

SOLUTION/ ANSWER KEY OF PRACTICE PAPER-2

CLASS XII MATHEMATICS

2019-20

Q NO	VALUE POINTS
1	(C) AB and BA both are defined
2	(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
3	(D)-5/2
4	(A)1/10
5	(D) $\sqrt{\alpha^2 + \gamma^2}$
6	(A) $\frac{\pi}{5}$
7	(C) 15/56
8	(A) $e^x \cos x + c$
9	(D) $(\alpha, \beta, -\gamma)$
10	(B) Parallel
11	Domain = $(-\infty, 1] \cup [2, \infty)$
12	2
13	Maximum value is $\frac{4}{3}$ OR $x+y=0$
14	$27 A $
15	$x+y-z=2$ OR $ \vec{a} ^2 \vec{b} ^2$
16	$\frac{-3}{\sqrt{1-x^2}}$
17	$\log 2 + \log x + c$
18	$3\log (x + \sin x) + c$ OR $\log x + \sin x + c$
19	$\tan x + \cot x + c$
20	$y = \frac{x^4 + c}{4x^2}$
21	$(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$ $\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c} = 0$ $\Rightarrow \vec{a} ^2 + \vec{b} ^2 + \vec{c} ^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

	$\Rightarrow \vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a} = \frac{-83}{2}$ <p>OR</p> <p>Given vectors are $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - \hat{k}$, therefore , $\vec{a} + \vec{b} = (3-2)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k}$ $\Rightarrow \vec{a} + \vec{b} = 1\hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k}$</p> <p>Hence unit vector in the direction of $(\vec{a} + \vec{b})$ is ,</p> $\frac{(\vec{a} + \vec{b})}{ (\vec{a} + \vec{b}) } = \frac{\hat{i} + \hat{k}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$		
22	$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$ $= \tan^{-1}\left[\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} - 2\sin \frac{x}{2}\cos \frac{x}{2}}\right]$ $= \tan^{-1}\left(\frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}}\right)$ $= \tan^{-1}\left[\frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}}\right]$ $= \tan^{-1}[\tan(\frac{\pi}{4} + \frac{x}{2})]$ $= \frac{\pi}{4} + \frac{x}{2}$ <p>OR</p> <p>$f(x) = \cos x$, $g(x) = 3x^2$</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px; width: 50%;"> $f(x) = \cos x$ $f(x) = \cos x \Rightarrow f(g(x)) = \cos g(x)$ $\Rightarrow f \circ g(x) = \cos 3x^2$ </td> <td style="padding: 5px; width: 50%;"> $g(x) = 3x^2 \Rightarrow g(f(x)) = 3f(x)^2$ $g \circ f(x) = 3\cos^2 x$ </td> </tr> </table> <p>Hence $g \circ f \neq f \circ g$</p>	$f(x) = \cos x$ $f(x) = \cos x \Rightarrow f(g(x)) = \cos g(x)$ $\Rightarrow f \circ g(x) = \cos 3x^2$	$g(x) = 3x^2 \Rightarrow g(f(x)) = 3f(x)^2$ $g \circ f(x) = 3\cos^2 x$
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23	$\frac{dy}{dx} = A \cos x + B(-\sin x) = A \cos x - B \sin x$ $\frac{d^2y}{dx^2} = A(-\sin x) - B \cos x = -A \sin x - B \cos x$		
24	<p>According to question,</p> <p>we have to find out the point on the curve at which the y coordinate is changing 2 times as fast as the x - coordinate.i.e ; $\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = 2$, now equation of the curve $6y = x^3 + 2$</p> $\Rightarrow 6 \frac{dy}{dx} = 3x^2 \Rightarrow 2 \frac{dy}{dx} = x^2$		

now, put $x = 4$ in $6y = x^3 + 2 \Rightarrow 2x^2 = x^2$

$$\Rightarrow x = \pm 2$$

When $x=2$

$$6y = 2^3 + 2 = 10$$

$$\Rightarrow y = 10/6$$

$$\Rightarrow y = 5/3$$

hence, a point on the curve is $(2, 5/3)$

When $x=-2$

$$6y = (-2)^3 + 2 = -8 + 2 = -6$$

$$\Rightarrow y = -1$$

\Rightarrow points on the curve will be $(-2, -1), (2, 5/3)$

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Given lines are $\vec{r} = (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = (7\hat{i} - 6\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

on comparing with $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{a}_2 + \mu \vec{b}_2$ we get $\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$ and $\vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

angle between the lines is given by

$$\cos \theta = \left| \frac{\overrightarrow{b_1} \cdot \overrightarrow{b_2}}{\|\overrightarrow{b_1}\| \|\overrightarrow{b_2}\|} \right| = \left| \frac{(3\hat{i} + 2\hat{j} + 6\hat{k})(\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{49}\sqrt{9}} \right| = \frac{19}{\sqrt{49}\sqrt{9}} = \frac{19}{7 \times 3} = \cos^{-1} \frac{19}{21}$$

26

No. of spade cards in a pack of 52 playing cards = 13

Let E: getting a spade

$$\therefore P(E) = \frac{13}{52}, P(E) = \frac{39}{52}$$

$$\text{Therefore, } P(\text{only 2 cards are spades}) = {}^4C_2 \frac{13}{52} \cdot \frac{13}{52} \cdot \frac{39}{52} \cdot \frac{39}{52} = \frac{54}{256}$$

27	<p>For $x_1, x_2 \in A$, let $f(x_1) = f(x_2)$</p> $\frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}$ $(4x_1 + 3)(6x_2 - 4) = (6x_1 - 4)(4x_2 + 3)$ $\Rightarrow x_1 = x_2$ <p>Hence, f is one-one.</p> <p>For any $y \in A$ s.t $y = \frac{4x+3}{6x-4} \exists x$ such that</p> $6xy - 4y = 4x + 3$ $\Rightarrow x = \frac{4y+3}{6y-4} \in A$ <p>Also, $f(x) = f\left(\frac{4y+3}{6y-4}\right) = \frac{4\left(\frac{4y+3}{6y-4}\right)+3}{6\left(\frac{4y+3}{6y-4}\right)-4} = y$</p> <p>$\Rightarrow f(x)$ is onto. Since, $f(x)$ is one-one and onto, therefore f^{-1} exists and $f^{-1}(y) = \frac{4y+3}{6y-4}$.</p>
28	<p>Here $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \cot^{-1} a$</p> $\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \cos(\cot^{-1} a)$ <p>Applying componendo and dividendo, we get: $\frac{x^2-y^2+x^2+y^2}{x^2-y^2+x^2-y^2} = \frac{\cos(\cot^{-1} a)+1}{\cos(\cot^{-1} a)-1}$</p> $\Rightarrow \frac{2x^2}{-2y^2} = \frac{\cos(\cot^{-1} a)+1}{\cos(\cot^{-1} a)-1}$ $\Rightarrow \frac{d}{dx}\left(\frac{x^2}{y^2}\right) = \frac{d}{dx}\left(-\frac{\cos(\cot^{-1} a)+1}{\cos(\cot^{-1} a)-1}\right)$ $\Rightarrow \frac{y^2x - 2x^2y^2}{(y^2)^2} = 0$

$$\Rightarrow y - x \frac{dy}{dx} = 0 \therefore \frac{dy}{dx} = \frac{y}{x} \quad \text{---(i)}$$

$$\text{Now } \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{y}{x} \right) \Rightarrow \frac{d^2 y}{dx^2} = \frac{\frac{dy}{dx} - y/x}{x^2}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{\frac{y}{x} - y}{x^2} = \frac{0}{x^2} \text{ by (i)}$$

$$\text{hence } \frac{d^2 y}{dx^2} = 0$$

OR

$$\sin y = x \sin(a+y) \Rightarrow x = \frac{\sin y}{\sin(a+y)}$$

Differentiating w.r.t y, we get,

$$\frac{dx}{dy} = \frac{\sin(a+y) \cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$$

$$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

29

The given differential equation can be written as

$$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$$

$$\text{Put } \frac{y}{x} = v \text{ and } \frac{dy}{dx} = v + x \frac{dv}{dx}, \text{ to get}$$

$$v + x \frac{dv}{dx} = v - \tan v \Rightarrow x \frac{dv}{dx} = -\tan v$$

$$\Rightarrow \cot v dv = \frac{-1}{x} dx, \text{ integrating both sides we get}$$

$$\log|\sin v| = -\log|x| + \log c$$

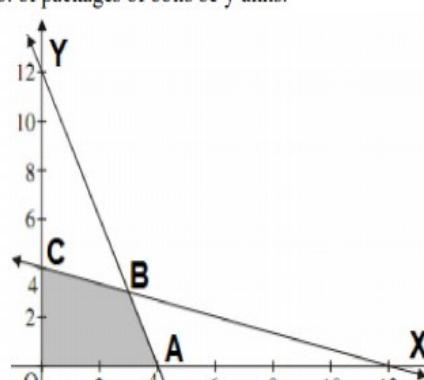
$$\Rightarrow \log|sinv| = \log \left| \frac{c}{x} \right|$$

Solution of differential equation is:

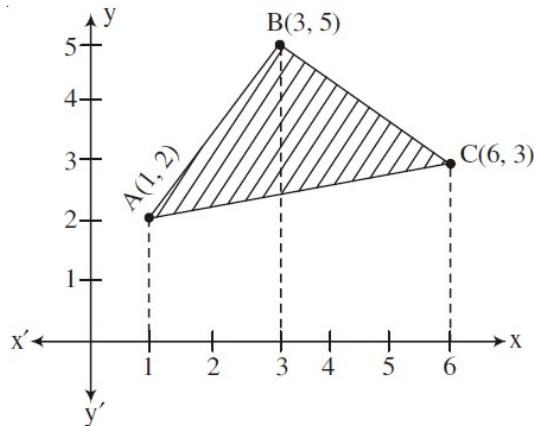
$$\sin\left(\frac{y}{x}\right) = \frac{c}{x} \text{ or } x\sin\left(\frac{y}{x}\right) = c$$

$$\begin{aligned}
 30 \quad &= \int \frac{3x+5}{x^2+3x-18} dx = \frac{3}{2} \int \frac{2x+3}{x^2+3x-18} dx + \frac{1}{2} \int \frac{1}{x^2+3x-18} dx \\
 &= \frac{3}{2} \int \frac{2x+3}{x^2+3x-18} dx + \frac{1}{2} \int \frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} dx \\
 &= \frac{3}{2} \log|x^2+3x-18| + \frac{1}{18} \log \left| \frac{x-3}{x+6} \right| + c
 \end{aligned}$$

$$\begin{aligned}
 31 \quad &\text{P (at least 3 are diamonds)} \\
 &\text{P(3)+ P(4)= } {}^4C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right) + {}^4C_4 \left(\frac{1}{4}\right)^4 \\
 &= \left(\frac{1}{4}\right)^4 (12 + 1) = \frac{13}{256} \\
 &\text{OR} \\
 &\text{P (only one on time)} = \text{P (A)} \text{ P}(\bar{B}) + \text{P}(\bar{A}) \text{ P(B)} \\
 &= \frac{2}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7} \\
 &= \frac{26}{49}
 \end{aligned}$$

32	<p>Let no. of packages of nuts be x units and no. of packages of bolts be y units.</p> <p>To maximize : $Z = ₹(35x + 14y)$</p> <p>Subject to constraints :</p> <p>$x \geq 0, y \geq 0,$ $x + 3y \leq 12,$ $3x + y \leq 12$</p> <table border="0" style="width: 100%;"> <thead> <tr> <th style="text-align: left;"><u>Corner Points</u></th><th style="text-align: left;"><u>Value of Z (in ₹)</u></th></tr> </thead> <tbody> <tr> <td>A(4, 0)</td><td>140</td></tr> <tr> <td>B(3, 3)</td><td>147 ← Maximum</td></tr> <tr> <td>C(0, 4)</td><td>56</td></tr> </tbody> </table>  <p>Hence, maximum profit of ₹147 is obtained when no. of packages of nuts = $x = 3$ units and, no. of packages of bolts = $y = 3$ units are produced.</p>	<u>Corner Points</u>	<u>Value of Z (in ₹)</u>	A(4, 0)	140	B(3, 3)	147 ← Maximum	C(0, 4)	56
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33	<p>Let $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$</p> <p>Then $A = IA$</p> $\Rightarrow \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad \therefore A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 9 & -12 & 9 \end{bmatrix}$ <p>OR</p> $\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2) \quad \text{The given system of equations is}$ <p>AX = B,</p> <p>where $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$</p> $X = A^{-1}B = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ <p>$A = 1200 \neq 0$</p> <p>$\Rightarrow A^{-1}$ exists.</p> <p>$X = A^{-1}B$</p> <p>$\text{adj } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$</p> <p>$A^{-1} = \frac{1}{ A } \text{adj } A = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$</p> <p>$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$</p> <p>$\therefore x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$</p>								

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$$\text{Required Area} = \int_1^3 \frac{3x+1}{2} dx + \int_3^6 \frac{21-2x}{3} dx - \int_1^6 \frac{x+9}{5} dx$$

$$= \left(\frac{3x^2}{4} + \frac{x}{2} \right) \Big|_1^3 + \left(7x - \frac{x^2}{3} \right) \Big|_3^6 - \left(\frac{x^2}{10} + \frac{9x}{5} \right) \Big|_1^6$$

$$= 7 + 12 - \frac{25}{2}$$

$$= \frac{13}{2}$$

35

Let Given surface area of open cylinder be S.

$$\text{Then } S = 2\pi rh + \pi r^2$$

$$\Rightarrow h = \frac{S - \pi r^2}{2\pi r}$$

$$\text{Volume } V = \pi r^2 h$$

$$V = \pi r^2 \left[\frac{S - \pi r^2}{2\pi r} \right] = \frac{1}{2} [Sr - \pi r^3]$$

$$\frac{dV}{dr} = \frac{1}{2} [S - 3\pi r^2]$$

$$\frac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2 \text{ or } 2\pi rh + \pi r^2 = 3\pi r^2$$

$$\Rightarrow 2\pi rh = 2\pi r^2 \Rightarrow h = r$$

$$\frac{d^2V}{dr^2} = -6\pi r < 0$$

\therefore For volume to be maximum, height = radius

Let x be the radius of circle and y be the side of square

$$2\pi x + 4y = k$$

$$A = \pi x^2 + y^2$$

$$A = \pi x^2 + \left(\frac{k - 2\pi x}{4} \right)^2 = \frac{16\pi x^2 + k^2 + 4\pi^2 x^2 - 4\pi kx}{16}$$

$$\frac{dA}{dx} = \frac{1}{16}(32\pi x + 8\pi^2 x - 4\pi k)$$

$$\frac{dA}{dx} = 0 \Rightarrow 32\pi x + 8\pi^2 x - 4\pi k = 0$$

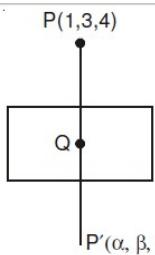
$$\Rightarrow x = \frac{k}{8+2\pi}$$

$$\left. \frac{d^2A}{dx^2} \right|_{x=\frac{k}{8+2\pi}} = \frac{1}{16}[32\pi + 8\pi^2] > 0 \Rightarrow \text{Sum of areas is minimum}$$

OR

$$2\pi \left(\frac{k}{8+2\pi} \right) + 4y = k \Rightarrow y = \frac{k}{4+\pi} \Rightarrow y = 2x$$

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Equation of line PQ is

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda$$

The coordinates of Q are $(2\lambda + 1, -\lambda + 3, \lambda + 4)$

$\therefore Q$ lies on plane $2x - y + z + 3 = 0$

$$\therefore 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0$$

$$\Rightarrow 6\lambda + 6 = 0 \text{ i.e., } \lambda = -1$$

The coordinates of Q are $(-1, 4, 3)$

Let $P'(\alpha, \beta, \gamma)$ be the image of P .

$$\text{then } \frac{\alpha+1}{2} = -1, \frac{\beta+3}{2} = 4, \frac{\gamma+4}{2} = 3$$

$$\Rightarrow \alpha = -3, \beta = 5, \gamma = 2$$

\therefore the image P' is $(-3, 5, 2)$