} \), \( 6^{th} \) and \( 7^{th} \) terms in the expansion of \( (1 + x)^n \) are in A.P., then find the values of \( n \).

16. Find three numbers in G.P. whose sum is 21 and sum of their squares is 189.

   OR

   Find the sum of the series \( (3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \ldots \) to \( n \) terms.

17. A ray of light passing through the point \( P(1, 2) \) reflects on the x-axis at the point \( A \) and the reflected ray passes through the point \( Q(5, 3) \). Find the coordinate of \( A \).

   OR

   A straight line is drawn through the point \( A(2, 1) \) making an angle of \( 30^\circ \) with the positive direction of x-axis. Find the coordinates of two points on it at a distance of 2 units from \( A \) on either side of it.
18. Find the equation of the circle circumscribing the triangle formed by the straight lines \( x + y = 6 \), \( 2x + y = 4 \) and \( x + 2y = 5 \).

OR

Find the equation of the hyperbola with centre at origin, transverse axis along x-axis, eccentricity \( \sqrt{5} \) and sum of whose semi-axes is 9.

19. If the points A(1, 0, –6), B(–3, p, q) and C(–5, 9, 6) are collinear, find the values of p and q.

SECTION C

20. A survey shows that 63% people watch news channel A whereas 76% people watch news channel B. If \( x \)% of the people watch both news channels, the prove that 39 \( \leq \) \( x \) \( \leq \) 63.

21. In a \( \triangle ABC \), if \( \frac{b + c}{11} = \frac{c + a}{12} = \frac{a + b}{13} \), then prove that \( \frac{\cos A}{7} = \frac{\cos B}{19} = \frac{\cos C}{25} \).

22. Solve the following quadratic equation \( x^2 - (2 + 5i) x + (-6 + 4i) = 0 \).

23. A solution of 8% boric acid is to be diluted by adding 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of 8% solution, how many litres of 2% solution will have to be added?

24. Evaluate \( \lim_{y \to 0} \frac{(x + y) \sec (x + y) - x \sec x}{y} \)

OR

Find the derivative of \( \frac{\cos x}{x} \) from first principle.

25. A four digit numbers greater than 5000 are randomly formed from the digits 0, 1, 3, 5 and 7, what is probability of forming a number divisible by 5 when

(i) the digits may be repeated

(ii) the repetition of digits not allowed.
26. Find the mean, variance and standard deviation for the following data:

<table>
<thead>
<tr>
<th>Classes</th>
<th>10-20</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
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<td>9</td>
<td>6</td>
<td>6</td>
<td>3</td>
</tr>
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</table>

OR

From the data given below, find which group is more variable A or B.

<table>
<thead>
<tr>
<th>Classes</th>
<th>10-20</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
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<tr>
<td>Group A</td>
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<tr>
<td>Group B</td>
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<td>20</td>
<td>30</td>
<td>25</td>
<td>43</td>
<td>15</td>
<td>7</td>
</tr>
</tbody>
</table>

ANSWERS

1. \( \{x : x \in \mathbb{R}, x \leq -1\} \)
2. 7
3. \((2, 4) \) and \((2, -4)\)
4. \( P : \) A square is a quadrilateral, \( q : \) A square has all its sides equal.
5. If 8 is not greater than 6 then 7 is not greater than 5.
6. There exist a positive real number \( x \) such that \( x - 1 \) is not positive.
7. Domain = \( \mathbb{R} \), Range \([-5, \infty)\), Number = -4, 4
8. \( R = \{(2, 4), (2, 6), (2, 18), (2, 54), (6, 18), (6, 54), (9, 18), (9, 27), (9, 54)\} \)

Domain = \{2, 6, 9\}

Range = \{4, 6, 18, 27, 54\}
9. \[ \frac{23}{17} \left( \frac{\sqrt{3}}{2} - \frac{1}{2} + \frac{1}{\sqrt{2}} \right) \]
10. \( \frac{\pi}{6}, \frac{\pi}{4} \)

14. 1422
15. 7, 14

16. 3, 6, 12, or 12, 6, 3 or \( n(4n^2 + 9n + 6) \)

17. \( \left( \frac{13}{5}, 0 \right) \) or \( (2 + \sqrt{3}, 2) \) and \( (2 - \sqrt{3}, 0) \)

18. \( x^2 + y^2 - 17x - 19y + 50 = 0 \) or \( 4x^2 - y^2 = 36 \)

19. \( P = 6, q = 2 \)

22. 2 + 3i, 2i

23. \( 320 \leq x \leq 1280 \) where \( x = 2\% \) boric acid solution

24. \( \sec x \left( x \tan x + 1 \right) \) or \( \frac{-(x \sin x + \cos x)}{x^2} \)

25. \( \frac{99}{249} \) or \( \frac{33}{83}, \frac{3}{8} \)

26. 56, 422.33, 20.55 or Group B is more variable