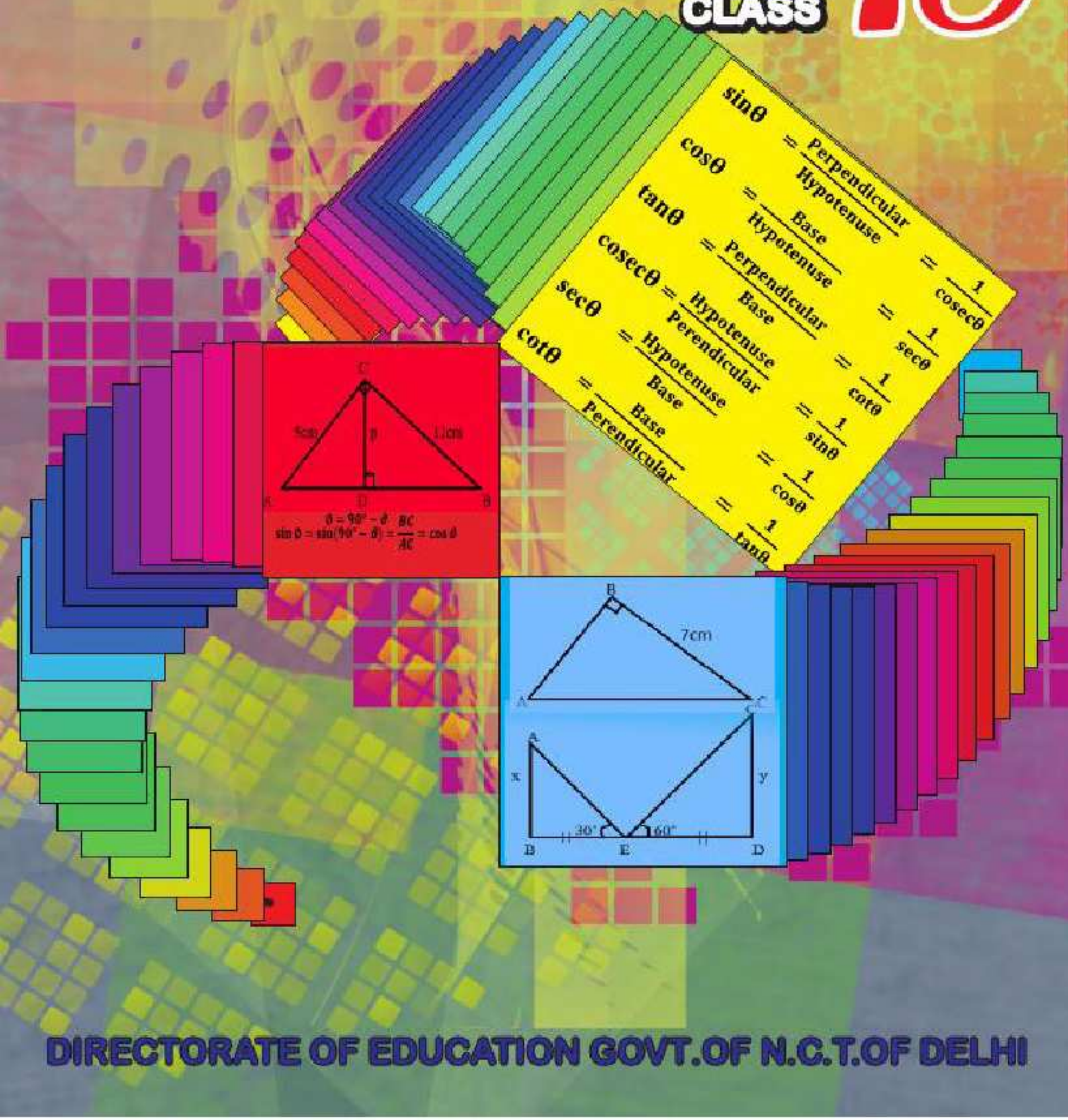
MENTAL MATHS

QUESTION BANK CLASS 10



DIRECTORATE OF EDUCATION GOVT.OF N.C.T.OF DELHI

MENTAL MATHS CLASS X

2024-25

DIRECTORATE OF EDUCATION

GOVT. OF NCT OF DELHI

ASHOK KUMAR
IAS

सत्यमेव जयते

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MESSAGE

The eloquent words of Galileo Galilei resonate: 'The laws of nature are written by the hand of God in the language of mathematics.' In this profound observation, the great astronomer awakened humanity to the paramount importance of mathematics. Within our school education system, mathematics holds a pivotal role, with a dedicated focus on foundational numeracy and literacy.

This year marks a significant milestone, as the project extends its reach to Government-Aided schools and introduces Level IV for classes 11th and 12th as well.

In the competitive arena, where time is of the essence, a strong command over mathematics is indispensable. These skills are not only prized in competitive exams but also wield significant influence in the realms of entrepreneurship and innovation. Mental Maths, with its transformative impact, enhances students' number sense, fosters an understanding of relationships between quantities, and cultivates logical thinking for problem-solving.

The meticulously crafted Mental Maths Question Banks recognize the diverse abilities, needs, and interests of students. As the saying goes, 'Nothing great can be achieved without consistent and persistent hard work'. Heartfelt congratulations to the State Core Team members, District Cordinators and Subject Experts for their silent and steadfast dedication to bring forth these impactful publications.

(Ashok Kumar)

BHUPESH CHAUDHARY, IAS
DIRECTOR (EDUCATION & SPORTS)

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MESSAGE

Beyond mere numbers and equations, Mathematics serves as a foundational language, intricately woven into the fabric of everything from the technology we rely on to the scientific principles shaping our understanding of the cosmos.

Enter Mental Maths – a captivating art of calculation sans paper or tools, a dance of numbers performed within the confines of the mind. It's not just about crunching numbers; it's about empowerment. Mental Maths nurtures the comprehension of place value, fortifies basic operations, and establishes a robust foundation for grappling with more complex mathematical concepts in the future.

Engaging in Mental Maths includes exercising multiple cognitive processes – memory, attention, and critical thinking. Studies reveal that regular Mental Maths exercises contribute to maintaining cognitive reserve, postponing the onset of age-related memory loss, and fending off other cognitive declines. In essence, Mental Maths keeps our minds agile and adaptable, akin to the benefits of physical activity for our bodies. It becomes the catalyst for swift decision-making and adept situational adaptation.

A heartfelt commendation goes to the dedicated State Core Team members and subject experts who meticulously crafted the Mental Maths Question Banks. These resources, tailored for students in Government and Government-Aided Schools of the Directorate of Education are a testament to their sincere efforts and the wise guidance of the Project Director of Mental Maths. It brings me immense pleasure to present this Mental Maths Question Bank to students, encouraging them to weave the magic of Mental Maths into the tapestry of their daily lives.

(BHUPESH CHAUDHARY)

विकास कालिया
क्षेत्रीय शिक्षा निदेशक
उत्तर एवं मध्य क्षेत्र,
पुरस्कार एवं कल्याण शाखाएँ,
पत्राचार विद्यालय एवं
रा. मुक्त विद्यालयी शिक्षा शाखाएँ
परियोजना निदेशक: मेंटल मैथ्स

सत्यमेव जयते

VIKAS KALIA
Regional Director of Education
Central & North,
Awards & Welfare Branches,
Patrachar Vidyalaya &
NIOS (Branches)
Project Director: Mental Maths

MESSAGE

At the tender age of 16, RPraggnanandhaa, the prodigious talent in Indian chess, sent waves through the global chess community by outsmarting Chess Grandmaster Magnus Carlsen in a lightning-fast game at the Airthings Masters Rapid Chess Tournament. His secret weapon was the remarkable ability for mental calculations. This young genius effortlessly combines his exceptional talent with lightning-quick numerical intuition, fortifying his strategic thinking skills.

At the age of 20, Neelakanta Bhanu Prakash of Hyderabad secures his place as the fastest human calculator on the planet, clinching India's first gold in the Mental Calculation World Championship at the Mind Sports Olympiad in London. Holding an impressive tally of 4 world records and 50 Limca records for speed calculation, his journey is even more remarkable considering a childhood setback. A skull fracture at the age of 5 kept him away from school for a year, but he turned adversity into opportunity, delving into puzzle-solving and mathematics games to hone his cognitive skills.

Mental Mathematics isn't just about acing exams; it's a cognitive superpower that equips the brain to think strategically, break down challenges into manageable steps, and devise creative solutions. This skill transcends academic boundaries, proving invaluable when estimating shopping costs, calculating expenses, or planning a trip. Imagine confidently tallying a shopping bill without reaching for any gadgets.

Recognizing that each student has a unique learning style, Mental Maths Question Banks cater to diverse needs, offering a plethora of materials. Through collaborative efforts, students engage in exhilarating Mental Maths competitions, learning from one another and building self-confidence.

A heartfelt acknowledgment goes to the Mental Maths State Core Team, District and Zonal Coordinators, and HOSs for their unwavering dedication to bringing the Mental Maths superpower to students across all Government and government-aided schools of the Directorate of Education. Gratitude extends to the esteemed Secretary Education and the Director of Education for their guidance and constructive feedback, steering the Mental Maths Project toward continuous improvement.

(VIKAS KALIA)
PROJECT DIRECTOR (MMP)

ACKNOWLEDGEMENT

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STATE LEVEL MENTAL MATH QUIZ COMPETITION RESULT 2022-2023
LEVEL-3
REGION EAST (1st POSITION)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
1	X	SHREYANSH GUPTA	MUNNA GUPTA	20180021877	SOSE KHICHRIPUR	1002401	KAMALJEET
2	X	PIYUSH KUMAR	BANWARI LAL	20180015820	RPVV B BLOCK YAMUNA VIHAR	1104149	SANGEETA
3	X	PIYUSH THAKUR	UDAYKANT THAKUR	20210047830	GBSSS TAJPUR PAHARI BADARPUR	1925360	SURAJ PAL

REGION SOUTH (1ST RUNNER UP)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
1	X	PUNEET KUMAR	AMRITESH KUMAR	20180149516	SOSE SEC.10 DWARKA	1821291	ANJU KUMARI
2	X	SHAILESH	DEVENDER	2018012318	GBSSS DEVLI	1923018	MANOJ KUMAR
3	X	PRABHASH KUMAR JHA	MANOJ KUMAR JHA	20180204788	GBSSS NO-1 SAGARPUR	1821006	HARIOM DUTT

REGION WEST (2ND RUNNER UP)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
1	X	RANJIT KUMAR	RAJENDER KUMAR MAHTO	20190319052	GBSSS BAPROLA	1617258	RAHUL YADAV
2	X	AADARSH KUMAR	RAJESH KUMAR SAHNI	20180003243	DBRA SOSE HARI NAGAR	1514116	RITIKA GUPTA
3	X	PRIYANSHU	NEERAJ KUMAR	20200166777	SBV A BLOCK VIKASPURI	1618002	DEVINDER SINGH

REGION NORTH (4TH POSITION)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
1	X	RUPESH KUMAR	RAM KUMAR	20180077536	SOSE SEC 23 ROHINI	1413345	DEEPAK
2	X	ANSH BANSAL	DEEPAK KUMAR	20210524837	SOSE BT BLOCK SHALIMAR BAGH	1309305	ANURADHA
3	X	LAKSHAY SINGH	SURENDER SINGH	20130207409	SOSE BT BLOCK SHALIMAR BAGH	1309305	ANURADHA

REGION CENTRAL (5TH POSITION)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
1	X	ANSH	AKHILESH KUMAR GAUR	20160276189	DBRA SOSE CIVIL LINES	1207259	MANEETA SHARMA
2	X	SHUBHAM KUMAR BHARADWAJ	AJAY KUMAR THAKUR	20130256181	DBRA SOSE CIVIL LINES	1207259	MANEETA SHARMA
3	X	VISHAL SINGH	LAXMAN SINGH	20170190098	SBV ROUSE AVENUE	2127001	KAMLESH

STATE LEVEL MENTAL MATH QUIZ COMPETITION RESULT 2023-2024
LEVEL-3
REGION SOUTH (1st POSITION)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
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2	X	RITIK	BHASKAR	20220250638	SBV DEENDAR PUR	1822247	LOKESH CHAUHAN
3	X	LAKSHITA	RAGHUVINDER SINGH	20220284215	DBRA SOSE SEC 10 DWARKA	1821291	KAMAL YADAV

REGION CENTRAL (2nd POSITION)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
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3	X	KARISHMA	SHANKER KUMAR SHARMA	20210362771	GGSSS BURARI	1207117	ARTI CHAUHAN

REGION WEST (3rd POSITION)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
1	X	HARSH KUMAR SINGH	NAND KUMAR SINGH	20130255505	GBSSS NO 1, PUNJABI BAGH	1515010	SANJEEV
2	X	NEEV AGGARWAL	SACHIN AGGARWAL	20190176911	AES NT RR MEMORIAL SR SEC SCHOOL JANAKPURI	1618080	P MALLESWARI
3	X	PRINCE KUMAR	DASHRATH PRASAD	20160111348	SBV NO 1 TILAK NAGAR	1514005	TARUN KUMAR

REGION- EAST (4th POSITION)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
1	X	SIDDHANT KUSHWAHA	SANTOSH KUSHWAHA	20190034262	DBRA SOSE KAKAJI	1925438	AJAY VIR SINGH
2	X	AVIN JANGID	VINOD KUMAR	20170290248	DBRA SOSE KHICHRIPUR	1002401	KAMAL JEET SINGH
3	X	ANSH KUMAR TIWARI	MANJAY KISHOR TIWARI	20190063912	SBV KALYANVAS	1002003	AJAY KUMAR SHARMA

REGION NORTH (5th POSITION)

S. No.	CLASS	NAME OF STUDENT	FATHER'S NAME	STUDENT ID	SCHOOL NAME	SCHOOL CODE	NAME OF GUIDE TEACHER
1	X	ARYAN THAKUR	MANOJ KUMAR THAKUR	20180195029	DBRA SOSE SEC 18, ROHINI	1310469	AJAY SAHA
2	X	HIMANSHU	KASHMIRI LAL	20210184012	GBSS NITHARI	1412259	RAJESH SHARMA
3	X	SAMYAK CHOUDHARY	SANJEEV KUMAR CHAUDHARY	20220246120	DBRA SOSE SEC 23, ROHINI	1413345	DEEPAK

CONSTITUTION OF INDIA

[1][PART IV A

FUNDAMENTAL DUTIES

Article 51A. Fundamental duties. — It shall be the duty of every citizen of India—

a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;
b) to cherish and follow the noble ideals which inspired our national struggle for freedom;
c) to uphold and protect the sovereignty, unity and integrity of India;
d) to defend the country and render national service when called upon to do so;
e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;
f) to value and preserve the rich heritage of our composite culture;
g) to protect and improve the natural environment including forests, lakes, rivers and wildlife, and to have compassion for living creatures;
h) to develop the scientific temper, humanism and the spirit of inquiry and reform;
i) to safeguard public property and to abjure violence;
j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;]

[2][(k) who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.]

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1. Ins. by the Constitution (Forty-second Amendment) Act, 1976, Sec. 11 (w.e.f. 3-1-1977).
2. Ins. by the Constitution (Eighty-sixth Amendment) Act, 2002, Sec. 4 (w.e.f. 1-4-2010).

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens:

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity;

and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty- sixth day of November, 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

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1. Subs. by the Constitution (Forty-second Amendment Act,1976, Sec. 2, for "SOVEREIGN DEMOCRATIC REPUBLIC" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment Act,1976, Sec. 2, for "Unity of the Nation" (w.e.f. 3.1.1977)

SCHEDULE OF MENTAL MATHS QUIZ COMPETITIONS

FOR THE YEAR 2024-2025

DIRECTORATE OF EDUCATION

GOVT OF NCT OF DELHI

❖ **Practice to students from Question Bank**	:	01.04.2024 to 19.10.2024
❖ **School Level Quiz Competitions**	:	21.10.2024 to 30.10.2024
❖ **Cluster Level Quiz Competition**	:	14.11.2024 to 20.11.2024
❖ **Zonal Level Quiz Competition**	:	25.11.2024 to 30.11.2024
❖ **District Level Quiz Competition**	:	07.12.2024 to 13.12.2024
❖ **Regional Level Quiz Competition**	:	26.12.2024 to 31.12.2024
❖ **State Level Quiz Competition**	:	18.01.2025 to 31.01.2025

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CHAPTER -1

REAL NUMBERS

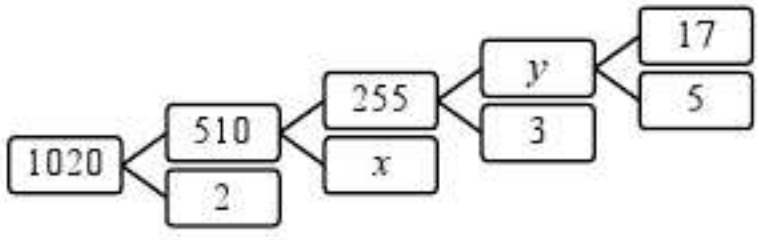
POINTS TO REMEMBER

- Euclid's Division Lemma: For any two given positive integers a and b ($a \geq b$), there exists unique whole numbers q and r such that $a = bq + r$, where $0 \leq r < b$
- If p is a prime number and p divides a^n, then p divides a where a is any positive integer.
- If p is a prime number, then $\sqrt{p}$ is an irrational number.
- Let $x = \frac{p}{q}$ be a rational number, where p and q are co-prime integers and $q \neq 0$, then
  a) x has a terminating decimal representation when q is of the form $2^m \times 5^n$ for some non – negative integers m and n.
  b) x has a non – terminating repeating decimal representation, if q is not of the form of $2^m \times 5^n$.
  c) If the denominator of a rational number is of the form $2^m \times 5^n$, then it will terminate after m places if $m \geq n$ or after n places if $n > m$.
- If a and b are two positive integers, then HCF (a, b) × LCM (a, b) = a × b.
- HCF of any given positive numbers is always a factor of its LCM.

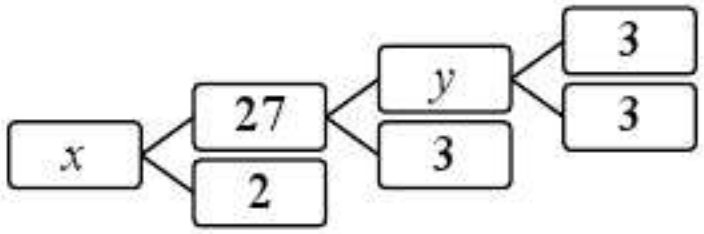
QUESTIONS

1. Find the exponent of 5 in the prime factorization of 3125.
2. Find the sum of the exponents of prime factors in the prime factorization of 98.
3. Express $0.\overline{6}$ in the form of $\frac{p}{q}$.
4. If the prime factorization of a natural number n is $2^5 \times 3^6 \times 5^4 \times 7$, then find the number of consecutive zeroes in n.

5. If $a = p^2q^3r$, $b = p^4q^5r^3$ and $c = p^2q^2r^2$, where p, q and r are prime numbers, then find HCF (a, b, c).
6. If $a = p^2q^3r$, $b = p^4q^5r^3$ and $c = p^2q^2r^2$, where p, q and r are prime numbers, then find LCM (a, b, c).
7. Find after how many places will the decimal expansion of the rational number $\frac{1237}{1250}$ terminate?
8. After how many places the decimal expansion of $\left[\frac{116}{2^5 \times 5^2}\right]$ will terminate?
9. Find the smallest positive number by which $\sqrt{125}$ should be divided to get a rational number.
10. Find the smallest positive number by which $\sqrt[3]{81}$ should be multiplied to get a rational number.
11. If m and n are two prime numbers, then find HCF (m, n).
12. If m and n are two prime numbers, then find LCM (m, n).
13. Find the HCF and LCM of the smallest composite number and single digit largest prime number.
14. Find the value of $\sqrt{(199)(201)+1}$.
15. Find the value of $\sqrt{(999)(1001)+1}$.
16. Find the digit at the unit's place of the number $7^{2019} \times 3^{2019}$.
17. Find the digit at the unit's place of the number $44^{11} \times 66^{11} \times 99^{11} + 11^{11}$.
18. Find the digit at the unit's place of the number
$4^1 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 \ldots \times 4^{99} \times 9^{100}$.
19. Find the value of 2xy with the help of factor tree given below:



20. Find the value of $\left(\frac{x}{y}\right)^2$ with the help of factor tree given below:



21. Find the value of a for which $\mathbf{p^n = (a \times 5)^n}$ ends with digit 0.

22. Find the smallest positive rational number by which $\frac{1}{3}$ should be multiplied so that its decimal expansion terminates after one place of decimal.

23. Find the difference between the largest two digit prime number and the smallest three digit prime number.

24. If $\mathbf{7560 = 2^3 \times 3^n \times q \times 7}$, then find the value of n + q.

25. Find the smallest prime factor of $11 \times 13 \times 19 \times 23 + 23$.

26. If $\left(\frac{15}{2^3} \times 5^2 \times 3^\beta \times 7^n\right)$ has a terminating decimal representation, then find the least possible values of n and β.

27. Two numbers are in the ratio 17: 13. If their HCF is 15, then find the sum of the numbers.

28. Find the HCF of $(2^{125} - 1)$ and $(2^{15} - 1)$.
(Hint: HCF $[(a^m - 1), (a^n - 1)] = a^{HCF(m,n)} - 1$)

29. The HCF and LCM of two numbers are 33 and 264 respectively. When the first number is divided by 2, the quotient is 33. Find the other number.

30. The LCM of two numbers is 1890 and their HCF is 30. If one of the numbers is 270, then find the other number.

31. The HCF of two numbers is 11 and their LCM is 616. If one of the numbers is 88, then find the other number.

32. Given that HCF (2730, 4400) = 110 and LCM (2730, 4400) = 273 k, find the value of k.

33. If HCF (144,180) = 13m – 3, then find the value of m.

34. The HCF (a, b) = 29, where a, b > 29 and LCM (a, b) = 4147. Find the value of $|a - b|$.

35. The LCM of two numbers is 45 times their HCF. If one of the numbers is 125 and sum of HCF and LCM is 1150, then find the other number.
36. Three numbers are in the ratio of 3: 4: 5 and their LCM is 2400. Find their HCF.
37. Find the HCF of smallest 3 digit positive number obtained by using three different digits and the greatest two digit composite number.
38. If HCF (a, b) = LCM (a, b), then find the relation between a and b.
39. If HCF (20, p) = 2 and LCM (20, p) = 60, then find the value of p.
40. Find the total number of factors of an even prime number.
41. If $(-1)^n + (-1)^{8n} = 0$, then find the least positive value of n.
42. If a = bq + r, then what are the possible factors of (a – r)?
43. Find the least positive integer which is divisible by first five natural numbers.
44. If the adjacent sides 'a' and 'b' of a rectangle are in the ratio 3: 5 such that HCF (a, b) = 11, then find the perimeter of the rectangle.
45. If the length of a rectangle is LCM (a, b) and breadth is HCF (a, b), then find its area.
46. If r is the remainder when (5m +1) (5m+3) (5m+4) is divided by 5, then find the possible values of r, if r is a natural number.
47. Two equilateral triangles have sides of lengths 51 cm and 85 cm respectively. Find the greatest length of tape that can exactly measure both the sides.
48. Six bells commence tolling together. They toll at the intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together?
49. In a seminar the number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required where in each room, the same number of participants are to be seated and all of them being of the same subject.
50. Find the least number of soldiers in a regiment such that they can stand equally in rows of 15, 20 and 25 and form a perfect square.

ANSWERS

Q. No	Answer	Q. No	Answer
1	5	26	$n = 0, \beta = 0$
2	3	27	450
3	$\frac{2}{3}$	28	31
4	4	29	132
5	p^2q^2r	30	210
6	$p^4q^5r^3$	31	77
7	4 places	32	400
8	3 places	33	3
9	$\sqrt{5}$	34	58
10	$\sqrt[3]{9}$	35	225
11	1	36	40
12	$m \times n$	37	3
13	HCF = 1, LCM = 28	38	a = b
14	200	39	6
15	1000	40	2
16	1	41	1
17	7	42	1, b, q, bq
18	6	43	60
19	340	44	176 units
20	36	45	ab sq. units
21	2	46	2
22	$\frac{3}{10}$	47	17 cm
23	-4	48	16
24	8	49	21
25	2	50	900 soldiers

CHAPTER 2

POLYNOMIALS

POINTS TO REMEMBER

- Algebraic expressions in which power (exponent) of the variable of each term is a whole number are called polynomials e.g. $2x + 3$, $5t^2 + 7t + 8$
- Degree of the polynomial in one variable: The highest power of the variable of any term in a polynomial is called its degree.
- Following are the forms of various degree polynomials.

Examples	Name of the polynomial	Degree of polynomial
5	Constant polynomial	0
$2x+3$	Linear polynomial	1
$5x^2 + 7x + 8$	Quadratic polynomial	2
$3x^3 + 2x^2 + 5x + 7$	Cubic polynomial	3
$t^4 + 8t^3 + 7t^1 + 4t + 5$	Biquadratic polynomial	4
0	Zero polynomial	Not defined

- If for a polynomial p(x), $p(\alpha) = 0$, then α is called a zero of the polynomial p(x).
- A polynomial of degree 'n' has atmost 'n' zeroes.
- If α and β are the zeroes of the polynomial $ax^2 + bx + c$, $a \neq 0$ then

  $$\text{Sum of the zeroes } (\alpha + \beta) = -\frac{\text{coefficient of } x}{\text{coefficient of } x^1} = -\frac{b}{a}$$

  $$\text{Product of zeroes } (\alpha.\beta) = \frac{\text{constant term}}{\text{coefficient of } x^1} = \frac{c}{a}$$

- If α and β are the zeroes of the quadratic polynomial p(x) then

  $$p(x) = k[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$$

  i.e. $p(x) = k[x^2 - (\alpha + \beta)x + \alpha\beta]$, where k is any non-zero real number.
- If α, β and γ are the zeroes of the cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ where $a \neq 0$, then

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{coefficient of x}}{\text{coefficient of x}^3}$$

$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{\text{constant term}}{\text{coefficient of x}^3}$$

- If α, β and γ are the zeroes of the cubic polynomial

  $p(x) = ax^3 + bx^2 + cx + d$, $(a \neq 0)$ then

  $p(x) = k[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma]$, where k is any non- zero real number.

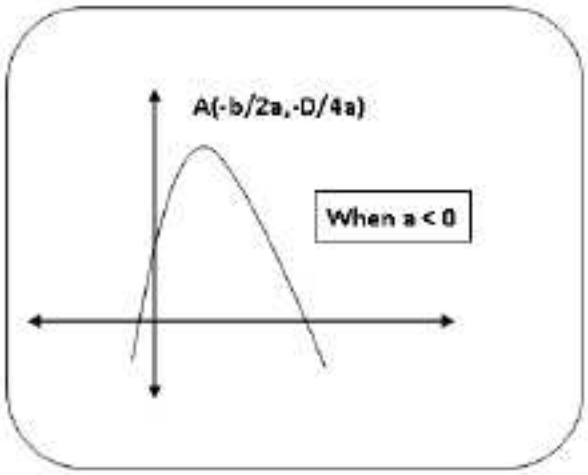
- Geometrically, the zeroes of a polynomial $f(x)$ are x – coordinates of the point where the graph of $y = f(x)$ intersects x – axis.

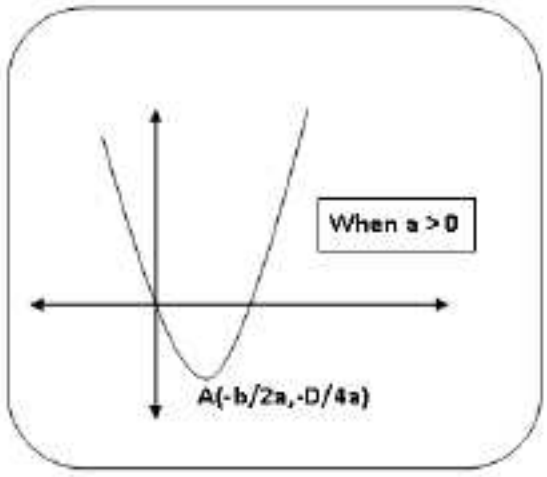
- Coordinates of vertex 'A' of graph of $y = ax^2 + bx + c$ is

  $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ i.e $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$, where $D = b^2 - 4ac$

  e.g.: if $y = x^2 - 2x + 4$, then coordinates of its vertex =

  $\left(-\frac{b}{2a}, -\frac{D}{4a}\right) = (1, 3)$





- Division algorithm for polynomials: Given any polynomial $p(x)$ and any non-zero polynomial $g(x)$, where $deg\ p(x) \geq deg\ g(x)$, then there exists polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x)\ q(x) + r(x)$, where $r(x) = 0$ or degree $r(x) <$ *degree* $g(x)$.

QUESTIONS

1. Find the zeroes of $y^2 - 8 - 2y$.

2. Find the zeroes of f (t) $= t^2 - 27$.

3. Find the zeroes of p(y) $= 6y^2 - 3$.

4. Find the degree of the polynomial $(z^4 - 2 + z^2)^2$.

5. Find the degree of the polynomial $(x^2 - 1)^2(x^3 + x^2 + x + 1)$.

6. Find the coordinates of the point where the graph of the polynomial $f(x) = 3x + 2$ intersects x – axis.

7. Find the value of 'a' for which the graph of the polynomial $ax^2 + bx + c$, $a \neq 0$ is an upward parabola.

8. If m – n, m and m + n are the zeroes of the polynomial $p(y) = 2y^3 - 6y^2 + 5y - 7$, then find the value of m.

9. Find the sum and the product of zeroes of the quadratic polynomial $3y^2 + 1 - 3\sqrt{3}y$.

10. Find the quadratic polynomial whose zeroes are $4\sqrt{2}$ and $-2\sqrt{2}$.

11. Find a quadratic polynomial whose zeroes are $(5 + \sqrt{2})$ and $(5 - \sqrt{2})$.

12. Find a quadratic polynomial whose zeroes are $\frac{2+\sqrt{3}}{3}$ and $\frac{2-\sqrt{3}}{3}$.

13. Find a quadratic polynomial whose sum of the zeroes is 0 and one zero is 5.

14. Form a quadratic polynomial whose one of the zeroes is $(2 + \sqrt{5})$ and sum of the zeroes is 4.

15. Form a quadratic polynomial whose zeroes are reciprocal of the zeroes of the polynomial $ax^2 + bx + c$ ($a \neq 0$).

16. If α and β are the zeroes of $x^2 - 3x + 2$, form a quadratic polynomial whose zeroes are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$.

17. If α and β are the zeroes of the polynomial $y^2 - 2y - 2$, then find the polynomial whose zeroes are $(2\alpha + 1)$ and $(2\beta + 1)$.

18. Find the cubic polynomial whose zeroes are 0, 5 and -5.

19. If α and β are the zeroes of the quadratic polynomial $2x^2 + 5x + 1$, then find the value of $(\alpha + \beta + \alpha\beta)$.

20. If α and β are the zeroes of the quadratic polynomial $2x^2 - 5x + 8$, then find

the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$.

21. If α and β are the zeroes of the quadratic polynomial $p(t) = t^2 + t - 2$, then find the value of $\frac{1}{\alpha}-\frac{1}{\beta}$.

22. If α and β are the zeroes of the quadratic polynomial $f(y) = 6y^2 + y - 2$, then find the value of $\alpha^2\beta + \beta^2\alpha$.

23. If α, β and γ are the zeroes of the polynomial x^3+bx^2+cx+d, then find the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

24. If α, β and γ are the zeroes of the polynomial x^3-px^2+qx-r, then find the value of $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}$.

25. If 1 is a zero of the polynomial ax^2+bx+c $(a \neq 0)$, then find the value of $\frac{b+c}{a}$.

26. If sum of the zeroes of the polynomial $(a + 1)x^2 + (2a + 3)x + (3a + 4)$ where $a \neq -1$ is -1, then find a.

27. If the sum of zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.

28. If one zero of the polynomial $z^2 + 13z - p$ is reciprocal of the other, then find the value of p.

29. For what value of p, (-4) is a zero of the polynomial $x^2 - 2x - (7p + 3)$.

30. If 1 is a zero of the polynomial $ax^2 - 3(a - 1)x - 1$ $(a \neq 0)$, then find the value of a.

31. If sum and product of the zeroes of the polynomial $ax^2 - 5x + c$ $(a \neq 0)$ is equal to 10 each, then find the value of a + c.

32. If α and β are zeroes of the polynomial $x^2 - 3x + p$ and $\alpha - \beta = 1$, then find the value of p.

33. If $x+2$ is a factor of $x^2 + ax + 2b$ and $a + b = 4$, then find the values of a and b.

34. If α and β are the zeroes of the quadratic polynomial $p(y) = 2y^2 + 5y + m$ such that $\alpha^2+\beta^2+\alpha\beta = \frac{17}{4}$, then find the value of m.

35. If α and β are the zeroes of the quadratic polynomial $f(t) = t^2 - k(t+1) - c$ such that $(\alpha+1)(\beta+1) = 0$, then find the value of c.

36. If the sum of zeroes of the polynomial $5x^2 + (p+q+r)x + pqr$ is 0, then find the value of $p^3 + q^3 + r^3$.

37. If one of the zeroes of the polynomial $x^2 - 9x + (7k+4)$ is double of the other, then find the value of k.

38. If $f(y) = y^3 - 2y^2 - y + 2 = (y+1)(y-2)(y-k)$, then find the value of k.

39. Find the common factor of $x^2 - 1, x^4 - 1$ and $(x-1)^2$.

40. Find the common zero of $x^2 + 2x + 1$, $x^2 - 1$ and $x^3 + 1$.

41. If $x + a$ is a common factor of the polynomials $x^2 - 3x - 10$ and $x^2 - 8x + 15$, then find the value of a.

42. Find the common factor in $x^2 + x - 12$ and $x^2 + 9x + 20$.

43. Find the value of x for which both the polynomials $2x^2 + 8x + 8$ and $x^2 - 3x - 10$ becomes zero.

44. If a polynomial of degree 5 is divided by a quadratic polynomial, then find the degree of the remainder polynomial.

45. Find the quotient when $x^2 - 9x + 20$ is divided by $x - 5$.

46. If $x^3 + x^2 - ax + b$ is completely divisible by $x^2 - x$, then find the values of a and b.

47. What should be added to the polynomial $x^2 - 8x + 6$ so that 4 becomes a zero of the polynomial?

48. What should be subtracted from $x^3 - 3x^2 + 6x - 15$ so that it is completely divisible by $(x - 3)$?

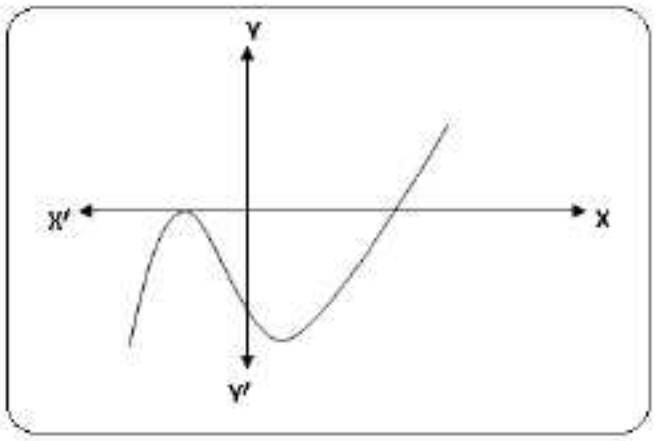
49. If two zeroes of the polynomial $x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find its third zero.

50. If two zeroes of the polynomial $px^3 + qx^2 + rx + s, p \neq 0$ are 0 each, then find its third zero.

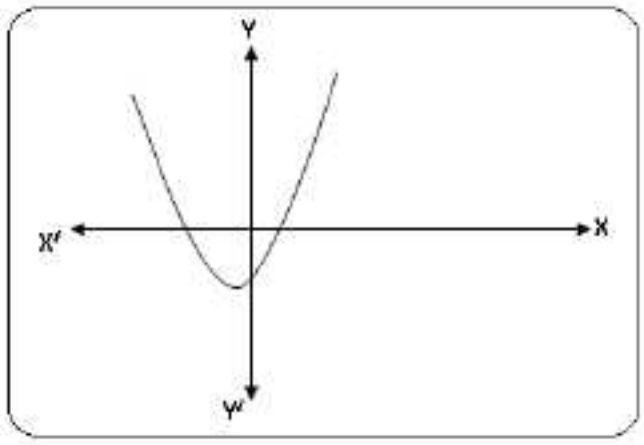
51. Find the zeroes of the polynomial $x^3 - 5x^2 - 16x + 80$, if its two zeroes are equal in magnitude but opposite in sign.

52. **Find the coordinates of the vertex of the figure obtained by drawing the graph of $2x^2 - 4x + 5$.**

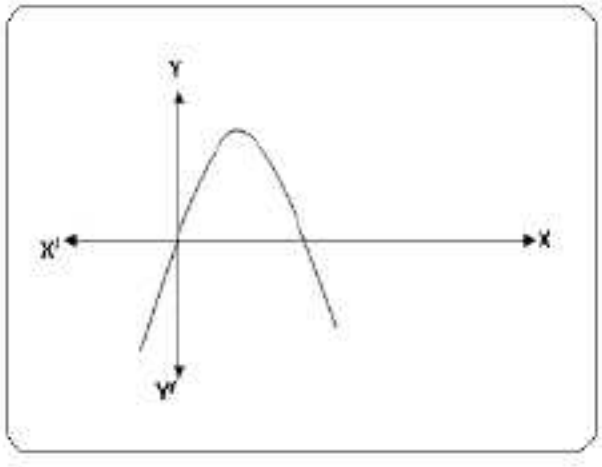
53. **Find the number of zeroes of the polynomial p(x) represented in the given graph.**



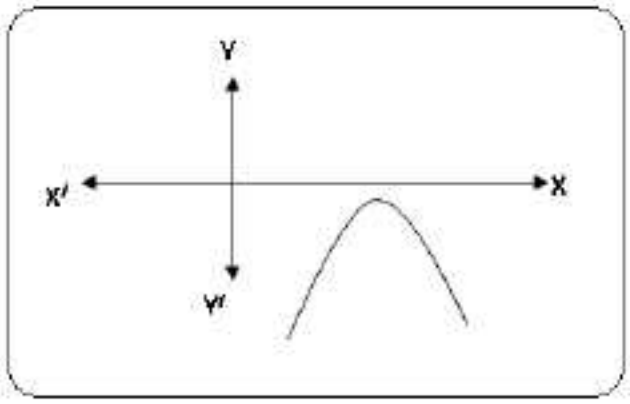
54. **Find the number of zeroes of the polynomial p(x) represented in the given graph.**



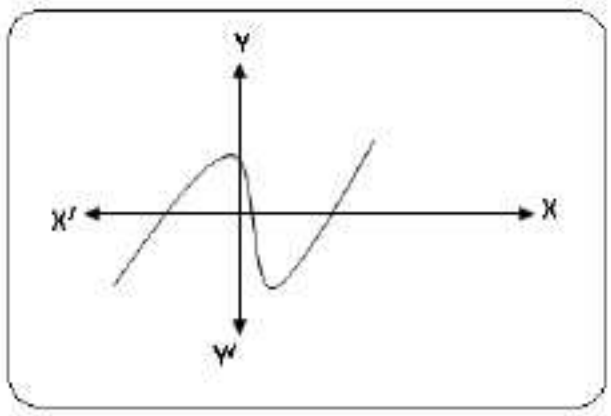
55. **Find the number of zeroes of the polynomial p(x) represented in the given graph.**



56. **Find the number of zeroes of the polynomial p(x) represented in the given graph.**



57. **Find the number of zeroes of the polynomial p(x) represented in the given graph.**



58. **If $(y + k)$ is a factor of the polynomial $2y^2 + 2ky + 5y + 10$, then find the value of k.**

59. **If one of the zero of the polynomial $t^2 - 4t + 1$ is $2 + \sqrt{3}$, then find the other zero.**

60. **If $p(t) = t^2 + 5t + 10$, then find the value of $p(20) + p(10)$.**

ANSWERS

Q.No	Answer	Q.No	Answer
1.	$y = -2, 4$	21.	$\pm\frac{3}{2}$
2.	$t = \pm 3\sqrt{3}$	22.	$\frac{1}{18}$
3.	$y = \pm\frac{1}{\sqrt{2}}$	23.	$-\frac{c}{d}$
4.	8	24.	$\frac{p}{r}$
5.	7	25.	-1
6.	$\left(-\frac{2}{3}, 0\right)$	26.	$a = -2$
7.	$a > 0$	27.	$k = 9$
8.	$m = 1$	28.	-1
9.	$\sqrt{3}, \frac{1}{3}$	29.	3
10.	$k(x^2 - 2\sqrt{2}x - 16), k \neq 0$	30.	1
11.	$k(x^2 - 10x + 23),\ k \neq 0$	31.	$5\frac{1}{2}$
12.	$k\left(x^2 - \frac{4}{3}x + \frac{1}{9}\right),\ k \neq 0$	32.	2
13.	$k(x^2 - 25),\ k \neq 0$	33.	$a = 3,\ b = 1$
14.	$k(x^2 - 4x - 1),\ k \neq 0$	34.	$m = 4$
15.	$k\left(x^2 + \frac{b}{c}x + \frac{a}{c}\right),\ k \neq 0$	35.	$c = 1$
16.	$k(x^2 - 10x + 9),\ k \neq 0$	36.	$3pqr$
17.	$k(y^2 - 6y - 3),\ k \neq 0$	37.	$k = 2$
18.	$k(x^3 - 25x),\ k \neq 0$	38.	$k = 1$
19.	-2	39.	$x - 1$
20.	$-\frac{7}{16}$	40.	$x = -1$

Q.No	Answer	Q.No	Answer
41.	$a = -5$	51.	*4, -4 and 5*
42.	$x + 4$	52.	*(1, 3)*
43.	$x = -2$	53.	*2*
44.	*1 or 0 or not defined*	54.	*2*
45.	$x - 4$	55.	**2**
46.	$a = 2, b = 0$	56.	*0*
47.	**10**	57.	*3*
48.	**3**	58.	$k = 2$
49.	*4*	59.	$2 - \sqrt{3}$
50.	$-\frac{q}{p}$	60.	*670*

CHAPTER – 3

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

POINTS TO REMEMBER

- General form of pair of linear equations in two variables is

$$a_1x + b_1y + c_1 = 0 \text{ and}$$

$$a_2x + b_2y + c_2 = 0$$

where $a_1, b_1, a_2, b_2, c_1, c_2$ are real numbers such that $(a_1)^2 + (b_1)^2 \neq 0$ and $(a_2)^2 + (b_2)^2 \neq 0$

- Methods to solve a pair of linear equations in two variables:
  a) Graphical method
  b) Algebraic methods:
     i) Substitution method
     ii) Elimination method
     iii) Cross Multiplication method

- In above equations if $\frac{a_1}{a_1} \neq \frac{b_1}{b_1}$, then
  i) The pair of linear equations is consistent.
  ii) The pair of linear equations represents intersecting lines.
  iii) The pair of linear equations has a unique solution (exactly one solution).

- In above equations if $\frac{a_1}{a_1} = \frac{b_1}{b_1} = \frac{c_1}{c_1}$, then
  i) The pair of linear equations is dependent and consistent.
  ii) The pair of linear equations represents coincident lines.
  iii) The pair of linear equations have infinitely many solutions.

- In above equations if $\frac{a_1}{a_1} = \frac{b_1}{b_1} \neq \frac{c_1}{c_1}$, then
  - i) The pair of linear equations is inconsistent.
  - ii) The pair of linear equations represents parallel lines.
  - iii) The pair of linear equations has no solution.
- Area of a triangle $= \frac{1}{2} \times$ base $\times$ corresponding height
- If area of a triangle is zero, then the three given points are collinear and vice versa.
- Special case: When coefficient of x and y are interchanged in two equations i.e. $ax + by = c$, $bx + ay = d$, then $x + y = \frac{c+d}{a+b}$ and $x - y = \frac{c-d}{a-b}$.

  By solving above two equations we can get the values of x and y
- Special case: If $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$ and $\frac{a_1}{a_1} \neq \frac{b_1}{b_1}$, then $x = 0$ and $y = 0$

QUESTIONS

1. Find y in terms of x for the equation: $3x + 4y = 12$.
2. Find the coordinates of the points of intersection of $px + qy = r$ with the coordinate axes.
3. At what point the linear equation $2x + 3y = -7$ intersects x axis?
4. At what point the linear equation $3x - 7y = 5$ intersects y axis?
5. Find the number of solutions of the following pair of linear equations: $2x + 4y - 16 = 0$, $6x + 12y = 48$.
6. Find the number of solutions of the following pair of linear equations: $5x + 15y = 20$, $4x + 12y - 14 = 0$.
7. If $3^{x+y} = 3^{x-y} = \sqrt{27}$, then find the value of x.
8. If a pair of linear equations in two variables is inconsistent, then what is the nature of their graph?
9. Find the number of solutions for the pair of equations $x = 0$ and $x = 5$.
10. The graph of y = 5 is parallel to which axis?

11. Find the sum of intercepts cut off by the line $5x + 10y = 20$ on coordinate axes.

12. Find the sum of intercepts cut off by the line $3x + 2y = 5$ on coordinate axes.

13. For what value of k, (6, k) is a solution of the equation $3x + y = 22$.

14. If $x = 4$ and $y = 3p - 1$ is a solution of $x + y = 6$, then find the value of $\frac{p+y}{2}$.

15. If $x = a$ *and* $y = b$ is a solution of the equations $x + y = 8$, $x - y = 2$, then find the values of a *and* b.

16. Find the point of intersection of $y = 2$ *and* $2x + 3y = 5$.

17. Find the value of $x - y$ for which $x + 2y = 9$, $x - 2y = 1$.

18. If $2x + 3y = 0$ and $4x - 3y = 0$, then find the value of $x + y$.

19. Solve for x and y: $\sqrt{5}x + \sqrt{7}y = 0, \sqrt{3}x - \sqrt{2}y = 0$.

20. If $\sqrt{a}x - \sqrt{b}y = 0, \sqrt{b}x - \sqrt{a}y = 0$, then find the value of xy.

21. If $2^x = 8^{y-1}$ *and* $9^y = 3^{x-6}$, then find the value of $x - y$.

22. Find the value of $x + y$, *if* $\sqrt{x} + \sqrt{y} = 7$ *and* $\sqrt{x} - \sqrt{y} = 1$

23. Solve for x *and* y: $\frac{2x}{a} + \frac{y}{b} = 2, \frac{x}{a} - \frac{y}{b} = 4$.

24. Solve for x *and* y: $x - y = 0.9, \frac{11}{2(x+y)} = 1$.

25. Solve for x *and* y: $31x + 29y = 89, 29x + 31y = 91$.

26. Find the value of $x + y$ for the following pair of linear equations :

    $152x - 378y = -74$ *and* $-378x + 152y = -604$

27. Find the area of the triangle formed by the lines $x = y$, $y = 4$ *and* y*-axis*.

28. Find the area of the triangle formed by the lines $y = x$, $x = a$ *and* $y = b$.

29. Find the area of the triangle formed by the line $x + y = 6$ and coordinate axes.

30. Find the area of the triangle whose vertices are (3, 2), (5, 2) and (-7, 2).

31. Find the area of the triangle whose vertices are (3, 5), (3, -7) and (3, 0).

32. Find the area of the triangle whose vertices are (0, 0), (2, 2) and (4, 0).

33. Find the area of the triangle whose vertices are (2, 2), (4, 4) and (6, 2).

34. Find the value of k for which the given system of equations has no solution:

    $kx + 2y - 1 = 0, 5x - 3y + 2 = 0$

35. Find the value of k for which the given system of equations have infinitely many solutions:

$2x - 3y = 7,\ (k+2)x - (2k+1)y = 3(2k-1)$

36. Find the value of m for which the given system of equations has unique solution:

$2x + 3y - 5 = 0,\quad mx - 6y = 8$

37. In triangle ABC, $\angle A = x$, $\angle B = y$ and $\angle C = y + 20°$. If $y - x = 50°$, then what type of triangle is ABC?

38. In a cyclic quadrilateral ABCD, $\angle A = (6x + 10)°$, $\angle B = (5x)°$, $\angle C = (x + y)°$, $\angle D = (3y - 10)°$. Find the value of $x + y$.

39. The angles of a triangle are x, y and $40°$. If $x - y = 30°$, then find the value of x and y.

40. The larger of two supplementary angles exceeds the smaller by 18°. Find the angles.

41. Megha has only one rupee and two rupee coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹75, then find the number of ₹1 and ₹2 coins.

42. Meena went to the bank to withdraw ₹2000. She asks the cashier to give her ₹50 and ₹100 notes only. She receives 25 notes in all, find how many notes of ₹50 and ₹100 she received?

43. The sum of digits of a two digit number is 9. If 27 is added to it, then the digits of the number get reversed, then find the number.

44. Sum of two numbers is 35 and their difference is 13. Find the smaller number.

45. If one number is twice the other and their sum is 117, then find the larger number.

46. The sum of two numbers is 20 and their product is 75. Find the sum of their reciprocals.

47. The sum of two numbers is 20 and their product is 19. Find their difference.

48. The sum of numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Find the fraction.

49. If 2 is added to the numerator of a fraction, it becomes $\frac{1}{2}$ and if 1 is subtracted from the denominator it becomes $\frac{1}{3}$. Find the fraction.

50. The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator. Find the fraction.

51. Two numbers are in the ratio 3:4. If 8 is added to each of the number, the ratio becomes 4:5. Find the sum of the numbers.

52. Cost of 3 books and 4 pens together is ₹257 and the cost of 4 books and 3 pens together is ₹324. Find the total cost of two books and two pens.

53. 37 markers and 53 fountain pens together cost ₹3200, while 53 markers and 37 fountain pens together cost ₹4000. Find the cost of 1 marker and 3 fountain pens.

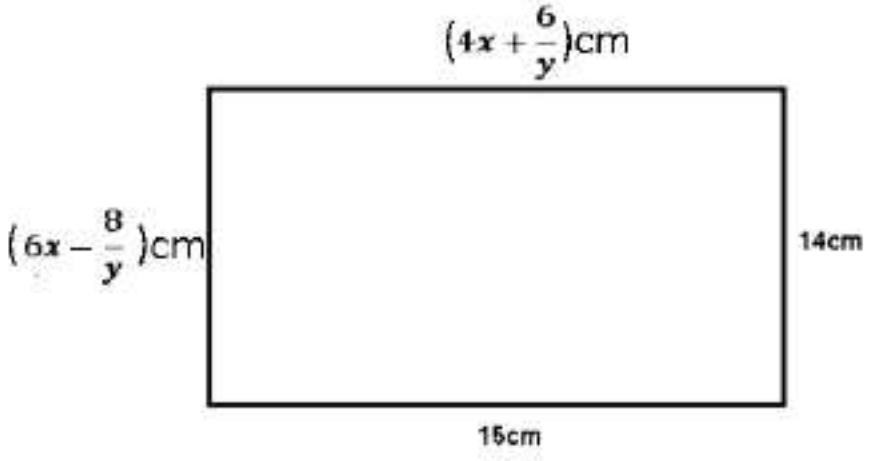
54. A father is three times as old as his son. After 12 years, he will be twice as old as his son. Find the present age of son.

55. The age of a father is 3 times the sum of the ages of his two children. After 6 years his age will be twice the sum of the ages of his two children. Find the present age of the father.

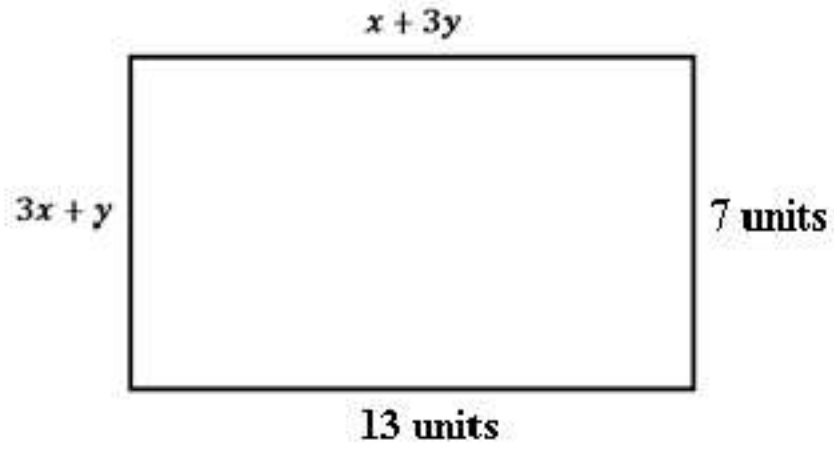
56. The monthly income of A and B are in the ratio 9: 7 and their monthly expenditure are in the ratio 4:3. If each of them saves ₹1600 per month, find their monthly income.

57. Half the perimeter of a rectangular garden whose length is 12m more than its width is 60m. Find the length of the garden.

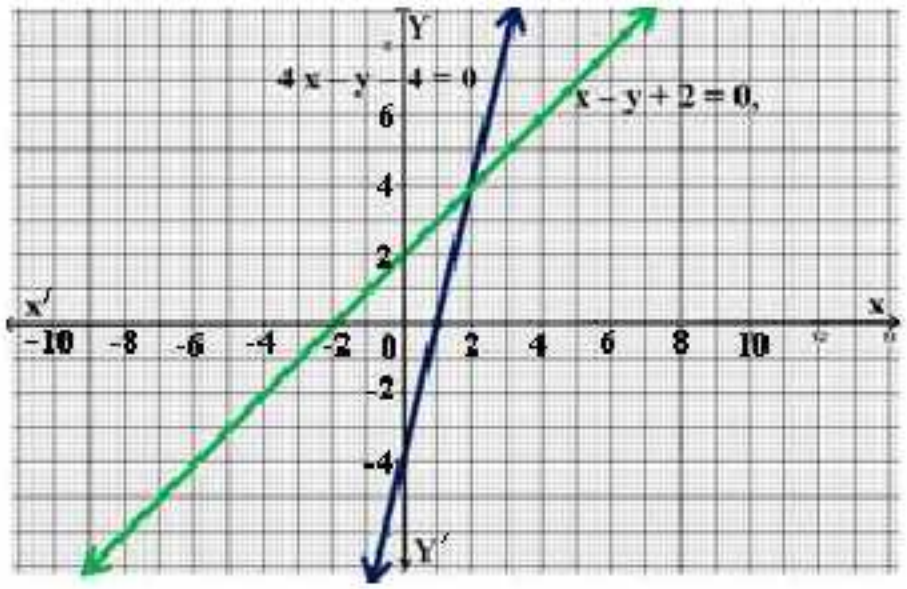
58. In the given rectangle, find the values of x and y.



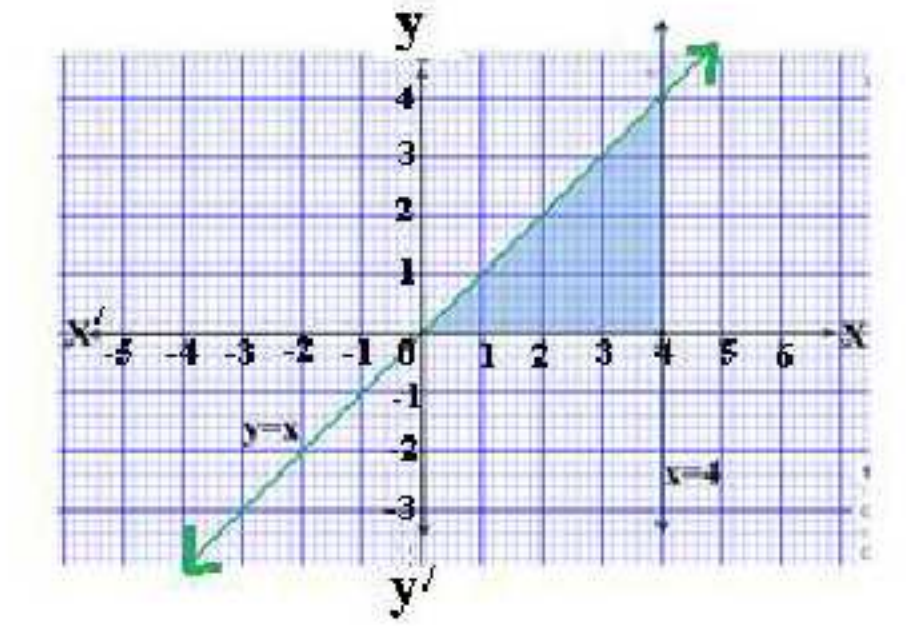
59. **Find the values of x *and* y in the following rectangle:**



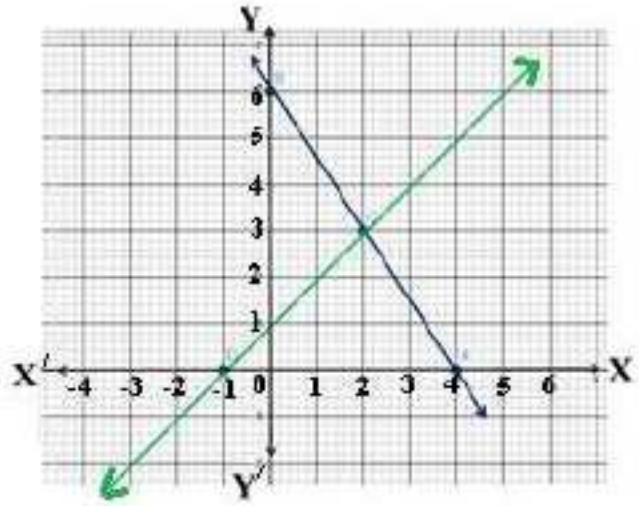
60. **From the figure, find area of the triangle formed by the pair of linear equations: $x - y + 2 = 0$, $4x - y - 4 = 0$ and x axis.**



61. **From the figure, find area of the shaded triangle.**



62. **Find the ratio of area of the triangle formed by given lines with x - axis and y- axis in the given figure.**



ANSWERS

Q.No	Answer	Q.No	Answer
1.	$y = \frac{12 - 3x}{4}$	21.	*15*
2.	$\left(\frac{r}{p}, 0\right), \left(0, \frac{r}{q}\right)$	22.	*25*
3.	$\left(-\frac{7}{2}, 0\right)$	23.	$x = 2a, y = -2b$
4.	$\left(0, -\frac{5}{7}\right)$	24.	$x = 3.2, y = 2.3$
5.	*Infinitely many solutions*	25.	$x = 1, y = 2$
6.	*No solution*	26.	$x + y = 3$
7.	$x = 1\frac{1}{2}$	27.	*8 sq. units*
8.	*Parallel lines*	28.	$\frac{1}{2}(a - b)^2$ *sq. units*
9.	*No solution*	29.	*18 sq. units*
10.	*x axis*	30.	*0*
11.	*6 units*	31.	*0*
12.	$4\frac{1}{6}$ *units*	32.	*4 sq. units*
13.	$k = 4$	33.	*4 sq. units*
14.	$1\frac{1}{2}$	34.	$k = -3\frac{1}{3}$
15.	$a = 5, b = 3$	35.	$k = 4$
16.	$\left(-\frac{1}{2}, 2\right)$	36.	$m \neq -4$ *(m is real)*
17.	*3*	37.	*Right Angled Triangle*
18.	*0*	38.	$x + y = 50$
19.	$x = 0, y = 0$	39.	$x = 85°, y = 55°$
20.	*0*	40.	*99° and 81°*

Q.No	Answer	Q.No	Answer
41.	*No. of ₹ 1 coins = 25* *No. of ₹ 2 coins = 25*	52.	*₹ 166*
42.	*10 notes of ₹ 50* *15 notes of ₹ 100*	53.	*₹ 110*
43.	*36*	54.	*12 years*
44.	*11*	55.	*54 years*
45.	*78*	56.	*A = ₹14,400* *B = ₹11,200*
46.	$\frac{4}{15}$	57.	*36 m*
47.	*18*	58.	*x = 3, y = 2*
48.	$\frac{5}{7}$	59.	*x = 1 unit, y = 4 units*
49.	$\frac{3}{10}$	60.	*6 sq. units*
50.	$\frac{7}{18}$	61.	*8 sq. units*
51.	*56*	62.	*3 : 2*

CHAPTER – 4

QUADRATIC EQUATIONS

POINTS TO REMEMBER

❖ **Quadratic Equation :**

An equation of degree 2 is called a quadratic equation. The general form of a quadratic equation in one variable x is $ax^2 + bx + c = 0$ where a, b and c are real numbers and $a \neq 0$.

❖ **Methods for solving quadratic equations are :**

- **Factorization method**
- **Completing the square method**
- **Quadratic formula**

❖ **Discriminant: For the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$**

$$D = b^2 - 4ac$$ **is called discriminant.**

❖ **Nature of roots**

If D = 0, Real and equal roots

If D > 0, Real and distinct roots

If D < 0, No real roots

❖ **If $D \geq 0$, then real roots α and β of the quadratic equation $ax^2 + bx + c = 0$ are given by $\alpha = \frac{-b + \sqrt{D}}{2a}$ and $\beta = \frac{-b - \sqrt{D}}{2a}$ $(a \neq 0)$**

❖ **Relationship between roots and coefficients :**

If α and β are two roots of $ax^2 + bx + c = 0$, then

$$\text{Sum of the roots} = \alpha + \beta = -\frac{b}{a}$$

$$\text{Product of the roots} = \alpha\beta = \frac{c}{a}$$

❖ **Quadratic Equation : $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$**

❖ ***If $x^2 - a^2 \leq 0$ then $-a \leq x \leq a$.***

If $x^2 - a^2 \geq 0$ then $x \leq -a$ or $x \geq a$

QUESTIONS

1. Find the discriminant of the equation: $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$.
2. If the sum of roots of the equation $x^2 - (k + 6)x + 2(2k - 3) = 0$ is equal to half of their product, then find k.
3. Find the values of x:

   $x + \frac{1}{x} = 11\frac{1}{11}$
4. If $x^2 - x - 56 = (x + k)(x + 7)$, then find k.
5. Find the value(s) of k for which the quadratic equation: $kx(x - 3) + 9 = 0$ has equal roots.
6. Find the value(s) of k for which the given equation $4x^2 - 3kx + 9 = 0$ has real roots.
7. Find the value(s) of k for which the given equation $x^2 - kx + 9 = 0$ has no real roots.
8. If $2x^2 + 3px + 4$ is exactly divisible by $x - 2$, then find p.
9. Find the value of k for which roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other.
10. Find the value(s) of z if $z^2 + \frac{1}{z^1} = 2$, $z \neq 0$.
11. Find the value of x which satisfies the equation $\frac{x}{17} + 1 = \sqrt{1 + \frac{35}{289}}$.
12. The roots of the equation $x^2 - 12x + p = 0$ are in the ratio 1:2, find the value of p.
13. If sum and product of roots of equation $kx^2 + 6x + 4k = 0$ are equal, then find the value of k.
14. Find the value of k, if given equations $3x^2 + 4kx + 2 = 0$ and $x^2 + 2x + 1 = 0$ have a common root.
15. Find the values of a and b, if both roots of $4x^2 + 6x + b = 0$ and $2x^2 + ax - 4 = 0$ are same.
16. If $x = 1$ is a common root of the equations $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$, then find the value of ab.

17. If the sum of roots of the equation $x^2 - x = \lambda(2x - 1)$ is zero, then find the value of λ.

18. Find the condition for which one root of the equation $ax^2 + bx + c = 0$, $a \neq 0$ is twice the other.

19. Find a quadratic equation whose roots are half of the roots of the equation $x^2 - 3x - 10 = 0$

20. If α and β are the roots of $2x^2 + x - 6 = 0$, then find a quadratic equation whose roots are 3α and 3β.

21. Form a quadratic equation whose one root is $2 + \sqrt{7}$.

22. Find the value of

$$\sqrt{10 + 3\sqrt{10 + 3\sqrt{10 + 3\sqrt{10 \ldots\ldots\ldots\ldots\ldots}}}}$$

23. Find x:

$$x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 \ldots\ldots\ldots\ldots}}}}$$

24. Find x:

$$\sqrt{13 - x^2} = x + 5$$

25. If one root of a quadratic equation is negative of the other, then find the coefficient of the middle term of the quadratic equation.

26. For what value of 'p', the equation $9x^2 - 12x + p = 0$ will be in the form of a perfect square?

27. Find the value of x which satisfies the equation $x + \frac{4}{x} = -4, x \neq 0$.

28. If the roots of the equation $2x^2 + (4m + 1)x + 2(2m - 1) = 0$ are reciprocals of each other, then find the value of m.

29. Find the ratio of the product and sum of the roots of the equation $5x^2 - 18x + 12 = 0$.

30. Find the sum of reciprocal of the roots of the equation $x^2 - 7x + 12 = 0$.

31. If roots of the equation $ax^2 + bx + c = 0$ are $5 \pm \sqrt{5}$, then find the value of a:c.

32. If α and β are the roots of $x^2 - 4x + k = 0$ and $\alpha^2 + \beta^2 = 40$, then find the value of k.

33. Find the values of x:

$$x^2 - 2ax + a^2 - b^2 = 0$$

34. Find the positive value of k for which the equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have real roots.

35. If -4 is the root of the equation $x^2 + px - 4 = 0$ and the equation $x^2 + px + k = 0$ has equal roots, then find the value of k.

36. Find a quadratic equation whose roots are reciprocal of the roots of the equation $2x^2 - 3x + 1 = 0$.

37. If one root of the equation $kx^2 - 14x + 8 = 0$ is six times the other, then find the value of k.

38. If α and β are roots of the equation $x^2 - 4x + 3 = 0$, then find value of $\alpha^4\beta^2 + \alpha^2\beta^4$.

39. If one root of the quadratic equation $2x^2 + kx + 4 = 0$ is 2, then find the other root.

40. If the sum of the roots of $kx^2 - 3x + 9 = 0$ is $-\frac{3}{7}$, then find the product of the roots of this equation.

41. If the sum of first n natural numbers is given by $S = \frac{n(n+1)}{2} = 66$, then find the value of n.

42. Find the value of k, if the difference of roots of quadratic equation $x^2 - 5x + (3k - 3) = 0$ is 11.

43. Find the value of x, *if* $x = \cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cdots\infty}}}$

44. The sum of ages (in years) of a boy and his brother is 15 and the product of their ages (in years) is 56. Find their ages.

45. The sum of a natural number and its reciprocal is $\frac{37}{6}$. Find the number.

46. Divide 29 into two parts such that their product is 198.

47. The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}$, then find the number.

48. Find two consecutive even integers sum of whose squares is 340.

49. A two digit number is 4 times the sum of its digits and twice the product of its digits. Find the number.

50. If sum of first n odd natural numbers is 324, then find the value of n.

51. If an integer is added to its square, the sum is 90, then find the integers.

52. What is the condition to be satisfied for which quadratic equations $ax^2 + 2bx + c = 0$ *and* $bx^2 - 2\sqrt{ac}x + b = 0$ have equal roots?

53. Find the values of x: $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

54. Find x:

$$\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6};\ x \neq 3, -5$$

55. If the equation $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, then find the value of $(1 + m^2)$.

ANSWERS

Q. No.	Answer	Q. No.	Answer
1.	*64*	16.	*3*
2.	*9*	17.	$-\frac{1}{2}$
3.	*11*, $\frac{1}{11}$	18.	$2b^2 = 9ac$
4.	*-8*	19.	$2x^2 - 3x - 5 = 0$
5.	$k = 4$	20.	$2x^2 + 3x - 54 = 0$
6.	$k \geq 4$ *or* $k \leq -4$	21.	$x^2 - 4x - 3 = 0$
7.	$-6 < k < 6$	22.	5
8.	$p = -2$	23.	$\frac{1+\sqrt{5}}{2}$
9.	$k = 3$	24.	*-3, -2*
10.	*1 , -1*	25.	*0*
11.	*1*	26.	*4*
12.	*32*	27.	*-2*
13.	$-1\frac{1}{2}$	28.	1
14.	$1\frac{1}{4}$	29.	2: 3
15.	$a = 3, b = -8$	30.	$\frac{7}{12}$

Q. No.	Answer	Q. No.	Answer
31.	$\mathbf{1:20}$	44.	*7 years and 8 years*
32.	$k = -12$	45.	*6*
33.	$a + b, a - b$	46.	*11,18*
34.	*16*	47.	*5,10*
35.	$2\frac{1}{4}$	48.	*12,14*
36.	$x^2 - 3x + 2 = 0$	49.	*36*
37.	*3*	50.	*18*
38.	*90*	51.	*-10, 9*
39.	*1*	52.	$b^2 = ac$
40.	$-1\frac{2}{7}$	53.	$-\frac{2b}{3a}, \frac{3a}{4b}$
41.	*11*	54.	*−9,7*
42.	*-7*	55.	$\frac{c^2}{a^2}$
43.	$\sqrt{2}\ -1$		

CHAPTER – 5

ARITHMETIC PROGRESSIONS

POINTS TO REMEMBER

- General A.P. with n terms is $a, a + d, a + 2d, \ldots, a + (n-1)\,d$ where a is the first term and d is the common difference.
- n^{th} term of an A.P. is

$$a_n \text{ or } t_n = a + (n-1)d$$

- Sum of n terms of an A.P. : $S_n = \frac{n}{2}[2a + (n-1)d]$

Or

For an A.P. having n terms: $S_n = \frac{n}{2}[a + l]$ where l is the last term

- r^{th} term from the end of an A.P. having n terms

$$= (n - r + 1)^{th} \text{ term from the beginning}$$

$$= a + (n - r + 1 - 1)d \quad = a + (n - r)d$$

- r^{th} term of an A.P. from the end is $T_r = a_n - (r-1)d$ where a_n is the last term.
- If a, b and c are in A.P. then $2b = a + c$
- If sum of first three terms of an A.P. is given, then we can take the first three terms as $a - d, a, a + d$
- If sum of first four terms of an A.P. is given, then we can take the first four terms as $a - 3d, a - d, a + d, a + 3d$
- If sum of first five terms of an A.P. is given then, we take the first five terms as $a - 2d, a - d, a, a + d, a + 2d$
- To find a_n when S_n is given: $a_n = S_n - S_{n-1}$
- Common difference: $d = a_{n+1} - a_n$
- If a : b :: c : d then a and d are called extremes and b and c are called the means and $ad = bc$

QUESTIONS

1. Find the sum of the first 10 odd natural numbers.
2. Find the common difference of the A.P. : $8\frac{1}{8}, 8\frac{2}{8}, 8\frac{3}{8}, \ldots$.
3. Find the value of $a_{25} - a_{15}$ for the A.P. : $6, 9, 12, 15, \ldots$.
4. Find the common difference of the A.P. : $4, 9, 14, \ldots$. If the first term changes to 6 and the common difference remains the same, then find the 3rd term of the new A.P..
5. Find the value of x, if $8x + 9, 6x - 2, 2x - 7$ are three consecutive terms of an A.P.
6. An A.P. : $5, 8, 11, \ldots$ has 40 terms. Find the sum of the last 10 terms.
7. For what value of k will $k + 9, 2k - 1$ and $2k + 7$ be the consecutive terms of an A.P. ?
8. If the sum of first n terms of an A.P. is $5n^2 + 2n$, then find its 2nd term.
9. Find the common difference of the A.P.

$$\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$$

10. Find the middle term of the A.P. : $6, 13, 20, \ldots, 216$.
11. If $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ are in A.P., then find the value of x.
12. If $x, 13, y, 3$ are in A.P., then find the value of $x + 2y$.
13. Find 5^{th} term of an A.P., whose nth term is $3n-5$.
14. Find the value of p for which $\frac{4}{5}$, p and 2 are three consecutive terms of an A.P..
15. Find the sum of first n natural numbers.
16. If the sum of first n even natural numbers is equal to k times the sum of first n odd natural numbers, then find the value of k.
17. If 18, a, b and -3 are in A.P., then find $a+b$.
18. If 4, a_2, a_3, a_4 and 28 are in A.P., then find a_4.
19. The fourth term of an A.P. is 11. The sum of the fifth and the seventh terms of the A.P. is 34. Find its common difference.

20. If the numbers a, b, c, d and e form an A.P., then find the value of

$a - 4b + 6c - 4d + e$.

21. Three consecutive terms of an A.P. are $a-d$, a and $a+d$, their sum is 33 and d is 5, find the terms.

22. If a, b, c are in A.P., then find the value of

$$(a + 2b - c)(2b + c - a)(c + a - b).$$

23. If sides of a right angled triangle are in A.P., then find the ratio of its sides.

24. If sum of three consecutive terms of an A.P. is 24, then find its middle term.

25. If sum of first five consecutive terms of an A.P. is 115, then find its third term.

26. If angles of a triangle are in A.P. and smallest angle is $40°$, then find the largest angle.

27. The angles of a quadrilateral are in A.P. whose common difference is $10°$, Find sum of the smallest and the largest angle.

28. Find the sum of n terms of the series

$$\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \cdots$$

29. Find a, b and c such that the following numbers are in A.P. : a, 7, b, 23, c.

30. Divide 16 into 4 parts which are in A.P. such that the product of extremes is one less than the sum of means.

31. If the sum of the first 7 terms of an A.P. is 49 and that of the first 17 terms is 289, find the sum of first n terms of an A.P..

32. If the first three terms of an A.P. are b, c and 2b, then find the ratio of b and c.

33. If ratio of the sum of the first m and n terms of an A.P. is $m^2 : n^2$, then find the ratio of its m^{th} and n^{th} terms.

34. The sum of 3 consecutive terms of an A.P. is 12 and the sum of their cubes is 288, find the terms.

35. How many two digit natural numbers are there, which when divided by 3 yield 1 as remainder?

36. If sum of first m terms of an A.P. is same as the sum of its first n terms, then find the sum of its first (m + n) terms.

37. If $X = 1 - 6 + 2 - 7 + 3 - 8 + \cdots$ **to 100 terms**, then find the value of X.

38. Which term of the A.P.: 52, 48, 44, ... will be the first negative term?

39. From your pocket money you save ₹ 1 on day 1, ₹ 2 on day 2, ₹ 3 on day 3 and so on. How much money will you save in the month of February 2024?

40. The digits of a positive number of 3 digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

41. If the p^{th} term of an A.P. is q and q^{th} term is p, then find its n^{th} term.

42. If the m^{th} term of an A.P. is $\frac{1}{n}$ and its n^{th} term is $\frac{1}{m}$, then find its mn^{th} term.

43. The sum of first n terms of three Arithmetic Progressions are S_1, S_2 and S_3 respectively. The first terms of three A.P. are 1, 2 and 3 respectively and their common differences are 1, 2 and 3 respectively. Find $S_1 + S_2 - S_3$.

44. For the A.P.: 8, 10, 12... with 60 terms, find the sum of the last 10 terms.

45. The first and the last terms of an A.P. are 8 and 65 respectively. If the sum of all of its terms is 730, then find its common difference.

46. The sum of the first n terms of an A.P. is $5n - n^2$. Find the n^{th} term of this A.P..

47. If the seventh term of an A.P. is $\frac{1}{9}$ and its ninth term is $\frac{1}{7}$, find its 63^{rd} term.

48. Find the sum of all odd integers between 1 and 100 which are divisible by 3.

49. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

50. A sum of ₹ 1500 is to be used to give 10 cash prizes to students of a school for their academic performance. If each prize is ₹ 20 less than its preceding prize, then find the value of the first prize.

ANSWERS

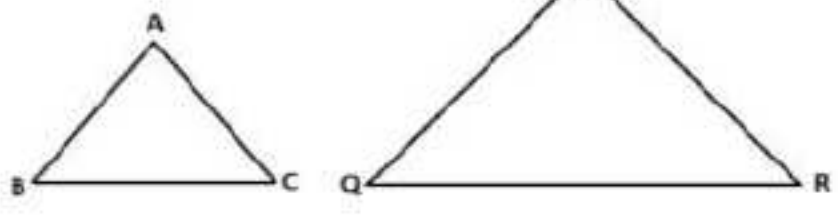
Q. No	Answer	Q. No	Answer
1	100	26	80°
2	$\frac{1}{8}$	27	180°
3	30	28	$\frac{1}{2}(7n-1)$
4	d=5,a_3=16	29	a = -1, b= 15, c = 31
5	3	30	1, 3, 5, 7
6	1085	31	n^2
7	18	32	2:3
8	17	33	$\frac{2n-1}{2n-1}$
9	-1	34	2, 4, 6
10	111	35	30
11	1	36	0
12	34	37	-250
13	10	38	15^{th} term
14	$1\frac{2}{5}$	39	₹ 435
15	$\frac{n(n+1)}{2}$	40	852
16	$\frac{n+1}{n}$	41	$p+q-n$
17	15	42	1
18	22	43	0
19	3	44	1170
20	0	45	3
21	6, 11, 16	46	6−2n
22	4abc	47	1
23	3:4:5	48	867
24	8	49	89
25	23	50	₹240

CHAPTER – 6

TRIANGLES

POINTS TO REMEMBER

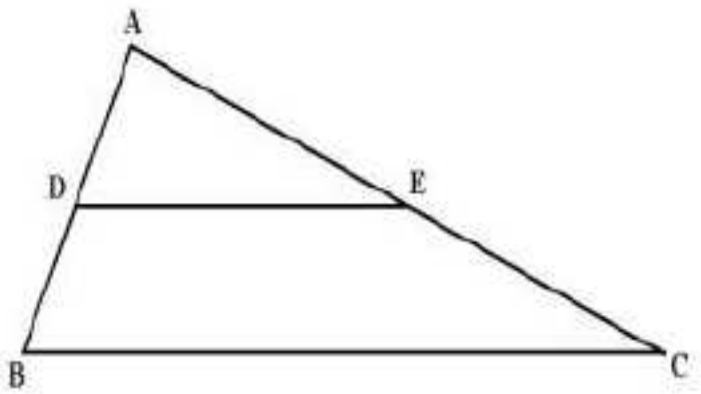
- **Two triangles are said to be similar if their corresponding angles are equal and their corresponding sides are proportional (in the same ratio)**



$$\Delta ABC \sim \Delta PQR \Rightarrow \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \;\&\; \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

- **Criteria of Similarity: (a) AAA or AA (b) SSS (c) SAS**
- **BASIC PROPORTIONALITY THEOREM(BPT) or THALES THEOREM**

If a line is drawn parallel to one side of a triangle to intersect the other two sides in two distinct points, then the other two sides are divided in same ratio.

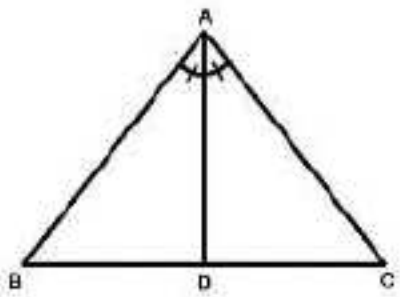


In ΔABC, $DE \parallel BC$, then

$$\frac{AD}{DB} = \frac{AE}{EC} \text{ and } \frac{AB}{DB} = \frac{AC}{EC} \text{ and } \frac{AB}{AD} = \frac{AC}{AE}$$

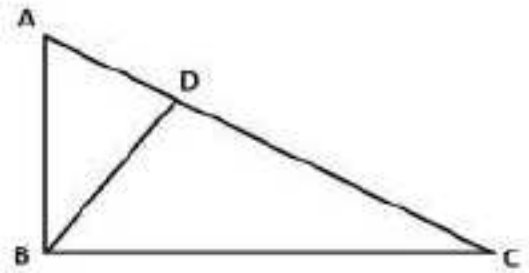
- **If $\Delta ABC \sim \Delta PQR \Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$**

- In ΔABC, if AD is the angle bisector, then $\frac{AB}{AC} = \frac{BD}{CD}$

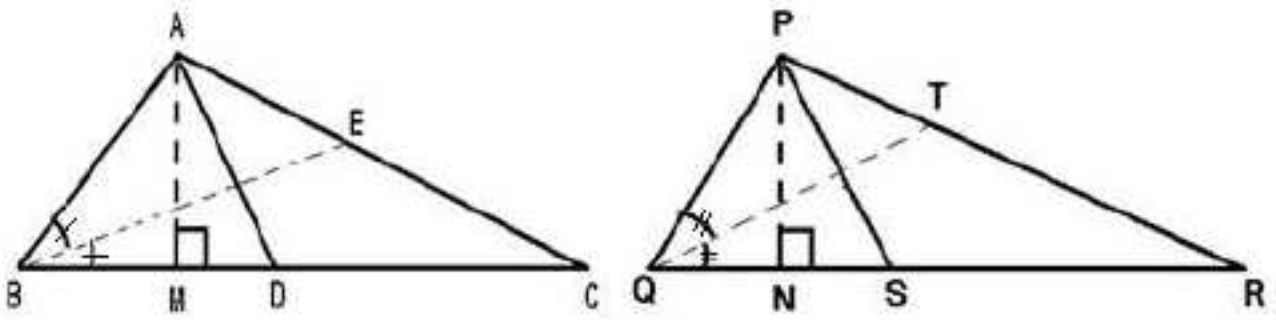


- In $\Delta ABC, \angle B = 90°$, then $AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

- In $\Delta ABC, \angle B = 90°$ and $BD \perp AC$, then $\Delta ABD \sim \Delta ACB \sim \Delta BCD$.



- If two triangles are similar then their perimeters, medians, altitudes and angle bisectors are in the same ratio.
- The areas of two similar triangles are in the ratio of squares of their corresponding sides, altitudes, medians, perimeter and angle bisectors.

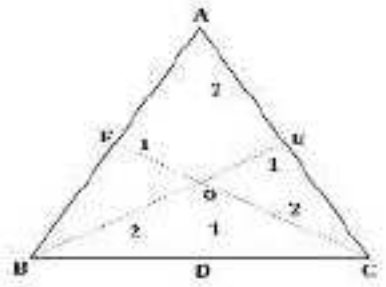


- E.g. If $\Delta ABC \sim \Delta PQR \Rightarrow \frac{\text{perimeter}(\Delta ABC)}{\text{perimeter}(\Delta PQR)} = \frac{AD}{PS} = \frac{AM}{PN} = \frac{BE}{QT}$ and

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{AD^2}{PS^2} = \frac{AM^2}{PN^2} = \frac{(\text{perimeter of } \Delta ABC)^2}{(\text{perimeter of } \Delta PQR)^2} = \frac{BE^2}{QT^2}$$

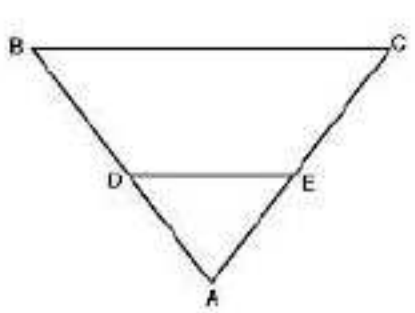
- **Medians are divided by the centroid in the ratio 2:1**

$$\frac{AO}{OD} = \frac{BO}{OE} = \frac{CO}{OF} = \frac{2}{1}$$

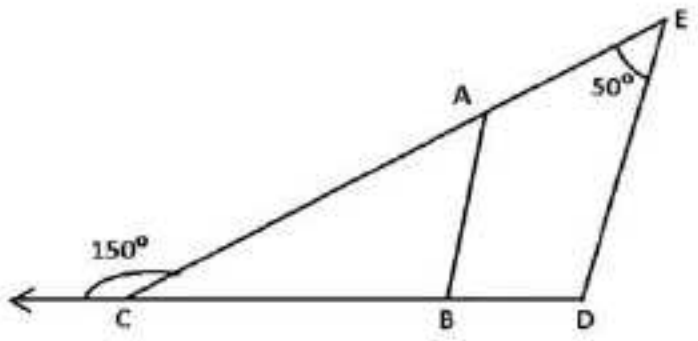


QUESTIONS

1. **In ΔABC, $DE \parallel BC$ such that BC = 8cm, AB=6 cm and DA=1.5 cm. Find DE.**

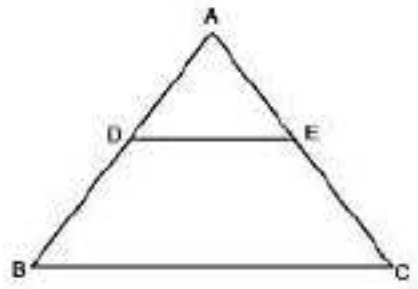


2. **In fig. $\Delta ACB \sim \Delta ECD$, find $\angle ABC$.**

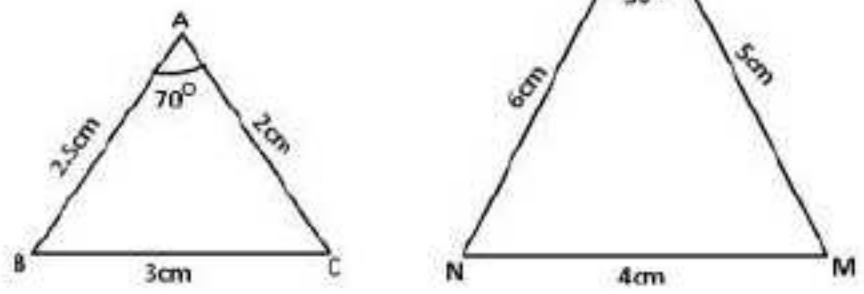


3. **In ΔABC, D and E are points on AB and AC respectively such that $DE \parallel BC$.**

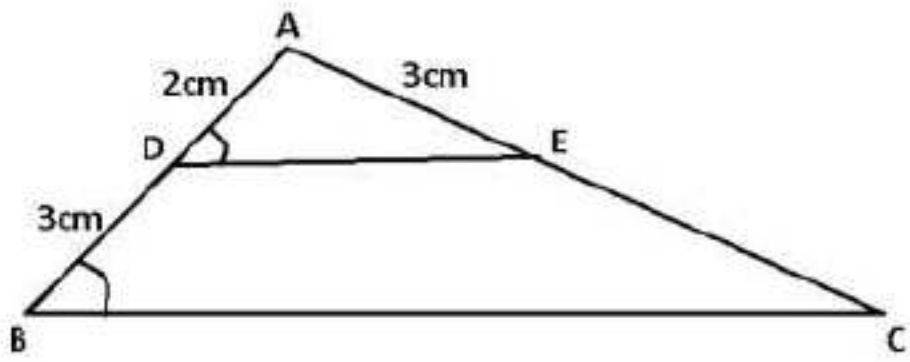
If $AD = \frac{1}{3}BD$ and AE= 4.5cm, then find AC.



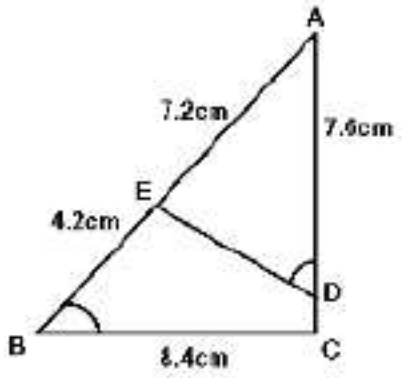
4. **In fig. find $\angle M$ and $\angle N$.**



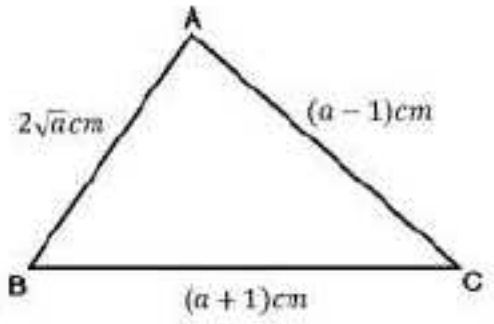
5. **In fig., if $\angle ADE = \angle ABC$, then find CE.**



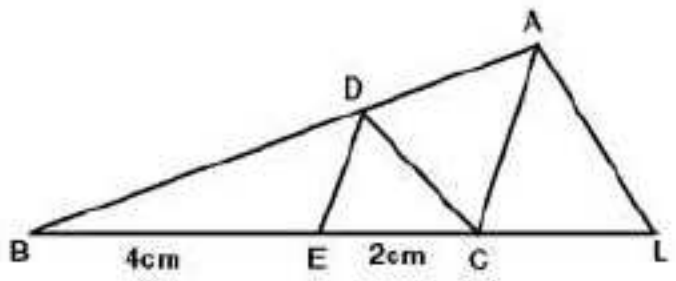
6. **ΔABC and ΔBDE are two equilateral triangles such that D is the mid-point of BC. Find ratio of the areas of Δ ABC and Δ BDE.**
7. **Find the perimeter of a square whose diagonal is 10 cm.**
8. **$\Delta ABC \sim \Delta DEF$, AB = 4cm, DE = 6cm, EF = 9cm and FD = 12cm, find the perimeter of ΔABC.**
9. **Find the altitude of an equilateral triangle having side 24 cm.**
10. **In ΔABC, $\angle ADE = \angle ABC$, find DE.**



11. In ΔABC, find $\angle BAC$.



12. In the given figure, CD || LA and DE || AC. Find CL.

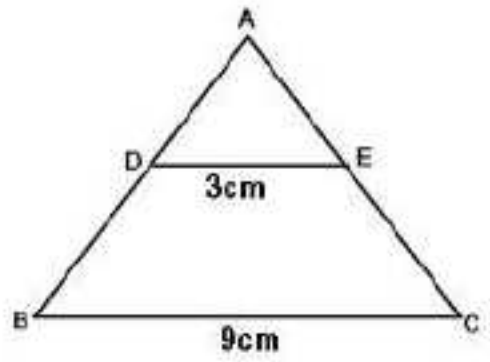


13. Find the value of a if a cm, (a−1) cm and (a + 8) cm are the lengths of the sides of a right angled triangle where a is a natural number.

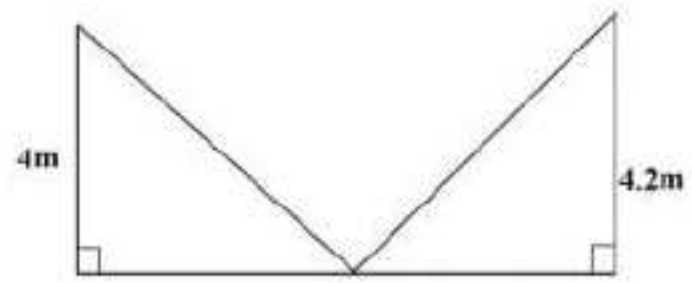
14. Find the distance between a tower and a building of heights 55m and 34m respectively, such that the distance between their tops is 29m.

15. $\Delta PQR \sim \Delta ABC$, if PQ:AB=3:4 and $ar(\Delta PQR) = 216 sq.units$, find the area of ΔABC.

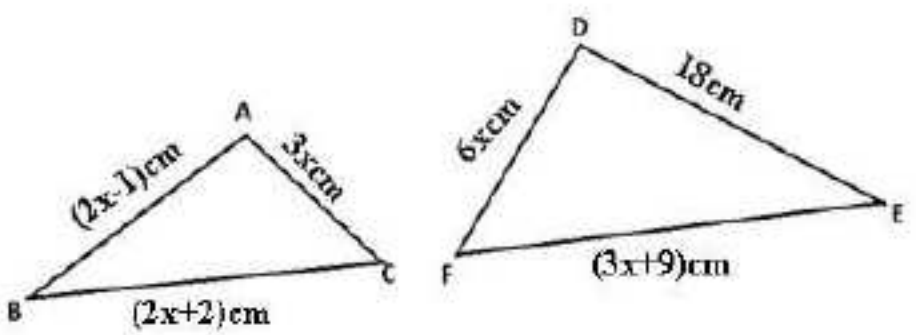
16. In ΔABC, $DE \parallel BC$, $ar(\Delta ADE) = 30$ sq. cm, find the ar(trap. BCED).



17. **Walls of two buildings on either side of a street are parallel to each other. A ladder 5.8m long is placed on the street such that its top just reaches the window of a building at the height of 4 m. On turning the ladder over to the other side of the street, its top touches the window of the other building at a height 4.2m. Find the width of the street.**

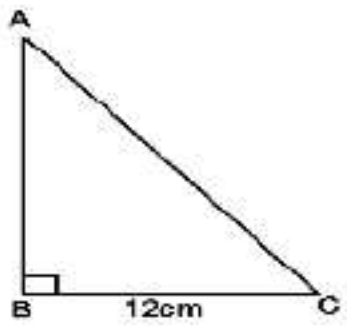


18. **If $\Delta ABC \sim \Delta DEF$, then find the value of x.**

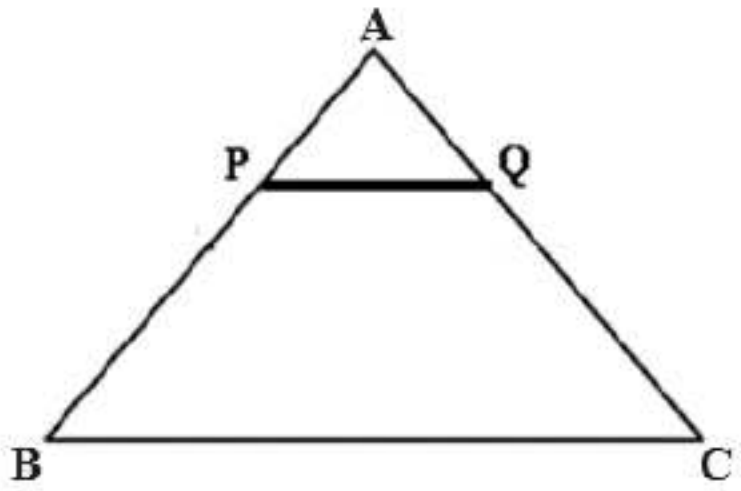


19. **In an isosceles right angled triangle, if the hypotenuse is $5\sqrt{2}$cm, then find the length of each equal side of the triangle.**

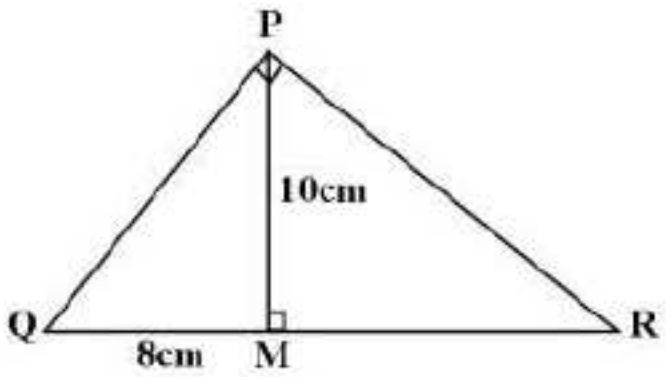
20. If ar(ΔABC) = **210 sq. cm**, then find AC.



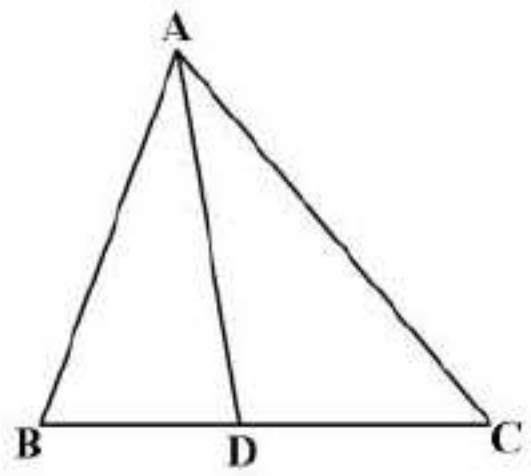
21. In fig. PQ∥BC and AP : PB=1:2, find $\frac{\text{perimeter of } \Delta APQ}{\text{perimeter of } \Delta ABC}$.



22. In $\Delta PQR, \angle QPR = 90^{\circ}$ and PM ⊥QR, QM = 8cm. Find QR.

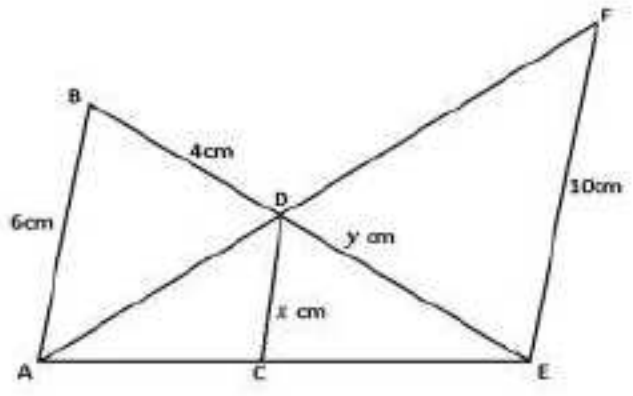


23. In ΔABC, it is given that $\frac{AB}{AC} = \frac{BD}{DC}$, if $\angle B = 70^{\circ}$ and $\angle C = 50^{\circ}$, find $\angle BAD$.

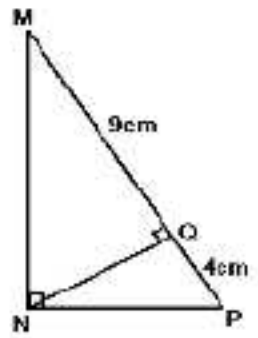


24. **Find the perimeter of an equilateral triangle whose height is $\sqrt{3}$ cm.**

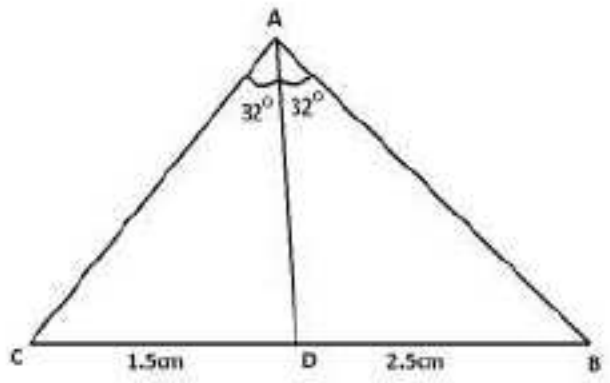
25. **In fig. AB∥CD∥EF, if AB= 6cm, CD= x cm, EF = 10cm, BD = 4cm and DE = y cm, then find the value of x and y.**



26. **In the given figure, if $\angle MNP = 90^{\circ}$, NQ⊥ MP, then find NQ.**

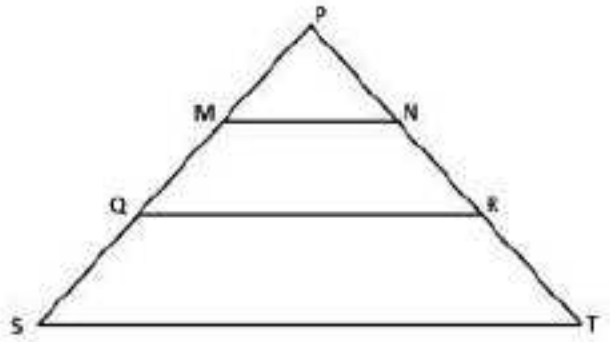


27. **In fig. find $\frac{AB}{AC}$.**

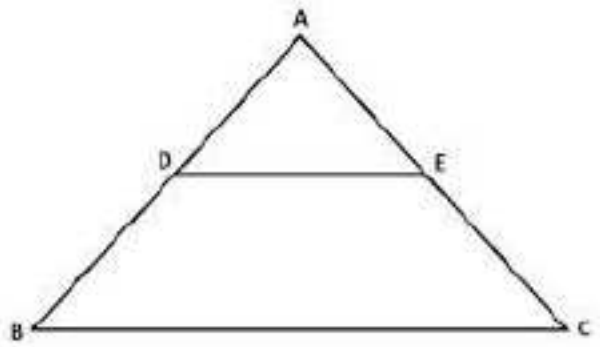


28. **ΔABC is an isosceles right angled triangle, right angled at B. Point P is on base BC such that PC= $\frac{1}{3}$BC, if AB=6 cm, then find AP.**

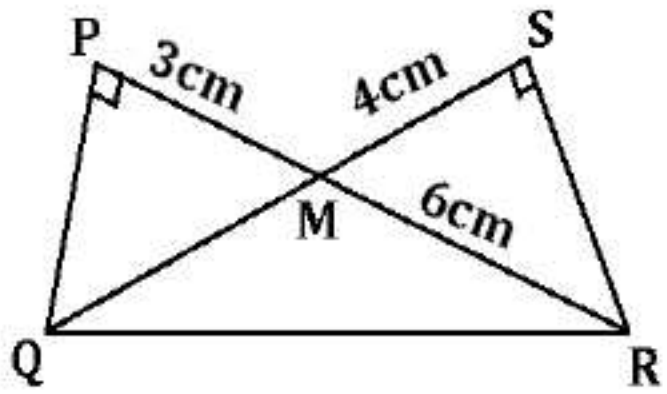
29. In fig. $\angle S = \angle PRQ$, MN || QR, if $\frac{PS}{PT} = \frac{2}{3}$, then find $\frac{PN}{PM}$.



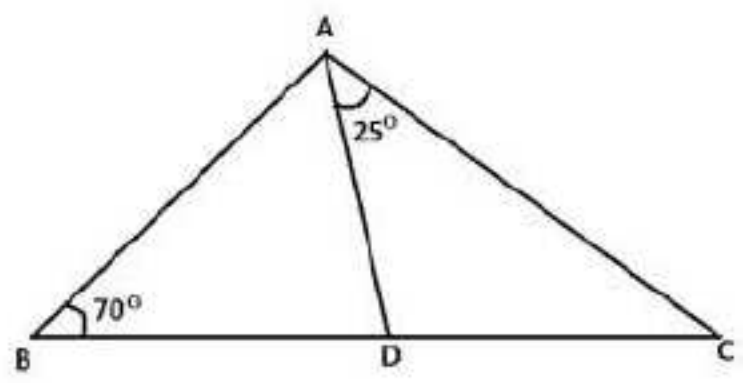
30. In fig. DE ‖ BC and AD: DB= 2:3. Find $ar(\Delta ADE) : ar(\Delta ABC)$.



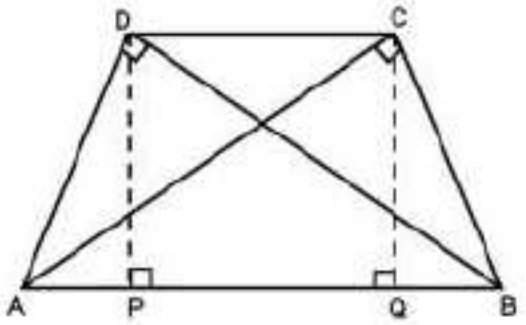
31. In the given figure, find the length of MQ.



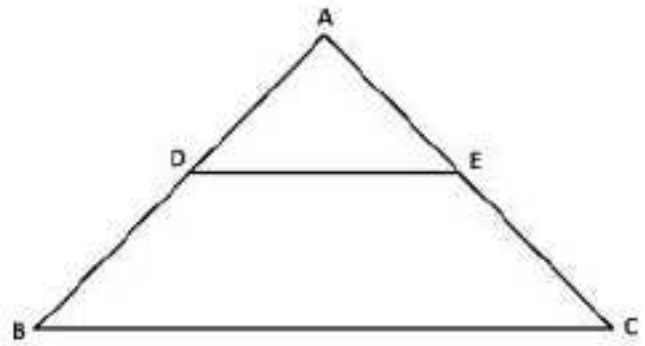
32. In the given fig. if $\frac{AB}{AC} = \frac{BD}{CD}$, then find $\angle ACD$.



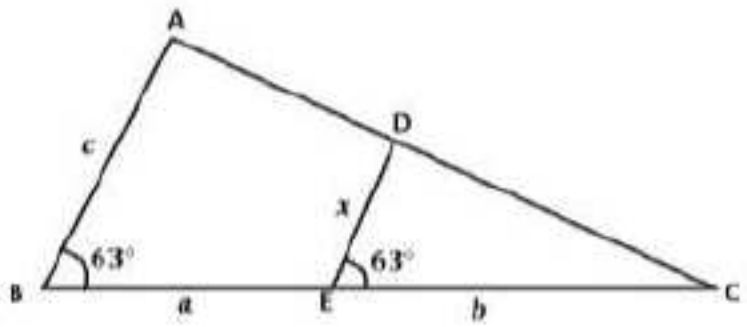
33. In a trapezium ABCD, AB∥DC, BD ⊥ AD and AC ⊥ BC. If AD=15cm, BC=15cm, AB=25cm. Find ar (trapezium ABCD).



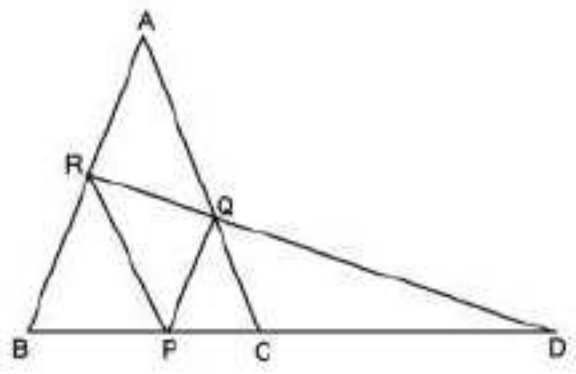
34. In the given fig., if DE∥BC, AD = 4BD and AC=1.6 cm, then find AE.



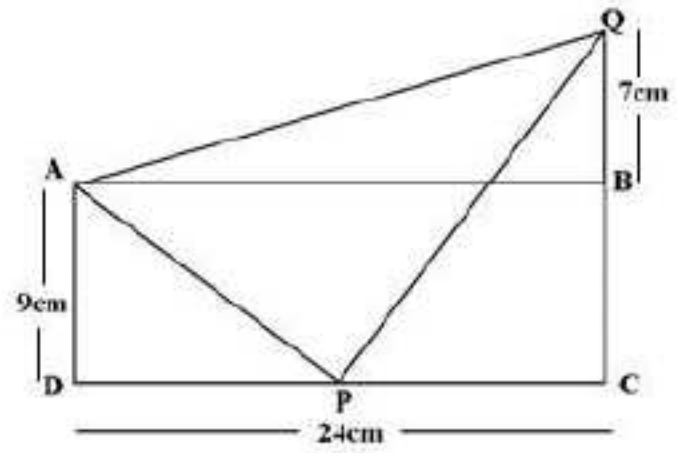
35. Express x in terms of a, b and c from the given figure.



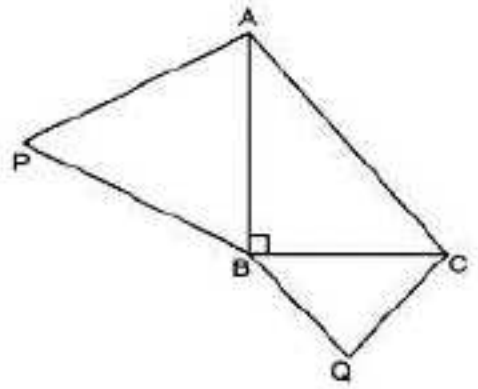
36. In the given figure, PQ ∥ BA and PR ∥ CA. If PD=12cm, then find BD × CD.



37. In the given figure, ABCD is a rectangle, P is the mid-point of DC. Find the measure of ∠APQ.

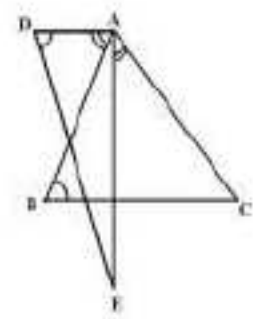


38. **Equilateral triangles are drawn on the sides of a right angled triangle ABC. If ar(ΔPAB) = $9\sqrt{3}$ sq. cm and ar(ΔBQC) = $16\sqrt{3}$ sq. cm, then find AC.**

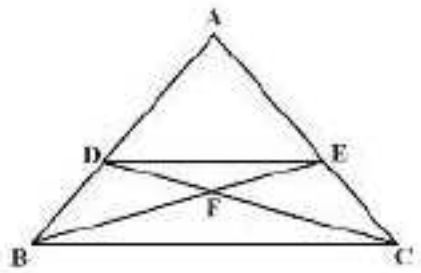


39. **If in an equilateral triangle, the length of the median is $\sqrt{3}$cm, then find the length of side of the equilateral triangle.**

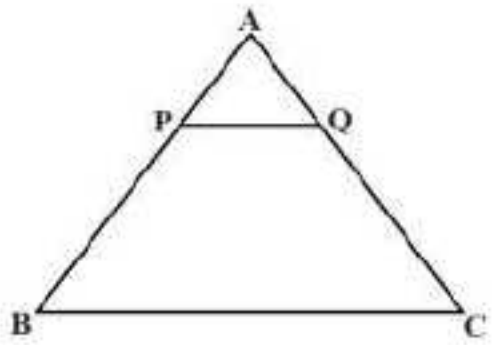
40. **In fig. ∠BAD = ∠CAE and ∠ADE = ∠ABC. If AC: BC = 3:2, then find the ratio DE: AE.**



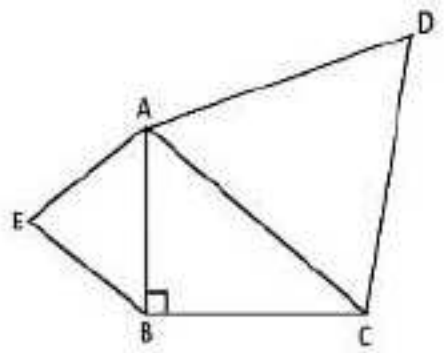
41. **In fig. DE ∥ BC and AD: DB = 5:4, then find $\frac{ar(\Delta DFE)}{ar(\Delta CFB)}$.**



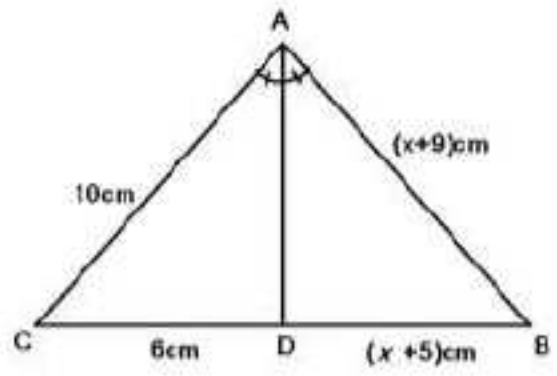
42. In the given figure, PQ∥BC and AP : PB = 3:7. If ar(ΔABC) is 100 sq.cm, then find ar(trapezium PQCB) .



43. ΔABC is an isosceles right triangle right angled at B. Two equilateral triangles are constructed on sides AB and AC. Find $\frac{ar(\Delta ABE)}{ar(\Delta ACD)}$.



44. In ΔABC, if∠CAD = ∠BAD, then find x.



45. If ΔABC~ΔDEF such that AB=9.1cm and DE=6.5cm. If the perimeter of ΔDEF is 25cm, then find the perimeter of ΔABC.
46. A vertical pole of length 6m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28m long. Find the height of the tower.
47. ΔABC~ΔPQR such that ar (ΔABC) = 4ar(ΔPQR). If BC=12cm, then find QR.
48. The foot of the ladder is 6m away from the wall and its top reaches a window 8m above the ground. If the ladder is shifted in such a way that its foot is 8m away from the wall, to what height does it tip reaches?

49. In an isosceles ΔABC, if AB=AC=25cm and BC=14cm, then find the measure of altitude from A on BC.

50. In ΔABC, $\angle A = 90°$, AB=5cm and AC=12cm. if AD$\perp$BC, then find AD.

ANSWERS

Q. No	Answer	Q. No	Answer
1	2cm	26	6cm
2	100°	27	$1\frac{2}{3}$
3	18cm	28	$2\sqrt{13}$ cm
4	$\angle M=70°, \angle N=60°$	29	$\frac{2}{3}$
5	4.5cm	30	4:25
6	4:1	31	4.5cm
7	$20\sqrt{2}$ cm	32	60°
8	18cm	33	192sq.cm
9	$12\sqrt{3}$ cm	34	1.28cm
10	5.6cm	35	$\frac{bc}{a+b}$ units
11	90°	36	144 sq.cm
12	3cm	37	90°
13	21	38	10cm
14	20m	39	2cm
15	384sq.units	40	2:3
16	240sq.cm	41	$\frac{25}{81}$
17	8.2m	42	91 sq.cm
18	x=5	43	1:2
19	5cm	44	1
20	37cm	45	35cm
21	$\frac{1}{3}$	46	42cm
22	20.5cm	47	6cm
23	30°	48	6 m
24	6cm	49	24 cm
25	x=3.75 , y= 6.67	50	$4\frac{8}{13}$ cm

CHAPTER -7

COORDINATE GEOMETRY

POINTS TO REMEMBER

- To locate the position of a point in a plane, we require a pair of coordinates.
- Coordinate axes divide the plane into four quadrants.
- The perpendicular distance of a point from the y - axis measured along the x - axis is called its x coordinate (abscissa).
- The perpendicular distance of a point from the x - axis measured along the y axis is called its y coordinate (ordinate).
- The coordinate of a point on x - axis is of the form $(x, 0)$.
- The coordinate of a point on y - axis is of the form $(0, y)$.
- The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- The distance of a point P(x, y) from the origin O $(0, 0)$ is given by

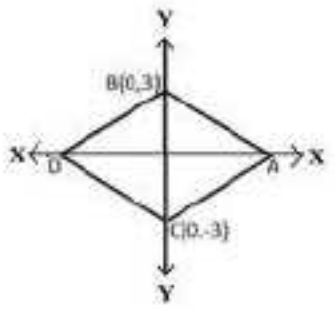
$$OP = \sqrt{x^2 + y^2}$$

- Coordinates of the point P(x, y) which divides the line segment joining the points A (x_1, y_1), and B (x_2, y_2) internally in the ratio $m_1 : m_2$ is $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$. It is called section formula.
- The coordinates of the midpoint of the line segment joining the points P (x_1, y_1) and Q(x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- The coordinates of the centroid of the triangle formed by points A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.
- The area of the triangle formed by points A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) is given by $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$ sq.units.
- If points A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are collinear, then the area of triangle formed by these three points is 0 and vice versa.

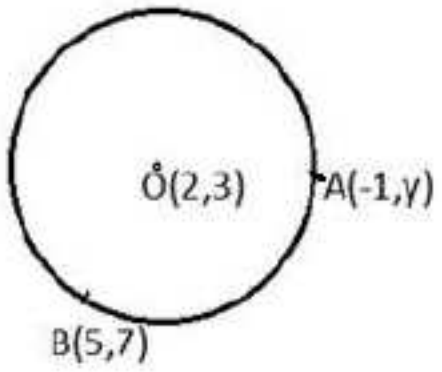
QUESTIONS

1. If the distance between the points (x, 0) and (0, 3) is 5 units, then find the value(s) of x.
2. Find the distance between the points (10 cos 30°, 0) and (0, 10 cos 60°).
3. Find the distance between the points $(-\frac{8}{5}, 2)$ and $(\frac{2}{5}, 2)$.
4. If the distance between the points (4, k) and (1, 0) is 5 units, then find the value(s) of k.
5. The x coordinate of a point P is twice its y coordinate. If P is equidistant from Q (2, -5) and R (-3, 6), then find the coordinates of P.
6. If P (5, 2), Q (2, -2) and R (-2, a) are the vertices of a right angled triangle with $\angle Q = 90°$, then find the value of a.
7. Find the distance between the points A (cosθ, sinθ) and B (sinθ, - cos θ).
8. Find the perimeter of the triangle with vertices (0, 4), (0, 0) and (3, 0).
9. A circle with centre C (3, 5) passes through a point (-2, 4). Find the diameter of the circle.
10. Find the lengths of sides AB, BC and CA of a triangle formed by joining the points A (-5, 6), B (-4, -2) and C (7, 5).
11. If the ordinate of a point A on y axis is 5 and point B has coordinates (-3, 1), then find the length of AB.
12. If the centre of circle is (3, 4), then find the distance of the point A (5, 8) from the centre of the circle.
13. Find the distance between the points (a cosb, a sinb) from the origin.
14. Find the value of k, if point (0, 4) is equidistant from the points (10, k) and (k, 8).
15. Find x and y, if O (0, 0), A (0, 2), B (x, y) and C (3, 0) form a rectangle OABC.
16. Find the value of y such that points A (5, y), B (5, 2), C (2, 2) and D (2, 5) form a square ABCD.

17. In the given figure, find the coordinates of the points A and D, if BACD is a rhombus and the base BC of an equilateral ΔABC lies on y axis.



18. In the given figure, O is the centre of circle and A and B are any two points on circle, find y.



19. Find coordinates of point P, if P and Q trisect the line segment joining the points (5, -3) and (-1, 3).
20. The line segment joining the points (2, 1) and (5, -8) is trisected at points P and Q. If point P lies on the line $2x - y + k = 0$, then find k.
21. Find the coordinates of vertex C, if length of one of the sides of an equilateral triangle is 'a' units and base BC lies on x-axis with B at the origin.
22. Find the coordinates of P, if the distance of the point P from the point (3, 4) is $\sqrt{10}$ units and abscissa of P is double of its ordinate.
23. Find the value of x, if the distance of the point (0, x) from (3, 5) is 5 units.
24. Find the value of y, if the points A (5, y), B (5, 5), C (1, 5) and D (1, 2) are the vertices of a rectangle.
25. Find the coordinates of point P which lies on x axis and equidistant from (-2, 5) and (2, -3).
26. Find the coordinates of point P that lies on y axis and equidistant from (3, 4) and (-2, 5).

27. Find the ratio in which the line segment joining the points P(3, -6) and Q (5, 3) is divided by x axis.

28. Let A and B be the points of trisection of the line segment joining the points P (2, -2) and Q (-7, 4) such that A is nearer to P. Find the coordinates of A.

29. If the point R (-1, 2) divides the line segment joining the points P(2, 5) and Q(x, y) internally in the ratio 3: 4, then find the value of $x^2 + y^2$.

30. Find the ratio in which S (4, m) divides the line segment joining the points P (2, 3) and Q (6, -3).

31. Find the coordinates of the point, which divides the line segment joining the points A (3, -6) and B (-2, 7) in the ratio 2:3.

32. Find the value of y, if P(x, y) divides the line segment joining points (-3, 3) and (1, -2) in the ratio 2:3.

33. Find the value of x, if P(x, y) divides the line segment joining the points A (7, -5) and B (2, -1) in the ratio 4:1.

34. Find the value of k, if x axis divides the line segment joining the points (-4, -6) and (5, 2) in the ratio k: 1.

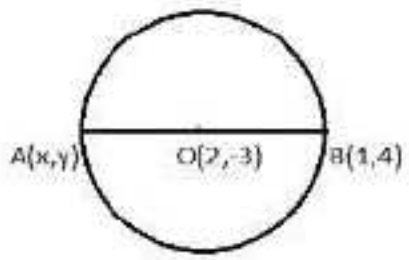
35. Find the ratio in which the line segment joining the points A (-4, 4) and B (8, 8) is divided by (-1, 5).

36. If P (2, q) is the mid-point of the line segment joining the points A (6, -5) and B (-2, 11), then find the value of q.

37. If M (0, 4) is the mid-point of the line segment joining the points P (-2, 3) and Q (a, b), then find the values of a and b.

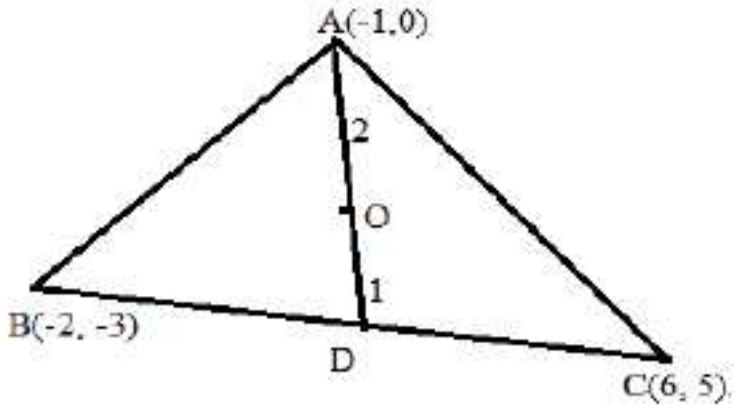
38. If the end points of diameter of a circle are (2, 4) and (-3, -1), then find the radius of the circle.

39. In the given figure, find the coordinates of A where O is the centre of the circle.

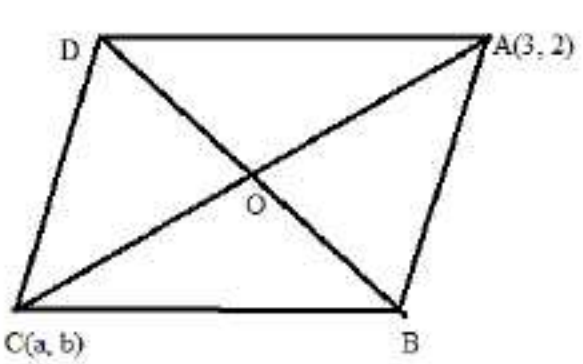


40. **Find the distance of the point (0, 2) from the midpoint of the line segment joining the points (4, 10) and (2, 2).**

41. **In the given figure, A (-1, 0), B (-2, -3) and C (6, 5) are the coordinates of vertices of ΔABC. If D is the mid-point of BC, then find the coordinates of O which divides AD in the ratio 2:1.**



42. **In the parallelogram ABCD, coordinates of A and C are (3, 2) and (a, b) respectively. If AC and BD intersects at O (0, 0), then find the values of 'a' and 'b'.**



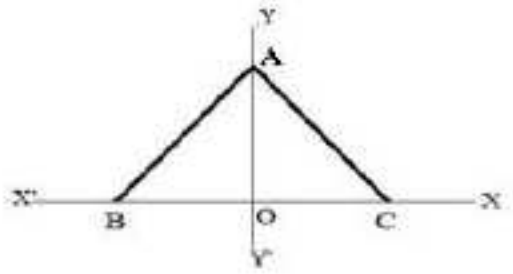
43. **Q is the midpoint of the line segment PR where coordinates of P, Q and R are (6, -2), (1, 3) and (x, 8) respectively. Find 'x'.**

44. **If $(\frac{a}{3}, 4)$ is the midpoint of the line segment joining the points P (-6, 5) and R (-2, 3), then find the value of a.**

45. **Find the value of p + q, if A (p, q) is the midpoint of the line segment joining the points (5, 3) and (-2, 4).**

46. **Find the value of p - q, if the midpoints of the line segment joining the points (3p, 4) and (-2, 2q) is (2, 6).**

47. The base BC of an equilateral ΔABC with side 24cm lies along the x-axis such that the midpoint of the base is at origin. Find the coordinates of B.



48. Find the coordinates of the centroid of the triangle whose vertices are $(3, -6)$, $(-3, -2)$ and $(6, -4)$.

49. If the centroid of the triangle formed by the points $(7, x)$, $(y, -6)$ and $(9, 10)$ is at $(6, 3)$, then find the value of $x + y$.

50. If the centroid of the triangle formed by the points (a, b), (b, c) and (c, a) is at the origin, then find the value of $a^3 + b^3 + c^3$.

51. If the points A $(-2, 1)$, B (a, b) and C $(4, -1)$ are collinear and $a - b = 1$, then find the values of a and b.

52. Find area of the triangle formed by the points $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$.

53. Find the area of the triangle formed by joining the mid points of the sides of a triangle, whose vertices are $(3, 2)$, $(5, 4)$, and $(3, 6)$.

54. Find the value of a so that the points A $(a, -6)$, B $(-3, -4)$ and C $(-1, 0)$ lie on a line.

55. Find the area of the triangle ABC with A $(1, -4)$ and mid points of sides through A being $(2, -1)$ and $(0, -1)$ respectively.

ANSWERS

Q. No	Answer	Q. No	Answer
1	$x = 4, -4$	29	29
2	10 units	30	1:1
3	2 units	31	$\left(1, -\frac{4}{5}\right)$
4	$k = \pm 4$	32	$y = 1$
5	(16, 8)	33	$x = 3$
6	$a = 1$	34	$k = 3$
7	$\sqrt{2}$ units	35	1:3
8	12 units	36	$q = 3$
9	$2\sqrt{26}$ units	37	$a = 2, b = 5$
10	AB = $\sqrt{65}$ units, BC = $\sqrt{170}$ units, CA = $\sqrt{145}$ units	38	$\frac{5\sqrt{7}}{7}$ units
11	5 units	39	A(3, −10)
12	$2\sqrt{5}$ units	40	5 units
13	a units	41	$\left(1, \frac{2}{3}\right)$
14	$k = 12.5$	42	$a = -3, b = -2$
15	$x = 3, y = 2$	43	$x = -4$
16	$y = 5$	44	$a = -12$
17	A = $(3\sqrt{3}, 0)$, D $(-3\sqrt{3}, 0)$	45	5
18	$y = -1$ or 7	46	-2
19	$(3, -1)$	47	$(-12, 0)$
20	−8	48	$(2, -4)$
21	$(a, 0)$ or $(-a, 0)$	49	7
22	(6, 3), (2, 1)	50	3abc
23	$x = 1, 9$	51	$a = 1, b = 0$
24	$y = 2$	52	0
25	$(-2, 0)$	53	1 sq. unit
26	$(0, 2)$	54	$a = -4$
27	2:1	55	12 sq. units
28	A(−1, 0)		

CHAPTER – 8

TRIGONOMETRY AND ITS APPLICATIONS

POINTS TO REMEMBER

1) $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{\text{cosec}\theta}$

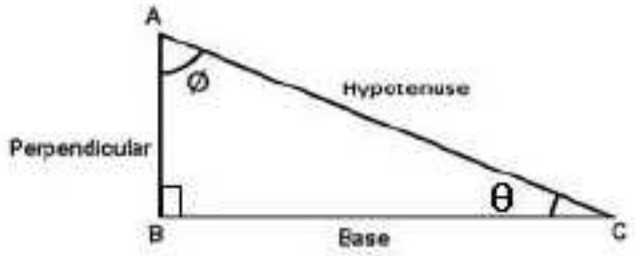
2) $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{1}{\sec\theta}$

3) $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{1}{\cot\theta}$

4) $\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perendicular}} = \frac{1}{\sin\theta}$

5) $\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{1}{\cos\theta}$

6) $\cot\theta = \frac{\text{Base}}{\text{Perendicular}} = \frac{1}{\tan\theta}$



$$\emptyset = 90° - \theta$$

$$\sin\emptyset = \sin(90° - \theta) = \frac{BC}{AC} = \cos\theta$$

Trigonometric ratios of complementary angles

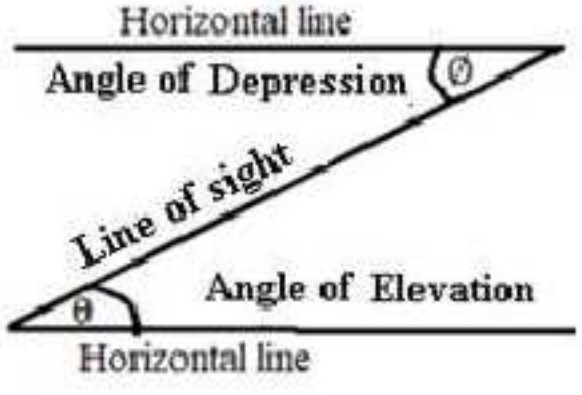
1) $\sin(90^\circ - \theta) = \cos\theta$ 2) $\cos(90^\circ - \theta) = \sin\theta$ 3) $\tan(90^\circ - \theta) = \cot\theta$

4) $\cot(90^\circ - \theta) = \tan\theta$ 5) $\sec(90^\circ - \theta) = \text{cosec}\,\theta$ 6) $\text{cosec}(90^\circ - \theta) = \sec\theta$

Trigonometric Identities

1) $\sin^2\theta + \cos^2\theta = 1$ 2)) $\sec^2\theta - \tan^2\theta = 1$ 3) $\text{cosec}^2\theta - \cot^2\theta = 1$

Angles of elevation and angles of depression: -



1) Angle of elevation is the angle that is formed between the horizontal line and the line of sight when an observer looks upwards
2) Angle of depression is the angle that is formed between the horizontal line and the line of sight when an observer looks downwards
3) Angle of Elevation = Angle of Depression

QUESTIONS

1. If $\sec\theta + \tan\theta = 2$, then find $\tan\theta$.
2. If $\sin\theta - \cos\theta = \frac{3}{5}$, then find $\sin\theta\cos\theta$.
3. If $\sin\theta = \frac{a}{b}$, then find $\cos\theta$.
4. If $\tan x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$ and x is acute, find x.
5. If $\cos\theta + \sec\theta = 2$, then find $\cos^3\theta + \sec^3\theta$.
6. If $\cot\theta = \frac{17}{4}$, then find $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$.
7. If θ is acute and $\sin\theta - \cos\theta = \frac{1}{2}$, then find $\sin\theta + \cos\theta$.
8. If $\theta = 45^\circ$, then find the value of $2\sin\theta\cos\theta$.
9. Find the value of $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$.
10. If $5\cos\theta = 3$, then find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}$.
11. If $\tan\theta = \frac{12}{5}$, then find the value of $\frac{13\sin\theta}{3}$.
12. Find the value of $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2$.
13. Find the value of $(\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta)$.
14. If $\tan\theta + \cot\theta = 2$, then find the value of $\tan^2\theta + \cot^2\theta$.
15. If $\cos A + \cos^2 A = 1$ then find the value of $\sin^2 A + \sin^4 A$.
16. Find θ (θ is acute) if $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$
17. If $\sin A + \operatorname{cosec} A = 3$, then find $\frac{\sin^4 A + 1}{\sin^2 A}$.
18. If $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 14k + \tan^2\theta + \cot^2\theta$, then find k.
19. If $\cos\theta - \cos(90^\circ - \theta) = 0$, then find the value of θ (θ is acute).
20. If $x = 15^\circ$, then find the value of $4\sin 2x \cos 4x \sin 6x$.
21. If $k - 2 = \sec^2 A (1 + \sin A)(1 - \sin A)$, then find the value of k.
22. If $\sin^4\theta - \cos^4\theta = k^4$, then find $\sin^2\theta - \cos^2\theta$.
23. If $x = 4\sin\theta$ and $y = 3\cos\theta$, then find the value of $\sqrt{16y^2 + 9x^2}$.
24. Find the value of $\frac{1+\cot^2\theta}{1-\cos^2\theta} - \operatorname{cosec}^4\theta$.
25. Find the value of $\frac{\cos^2 A + \sin^2 A - 1}{\sin^2 A \tan^2 A}$.
26. Find the value of $\sin^2 10^\circ + \sin^2 20^\circ + \sin^2 30^\circ + \ldots + \sin^2 80^\circ$.

27. If $\tan A + \cot A = 4$, then find the value of $\frac{\tan^2 A + \cot^2 A}{\tan^2 A + \cot^2 A + 20 \tan A \cot A}$.

28. Find the value of $\cot^4 A - \operatorname{cosec}^4 A + \cot^2 A + \operatorname{cosec}^2 A$.

29. If $7\sin^2 A + 3\cos^2 A = 4$ and $0° < A < 90°$, then find the value of $\tan A$.

30. If $\cos\theta + \sec\theta = 2$, then find the value of $\cos^{68}\theta + \sec^{68}\theta$.

31. If $x = a\cos^3\theta$ and $y = b\sin^3\theta$, then find the value of $\left(\frac{x}{a}\right)^{\frac{1}{3}} + \left(\frac{y}{b}\right)^{\frac{1}{3}}$.

32. Find the value of $3(\sec^2\theta + \tan^2\theta)$, if $\sec^2\theta = \frac{4}{3}$.

33. If $x = a(\sin\theta + \cos\theta)$ and $y = b(\sin\theta - \cos\theta)$, then find the value of

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2.$$

34. If $x\sin 45° = y\operatorname{cosec} 30°$, then find the value of $\left(\frac{x}{y}\right)^4$.

35. If $\cos x + \cos y = 2$, then find the value of $\sin x + \sin y$.

36. Find the value of $\tan\theta \times \frac{\sin\theta - \sin^3\theta}{\cos\theta - \cos^3\theta}$.

37. Find the value of x $(0 < x < 90)$ if $\cos x° = \cos 60° \cos 30° + \sin 60° \sin 30°$

38. If $\sin(A - B) = \sin A \cos B - \cos A \sin B$, then find $\sin 15°$.

39. Find $\tan A$ in terms of $\sin A$.

40. Simplify: $(\sec\theta + \tan\theta)(1 - \sin\theta)$.

41. Find the value of $\frac{\sin^4\theta + \cos^4\theta}{1 - 2\sin^2\theta\cos^2\theta}$.

42. Find the value of $\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} - \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$.

43. Find the value of $(1 + \cot^2\theta)(3 - 3\cos\theta)(1 + \cos\theta)$.

44. Find the value of

$$\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

45. $4\sin\theta = 3$, find the value of x, if

$$\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \frac{\sqrt{7}}{x} + \cos\theta$$

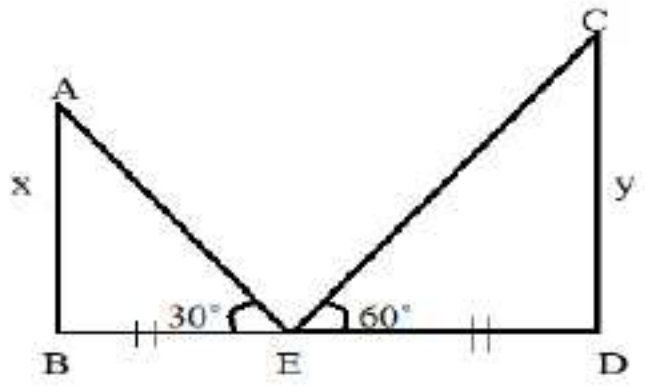
46. Find the angle of elevation of the sun at an instant when the length of the shadow of a pole is $\sqrt{3}$ times its height.

47. A ladder was placed against a wall in such a way that it makes an angle of 30° with the ground. If top of the ladder is 10m above the ground, find the distance between wall and foot of the ladder.

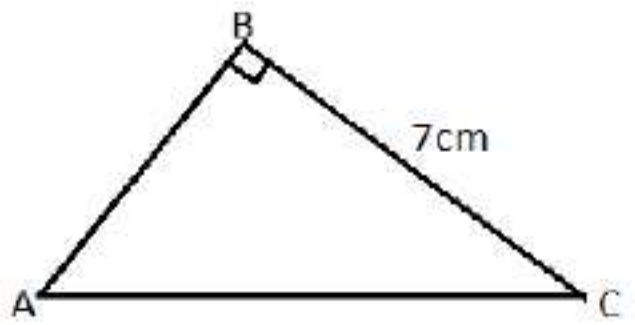
48. Two posts are 'k' metre apart and the height of the one is double that of the other. If from the middle point of the line segment joining their feet, an observer finds the angular elevation of their tops to be complementary, then find the height (in m) of shorter post.

49. If a tower of 6 metre height casts a shadow of $2\sqrt{3}$ metre along the ground, then find the angle of elevation of the sun with the ground at that time.

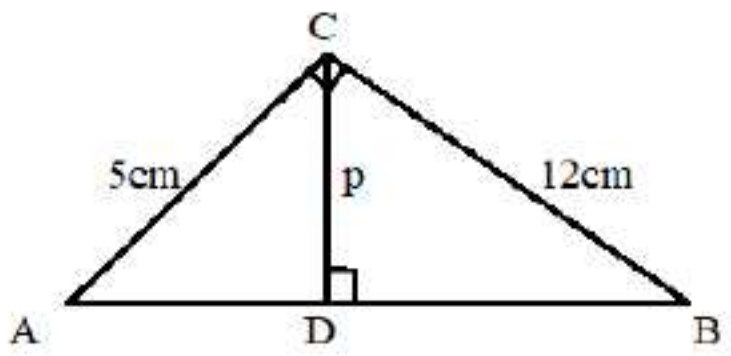
50. In the given figure, if BE = ED then find x:y



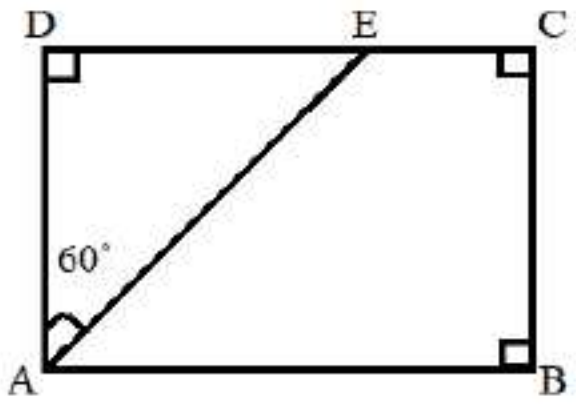
51. In right angled ΔABC, $\angle B = 90°$ and $AC - AB = 1$ cm, then find the value of $\cos A + \cos B + \cos C$



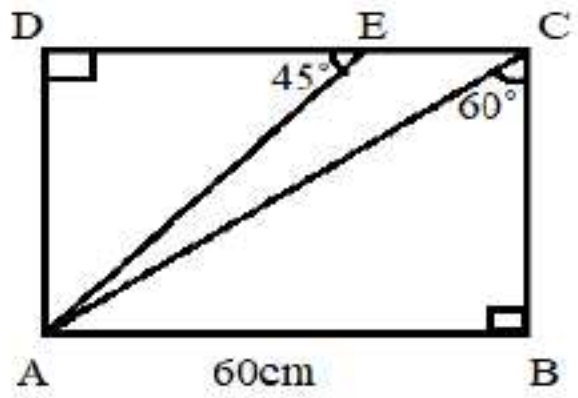
52. In the given figure, find the value of 'p'.



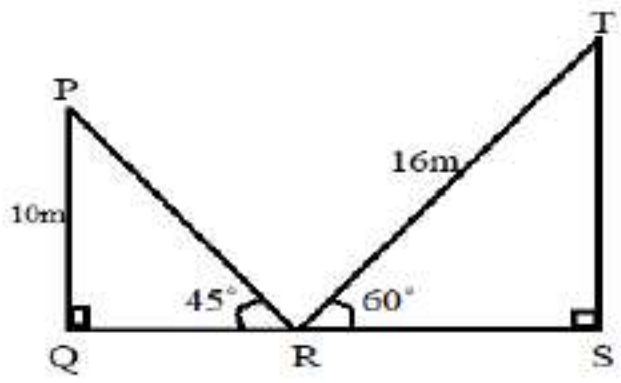
53. In the given figure, if AD = 40 cm, then find AE.



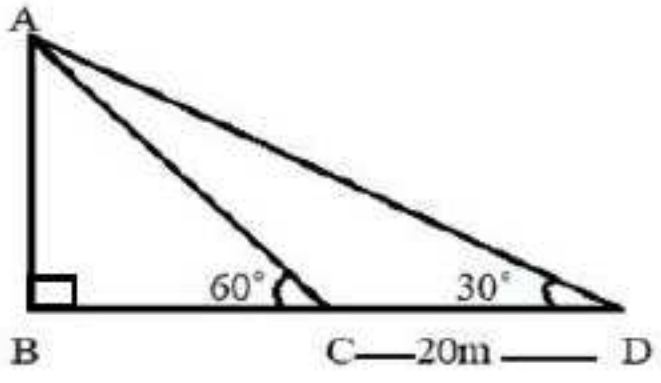
54. In the given figure, find AC + AD.



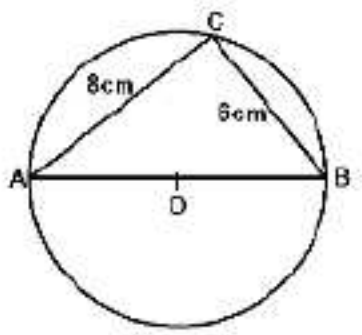
55. In the given figure, find the length of QS (in m).



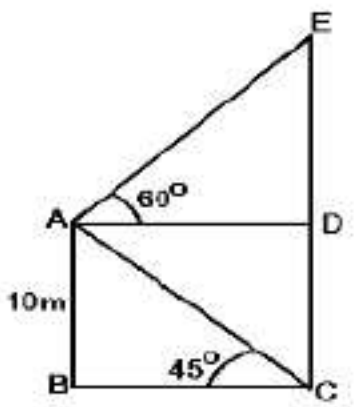
56. In the given figure, if ∠B = 90°, then find the height of the tower AB (in m)



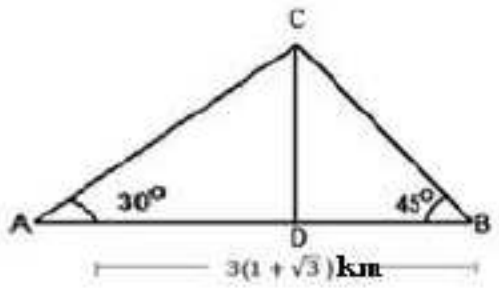
57. In the given fig. find **tanAtanB**, where O is centre of the circle.



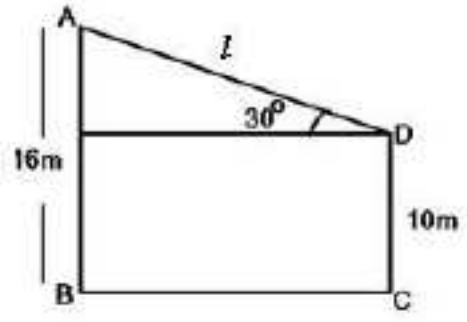
58. In the given figure, a man is standing on the deck of a ship at point A. CE is the hill. Calculate the height of the hill.



59. Stations A and B are $3(1+\sqrt{3})$ km apart. Each station sights an airplane at an angle of elevation $30°$ & $45°$ as shown in figure. Find the altitude of the airplane.



60. The tops of two poles of height **16m** and **10m** are connected by a wire of length l. If the wire makes an angle of $30°$ with the horizontal (see fig), then find l.



ANSWERS

Q. No.	Answer	Q. No.	Answer
1.	$\frac{3}{4}$	16.	$60°$
2.	$\frac{8}{25}$	17.	7
3.	$\frac{\sqrt{b^2 - a^2}}{b}$	18.	$\frac{1}{2}$
4.	$45°$	19.	$45°$
5.	2	20.	1
6.	$18\frac{1}{16}$	21.	3
7.	$\frac{\sqrt{7}}{2}$	22.	k^4
8.	1	23.	12
9.	1	24.	0
10.	$\frac{11}{29}$	25.	0
11.	4	26.	4
12.	2	27.	$\frac{7}{17}$
13.	-1	28.	0
14.	2	29.	$\frac{1}{\sqrt{3}}$
15.	1	30.	2

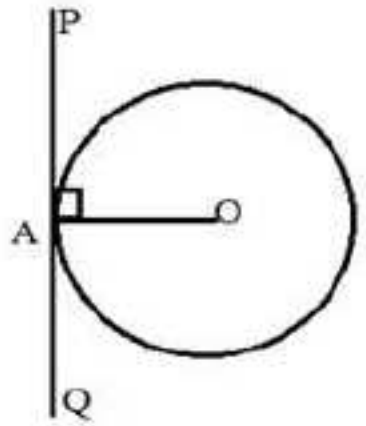
Q. No.	Answer	Q. No.	Answer
31.	1	46.	$30°$
32.	5	47.	$10\sqrt{3}$m
33.	2	48.	$\frac{k}{2\sqrt{2}}$ m
34.	64	49.	$60°$
35.	0	50.	1:3
36.	1	51.	$1\frac{6}{25}$
37.	30	52.	$4\frac{8}{13}$ cm
38.	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	53.	80cm
39.	$\frac{\sin A}{\sqrt{1-\sin^2 A}}$	54.	$60\sqrt{3}$ cm
40.	$\cos\theta$	55.	18m
41.	1	56.	$10\sqrt{3}$ m
42.	2	57.	1
43.	3	58.	$10(\sqrt{3}+1)$m
44.	2	59.	3km
45.	$1\frac{1}{3}$	60.	12 m

CHAPTER – 9

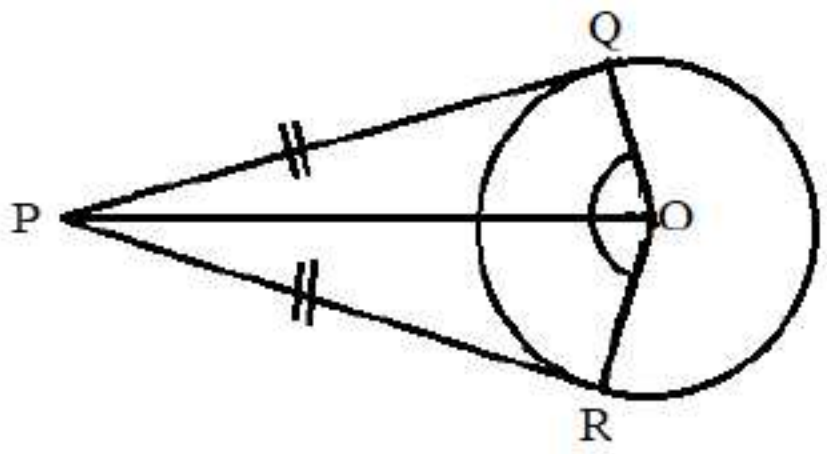
CIRCLES

POINTS TO REMEMBER

- **A tangent to a circle is a line that touches the circle at only one point.**



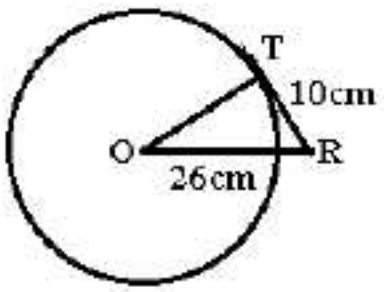
- **The tangent at any point of a circle is perpendicular to the radius through the point of contact. OA ⊥ PA. i.e. $\angle OAP = 90^\circ$.**
- **There are exactly two tangents to a circle through a point lying outside the circle. PQ and PR are two tangents from the external point P.**
- **The length of tangents drawn from an external point to a circle are equal Here, PQ = PR**



- **In the above figure, the sum of opposite angles of a Quadrilateral OQPR is 180°**

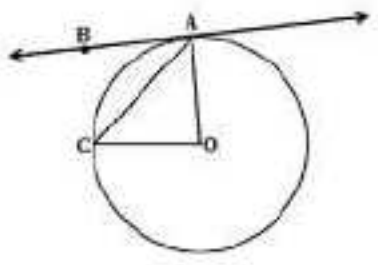
QUESTIONS

1. If a point R is 26 cm away from the centre O of a circle and the length of tangent RT is 10 cm, then find the radius OT.

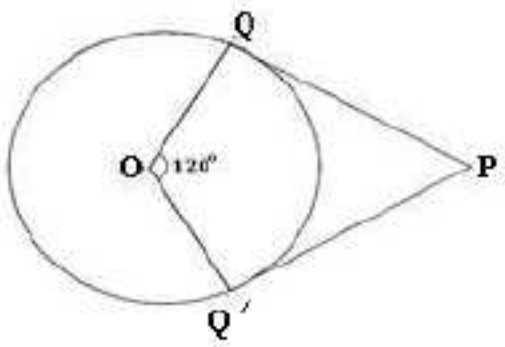


2. A chord of a circle of radius 8 cm subtends right angle at its centre. Find the length of the chord.

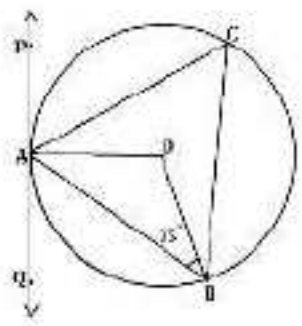
3. In the given figure, O is centre of the circle, AC is a chord and AB is a tangent at A. If $\angle AOC = 60°$, then find $\angle BAC$.



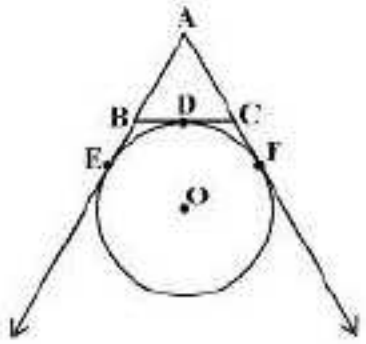
4. In the given figure, PQ and PQ′ are tangents from point P to the circle with centre O. If $\angle QOQ' = 120°$, then find $\angle OPQ$.



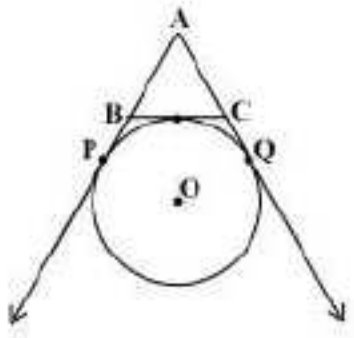
5. In the given figure, PAQ is a tangent to a circle with centre O at a point A. If $\angle OBA = 35°$, then find the value of $\angle BAQ$.



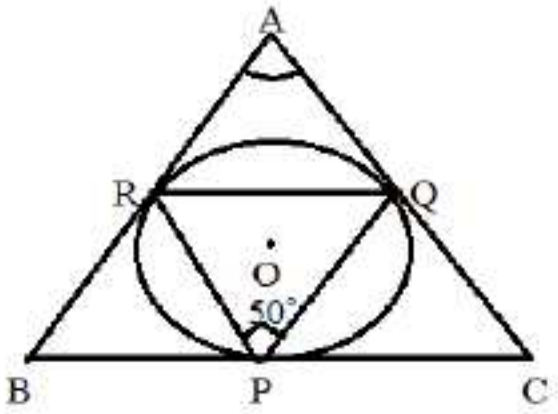
6. In the given figure, a circle touches the side BC of ΔABC and touches AB and AC produced at E and F respectively. If AE = 8cm, then find the perimeter of ΔABC.



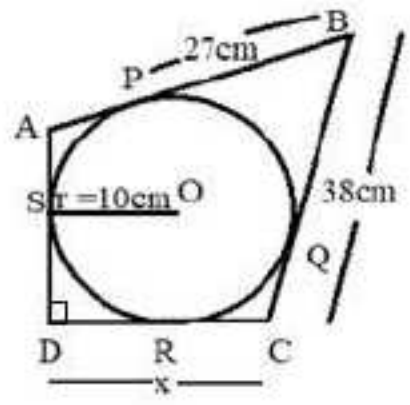
7. In the given figure, AP, AQ and BC are tangents to a circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then find the length of AP.



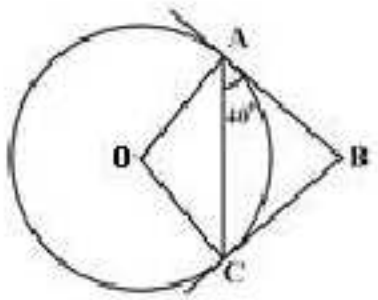
8. In the given figure, if ∠RPQ = 50° and O is the centre of circle, then find ∠BAC.



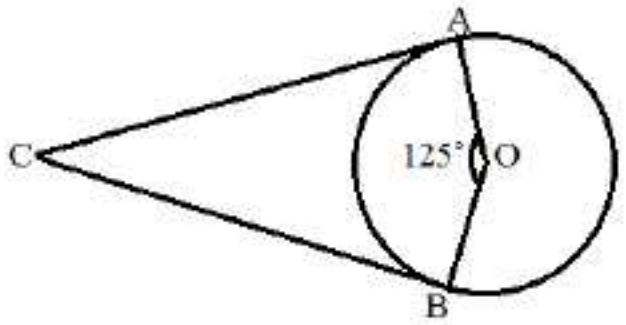
9. In the given figure, if O is the centre of circle, then find the value of x.



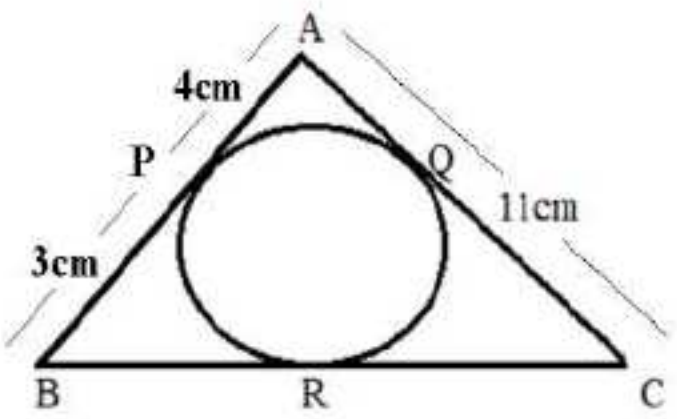
10. From an external point B, tangents BA and BC are drawn to a circle with centre O. If $\angle BAC = 40°$, then find $\angle AOC$.



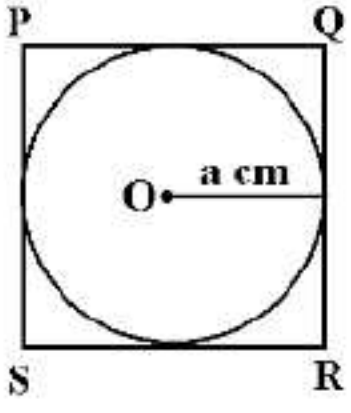
11. In the given figure, find the value of $(\angle ACB + \angle CAO)$.



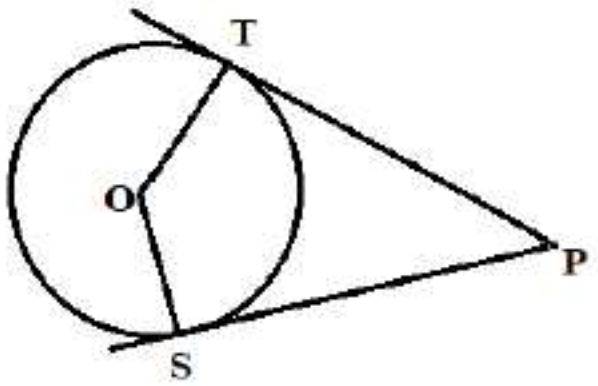
12. In the given figure, find the length of BC.



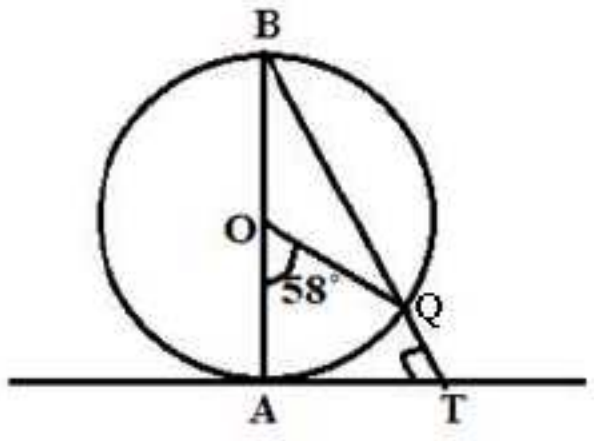
13. Find the perimeter of a square circumscribing a circle of radius 'a' cm.



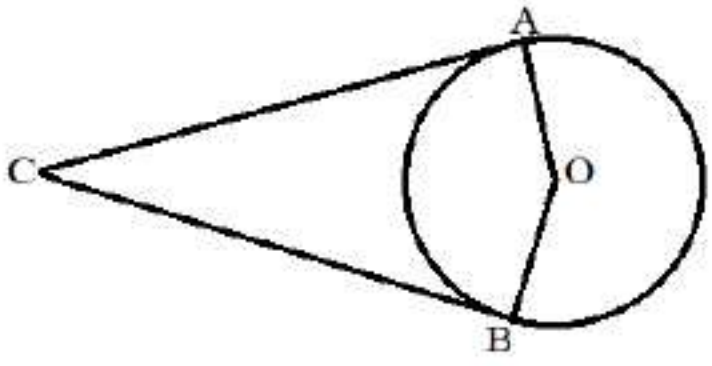
14. In the given figure, O is the centre of circle with radius r. If OP = 2r, then find the value of ∠OST.



15. In the given figure, AB is the diameter of circle with centre O and AT is tangent. Find the value of ∠ATQ.

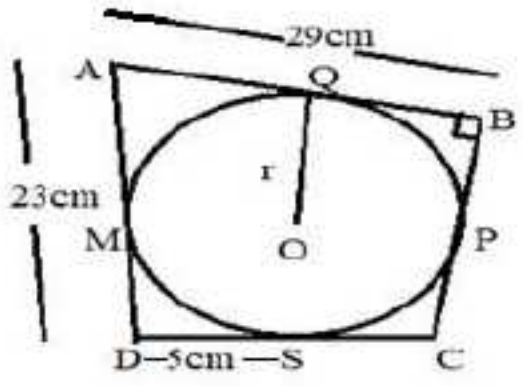


16. In the given figure, find the value of ∠AOB, if ∠ACB + ∠CBO = 120°.

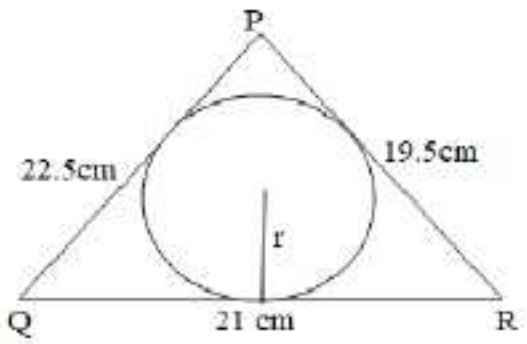


17. Find the radius of a circle whose circumference is equal to the sum of the circumferences of two circles of diameters 36 cm and 20 cm.

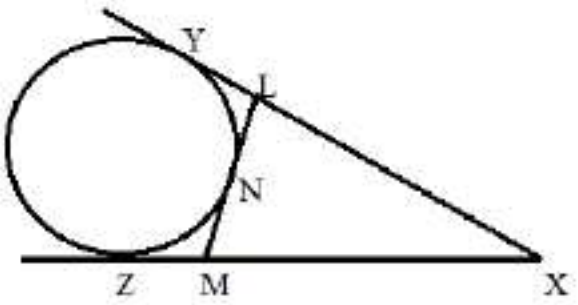
18. In the given figure, O is the centre of circle with radius r, find the radius of circle.



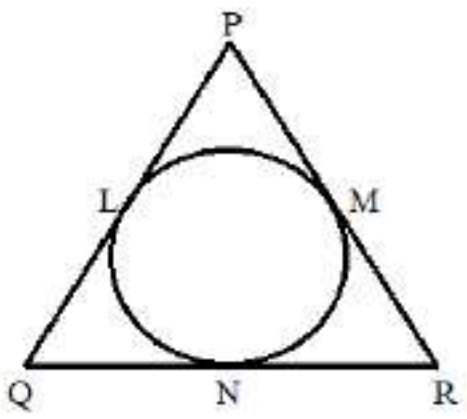
19. In the given figure, find radius of the circle, if area of $\Delta PQR = 189$ sq cm.



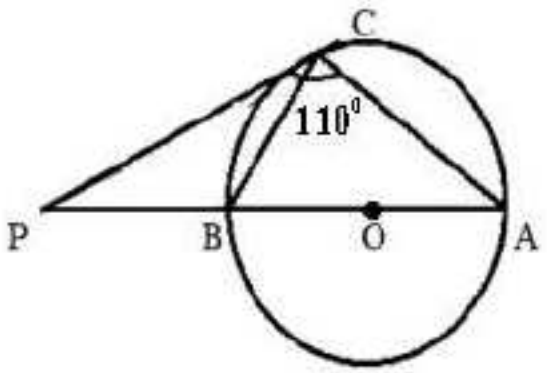
20. Find the perimeter of ΔXLM, if XY = 18cm.



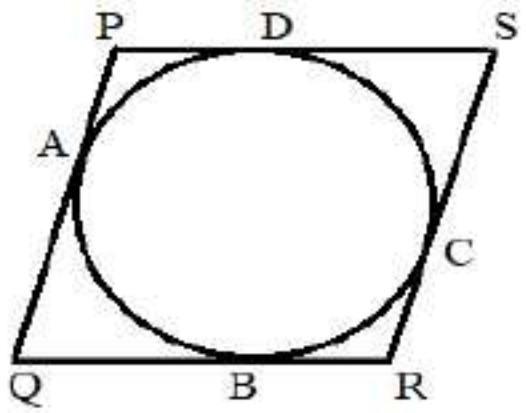
21. Find the perimeter of ΔPQR, where PM = a cm, RN= b cm, QL = c cm.



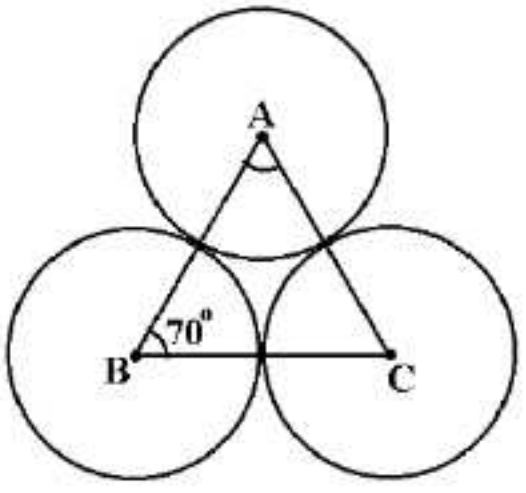
22. The tangent at a point 'C' of a circle and a diameter AB when extended intersect at 'P'. If $\angle PCA = 110°$, then find the value of $\angle CBA$.



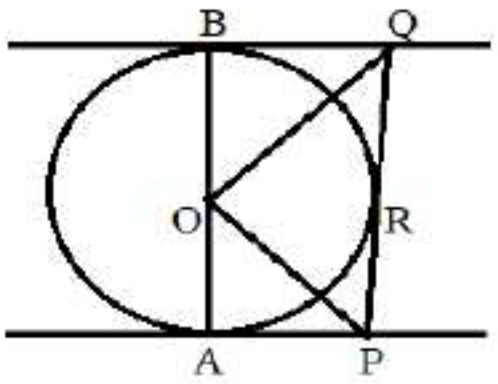
23. In the given figure, find the perimeter of quadrilateral PQRS, if PA = 3cm, DS = 4cm, SR = 7cm, QB = 4cm.



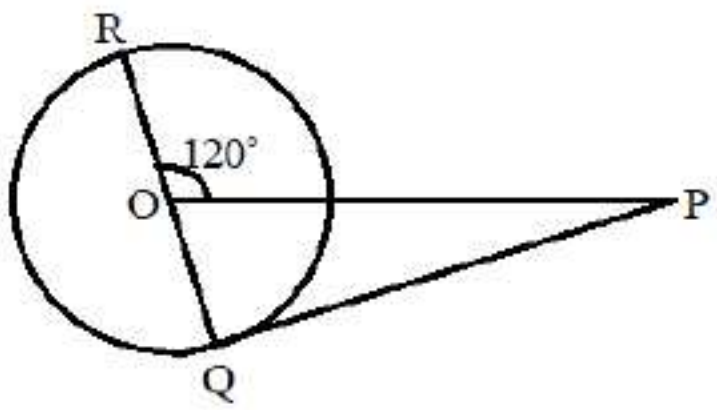
24. The following figure shows 3 touching circles. A is centre of the circle of radius 'r' cm. B and C are centres of circles of radius 3.8 cm each. If $\angle ABC = 70°$, then find $\angle BAC$.



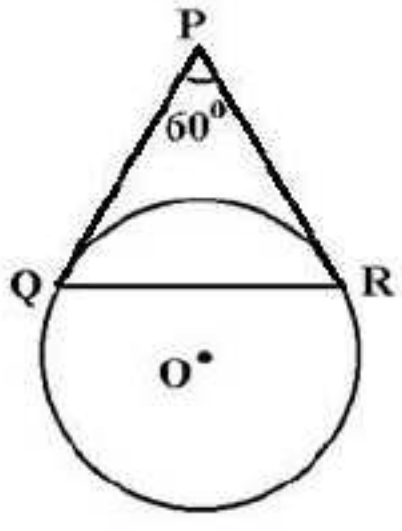
25. If BQ, QP and AP are tangents to the circle with centre O, then find the value of $\frac{4\angle QOP}{5}$.



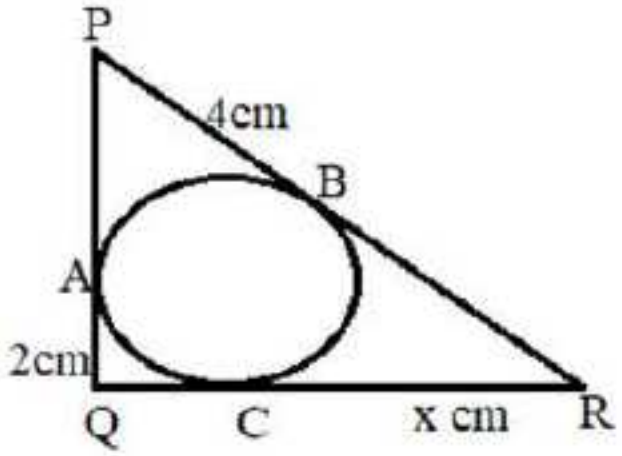
26. PQ is a tangent drawn from an external point P to a circle with centre O, QR is the diameter of circle. If $\angle POR = 120°$, then find the measure of $\angle OPQ$.



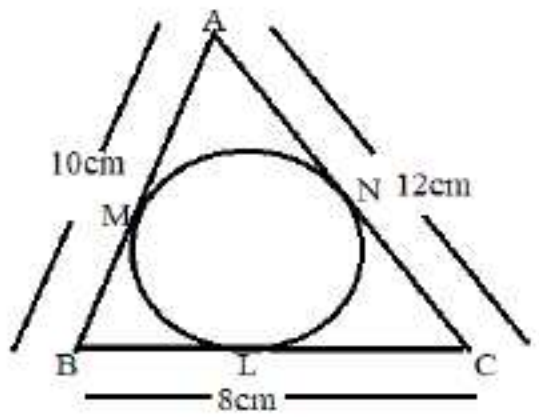
27. In the given figure, PQ and PR are tangents to a circle with centre O such that PQ = 5 cm and $\angle QPR = 60°$, then find the length of the chord QR.



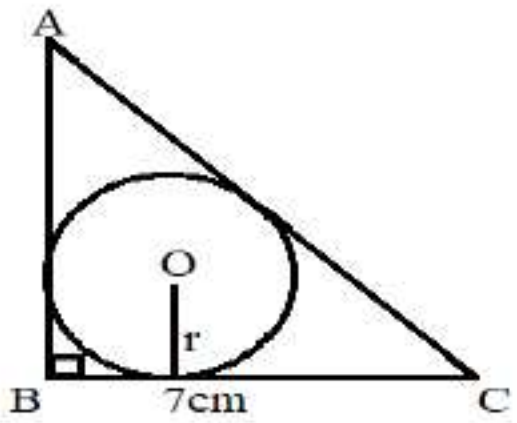
28. **In the given figure if perimeter of ΔPQR is 24cm, then find the value of x .**



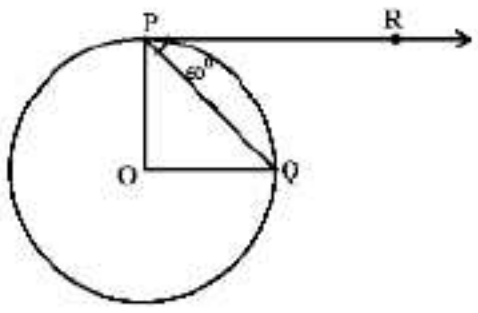
29. **In the given figure, find BL.**



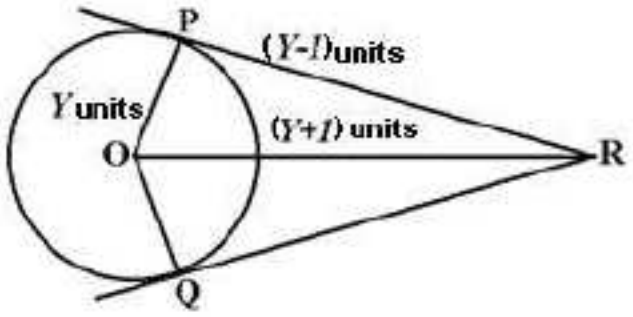
30. **In the given figure, find the value of r, if AC – AB = 1cm.**



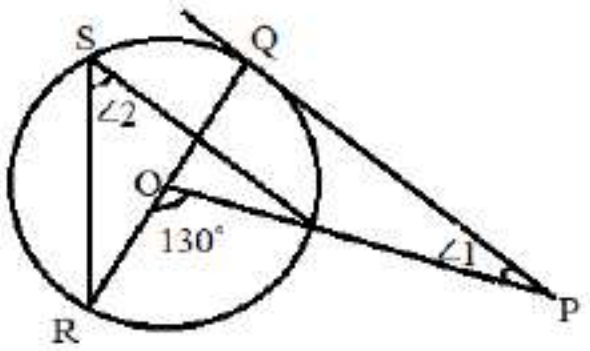
31. **In the given figure O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find ∠POQ.**



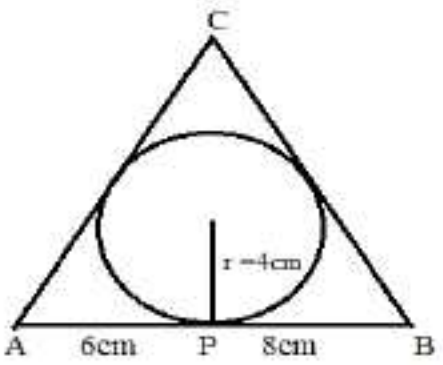
32. In the given figure, find the value of Y.



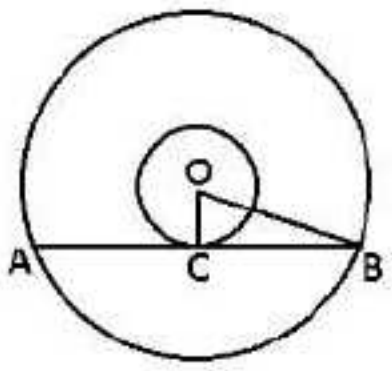
33. In the given figure PQ is a tangent from the external point P. If ∠POR = 130°, then find ∠1 + ∠2.



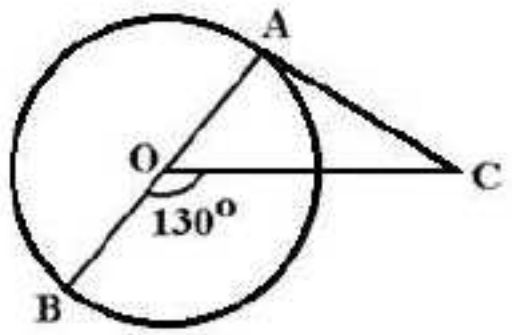
34. In the given figure, if area of ΔABC = 84sq cm, then find AC + BC.



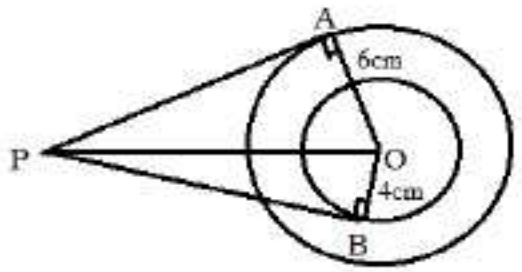
35. In the given figure, if AB = 48cm and OC = 7cm, then find the difference between the radii of two concentric circles.



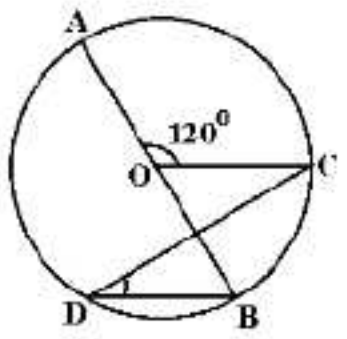
36. In the given figure, AB is the diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$, then find $\angle ACO$.



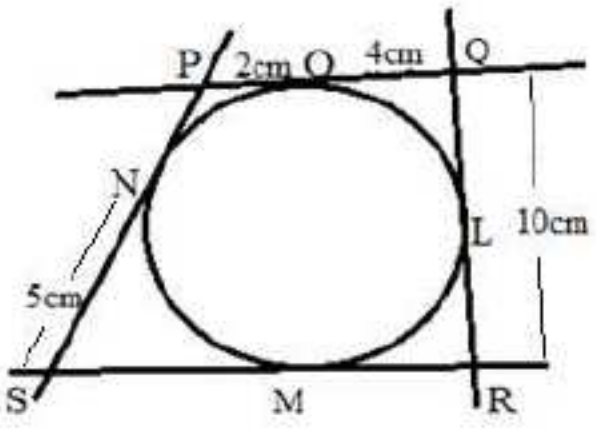
37. In the given figure, if PO – PA = 2cm, then find BP.



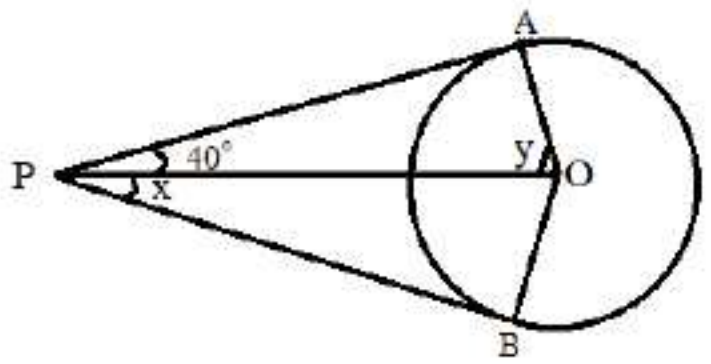
38. In the given figure, AB is the diameter of a circle with centre O. If $\angle AOC = 120°$, then find $\angle BDC$.



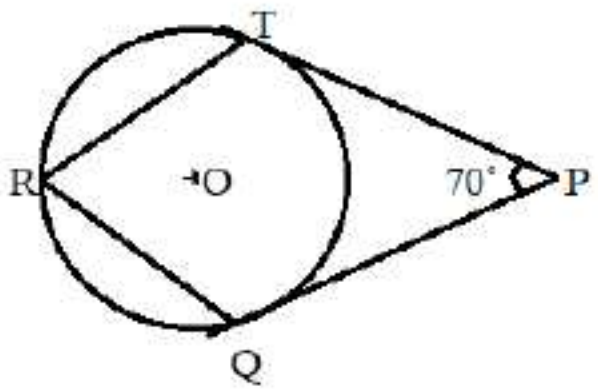
39. From the given figure, find the value of (PQ + QR + RS - SP).



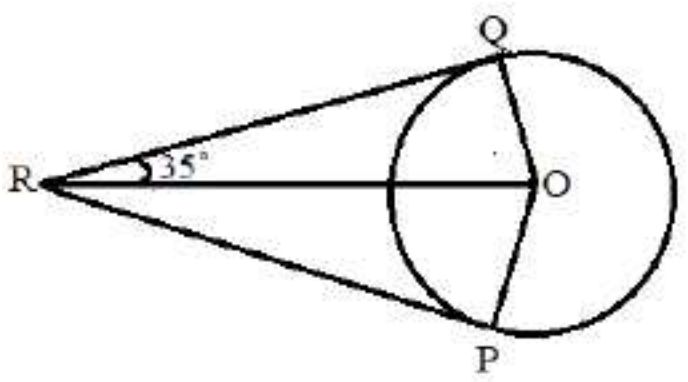
40. **From the given figure, find the value of (4y – 5x).**



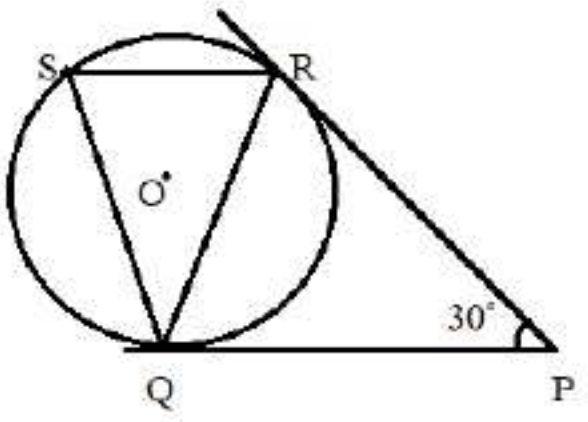
41. **In the given figure ∠TPQ = 70°, find ∠TRQ.**



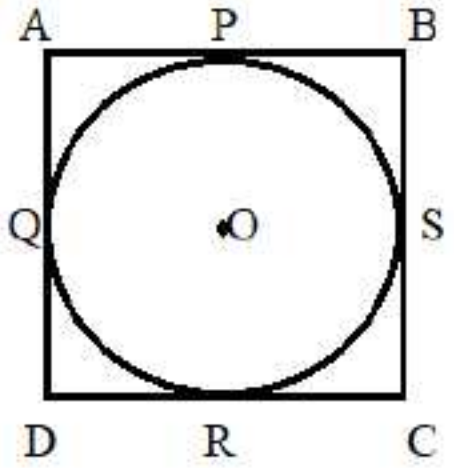
42. **From the given figure, find the value of (∠QOP - ∠ROP)**



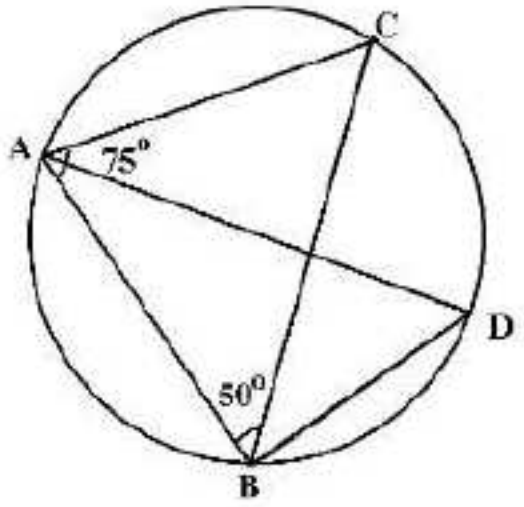
43. **In the given figure, SR∥QP, find the value of ∠RQS.**



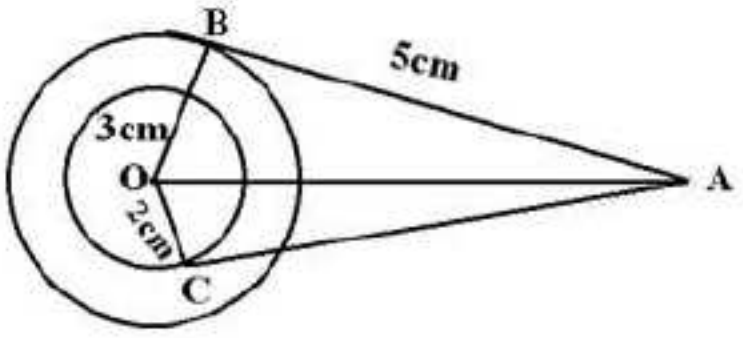
44. In the given figure, if AB = 15.5cm and CD = 16cm, find the perimeter of the Quadrilateral ABCD.



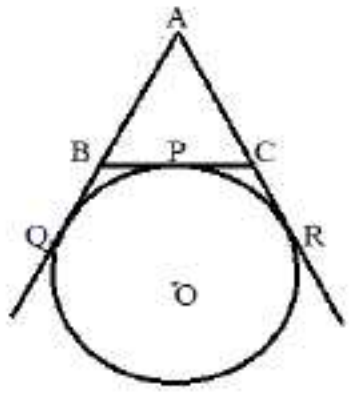
45. In the given figure ∠CAB = **75°** and ∠CBA = **50°**. Find the value of ∠DAB + ∠ABD.



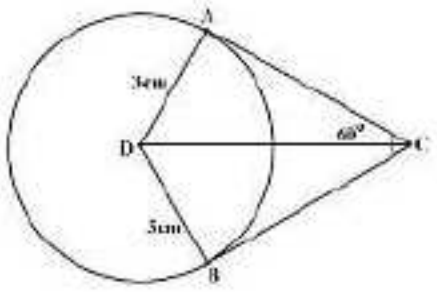
46. In the given figure, AB is a tangent to outer circle and AC is a tangent to inner circle. If AB = 5 cm, OB = 3 cm and OC = 2 cm, then find the value of AC.



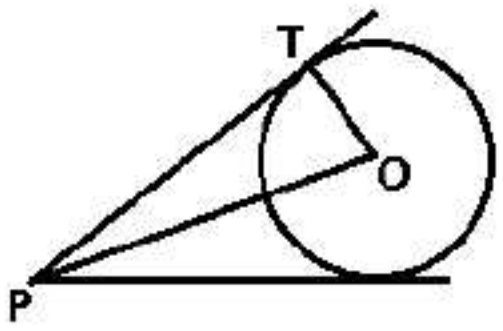
47. In the given figure, find AQ if AB = 2x cm, AC = 4y cm, BC = 6z cm.



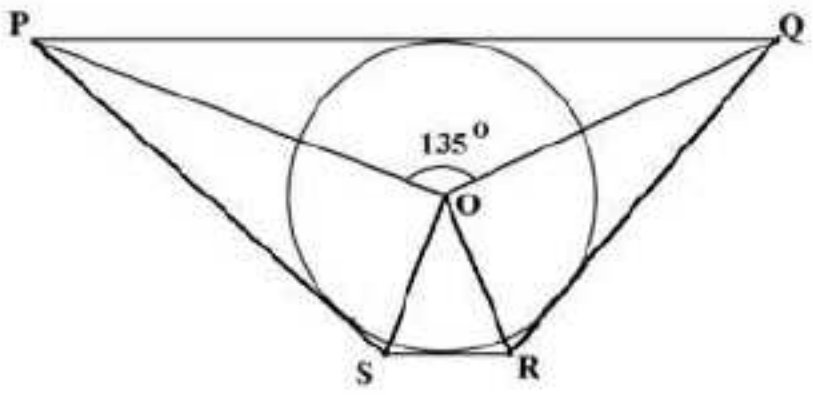
48. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.



49. In the given figure, OT: TP = 3: 4 and OP = 10cm, then find the radius of the circle.



50. In the given figure, O is the centre of the circle, if ∠POQ = 135°, then find ∠ROS.



ANSWERS

Q.No.	Answer	Q.No.	Answer
1	24 cm	26	30°
2	$8\sqrt{2}$ cm	27	5 cm
3	30°	28	6
4	30°	29	3 cm
5	55°	30	3 cm
6	16 cm	31	100°
7	7.5 cm	32	Y = 4
8	80°	33	105°
9	21 cm	34	28 cm
10	80°	35	18 cm
11	145°	36	40°
12	10 cm	37	$\sqrt{84}$ cm or $2\sqrt{21}$ cm
13	8a cm	38	30°
14	30°	39	20 cm
15	61°	40	0
16	150°	41	55°
17	28 cm	42	55°
18	11 cm	43	30°
19	6 cm	44	63 cm
20	36 cm	45	125°
21	2(a + b + c) cm	46	$\sqrt{30}$ cm
22	70°	47	(x + 2y + 3z) cm
23	28 cm	48	$3\sqrt{3}$ cm
24	40°	49	6 cm
25	72°	50	45°

CHAPTER 10

AREAS RELATED TO CIRCLES

POINTS TO REMEMBER

PERIMETER AND AREA OF A CIRCLE

- The area of a circle is the measurement of the region enclosed by its boundary.

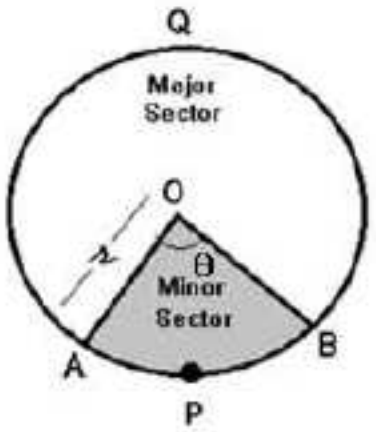
  Area of the circle = πr^2
- The perimeter of a circle is the length of its boundary. Perimeter of a circle is also known as circumference of a circle. Perimeter of the circle = $2\pi r$
- Area of sector of circle

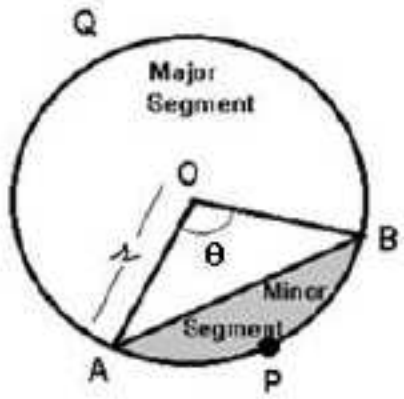
  The portion of circle enclosed between two radii and arc of a circle is called sector of a circle.

  Area of sector OAPB = $\frac{\theta}{360^\circ}\pi r^2$

  Length of an arc of sector OAPB = length of arc AB = $\frac{\theta}{360^\circ}2\pi r$

  Perimeter of the sector OAPB = $\frac{\theta}{360^\circ}2\pi r + 2r$





- Area of segment of circle

  Any chord AB divides the circle into two parts. The bigger part is known as major segment and smaller one is called minor segment.

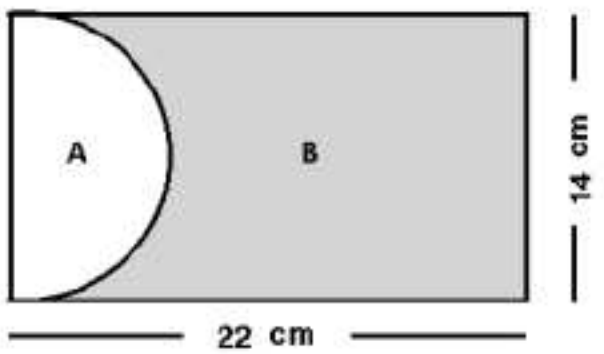
  Area of minor segment APB = Area of sector OAPB – Area of ΔOAB

  $= \frac{\theta}{360^\circ}\pi r^2 - \frac{1}{2}r^2 \sin\theta$

  Area of major segment = πr^2 – Area of minor segment

QUESTIONS

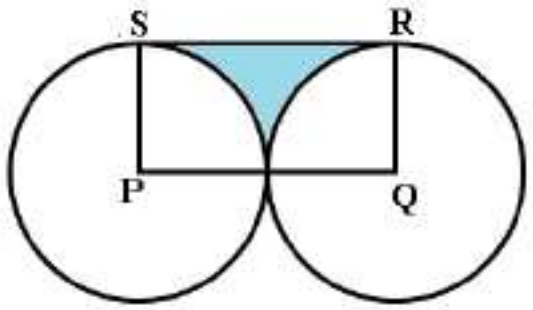
1. If area of a sector of a circle is $\frac{7}{18}$ of the area of the circle, then find the angle of the sector of the circle.
2. The difference between the circumference and the diameter of a circle is 30cm. Find the radius of the circle.
3. Find the area of a sector whose perimeter is 4 times its radius.
4. The hour hand of a clock is 6cm long. Find the area swept by it in between 11:20 am and 11: 55 am (in sq cm).
5. Find the area of the segment (in terms of π) of circle with central angle 90^{o} and radius 8 cm.
6. Find the area of a circle whose circumference is 44 cm.
7. Find the area of a sector with an arc of length of 30 cm and radius 10 cm.
8. In a circle of radius 21 cm an arc subtends an angle of 30° at centre. Find the length of arc.
9. If the perimeter of a circle is equal to that of a square, then find the ratio of their areas (in terms of π).
10. Find the area of the largest triangle that can be inscribed in a semi-circle of radius 85 m.
11. The perimeter of a semicircular protractor is 108cm, find the diameter of the protractor.
12. What will be the percentage increase in area of circle, if its radius is increased by 40%.
13. The side of a square is 14 cm. Find the area of circle circumscribed about this square.
14. In the given figure, find the area of the region A.



15. In figure given in Q.14, find the area of region B.

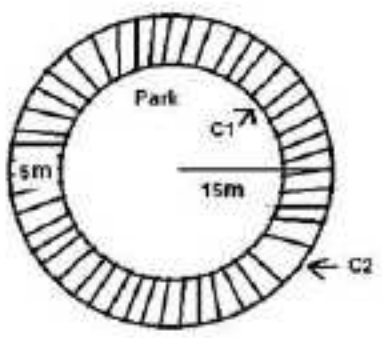
16. In figure given in Q.14, find the ratio of the areas of region B and region A.

17. In the given figure, two circles of radii 7cm each are shown. PQRS is a rectangle and PS and QR are radii. Find the area of shaded region.

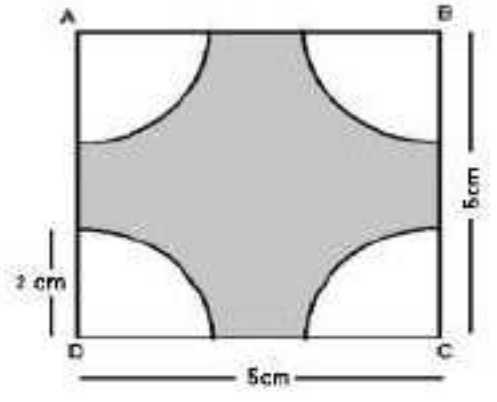


18. A path of width 5m is built around a circular park of radius 15m as shown in the figure. Find the

(i) Sum of the perimeters of the circles C_1 and C_2. (ii) The area of path.



19. In the given figure, ABCD is a square of side 5cm. A quadrant of a circle of radius 2cm is drawn at each vertex of the square. Find the area (in terms of π) of the shaded region.



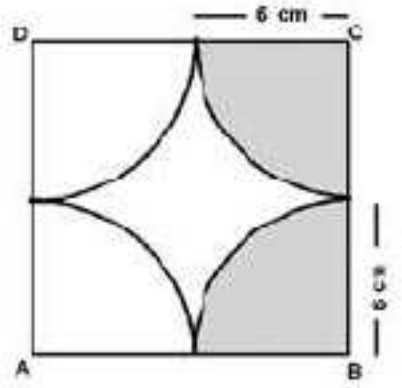
20. Find the perimeter (in terms ofπ) of the shaded region in Q.19.

21. In the given figure, ABCD is a square of side 12cm. A quadrant of a circle of radius 6cm is drawn at each vertex of the square. Find

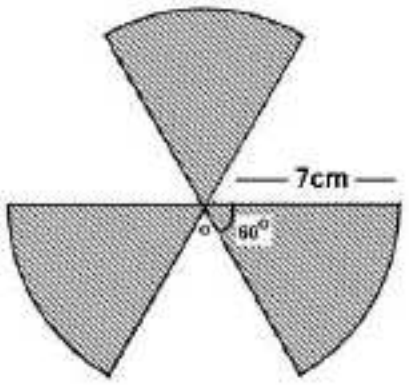
(i) Area (in terms of π) of the shaded region.

(ii) Perimeter (in terms of π) of the shaded region.

(iii) Area (in terms of π) of the unshaded region.

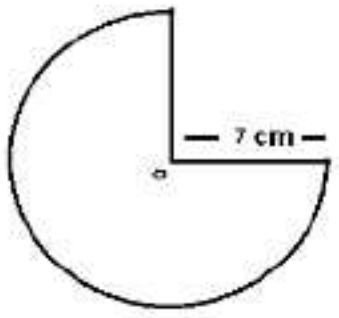


22. In the given figure, find the total area of 3 equal sectors of the given circle with centre O (use $\pi = \frac{22}{7}$).



23. Find the perimeter of the figure given in Q.23.

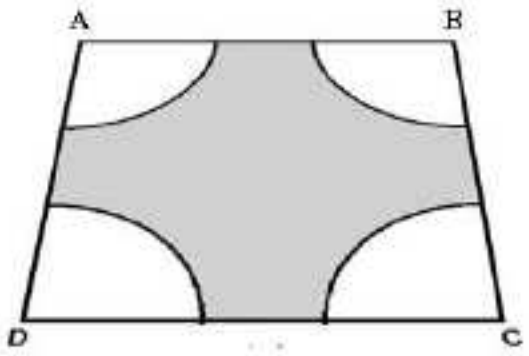
24. Find the perimeter of the figure given below, if O is the Centre of the circle from which a quadrant is cut (use $\pi = \frac{22}{7}$).



25. A steel wire when bent in the form of a square encloses an area of 121 sq cm. If the same wire is bent in the form of a circle, then find the area of circle.

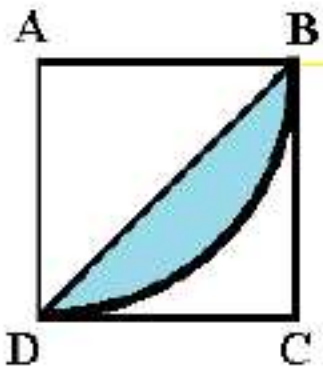
26. A bicycle wheel makes 500 revolutions in travelling 1.1 km. Find the diameter of the wheel (in cm).

27. In the given fig. ABCD is a trapezium in which AB‖ DC, AB = 18cm, DC= 32cm and the distance between AB and DC is 14 cm. If arcs of length of equal radii 7cm have been drawn with centres A, B, C and D, then find area of the shaded region.

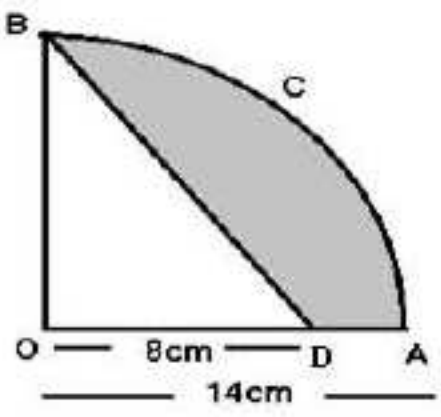


28. Two circles touches each other externally. The distance between the centre of the circles is 14cm and the sum of their areas is 308 square cm. Find the difference between radii of these circles (in cm).

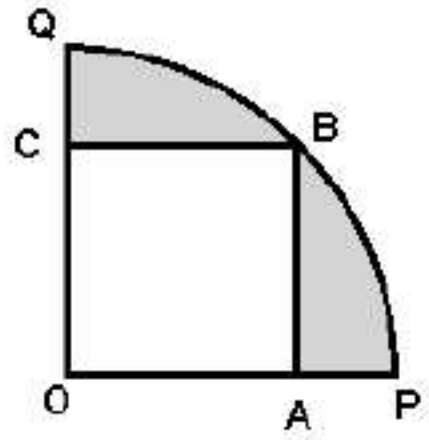
29. In the given figure, ABCD is a square of diagonal $7\sqrt{2}$ cm. With A as the centre, arc BD is drawn. Find area of the shaded region in terms of π.



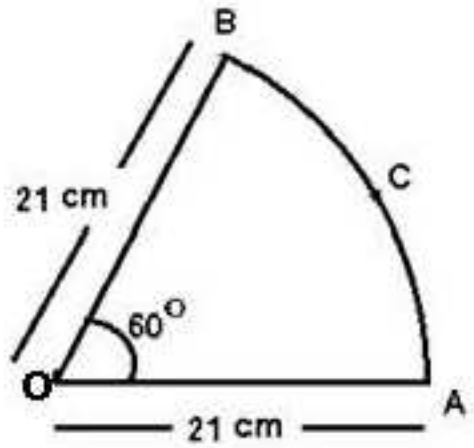
30. OACB is a quadrant of a circle with centre O and radius 14cm. If OD= 8cm, then find the area of shaded region (use $\pi = \frac{22}{7}$).



31. Square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20cm, find the area of shaded region (use $\pi = 3.14$).



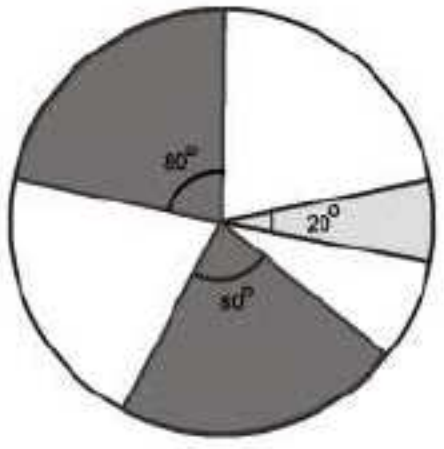
32. In the given figure, find the perimeter of OACB.



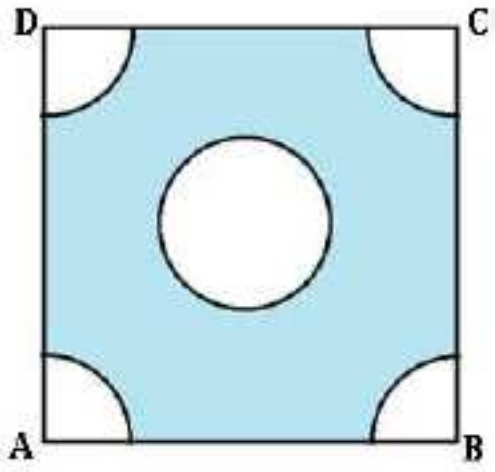
33. Find the perimeter of the protractor if its diameter is 14cm.

34. Find the diameter of a circle whose area is equal to the sum of the areas of two circles of diameters $2(a^2 - b^2)$ cm and 4ab cm.

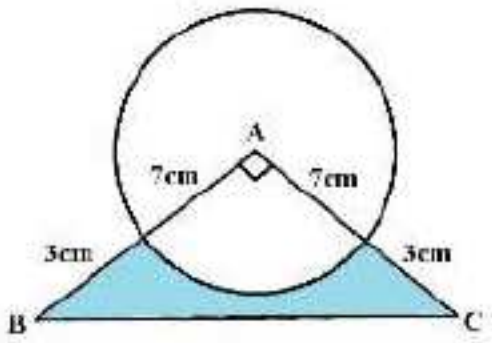
35. In the given figure, three sectors of a circle of diameter 7cm, making angles of 20°, 80° and 80° respectively at the centre are shaded. Find the area of the shaded region.



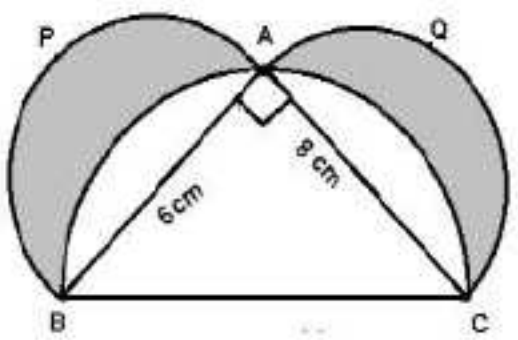
36. In the given figure ABCD is a square of side 5cm. A quadrant of a circle of radius 1cm is drawn at each vertex of the square and a circle of diameter 2cm is also drawn in the Centre. Find area of the shaded region. (use $\pi = 3.14$).



37. A design is made as shown in the figure. Find the area of shaded region.



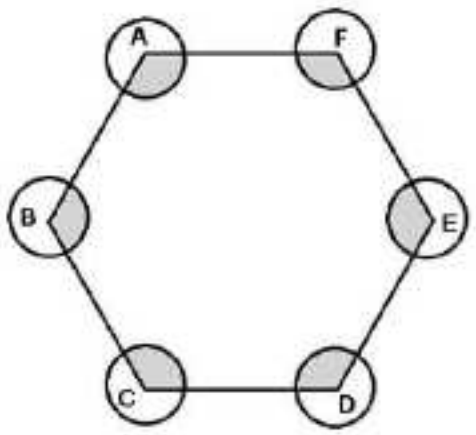
38. In the given fig. ΔABC is right angled at A. Semi circles are drawn with AB, AC and BC as diameters. If AB = 6cm and AC= 8cm, then find perimeter of the shaded region.



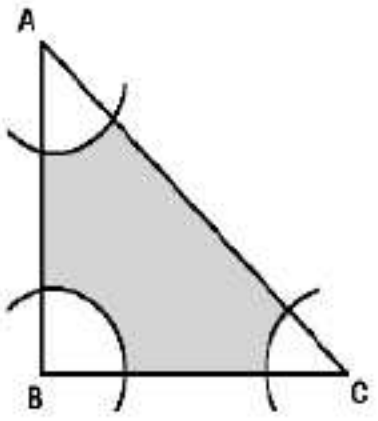
39. Two circles touch internally. The circumference of two circles are 88cm and 44cm. Find the distance between their centres.

40. Two circles touch internally. The sum of their areas is 116π square cm and distance between their centres is 6cm. Find the sum of radii of the circles.

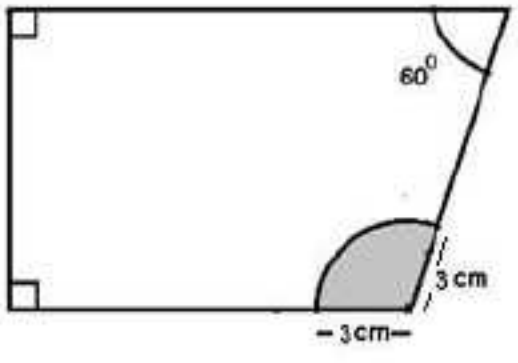
41. ABCDEF is a regular hexagon. With vertices A, B, C, D, E and F as the centres, circles of radius 7cm each are drawn as shown in fig. , find the area of shaded portion.



42. With vertices A, B and C of a ΔABC as centres, arcs are drawn with radius 4cm each as shown in figure. If AB=10cm, BC= 24cm and CA= 26cm, then find the area of shaded region(in terms of π).

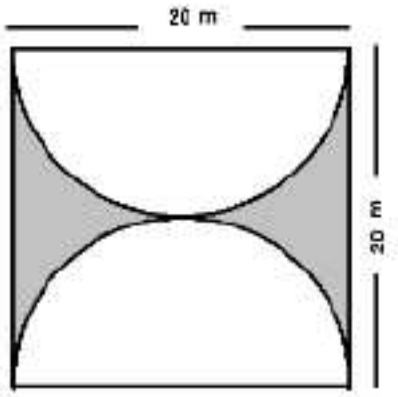


43. In the given figure, find the perimeter (in terms of π) of the shaded region.



44. In the figure given in Q.43, find the area (in terms of π) of the shaded region.

45. In the given figure, find the area (in terms of π) of the shaded region.

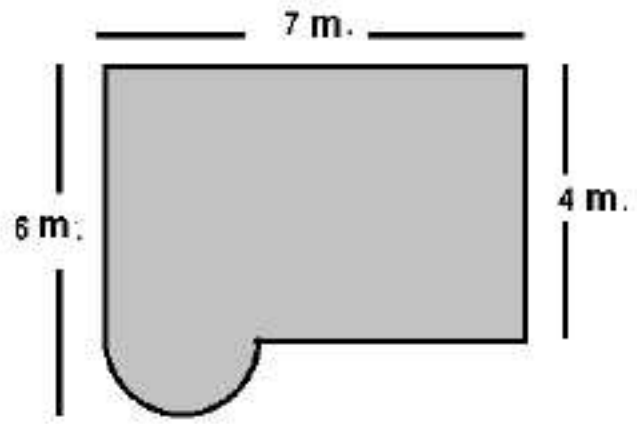


46. In the figure given in Q.45 find the perimeter (in terms of π) of the shaded region.

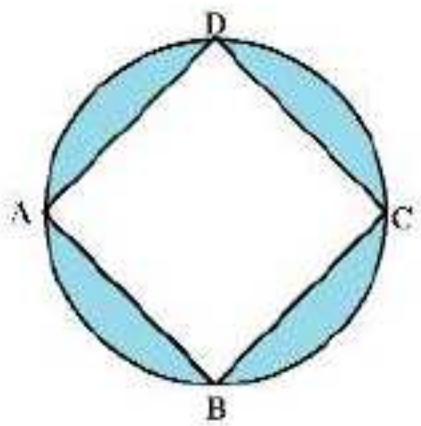
47. In the given figure, find the following (in terms of π):

(i) Area of shaded portion.

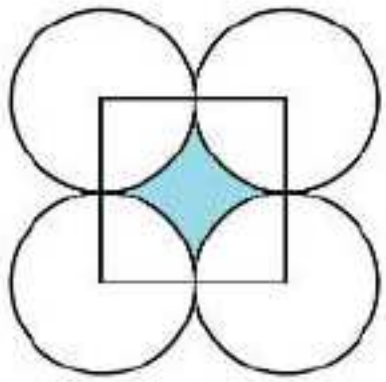
(ii) Perimeter of shaded portion



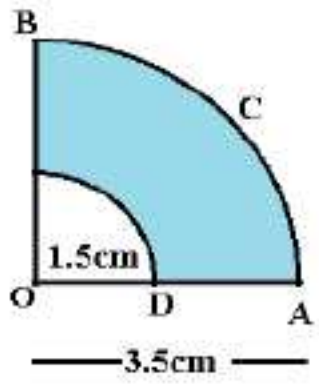
48. In the given figure, ABCD is square with side $7\sqrt{2}$ cm and inscribed in a circle. Find the area of the shaded region.



49. **Four equal circles each of radius 7 cm touch each other as shown in the figure. Find the area included between them. ($\pi = \frac{22}{7}$).**



50. **OACB is a quadrant of a circle with Centre O and radius 3.5 cm. If OD = 1.5 cm, then find the area of shaded region.**



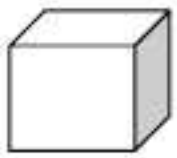
ANSWERS

1	140°	26	70 cm
2	7cm	27	196 sq cm
3	r^2 sq cm	28	0
4	5.5 sq cm	29	$\frac{49}{4}(\pi - 2)$sq cm
5	$(16\pi - 32)$sq cm	30	98 sq cm
6	154 sq cm	31	228 sq cm
7	150 sq cm	32	64cm
8	11 cm	33	36cm
9	$4:\pi$	34	$2(a^2 + b^2)$cm
10	7225 sq m	35	19.25 sq cm
11	42 cm	36	18.72 sq cm
12	96%	37	11.5 sq cm
13	308 sq cm	38	$37\frac{5}{7}$ cm
14	77 sq cm	39	7 cm
15	231 sq cm	40	14 cm
16	3:1	41	308 sq cm
17	21 sq cm	42	$(120 - 8\pi)$ sq cm
18	(i)220m (ii) 550 sq m	43	$(2\pi + 6)$cm
19	(25 - 4 π)sq cm	44	3π sq cm
20	$(4\pi + 4)$cm	45	$(400 - 100\pi)$ sq cm
21	i) 18 π sq cm (ii)[6(π + 4)]cm (iii)[144-18 π]sq cm	46	$(40 + 20\pi)$m
22	77 sq cm	47	(i)$(28 + 2\pi)$sq m (ii)$(18 + 2\pi)$m
23	64 cm	48	56 sq cm
24	47 cm	49	42 sq cm
25	154 sq cm	50	$7\frac{6}{7}$ sq cm

CHAPTER – 11

SURFACE AREAS AND VOLUMES

POINTS TO REMEMBER

Name of the Solid	Curved/ Lateral Surface Area	Total Surface Area	Volume
Cuboid 	$2(l+b)h$	$2(lb+bh+hl)$	lbh
Cube 	$4(edge)^2$	$6(edge)^2$	$(edge)^3$
Right Circular Cylinder 	$2\pi rh$	$2\pi r(h+r)$	πr^2h
Right Circular Cone 	πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2h$
Sphere 	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Hemisphere 	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$

QUESTIONS

1. Find the lateral surface area of a cube if length of its diagonal is $\sqrt{108}$m.
2. If each edge of a cube is increased by 50%, then find the percentage increase in its surface area.
3. Find the total surface area of a cube whose volume is $3\sqrt{3}a^3$ cu. units.
4. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water it can hold?
5. The radius of the base of a right circular cylinder is increased by 75% and its height is decreased by 50%. Find the percentage decrease or increase in its curved surface area.
6. Three solid spheres of diameters 6cm, 8cm and 10cm are melted to form a single solid sphere. Find the diameter of the new sphere.
7. A hollow pipe is 21cm long and its external diameter is 8cm. If the thickness of the pipe is 1cm, then find the volume of the pipe (in terms ofπ).
8. Find the volume (in terms ofπ) of the largest right circular cone that can be cut out of a cube whose edge is 12cm.
9. Find the percentage increase in the surface area of a cube when it's each edge is tripled.
10. 60 circular plates, each of radius 7cm and thickness one – third of a cm are placed one above another to form a solid circular cylinder. Find
    i) Total surface area of the cylinder so formed.
    ii) Volume of the cylinder so formed.
11. If the volumes of two cones are in the ratio 1:4 and their diameters are in the ratio 4:5, find the ratio of their heights.
12. If each edge of a cube is decreased by 25%, then find the ratio of volumes of original cube and the resulting new cube.
13. The dimensions of a metallic cuboid are 100cm$\times$ 80cm $\times$ 64cm. It is melted and recast into a cube. Find the total surface area of the cube so formed.
14. The radius and height of a right circular cone are in the ratio 2: 3. Find its slant height, if its volume is 100.48 cu.cm.$(Take\ \pi = 3.14)$

15. The radius of a spherical balloon decreases from 21 cm to 14cm when air is pumped out of it. Find the percentage decrease in its surface area.
16. How many cubes of edge 5cm can be put in a cubical box of 1m edge?
17. Using clay a student made a right circular cone of height 48 cm and base radius 12 cm. Another student reshapes it in the form of a sphere. Find the diameter of the sphere.
18. A shopkeeper has one spherical laddoo of radius 6 cm. How many spherical laddoos of radius 1.5 cm can be made from it?
19. Three solid cubes of sides 1cm, 6cm and 8cm are melted to form a new cube. Find the total surface area of the cube so formed.
20. A solid piece of metal is in the form of a cuboid of dimensions 11cm× **7 cm** × **7 cm**. It is melted to form 'n' number of solid spheres of radii $\frac{7}{2}$ cm each. Find the value of n.
21. An iron sphere of diameter 18cm is drawn into a wire of diameter 4mm. Find the length of the wire.
22. The sum of length, breadth and height of a cuboid is $6\sqrt{3}$cm and length of its diagonal is $2\sqrt{3}$ cm. find the total surface area of the cuboid.
23. The radii of the bases of a cylinder and a cone are in the ratio 3:4 and their heights are in the ratio 2:3, find the ratio of their volumes.
24. The radii of the base of two cylinders A and B are in the ratio 3:2 and their heights are in the ratio n: 1. If the volume of cylinder A is 3 times that of cylinder B, then find the value of n.
25. Two spherical iron balls each of diameter 6cm are immersed in the water contained in a cylindrical vessel(half filled) of radius 6cm. Find the level of the water that will be raised in the vessel.
26. The diameter of a car wheel is 42 cm. Find the number of complete revolutions it will make in moving a distance of 132km.
27. Find the mass of a solid cone of silver metal having base diameter 14cm and vertical height 30cm given that density of silver is 10g/cu.cm.
28. Garvit was making a mathematical model, in which he placed 4 cubes each of edge 20cm one above the other. Find the surface area of resulting cuboid.

29. A solid sphere of radius 'r' cm is cut into 4 equal parts. Find the total surface area of the 4 parts.

30. Find the weight of a hollow sphere of metal having internal and external diameters as 8cm and 10cm respectively if 1 cu.cm of metal weighs 21g.

31. The volume of cuboid is 36 times the volume of a cube. If the dimensions of the cuboid are 9cm, 18cm and 48cm, then find the total surface area of cube.

32. A solid spherical steel ball of radius 'r' was silver polished and then cut into 4 similar pieces.
    i) Find the non-polished area of each piece
    ii) Find the ratio of the polished area to the non-polished area of each piece.

33. The height of a circular cylinder is increased 6 times and base area is decreased to $\frac{1}{9}$ times of its value. By what factor its lateral surface area is increased/decreased?

34. Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.

35. 2.2 cu.dm. of copper is to be drawn into a cylindrical wire of diameter 0.5 cm. Find the length of the wire in metres.

36. A spherical lead ball of radius 15cm is melted and small lead balls of diameter 10mm are made. Find the total possible number of small lead balls so formed.

37. The radius of a metallic cylinder is 3cm and its height is 5cm. It is melted and moulded into small cones each of height 1cm and base radius 1 mm. Find the number of cones so formed.

38. If a solid cone of volume 27π cu.cm is kept inside a hollow cylinder whose radius and height are equal to that of cone, then find the volume of water needed to fill the empty space.

39. A conical flask is full of water having base radius r and height h. This water is poured into an empty cylindrical flask of base radius 'mr'. Find the height of water in the cylindrical flask.

40. A steel wire when bent in the form of a square encloses an area of 121 sq.cm.. If the same wire is bent in the form of a circle, then find area of the circle.

41. If the radius of a sphere is increased by 2cm, its surface area increases by 352 sq.cm, then find the radius of sphere before change.

42. If the perimeter of a circle is equal to that of a square, then find the ratio of their areas.

43. If the volume of a cube is 3375 cubic metre then, find its total surface area.

44. How many metallic circular discs of base diameter 1.5 cm and height 0.2 cm should be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

45. Two identical solid cubes of edge '$2l$' are joined end to end. Find the total surface area of the resulting cuboid.

46. If semi vertical angle of a cone of height 3cm is 60°. Find the following:
    i) Diameter of base of cone
    ii) Slant height of cone
    iii) Volume of cone (in terms of π)

47. The surface area of three adjacent faces of a cuboid are 36 sq.m, 27sq.m and 12 sq.m respectively. Find its volume.

48. A cone, hemisphere and cylinder stand on the same base and have equal height.
    i) Find the ratio of their curved surface areas.
    ii) Find the ratio of their volumes.

49. The radii of the base of a cylinder and a cone are in the ratio $\sqrt{3}:\sqrt{2}$ and their heights are in the ratio $\sqrt{2}:\sqrt{3}$. Find the ratio of their volumes.

50. A solid spherical ball of radius 3cm is melted and recast into three solid spherical balls. The radii of two of the balls are 1.5cm and 2cm. Find the radius of the third ball.

ANSWERS

1	144sq.m	26	100000
2	125%	27	15.4kg
3	$18a^2$ sq.units	28	7200 sq.cm
4	1,35,000 litres	29	$8\pi r^2$ sq. cm
5	Decrease of 12.5%	30	5368g or 5.368kg
6	12 cm	31	216sq.cm
7	147 π cu.cm	32	(i) πr^2 (ii) 1:1
8	144 π cu.cm	33	LSA is increased to 2 times
9	800%	34	7:9
10	i) 1,188 sq.cm ii) 3,080 cu.cm	35	112 m
11	25:64	36	27000
12	64:27	37	13500
13	38400 sq.cm	38	54 π cu.cm
14	$2\sqrt{13}$ cm	39	$\left(\frac{h}{3m^2}\right)$
15	$55\frac{5}{9}\%$	40	154 sq.cm
16	8000	41	6cm
17	24 cm	42	14:11
18	64	43	1350 sq.m
19	486 sq.cm	44	450
20	3	45	$40l^2$
21	243m	46	i) $6\sqrt{3}$cm ii) 6cm iii) 27π cu.cm
22	96 sq.cm	47	108 cu.m
23	9:8	48	(i) $1:\sqrt{2}:\sqrt{2}$ (ii) 1:2:3
24	$n=\frac{4}{3}$	49	$3\sqrt{3}:\sqrt{2}$
25	2cm	50	2.5 cm [hint : $1.5^3 + 2^3 + 2.5^3 = 3^3$]

CHAPTER -12

STATISTICS

POINTS TO REMEMBER

- **Mean :**

  Mean of ungrouped data: $\bar{x} = \frac{\text{Sum of all observations}}{\text{Total number of observations}}$

  Mean of grouped data:

  (i) By direct method, $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

  (ii) By assumed mean method, $\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}$ **where A = Assumed mean,**

  $d_i = x_i - A$

  (iii) By step deviation method, $\bar{x} = A + \frac{\sum f_i u_i}{\sum f_i} \times h$ **where A = Assumed mean,**

  $u_i = \frac{x_i - A}{h}$

- **Mode :**

Mode of ungrouped data = observation having maximum frequency

Mode of grouped data = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_1}\right) \times h$

Where l **= lower limit of the modal class**

f_1 **= frequency of the modal class**

f_0 **= frequency of class preceding the modal class**

f_2 **= frequency of class succeeding the modal class**

h **= class size of modal class**

- **Median :**

  For ungrouped data: Let there be n observations, arrange the data in ascending or descending order.

  If n is odd, Median = $\left(\frac{n+1}{2}\right)^{th}$ **observation,**

  If n is even, Median = $\frac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2}$

Median of grouped data = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

Where l = lower limit of median class

cf = Cumulative frequency of class preceding the median class

f = Frequency of the median class

h = Class size of the median class

- Empirical formula

  Mode = 3 Median – 2 Mean

QUESTIONS

1. If the mean of 5 observations x, x + 2, x + 4, x + 6 and x + 8 is 14, then find the mean of first 4 observations.
2. In three mathematics tests, Mishika scored 79, 94 and 87 marks respectively. How many marks must she secure in the fourth test to have an average of 89 marks for these tests?
3. The average weight of the following distribution is 58.5 kg. Find x.

Weight(in kg)	50	55	60	x +12.5	70
No. of men	1	4	2	2	1

4. Find p, if 20 is the mean of the following frequency distribution.

x_i	10	15	p	25	35
f_i	3	10	30	7	5

5. The weights (in kg) of 15 students are as follows:

   33, 27, 34, 37, 23, 31, 42, 44, 35, 38, 45, 40, 48, 39, 41.

   If the weight 45 kg is replaced by 54 kg and 31 kg is replaced by 21 kg, then find the new median.
6. Two distributions M and N with total number of observations 25 and 75, and mean 3 and 4 respectively are combined. What is the mean of the resulting distribution?
7. Find the median of the given data, if the mean is 4.5:

   5, 7, 7, 8, x, 5, 4, 3, 1, 2
8. If the median of $\frac{x}{7}, \frac{x}{6}, \frac{x}{5}, \frac{x}{3}, \frac{x}{2}, x$ and $\frac{x}{4}$ is 9 (where x > 0), find the value of x.

9. Find the upper limit of the median class for the following distribution:

Class	0 – 5	6–11	12 –17	18 – 23	24 – 29
Frequency	13	10	15	8	11

10. The mean of 3, 7, 5 and x is 5 and the mean of 12, 7, 6, x and y is 10. What is the value of y?

11. Find the median of a distribution whose mean is 169 and mode is 175.

12. Find the mean of all the factors of 42.

13. If the mean of first n natural numbers is $\frac{5n}{9}$, then find n.

14. Guddu was practicing for the pole vaulting event in the annual athletic meet. Find the mean height if he made vaults of the following heights:
3.4m, 3.5m, 3.4m, 3.5m, 3.8m, 3.5m, 3.4m

15. Find the difference between median and mean of the given data
17, 19, 20, 22, 23 and 25.

16. The rainfall (in mm) in a city on 7 days of a certain week was recorded as follows:

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Rainfall (in mm)	0.6	9.1	6.5	0.9	11.5	7.9	2.0

i) Find the mean rainfall.

ii) Find range of the rainfall.

iii) On how many days was the rainfall more than mean rainfall?

17. If $\sum f_i = 15, \sum f_i x_i = 3p + 36$ and mean of the distribution is 4, then find p.

18. The mean of 300 items was 50. Later on it was discovered that two items were misread as 67 and 26 instead of 76 and 62. Find the correct mean.

19. Consider the following data:

Class	65–85	85–105	105–125	125– 145	145- 165	165–185	185–205
Frequency	4	5	13	20	14	7	4

Find the sum of the upper limit of the median class and lower limit of the modal class.

20. Let m be the class mark and ℓ be the upper class limit of a class in a continuous frequency distribution. Find the lower limit of the class.

21. The mean of three numbers p, q and r is 9 and the mean of five numbers p, q, r, s and t is 14. Find the mean of s and t.

22. The width of each of five continuous classes in a frequency distribution is 5 and upper class limit of the lowest class is 10. Find the upper class limit of the highest class.

23. Find the mean of a grouped frequency distribution,

    where $\sum f_i u_i = 27, \sum f_i = 30$ and $u_i = \frac{x_i - 25}{10}$.

24. The mean of 5 numbers is 30. If one number is excluded their mean becomes 28, find the excluded number.

25. The mean monthly salary of 10 members of a group is ₹ 1445, one more member whose monthly salary is ₹1500 has joined the group. Find the mean monthly salary of 11 members of the group.

26. If $\bar{x}$ is the mean of $x_1, x_2, \ldots, x_n$, then for $a \neq 0$ find the mean of

$$ax_1, ax_2, \ldots, ax_n, \frac{x_1}{a}, \frac{x_1}{a}, \ldots, \frac{x_n}{a}.$$

27. The mean of 25 observations is 36. Out of these observations, if the mean of first 13 observations is 32 and that of the last 13 observations is 40. Find the 13^{th} observation.

28. Find the mean of the data given below :

$$x - 5, 2x + 5, x + 3, 3x + 7, 2x + 1, 2x + 9, 3x + 8.$$

29. If the mean of the observations x, x + 3, x + 5, x +7, x + 10 is 9, then find the mean of last three observations.

30. In a factory, the daily wages of 5 workers are ₹400, ₹500, ₹420, ₹350 and ₹300. If daily wages of each worker is increased by ₹50, find the mean wages.

31. There are 50 numbers. Each number is subtracted from 53 and mean of the numbers so obtained is found to be – 3.5. Find the mean of the numbers given in the beginning.

32. If the mode of the given data is 48, then find the value of $2x + 8$

$$36, 42, 48, 53, 36, 48, x + 2, 50, 42$$

33. Find the mean of first 10 composite numbers.

34. For the following distribution, find the product of lower limit of median class and upper limit of the modal class.

Class	0 – 5	5–10	10 –15	15 – 20	20 - 25
Frequency	10	15	12	20	9

35. If the difference of the mode and median of a data is 24, then find the difference of the median and mean of that data.

36. The mean and median of 100 observations are 50 and 52 respectively. The value of the largest observation is 100.It was later found that it is 120 instead of 100. Find the sum of correct mean and median.

37. For the following distribution, find the modal class:

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

38. If mode of the following distribution is 55, then find the value of x.

Class	0 – 15	15–30	30 –45	45 – 60	60 – 75	75–90
Frequency	10	7	x	15	10	12

39. The mean of 7 numbers is 24. If out of these 7 numbers, the mean of first four numbers is 23 and the mean of last four numbers is 25.5, then find the fourth number.

40. Find the median of all the factors of 108.

ANSWERS

Q. No	Answer	Q. No	Answer
1	13	21	21.5
2	96	22	30
3	50	23	34
4	19	24	38
5	38 kg	25	₹1450
6	3.75	26	$\left(a+\frac{1}{a}\right)\frac{\overline{x}}{2}$
7	4.5	27	36
8	36	28	2x + 4
9	17.5	29	$11\frac{1}{3}$
10	y = 20	30	₹444
11	171	31	56.5
12	12	32	100
13	9	33	11.2
14	3.5 m	34	200
15	0	35	12
16	(i) 5.5 mm (ii) 10.9mm (iii) 4 days	36	102.2
17	8	37	30 – 40
18	50.15	38	5
19	270	39	26
20	$2m - \ell$	40	10.5

CHAPTER -13

PROBABILITY

POINTS TO REMEMBER

- Probability of an event, $P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total no of outcomes}}$
- Probability of any event E lies from 0 to 1 i.e. $0 \leq P(E) \leq 1$
- P(Impossible event) = 0
- P(Sure event) = 1
- For any event E, P(E) + P(not E) = 1 or $P(E) + P(\bar{E}) = 1$

QUESTIONS

1. The table given below shows the marks obtained by students of a class in a test with maximum marks 100:

Marks	0 – 20	20–40	40 –60	60 – 80	Above 80
Frequency	8	16	40	10	6

A student is chosen at random. Find the probability that he gets

i) Less than 40 marks

ii) 60% or more marks

2. The probability of getting a bad egg in a lot of 400 eggs is 0.055. Find the number of bad eggs in the lot.
3. Find the probability of getting 53 Sundays or 53 Mondays in a leap year.
4. A card is drawn at random from a box containing cards numbered from 2 to 19. Find the probability of getting a prime numbered card.
5. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. Find the number of fresh apples in the heap.
6. A number is chosen at random from the numbers between 2 and 100. Find the probability that it is a perfect square number.

7. 17 cards numbered 1, 2, 3…, 17 are put in a box and mixed thoroughly. One person draws a card from the box. Find the probability that the number on the card is
   (i) Prime
   (ii) Odd
   (iii) Divisible by 3
   (iv) Divisible by 3 and 2 both.
8. Cards marked with the numbers 3 to 101 are placed in a box and mixed thoroughly. One card is drawn at random from this box. Find the probability of getting a perfect square or cube on the card drawn.
9. All the black face cards are removed from a pack of 52 playing cards. The remaining cards are well shuffled and then a card is drawn at random. Find the probability of getting
   (i) A face card
   (ii) A red card
   (iii) A black card
10. Find the probability that a leap year selected at random will contain 53 Sundays and 53 Mondays.
11. An integer is chosen between 0 and 100. Find the probability that it is:
   i) Divisible by 7
   ii) Not divisible by 7
12. Cards marked with numbers 1, 3, 5,… ,49 are placed in a box and mixed thoroughly. One card is drawn from the box. Find the probability that the number on the card is:
   i) Divisible by 3
   ii) A composite number
   iii) Not a perfect square
   iv) Multiple of 3 and 5

13. From two digit numbers, a number is chosen at random. Find the probability that it is a multiple of 5 or 7.
14. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability that the card bears a number greater than 3 and less than 10.
15. A box contains cards numbered 3, 5, 7, 9… 35, 37. A card is drawn at random from the box. Find the probability that the number on the card is a prime number.
16. In a single throw of 3 dice, find the probability of getting a total of 5.
17. A bag contains some red, blue and orange balls. The probabilities of selecting a red ball and a blue ball from this bag are $\frac{1}{4}$ and $\frac{1}{3}$ respectively. If this bag contains 10 orange balls, then find the total number of balls in the bag.
18. Two dice are thrown simultaneously. Find the probability of getting a prime number as a sum.
19. A jar contains 54 marbles out of which some are blue, some are red and some are yellow. The probability of selecting a blue marble at random is $\frac{1}{3}$ and probability of selecting a yellow marble at random is $\frac{5}{9}$. How many red marbles are there in the jar?
20. Two dice are thrown simultaneously. Find the probability of getting a multiple of 2 on one dice and a multiple of 3 on the other dice.
21. Two dice are rolled once. Find the probability of getting perfect square as product of numbers on both dice.
22. There are 100 cards in a bag on which numbers from 1 to 100 are written. A card is drawn at random from the bag. Find the probability that the number on the selected card is divisible by 9 and is a perfect square.
23. Two dice are rolled simultaneously. Find the probability of getting 3 at least once.

24. All kings, queens and aces are removed from a pack of 52 cards. The remaining cards are well shuffled and then a card is drawn from it. Find the probability that the card drawn is

    (i) A black face card

    (ii) A red card

25. What is the probability of winning a game if the probability of losing it is 0.029?

26. Out of 1000 small coloured bulbs 9^3 are of white color, 5^3 are of red color, 2^6 are of green color and rest are blue coloured. One bulb is choosen at random from the bag, find the probability that the bulb chosen is blue coloured.

27. A card is drawn from a well shuffled deck of 52 cards. Find the probability that the card drawn will be

    (i) A king or queen

    (ii) A red face card

    (iii) Neither a king nor a queen

    (iv) Either a red card or a black face card

    (v) Not a king

28. A bag contains slips with all natural numbers between 3 and 32. Find the probability that the slip chosen contains a multiple of 4.

29. A bag contains 12 balls out of which x are white.

    (i) If one ball is drawn at random, then find the probability that it will be a white ball?

    (ii) If 6 more white balls are put in the bag, then the probability of drawing a white ball will be double than that in [case (i)]. Find x.

30. Two fair dice are tossed simultaneously. Find the probability that the product of the numbers that turn up is 12.

31. Two fair dice are thrown together. Find the probability that the numbers obtained have a product less than 16.

32. A number 'x' is selected at random from the numbers 1, 4, 9, 16, 25 and another number 'y' is selected at random from the numbers 1, 2, 3, 4, 5. Find the probability that the value of 'xy' is more than 25.

33. Two dice are thrown simultaneously. Find the probability that:
    i) 5 will not come up on either of them.
    ii) 5 will come up on at least one.
    iii) 5 will come up on both dice.

34. A bag contains 6 red balls and some white balls. If the probability of drawing a white ball is double that of a red ball, then find the number of white balls in the bag.

35. A coin is tossed successively three times. Find the probability of getting exactly one head or two heads.

36. In a well shuffled pack of playing cards, find the probability of getting a card with even number.

37. There are three children in a family. Find the probability of having atmost one girl in the family.

38. A card is drawn from an ordinary pack of playing cards and a person bets that it is a heart or an ace. Find the probability of him not winning the bet.

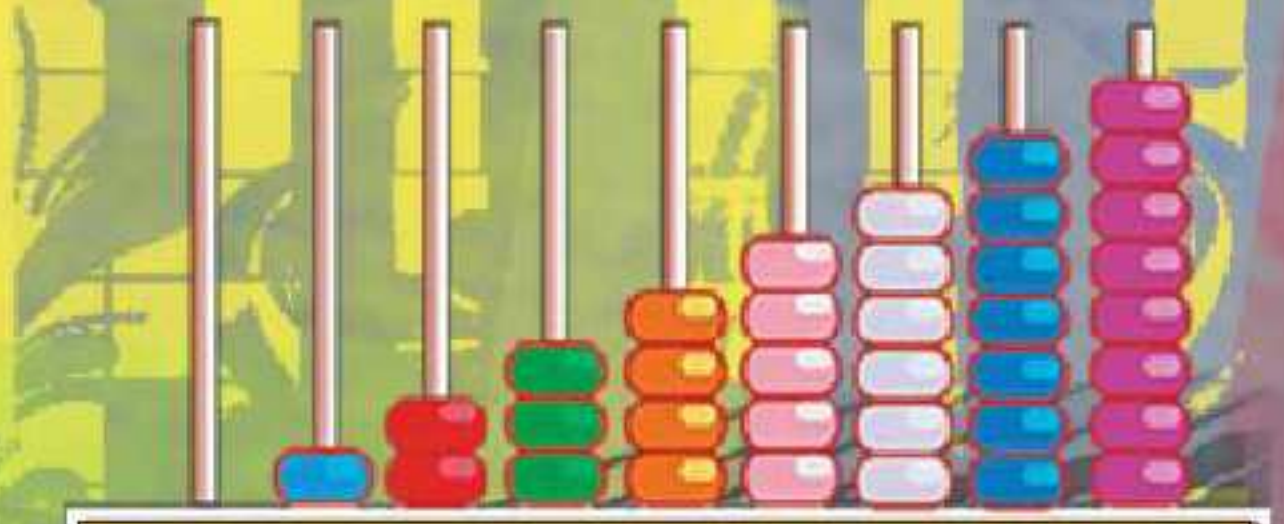
39. Komal and Amit were playing a game. Komal throws a dice and finds square of a number so obtained. Amit throws two different dice together and finds the product of the two numbers obtained. Find the difference in their probabilities to get the number 25.

40. A number is selected at random from the numbers 3, 5, 5, 7, 7, 7, 9, 9, 9, 9. Find the probability that the selected number is their average.

ANSWERS

Q. No	Answer	Q. No	Answer
1	i) $\frac{3}{10}$ ii) $\frac{1}{5}$	21	$\frac{2}{9}$
2	22	22	$\frac{3}{100}$
3	$\frac{3}{7}$	23	$\frac{11}{36}$
4	$\frac{4}{9}$	24	i) $\frac{1}{20}$ ii) $\frac{1}{2}$
5	738	25	0.971
6	$\frac{8}{97}$	26	$\frac{41}{500}$
7	i) $\frac{7}{17}$ ii) $\frac{9}{17}$ iii) $\frac{5}{17}$ iv) $\frac{2}{17}$	27	i) $\frac{2}{13}$ ii) $\frac{3}{26}$ iii) $\frac{11}{13}$ iv) $\frac{8}{13}$ v) $\frac{12}{13}$
8	$\frac{1}{9}$	28	$\frac{1}{4}$
9	i) $\frac{3}{23}$ ii) $\frac{13}{23}$ iii) $\frac{10}{23}$	29	i) $\frac{x}{12}$ ii) x = 3
10	$\frac{1}{7}$	30	$\frac{1}{9}$
11	i) $\frac{14}{99}$ ii) $\frac{85}{99}$	31	$\frac{25}{36}$
12	i) $\frac{8}{25}$ ii) $\frac{2}{5}$ iii) $\frac{21}{25}$ iv) $\frac{2}{5}$	32	$\frac{11}{25}$
13	$\frac{29}{90}$	33	i) $\frac{25}{36}$ ii) $\frac{11}{36}$ iii) $\frac{1}{36}$
14	$\frac{6}{13}$	34	12
15	$\frac{11}{18}$	35	$\frac{3}{4}$
16	$\frac{1}{36}$	36	$\frac{5}{13}$
17	24	37	$\frac{1}{2}$
18	$\frac{5}{12}$	38	$\frac{9}{13}$
19	6	39	$\frac{5}{36}$
20	$\frac{11}{36}$	40	$\frac{3}{10}$

MENTAL MATHS



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