DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
(2021-2022)

Class : XI

MATHEMATICS

Under the Guidance of

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IAS

सत्यमेव जयते

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MESSAGE

I would like to congratulate the members of Core Academic Unit and the subject experts of the Directorate of Education, who inspite of dire situation due to Corona Pandemic, have provided their valuable contributions and support in preparing the Support Material for classes IX to XII.

The Support Material of different subjects, like previous years, have been reviewed/ updated in accordance with the latest changes made by CBSE so that the students of classes IX to XII can update and equip themselves with these changes. I feel that the consistent use of the Support Material will definitely help the students and teachers to enrich their potential and capabilities.

Department of Education has taken initiative to impart education to all its students through online mode, despite the emergency of Corona Pandemic which has led the world to an unprecedented health crises. This initiative has not only helped the students to overcome their stress and anxiety but also assisted them to continue their education in absence of formal education. The support material will ensure an uninterrupted learning while supplementing the Online Classes.

(H. Rajesh Prasad)

UDIT PRAKASH RAI, IAS
Director, Education & Sports

सत्यमेव जयते

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MESSAGE

The main objective of the Directorate of Education is to provide quality education to all its students. Focusing on this objective, the Directorate is continuously in the endeavor to make available the best education material, for enriching and elevating the educational standard of its students. The expert faculty of various subjects undertook this responsibility and after deep discussions and persistent efforts, came up with Support Material to serve the purpose.

Every year the Support Material is revised/updated to incorporate the latest changes made by CBSE in the syllabus of classes IX to XII. The contents of each lesson/chapter are explained in such a way that the students can easily comprehend the concept and get their doubts solved.

I am sure, that the continuous and conscientious use of this Support Material will lead to enhancement in the educational standard of the students, which would definitely be reflected in their performance.

I would also like to commend the entire team members for their contributions in the preparation of this incomparable material.

I wish all the students a bright future.

(UDIT PRAKASH RAI)

Dr. RITA SHARMA
Additional Director of Education
(School/Exam)

सत्यमेव जयते

Govt. of NCT of Delhi
Directorate of Education
Old Secretariat, Delhi-110054
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D.O. No. PA/Addl.DE/Sch/31
Dated: 29.06.2021

MESSAGE

It gives me immense pleasure to present the revised edition of the Support Material. This material is the outcome of the tireless efforts of the subject experts, who have prepared it following profound study and extensive deliberations. It has been prepared keeping in mind the diverse educational level of the students and is in accordance with the most recent changes made by the Central Board of Secondary Education.

Each lesson/chapter, in the support material, has been explained in such a manner that students will not only be able to comprehend it on their own but also be able to find solution to their problems. At the end of each lesson/chapter, ample practice exercises have been given. The proper and consistent use of the support material will enable the students to attempt these exercises effectively and confidently. I am sure that students will take full advantage of this support material.

Before concluding my words, I would like to appreciate all the team members for their valuable contributions in preparing this unmatched material and also wish all the students a bright future.

(Rita Sharma)

भारत का संविधान

भाग 4क

नागरिकों के मूल कर्तव्य

अनुच्छेद 51क

मूल कर्तव्य - भारत के प्रत्येक नागरिक का यह कर्तव्य होगा कि वह -

(क) संविधान का पालन करे और उसके आदर्शों, संस्थाओं, राष्ट्रध्वज और राष्ट्रगान का आदर करे;

(ख) स्वतंत्रता के लिए हमारे राष्ट्रीय आंदोलन को प्रेरित करने वाले उच्च आदर्शों को हृदय में संजोए रखे और उनका पालन करे;

(ग) भारत की संप्रभुता, एकता और अखंडता की रक्षा करे और उसे अक्षुण्ण बनाए रखे;

(घ) देश की रक्षा करे और आह्वान किए जाने पर राष्ट्र की सेवा करे;

(ङ) भारत के सभी लोगों में समरसता और समान भ्रातृत्व की भावना का निर्माण करे जो धर्म, भाषा और प्रदेश या वर्ग पर आधारित सभी भेदभावों से परे हो, ऐसी प्रथाओं का त्याग करे जो महिलाओं के सम्मान के विरुद्ध हों;

(च) हमारी सामासिक संस्कृति की गौरवशाली परंपरा का महत्त्व समझे और उसका परिरक्षण करे;

(छ) प्राकृतिक पर्यावरण की, जिसके अंतर्गत वन, झील, नदी और वन्य जीव हैं, रक्षा करे और उसका संवर्धन करे तथा प्राणिमात्र के प्रति दयाभाव रखे;

(ज) वैज्ञानिक दृष्टिकोण, मानववाद और ज्ञानार्जन तथा सुधार की भावना का विकास करे;

(झ) सार्वजनिक संपत्ति को सुरक्षित रखे और हिंसा से दूर रहे;

(ञ) व्यक्तिगत और सामूहिक गतिविधियों के सभी क्षेत्रों में उत्कर्ष की ओर बढ़ने का सतत् प्रयास करे, जिससे राष्ट्र निरंतर बढ़ते हुए प्रयत्न और उपलब्धि की नई ऊँचाइयों को छू सके; और

(ट) यदि माता-पिता या संरक्षक है, छह वर्ष से चौदह वर्ष तक की आयु वाले अपने, यथास्थिति, बालक या प्रतिपाल्य को शिक्षा के अवसर प्रदान करे।

CONSTITUTION OF INDIA

Part IV A (Article 51 A)

Fundamental Duties

Fundamental Duties : It shall be the duty of every citizen of India —

1. to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;
2. to cherish and follow the noble ideals which inspired our national struggle for freedom;
3. to uphold and protect the sovereignty, unity and integrity of India;
4. to defend the country and render national service when called upon to do so;
5. to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;
6. to value and preserve the rich heritage of our composite culture;
7. to protect and improve the natural environment including forests, lakes, rivers and wild life, and to have compassion for living creatures.
8. to develop the scientific temper, humanism and the spirit of inquiry and reform;
9. to safeguard public property and to adjure violence;
10. to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement.
11. who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

भारत का संविधान

उद्देशिका

हम, भारत के लोग, भारत को एक [1]**[संपूर्ण प्रभुत्व-संपन्न समाजवादी पंथनिरपेक्ष लोकतंत्रात्मक गणराज्य]** बनाने के लिए, तथा उसके समस्त नागरिकों को **:**

सामाजिक, आर्थिक और राजनैतिक **न्याय,**

विचार, अभिव्यक्ति, विश्वास, धर्म

और उपासना की **स्वतंत्रता,**

प्रतिष्ठा और अवसर की **समता**

प्राप्त कराने के लिए,

तथा उन **सब में**

व्यक्ति की गरिमा और [2]**[राष्ट्र की एकता**

और अखंडता] सुनिश्चित करने वाली **बंधुता**

बढ़ाने के लिए

दृढ़संकल्प होकर **अपनी इस संविधान सभा में** आज तारीख 26 नवंबर, 1949 ई. को **एतदद्वारा इस संविधान को अंगीकृत, अधिनियमित और आत्मार्पित करते हैं।**

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1. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977 से) ''प्रभुत्व-संपन्न लोकतंत्रात्मक गणराज्य'' के स्थान पर प्रतिस्थापित।
2. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977 से) ''राष्ट्र की एकता'' के स्थान पर प्रतिस्थापित।

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

---

1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
(2021-2022)

MATHEMATICS
Class : XI

NOT FOR SALE

PUBLISHED BY : DELHI BUREAU OF TEXTBOOKS

2021-2022

SUPPORT MATERIAL

Class : XI

MATHEMATICS

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CLASS XI
MATHEMATICS (CODE 041)
TERM-WISE SYLLABUS
SESSION (2021-22)

TERM- I

One Paper
90 Minutes **Max Marks: 40**

NO	Units	Marks
I	Sets and Functions	11
II	Algebra	13
III	Coordinate Geometry	6
IV	Calculus	4
V	Statistics and Probability	6
	Total	40
	Internal Assessment	10
	Total	50

Unit-I: Sets and Functions

1. Sets

Sets and their representations. Empty set. Finite and Infinite sets. Equal sets. Subsets. Subsets of a set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets.

2. Relations &Functions

Ordered pairs. Cartesian product of sets. Number of elements in the Cartesian product of two finite sets. Cartesian product of the set of reals with itself (RxRonly).Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs.

Unit-II: Algebra

1. Complex Numbers and Quadratic Equations

Need for complex numbers, especially$\sqrt{-1}$, to be motivated by inability to solve some of the quardraticequations. Algebraic properties of complex numbers. Argand plane. Statement of Fundamental Theorem of Algebra, solution of quadratic equations (with real coefficients) in the complex number system.

2. Sequence and Series

Sequence and Series. Arithmetic Progression (A. P.). Arithmetic Mean (A.M.) Geometric Progression (G.P.),general term of a G.P.,sum of *n* terms of a G.P.,infinite G.P.and its sum,geometric mean(G.M.), relation between A.M. and G.M.

Unit-III: Coordinate Geometry

1. Straight Lines

Brief recall of two dimensional geometry from earlier classes. Slope of a line and angle between two lines. Various forms of equations of a line :parallel to axis ,point-slope form ,slope-intercept form ,two-point form, intercept form and normal form. General equation of a line. Distance of a point from a line.

Unit-IV: Calculus

1. Limits

Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions

Unit-V: Statistics and Probability

1. Statistics

Measures of Dispersion: Range, mean deviation, variance and standard deviation of ungrouped/grouped data.

INTERNAL ASSESSMENT MARKS	**10**
Periodic Test	**5 Marks**
Mathematics Activities: Activity file record +Term end assessment of one activity & Viva	**5 Marks**

Note: For activities NCERT Lab Manual may be referred

TERM – II

One Paper

90 Minutes **Max Marks: 40**

No.	Units	Marks
I	Sets and Functions (Cont.)	11
II	Algebra(Cont.)	13
III	Coordinate Geometry(Cont.)	6
IV	Calculus(Cont.)	4
V	Statistics and Probability(Cont.)	6
	Total	40
	Internal Assessment	10
	Total	50

Unit-I: Sets and Functions

1. **Trigonometric Functions**

Positive and negative angles, Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle .Truth of the identity $\sin^2x+\cos^2x=1$, for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing sin (x±y) and cos (x±y) in terms of sin x, sin y, cos x &cos y and their simple applications. Deducing the identities like the following:

$$\tan(x \pm y)=\frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}, \cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$$

$$\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha + \beta)\sin\frac{1}{2}(\alpha - \beta)$$

Identities related to sin 2x, cos2x, tan 2x, sin3x, cos3x and tan3x.

Unit-II: Algebra

1. **Linear Inequalities**

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Graphical method of finding a solution of system of linear inequalities in two variables.

2. **Permutations and Combinations**

Fundamental principle of counting. Factorial *n.* (n!) Permutations and combinations, formula for ^nP_r and ^nC_r, simple applications.

Unit-III: Coordinate Geometry

1. Conic Sections

Sections of a cone: circles, ellipse, parabola, hyperbola. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

2. Introduction to Three-dimensional Geometry

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

Unit-IV: Calculus

1. Derivatives

Derivative introduced as rate of change both as that of distance function and geometrically. Definition of Derivative, relate it to scope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

Unit-V: Statistics and Probability

1. Probability

Random experiments; outcomes, sample spaces (set representation).Events; occurrence of events, 'not', 'and' and 'or' events, exhaustive events, mutually exclusive events, Probability of an event, probability of 'not', 'and' and 'or' events.

INTERNAL ASSESSMENT	10 MARKS
Periodic Test	5 Marks
Mathematics Activities: Activity file record +Term end assessment of one activity & Viva	5Marks

Note: For activities NCERT Lab Manual may be referred

Assessment of Activity Work:

In first term any 4 activities and in second term any 4 activities shall be performed by the student from the activities given in the NCERT Laboratory Manual for the class (XI) which is available on the link: http://www.ncert.nic.in/exemplar/labmanuals.htmla record of the same may be kept by the student. A term end test on the activity is to beconducted.

The weightage are as under:

- The activities performed by the student in each term and record keeping : 3 marks
- Assessment of the activity performed during the term end test and Viva-voce : 2 marks
- **Prescribed Books:**

1) Mathematics Textbook for Class XI, NCERT Publications
2) Mathematics Exemplar Problem for Class XI, Published by NCERT
3) Mathematics Lab Manual class XI, ~~publish~~ed by NCERT

CONTENTS

CHAPTER - 1

SETS AND FUNCTIONS

KEY POINTS

- A set is a well-defined collection of objects.
- There are two methods of representing a set:
  - (a) Roster or Tabular form e.g. natural numbers less than 5 = {1, 2, 3, 4}
  - (b) Set-builder form or Rule method e.g.: Vowels in English alphabet= {x : x is a vowel in the English alphabet }

Georg Cantor
(1845-1918)

- **Types of sets:**
  - (i) Empty set or Null set or void set
  - (ii) Finite set
  - (iii) Infinite set
  - (iv) Singleton set
- **Subset:** A set A is said to be a subset of set B if $a \in A \Rightarrow a \in B, \ \forall a \in A$. We write it as $A \subseteq B$.
- **Equal sets:** Two sets A and B are equal if they have exactly the same elements i.e $A = B$ if $A \subset B$ and $B \subset A$.
- **Power set:** The collection of all subsets of a set A is called power set of A, denoted by P(A) i.e. $P(A) = \{ B : B \subset A \}$
- If A is a set with $n(A) = m$ then $n[P(A)] = 2^m$.
- **Equivalent sets:** Two finite sets *A* and *B* are equivalent, if their cardinal numbers are same i.e., $n(A) = n(B)$.

- **Proper subset and super set:** If $A \subset B$ then A is called the proper subset of B and B is called the superset of A.

Types of Intervals

Open Interval $(a, b) = \{ x \in R : a < x < b \}$

Closed Interval $[a, b] = \{ x \in R : a \leq x \leq b \}$

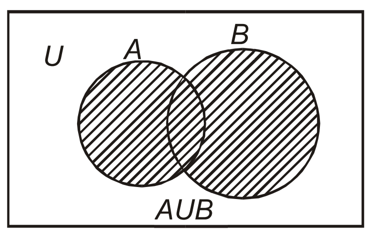
Semi open or Semi closed Interval,

$(a,b] = \{ x \in R : a < x \leq b\}$

$[a,b) = \{ x \in R : a \leq x < b\}$

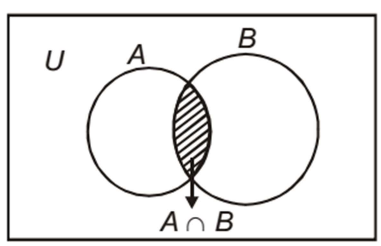
- Union of two sets A and B is,

$A \cup B = \{ x : x \in A \text{ or } x \in B \}$

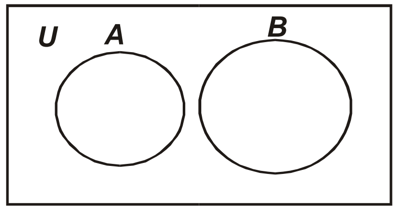


- Intersection of two sets A and B is,

$A \cap B = \{ x : x \in A \text{ and } x \in B\}$

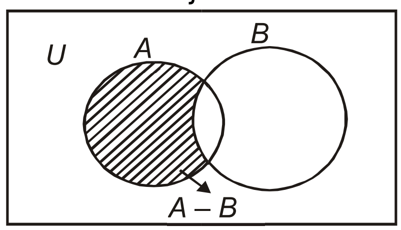


- Disjoint sets: Two sets A and B are said to be disjoint if $A \cap B = \phi$



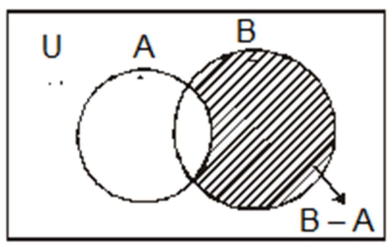
- Difference of sets A and B is,

  $A - B = \{ x : x \in A \text{ and } x \notin B\}$



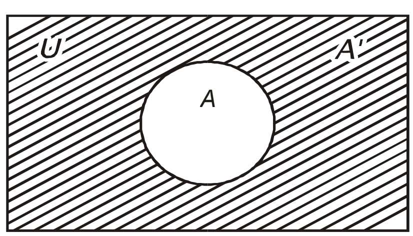
- Difference of sets B and A is,

  $B - A = \{ x : x \in B \text{ and } x \notin A \}$



- Complement of a set A, denoted by A′ or A^c is

  $A' = A^c = U - A = \{ x : x \in U \text{ and } x \notin A\}$



- Properties of complement sets :

  1. Complement laws

     (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$ (iii) $(A')' = A$

  2. De Morgan's Laws

     (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

  Note **:** This law can be extended to any number of sets.

  3. $\phi' = \bigcup$ and $\bigcup' = \phi$

  4. If $A \subset B$ then $B' \subset A'$

- Laws of Algebra of sets

  (i) $A \cup \phi = A$

  (ii) $A \cap \phi = \phi$

- $A - B = A \cap B' = A - (A \cap B)$

- Commutative Laws :–

  (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

- Associative Laws :–

  (i) $(A \cup B) \cup C = A \cup (B \cup C)$

  (ii) $(A \cap B) \cap C = A \cap (B \cap C)$

- Distributive Laws :–

  (i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

  (ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- If $A \subset B$, then $A \cap B = A$ and $A \cup B = B$

- When *A* and *B* are disjoint $n(A \cup B) = n(A) + n(B)$

- When A and B are not disjoint $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

VERY SHORT ANSWER TYPE QUESTIONS

Which of the following are sets? Justify your answer.

1. The collection of all the months of a year beginning with letter M.
2. The collection of difficult topics in Mathematics.

Let A = {1, 3, 5, 7, 9}. Insert the appropriate symbol $\in$ or $\notin$ in blank spaces: – (Question- 3, 4)

3. (i) 2 _____ A (ii) {3} _____ A (iii) {3, 5} _____ A
4. Write the set A = { x : x is an integer, $-1 \le x < 4$} in roster form
5. Write the set B = {3, 9, 27, 81} in set-builder form.

Which of the following are empty sets? Justify. (Question- 6, 7)

6. $A = \{ x : x \in N \text{ and } 3 < x < 4\}$
7. $B = \{ x : x \in N \text{ and } x^2 = x\}$

Which of the following sets are finite or infinite? Justify. (Question-8, 9)

8. The set of all the points on the circumference of a circle.
9. $B = \{ x : x \in N$ and x is an even prime number}
10. Are sets $A = \{ -2, 2\}$, $B = \{ x : x \in Z, x^2 - 4 = 0\}$ equal? Why?
11. Write $(-5,9]$ in set-builder form
12. Write $\{x : x \in R, -3 \le x < 7\}$ as interval.
13. If A = { 1, 3, 5}, how many elements has P(A)?

14. Write all the possible subsets of A = {5, 6}.

If A = Set of letters of the word 'DELHI' and B= the set of letters the words 'DOLL' find (Question- 17,18,19)

15. $A \cup B$

16. $A \cap B$

17. $A - B$

18. Describe the following sets in Roster form

    (i) The set of all letters in the word 'ARITHMETIC'.
    (ii) The set of all vowels in the word 'EQUATION'.

19. Write the set $A = \{x : x \in z, x^2 < 25\}$ in roster form.

20. Write the set B = {x : x is a two digit numbers such, that the sum of its digits is 7}

Fill in the blanks (21 – 23)

21. The total number of non-empty subsets of a finite set. Containing n elements is ___________.

22. The total number of proper subsets of finite set. Containing n elements is _________.

23. $n[P\{P(\phi)\}] =$ _________.

24. If A and B are two sets such that $A \subset B$, then find

    (i) $A \cap B$ (ii) $A \cup B$

25. Let A and B be two sets having 5 and 7 elements respectively. Write the minimum and maximum number of elements in

    (i) $A \cup B$ (ii) $A \cap B$

26. For any two sets A and B, $A \cap (A \cup B)'$ is equal to

    (a) A (b) B (c) ϕ (d) $A \cap B$.

27. In set builder method the null set is represented by

(a) { } (b) ϕ (c) $\{ x : x \neq x\}$ (d) $\{ x : x = x\}$.

28. If A and B are two given sets, then $A \cap (A \cap B)'$ is equal to

(a) A (b) B' (c) ϕ (d) $A - B$.

29. If A and B are two sets such that $A \subset B$ then $A \cap B'$ is

(a) A (b) B' (c) ϕ (d) $A \cap B$.

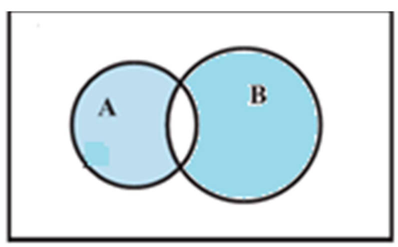
30. If $n(A \cup B) = 18$, $n(A - B) = 5$, $n(B - A) = 3$ then find $n(A \cap B)$

SHORT ANSWER TYPE QUESTIONS

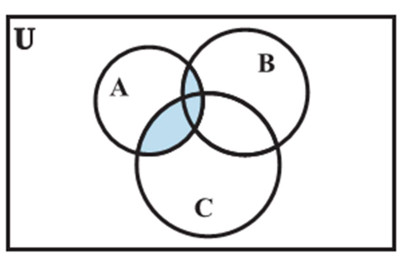
31. Are sets $A = \{1, 2, 3, 4\}$, $B = \{ x : x \in N \text{ and } 5 \le x \le 7\}$ disjoint? Justify?

What is represented by the shaded regions in each of the following Venn-diagrams? (Question 32, 33)

32.



33.



SHORT ANSWER TYPE QUESTIONS

34. If A = { 1, 3, 5, 7, 11, 13, 15, 17}

    B = { 2, 4, 6, 8 ... 18}

    And U is universal set then find $A' \cup [(A \cup B) \cap B']$

35. Two sets A and B are such that

    $n(A \cup B) = 21$ $n(A) = 10$ $n(B) = 15$ find $n(A \cap B)$ and $n(A - B)$

36. Let A = {1, 2, 4, 5} B = {2, 3, 5, 6} C = {4, 5, 6, 7} Verify the following identity

    $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

37. If $\bigcup = \{ x : x \in N \text{ and } x \leq 10\}$

    $A = \{ x : x \text{ is prime and } x \leq 10\}$
    $B = \{ x : x \text{ is a factor of } 24\}$

    Verify the following result

    (i) $A - B = A \cap B'$ (ii) $(A \cup B)' = A' \cap B'$ (iii) $(A \cap B)' = A' \cup B'$

38. Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$

39. For any sets A and B show that

    (i) $(A \cap B) \cup (A - B) = A$ (ii) $A \cup (B - A) = A \cup B$

40. On the Real axis, If A = [0, 3] and B = [2, 6], than find the following

    (i) A' (ii) $A \cup B$ (iii) $A \cap B$ (iv) $A - B$

41. In a survey of 450 people, it was found that 110 play cricket, 160 play tennis and 70 play both cricket as well as tennis. How many play neither cricket nor tennis?

42. In a group of students, 225 students know French, 100 know Spanish and 45 know both. Each student knows either French or Spanish. How many students are there in the group?

43. For all set A, B and C is $(A \cap B) \cup C = A \cap (B \cup C)$? Justify your answer.

44. Two sets A and B are such that $n(A \cup B) = 21$, $n(A' \cap B') = 9$, $n(A \cap B) = 7$ find $n(A \cap B)'$.

LONG ANSWER TYPE QUESTIONS

45. In a group of 84 persons, each plays at least one game out of three viz. tennis, badminton and cricket. 28 of them play cricket, 40 play tennis and 48 play badminton. If 6 play both cricket and badminton and 4 play tennis and badminton and no one plays all the three games, find the number of persons who play cricket but not tennis. What is the importance of sports in daily life?

46. Using properties of sets and their complements prove that

    (i) $(A \cup B) \cap (A \cup B') = A$

    (ii) $A - (A \cap B) = A - B$

    (iii) $(A \cup B) - C = (A - C) \cup (B - C)$

    (iv) $A - (B \cup C) = (A - B) \cap (A - C)$

    (v) $A \cap (B - C) = (A \cap B) - (A \cap C)$.

47. If A is the set of all divisors of the number 15. B is the set of prime numbers smaller than 10 and C is the set of even number smaller than 9, then find the value of $(A \cup C) \cap B$.

48. Two finite sets have m and n elements. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the value of m and n.

49. If $X = \{4^n - 3n - 1 \quad : \quad n \in N\}$
$Y = \{9(n-1) \quad : \quad n \in N\}$

Find the value of $X \cup Y$

50. A survey shows that 63% people watch news channel A whereas 76% people watch news channel B. If x% of people watch both news channels, then prove that $39 \leq x \leq 63$.

51. From 50 students taking examination in Mathematics, Physics and chemistry, each of the students has passed in at least one of the subject, 37 passes Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, almost 29 Mathematics and chemistry and at most 20 Physics and chemistry. What is the largest possible number that could have passes in all the three subjects?

CASE STUDY TYPE QUESTIONS

52. In a survey of of 600 students of class XI, 150 are using YouTube videos and 225 are consulting books (other than text book) as a learning resource. 100 were using both YouTube videos and books as a learning resource.

i. How many students are using either books or YouTube videos as the learning resource?

(a) 325 (b) 225 (c) 275 (d) 375

ii. How many students are neither using YouTube videos nor books as the learning resource?

(a) 350 (b) 325 (c) 225 (d) 250

iii. How many students are using YouTube videos only as the learning resource?

(a) 50 (b) 100 (c) 150 (d) 125

iv. How many students are using books only as the learning resource?

(a) 50 (b) 100 (c) 150 (d) 125

v. What can be the maximum number of students who will use YouTube video or books as learning resources?

(a) 600 (b) 375 (c) 275 (d) 500

53. In a class 18 students took Physics, 23 students took Chemistry and 24 students took Mathematics. Of these 13 took both Chemistry and Mathematics, 12 took both physics and chemistry and 11 took both Physics and mathematics. If 6 students were offered all the three subjects Find:

i. The total number of students are

(a) 47 (b) 37 (c) 35 (d) 49

ii. How many took Mathematics but not Chemistry?

(a) 11 (b) 1 (c) 6 (d) 12

iii. How many took exactly one of the three subjects?

(a) 12 (b) 11 (c) 13 (d) 1

iv. How many took exactly two of these subjects?

(a) 11 (b) 13 (c) 12 (d) 18

v. Number of students who took Physics or Mathematics but not Chemistry:

(a)12 (b) 13 (c) 11 (d) 18

54. In a town of 10,000 families, it was found that 40% families go to shop A for their home needs groceries, 20% families go to the shop B and 10% families go to shop C. 5% families go to shops

A and B, 3% go to B and C and 4% families go to A and C. 2% families go to all the three shops A, B and C. Find:

i. The number of families which go to shop A only;

(a) 4000 (b) 3300 (c) 3700 (d) 4200

ii. The number of families which don't visit/purchase from any of A, B and C.

(a) 4000 (b) 7000 (c) 3300 (d) 6000

iii. The number of families which don't visit/purchase from any of A, B and C.

(a) 300 (b) 200 (c) 100 (d) 600

iv. The number of families that purchase from exactly one shop.

(a) 4700 (b) 4000 (c) 5200 (d) 3800

v. The number of families that buy from at least one of the shops A, B or C.

(a) 4000 (b) 6000 (c) 7000 (d) 1000

ANSWERS

1. Set
2. Not a set
3. (i) $\notin$ (ii) $\notin$ (iii) $\notin$
4. A = {–1, 0, 1, 2, 3}
5. $B = \{ x : x = 3^n, n \in N \text{ and } 1 \leq n \leq 4\}$
6. Empty set because no natural number is lying between 3 and 4
7. Non-empty set because B = {1}

8. Infinite set because circle is a collection of infinite points whose distances from the centre is constant called radius.

9. Finite set because B = {2}

10. Yes, because $x^2 - 4 = 0$; $x = 2, -2$ both are integers

11. $\{x : x \in R, -5 < x \leq 9\}$

12. $[-3, 7)$

13. $2^3 = 8$

14. ϕ, {5}, {6}, {5, 6}

15. $A \cup B$ = {D, E, L, H, I, O}

16. $A \cap B$ = {D, L}

17. A – B = {E, H, I}

18. (i) {A, R, I, T, H, M, E, C} (ii) {E, U, A, I, O}

19. {–4, –3, –2, –1, 0, 1, 2, 3, 4}

20. {16, 25, 34, 43, 52, 61, 70}

21. $2^n - 1$

22. $2^n - 1$

23. 2

24. (i) A (ii) B

25. (i) min : 7, max : 12 (ii) min : 0, max : 5

26. (c) ϕ

27. (c) $\{ x : x \neq x \}$

28. (b) B'

29. (c) ϕ

30. 10

31. Yes, because $A \cap B = \phi$

32. $(A - B) \cup (B - A)$ or $A \Delta B$

33. $A \cap (B \cup C)$

34. $\cup$

35. $n(A \cap B) = 4$, $n(A - B) = 6$

38. $A = \{1, 2\}$, $B = \{1, 3\}$ $C = \{2, 3\}$

40. (i) $(-\infty, 0) \cup (3, \infty)$ (ii) $[0, 6]$ (iii) $[2, 3]$ (iv) $[0, 2)$

41. **Hint:** $\cup$ = set of people surveyed
A = set of people who play cricket
B = set of people who play tennis

Number of people who play neither cricket nor tennis

$$= n[(A \cup B)'] = n(U) - n(A \cup B)$$
$$= 450 - 200$$
$$= 250$$

42. There are 280 students in the group.

43. No, For example $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{3, 4\}$

44. 23

45. 6

47. $\{2, 3, 5\}$

48. $n = 3$ $m = 6$

49. Y

51. 14

52. i. (c) ii. (b) iii. (a) iv. (d) v. (b)

53. i. (c) ii. (a) iii. (b) iv. (d) v. (a)

54. i. (b) ii. (a) iii. (d) iv. (c) v. (b)

CHAPTER – 2

RELATIONS AND FUNCTIONS

CONCEPT MAP

- **Ordered Pair:** An ordered pair consists of two objects or elements in a given fixed order.

  Remarks: An ordered pair is not a set consisting of two elements. The ordering of two elements in an ordered pair is important and the two elements need not be distinct.

- **Equality of Ordered Pair:** Two ordered pairs (x_1, y_1) & (x_2, y_2) are equal if $x_1 = x_2$ and $y_1 = y_2$.

  i.e. $(x_1, y_1) = (x_2, y_2) \Leftrightarrow x_1 = x_2$ and $y_1 = y_2$

- **Cartesian product of two sets:** Cartesian product of two non-empty sets A and B is given by A × B and $A \times B = \{(x, y) : x \in A$ and $y \in B\}$.

- **Cartesian product of three sets:** Let A, B and C be three sets, then A × B × C is the set of all ordered triplet having first element from set A, 2nd element from set B and 3rd element from set C.

  i.e., $A \times B \times C = \{(x, y, z) : x \in A, y \in B \text{ and } z \in C\}$.

- **Number of elements in the Cartesian product of two sets:** If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

- **Relation:** Let A and B be two non-empty sets. Then a relation from set A to set B is a subset of A × B.

- **No. of relations:** If n(A) = p, n(B) = q then no. of relations from set A to set B is given by 2^{pq}.
- **Domain of a relation:** Domain of R = $\{a : (a, b) \in R\}$
- **Range of a relation:** Range of R = $\{ b : (a, b) \in R\}$
- Co-domain of R from set A to set B = set B.
- Range $\subseteq$ Co-domain
- **Relation an a set:** Let A be non-empty set. Then a relation from A to B itself. i.e., a subset of A × A, is called a relation on a set.
- **Inverse of a relation:** Let A, B be two sets and Let R be a relations from set A to set B.

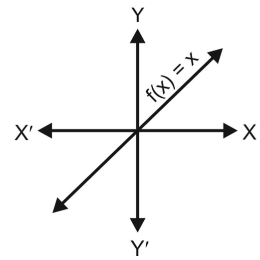
  Then the inverse of R denoted R^{-1} is a relation from set B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$
- **Function:** Let A and B be two non-empty sets. A relation from set A to set B is called a function (or a mapping or a map) if each element of set A has a unique image in set B.

  Remark: If $(a, b) \in f$ then 'b' is called the image of 'a' under f and 'a' is called pre-image of 'b'.
- **Domain and range of a function:** If a function 'f' is expressed as the set of ordered pairs, the domain of 'f' is the set of all the first components of members of f and range of 'f' is the set of second components of member of 'f'.

  i.e., $D_f = \{a : (a, b) \in f\}$ and $R_f = \{b : (a, b) \in D_f\}$
- **No. of functions:** Let A and B be two non-empty finite sets such that n(A) = p and n(B) = q then number of functions from A to B = q^p.
- **Real valued function:** A function $f : A \rightarrow B$ is called a real valued function if B is a subset of R (real numbers).

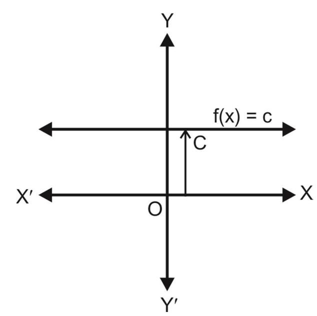
- **Identity function:** $f : R \rightarrow R$ given by $f(x) = x \ \forall \ x \in R$ (real number)

  Here, $D_f = R$ and $R_f = R$



- **Constant function:** $f : R \rightarrow R$ given by $f(x) = c$ for all $x \in R$ where c is any constant

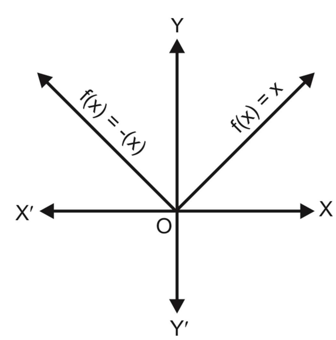
  Here, $D_f = R$ and $R_f = \{c\}$



- **Modulus function:** $f : R \rightarrow R$ given by $f(x) = |x| \ \forall \ x \in R$

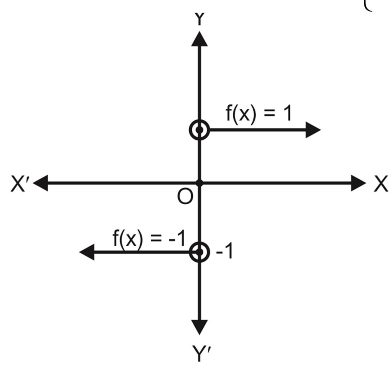
  Here, $D_f = R$ and $R_f = [0, \infty)$

  Remarks : $\sqrt{x^2} = |x|$



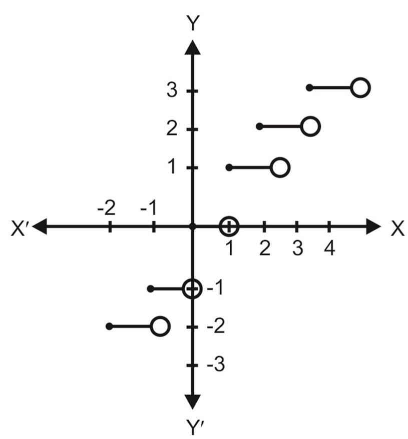
- **Signum function:** $f : R \to R$ defined by $f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

or $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$



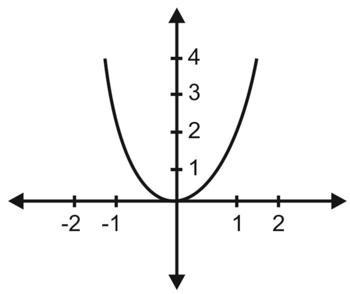
- **Greatest Integer function:** $f : R \to R$ defined by $f(x) = [x]$, $x \in R$ assumes the value of the greatest integer, less than or equal to x.

Here, $D_f = R$ and $R_f = Z$

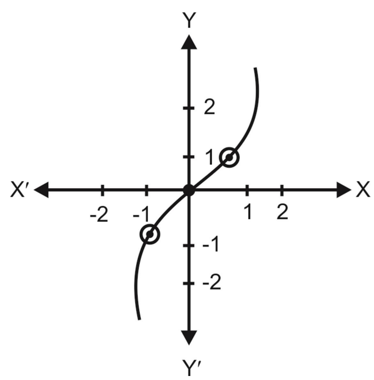


- Graph for $f : R \to R$, defined by $f(x) = x^2$

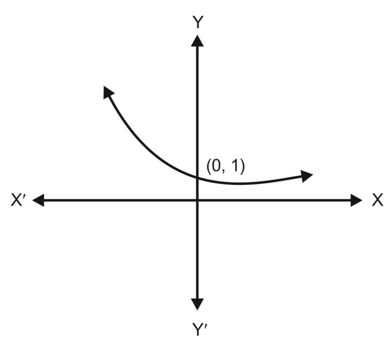
Here, $D_f = R$ and $R_f = [0, \infty)$



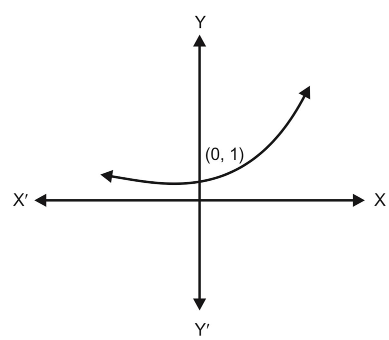
- Graph for f : R $\rightarrow$ R, defined by f(x) = x^3



- **Exponential function:** f : R $\rightarrow$ R, defined by f(x) = a^x, a > 0, a $\neq$ 1



When 0 < a < 1



When a > 1

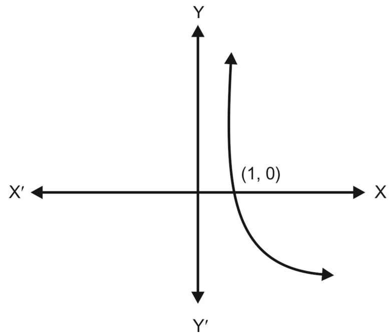
$$f(x) = a^x \begin{cases} > 1 & \text{for } x < 0 \\ = 1 & \text{for } x = 0 \\ < 1 & \text{for } x > 0 \end{cases}$$

$$f(x) = 0^x \begin{cases} < 1 & \text{for } x < 0 \\ = 1 & \text{for } x = 0 \\ > 1 & \text{for } x > 0 \end{cases}$$

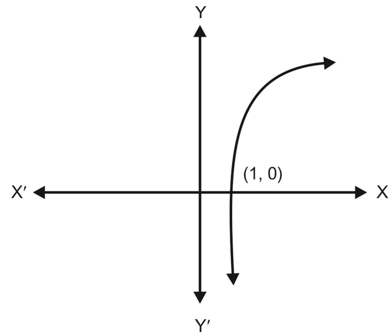
- Natural exponential function, $f(x) = e^x$

  $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots \infty, \quad 2 < e < 3$

- Logarithmic functions, $f : (0, \infty) \to R$; $f(x) \log_a x$, $a > 0$, $a \neq 1$



$f(x) = \log_a x$, $0 < a < 1$
$D_f = (0, \infty)$
$R_f = R$
Case I When $0 < a < 1$



$f(x) = \log_a x$, for $a > 1$
$D_f = (0, \infty)$
$R_f = R$
Case II When $a > 1$

- **Natural logarithm function:** $f(x) = \log_e x$ or $\ln(x)$.

- Let $f : X \to R$ and $g : X \to R$ be any two real functions where $x \subset R$ then

  $(f \pm g)(x) = f(x) \pm g(x) \ \forall \ x \in X$

  $(fg)(x) = f(x)\, g(x) \ \forall \ x \in X$

  $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \forall \ x \in X$ provided $g(x) \neq 0$

VERY SHORT ANSWER TYPE QUESTIONS

1. If A = {1, 2, 4}, B = {2, 4, 5}, C = {2, 5} then (A – B) × (B – C)

   (a) {(1, 2), (1, 5), (2, 5)} (b) {1, 4}

   (c) {1, 4} (d) None of these.

2. If R is a relation on set A = {1, 2, 3, 4, 5, 6, 7, 8} given by xRy $\Leftrightarrow$ y = 3x, then R = ?

   (a) {(3, 1), (6, 2), (8, 2), (9, 3)} (b) {(3, 1), (6, 2), (9, 3)}

   (c) {(3, 1), (2, 6), (3, 9)} (d) None of these.

3. Let A = {1, 2, 3}, B = {4, 6, 9} if relation R from A to B defined by x is greater then y. the range of R is -

   (a) {1, 4, 6, 9} (b) {4, 6, 9}

   (c) {1} (d) None of these.

4. If R be a relation from a set A to a set B then -

   (a) $R = A \cup B$ (b) $R = A \cap B$

   (c) $R \subseteq A \times B$ (d) $R \subseteq B \times A$.

5. If $2f(x) - 3f\left(\frac{1}{x}\right) = x^2$ $(x \neq 0)$, then f(2) is equal to -

   (a) $\frac{-7}{4}$ (b) $\frac{5}{2}$

   (c) –1 (d) None of these.

6. Range of the function f(x) = cos[x] for $\frac{-\pi}{2} < x < \frac{\pi}{2}$ is -

   (a) {–1, 1, 0} (b) {cos1, cos2, 1}

   (c) {cos1, –cos1, 1} (d) {–1, 1}.

7. If f(x) = $\log\left(\frac{1+x}{1-x}\right)$ and g(x) = $\frac{3x+x^3}{1+3x^2}$ then f{g(x)} is equal to -

(a) f(3x) (b) $\{f(x)\}^3$

(c) 3f(x) (d) –(f(x).

8. If f(x) = cos(logx) then value of f(x).f(y) – $\frac{1}{2}\left\{f\left(\frac{x}{y}\right)+f(xy)\right\}$ is -

(a) 1 (b) –1

(c) 0 (d) ±1.

9. Doman of f(x) = $\sqrt{4x-x^2}$ is -

(a) R – [0, 4] (b) R – (0, 4)

(c) (0, 4) (d) [0, 4].

10. If $[x]^2 - 5[x] + 6 = 0$, where [.] denote the greatest integer function then -

(a) $x \in [3, 4]$ (b) $x \in (2, 3]$

(c) $x \in [2, 3]$ (d) $x \in [2, 4)$.

11. Find a and b if (a – 1, b + 5) = (2, 3)

If A = {1,3,5}, B = {2,3}, find : (Question - 12, 13)

12. A × B

13. B × A

Let A = {1,2}, B = {2,3,4}, C = {4,5}, find (Question - 14, 15)

14. A × (B $\cap$ C)

15. A × (B $\cup$ C)

16. If $P = \{1,3\}$, $Q = \{2,3,5\}$, find the number of relations from P to Q

17. If $R = \{(x,y):\ x,y \in Z,\ x^2 + y^2 = 64\}$, then,

    Write R in roster form

    Which of the following relations are functions? Give reason. (Questions 18 to 20)

18. $R = \{ (1,1), (2,2), (3,3), (4,4), (4,5)\}$

19. $R = \{ (2,1), (2,2), (2,3), (2,4)\}$

20. $R = \{ (1,2), (2,5), (3,8), (4,10), (5,12), (6,12)\}$

SHORT ANSWER TYPE QUESTIONS

21. If A and B are finite sets such that $n(A) = 5$ and $n(B) = 7$, then find the number of functions from A to B.

22. If $f(x) = x^2 - 3x + 1$ find $x \in R$ such that $f(2x) = f(x)$

    Let f and g be two real valued functions, defined by, $f(x) = x$, $g(x) = |x|$.

    Find: (Question 23 to 26)

23. $f + g$

24. $f - g$

25. fg

26. $\dfrac{f}{g}$

27. If $f(x) = x^3$, find the value of, $\dfrac{f(5)-f(1)}{5-1}$

28. Find the domain of the real function, $f(x)=\sqrt{x^2-4}$

29. Find the domain of the function, $f(x)=\dfrac{x^2+2x+3}{x^2-5x+6}$

Find the range of the following functions. (Question- 30, 31)

30. $f(x)=\dfrac{1}{4-x^2}$

31. $f(x) = x^2 + 2$

32. Find the domain of the relation,
$R = \{(x, y): x, y \in Z, xy = 4\}$

Find the range of the following relations: (Question-33, 34)

33. $R = \{(a,b) : a, b \in N \text{ and } 2a + b = 10\}$

34. $R=\left\{\left(x,\dfrac{1}{x}\right): x\in Z, 0<x<6\right\}$

SHORT ANSWER TYPE QUESTIONS

35. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as,

$R = \{(x, y): x \in A, y \in B \text{ and } y = x^2\}$

(a) Depict this relation using arrow diagram.
(b) Find domain of R.
(c) Find range of R.
(d) Write co-domain of R.

36. If A = {2,4,6,9} B = {4,6,18,27,54} and a relation R from A to B is defined by $R = \{(a,b): a\in A, b\in B$, a is a factor of b and $a < b\}$, then find in Roster form. Also find its domain and range.

37. Let $f(x)=\begin{cases}x^2, & \text{when } 0 \le x \le 2\\ 2x, & \text{when } 2 \le x \le 5\end{cases}$

$g(x)=\begin{cases}x^2, & \text{when } 0 \le x \le 3\\ 2x, & \text{when } 3 \le x \le 5\end{cases}$

Show that f is a function while g is not a function.

38. Find the domain and range of,

$f(x) = |2x - 3| - 3$

39. Draw the graph of the Greatest Integer function

40. Draw the graph of the Constant function $f : R \to R$; $f(x) = 2\ \forall\ x \in R$. Also find its domain and range.

41. Draw the graph of the function $|x - 2|$

Find the domain and range of the following real functions (Question 42 to 47)

42. $f(x)=\sqrt{x^2+4}$

43. $f(x)=\dfrac{x+1}{x-2}$

44. $f(x)=\dfrac{|x+1|}{x+1}$

45. $f(x)=\dfrac{x^2-9}{x-3}$

46. $f(x)=\dfrac{4-x}{x-4}$

47. $f(x)=1-|x-3|$

48. Determine a quadratic function (f) is defined by f(x) = ax² + bx + c. If f(0) = 6; f(2) = 11, f(–3) = 6

49. Draw the graph of the function $f(x)=\begin{cases}1+2x & x<0\\ 3+5x & x\geq 0\end{cases}$ also find its range.

50. Draw the graph of following function

$$f(x)=\begin{cases}\dfrac{|x|}{x} & x\neq 0\\ 0 & x=0\end{cases}$$

Also find its range.

Find the domain of the following function.

51. $f(x)=\dfrac{1}{\sqrt{x+|x|}}$

52. $f(x)=\dfrac{1}{\sqrt{x-|x|}}$

53. $f(x)=\dfrac{1}{\sqrt{[x]^2-[x]-6}}$

54. $f(x)=\dfrac{1}{\sqrt{9-x^2}}$

55. $f(x)=\sqrt{4-x}+\dfrac{1}{\sqrt{x^2-1}}$

56. Find the domain for which the followings:
f(x) = $2x^2$ – 1 and g(x) = 1 – 3x are equal.

57. If $f(x)=x-\dfrac{1}{x}$ prove that $\left[f(x)\right]^3=f(x^3)+3f\left(\dfrac{1}{x}\right)$.

58. If [x] denotes the greatest integer function. Find the solution set of equation, $[x]^2 + 5[x] + 6 = 0$.

59. If $f(x) = \dfrac{ax - b}{bx - a} = y$. Find the value of f(y).

60. Draw the graph of following function and find range (R_f) of

$f(x) = |x - 2| + |2 + x| \quad \forall \quad -3 \le x \le 3$.

CASE STUDY TYPE QUESTIONS

61. To make himself self-dependent and to earn his living, a person decided to setup a small scale business of manufacturing hand sanitizers. He estimated a fixed cost of Rs. 15000 per month and a cost of Rs. 30 per unit to manufacture.

i. If x units of hand sanitizers are manufactured per month. What is the cost function?

(a) $15000 - 30x$ (b) $15000 + 30x$

(c) $15000 + x$ (d) $15000 + 31x$

ii. If each unit is sold for Rs. 45. What is the selling (revenue) function?

(a) $30x$ (b) $45 + x$ (c) $45x$ (d) $45 + 30x$

iii. What is the profit function?

(a) $15x + 15000$ (b) $15(x - 1000)$

(c) $15\,x$ (d) None of these

iv. For Break even (No Profit, no loss situation) in a month, how many units should be manufactured and sold?

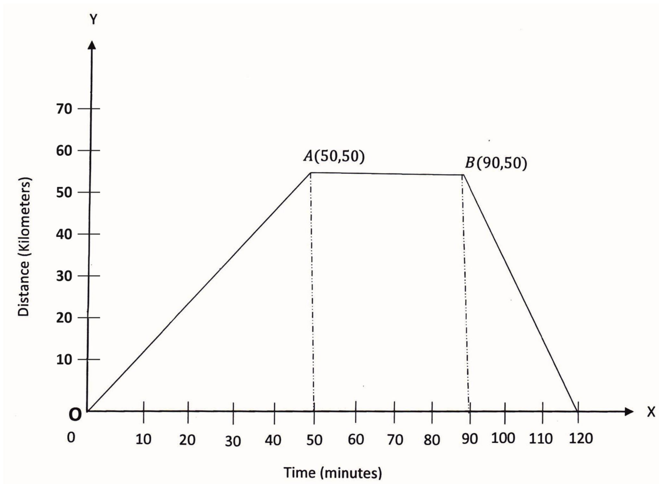
(a) 500 (b) 750 (c) 1000 (d) 1500

v. What is the monthly cost borne by the person if he decided to manufacture 1500 units in a month?

(a) 15000 (b) 30000 (c) 45000 (d) 60000

62. This is a graph showing how far the distances have been travelled by Sunita (in her car) in a given time.

She drove, stopped, does her work and returned back.



i. The line OA of the graph represents the function:

(a) $x = 10y$ (b) $y = 10x$ (c) $y = x$ (d) $y = \frac{x}{2}$

ii. At what distance from the starting point Sunita stopped to do her work?

(a) 30m (b) 40m (c) 50m (d) 60m

iii. How much time Sunita took to complete her work?

(a) 30 min (b) 40 min (c) 50 min (d) 60 min

iv. Line AB represents the constant function:

(a) $y = 50$ (b) $x = 50$ (c) $y = 10$ (d) $x = 9$

v. How much time Sunita took to reach at a distance of 40 km. from the initial point?

(a) 30 min (b) 40 min (c) 50 min (d) 1 hour

ANSWERS

1.	(b)	2.	(d)	3.	(c)	4.	(c)	5.	(a)
6.	(b)	7.	(c)	8.	(c)	9.	(d)	10.	(d)

11. $a = 3, b = -2$

12. $A \times B = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$

13. $B \times A = \{ (2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\}$

14. $\{(1,4), (2,4)\}$

15. $\{(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)\}$

16. $2^6 = 64$

17. $R = \{ (0,8), (0,-8), (8,0), (-8,0)\}$

18. Not a function because 4 has two images.

19. Not a function because 2 does not have a unique image.

20. Function because every element in the domain has its unique image.

21. 7^5

22. 0,1

23. $f+g=\begin{cases} 2x & x \geq 0 \\ 0 & x < 0 \end{cases}$

24. $f-g=\begin{cases} 0 & x \geq 0 \\ 2x & x < 0 \end{cases}$

25. $fg=\begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$

26. $\frac{f}{g}=\begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$ and Note:- $\frac{f}{g}$ is not defined at x = 0

27. 31

28. $(-\infty, -2] \cup [2, \infty)$

29. $R - \{2,3\}$

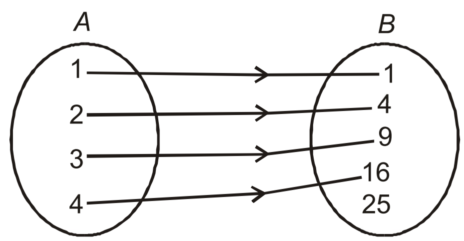
30. $(-\infty, 0) \cup [1/4, \infty)$

31. $[2,\infty)$

32. $\{-4, -2, -1, 1, 2, 4\}$

33. $\{2, 4, 6, 8\}$

34. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

35. (a) 

(b) $\{1, 2, 3, 4\}$

(c) $\{1, 4, 9, 16\}$

(d) $\{1, 4, 9, 16, 25\}$

36. R = { (2,4) (2,6) (2,18) (2,54) (6,18) (6,54) (9,18) (9,27) (9,54) }

Domain is R = {2,6,9}

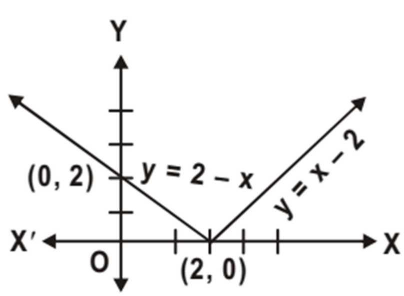
Range of R = { 4, 6, 18, 27, 54}

38. Domain is R

Range is $[-3, \infty)$

40. Domain = R, Range = {2}

41.



42. Domain = R,

Range = $[2, \infty)$

43. Domain = R – {2}

Range = R – {1}

44. Domain = R – {–1}

Range = {1, –1}

45. Domain = R – {3}

Range = R – {6}

46. Domain = R – {4}

Range = {–1}

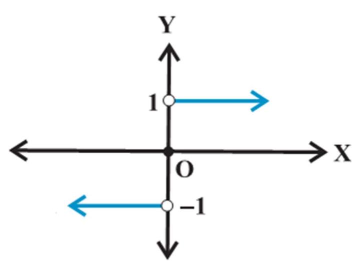
47. Domain = R

Range =$(-\infty, 1]$

48. $\frac{1}{2}x^2 + \frac{3}{2}x + 6$

49. $(-\infty, 1) \cup [3, \infty)$

50. Range of f = {–1,0,1}



51. $(0, \infty)$

52. ϕ (given function is not defined)

53. $(-\infty, -2) \cup (4, \infty)$

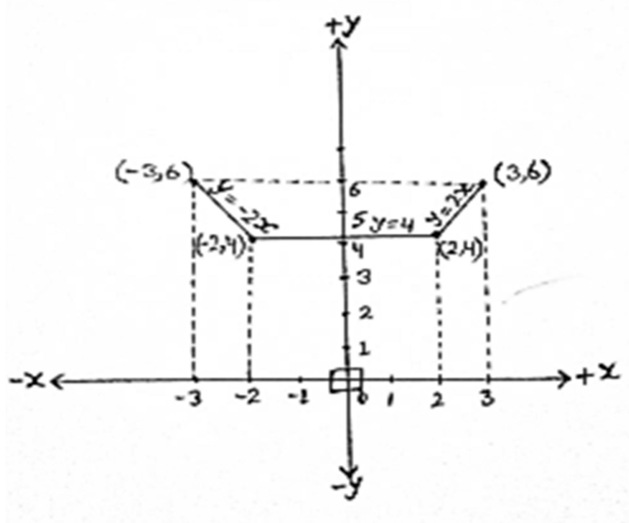
54. $(-3, 3)$

55. $(-\infty, -1) \cup (1, 4]$ 56. $\left\{-2, \frac{1}{2}\right\}$

58. $[-3, -1)$

59. x

60. $R_f = [4, 6]$ and graph is



61. i. (b) ii. (c) iii. (b) iv. (c) v. (d)

62. i. (c) ii. (c) iii. (b) iv. (a) v. (b)

CHAPTER - 3

TRIGONOMETRIC FUNCTIONS

KEY POINTS

- 1 radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

- π radian = 180 degree, $1^\circ = 60^\circ$

$$1 \text{ radian} = \left(\frac{180}{\pi}\right)^\circ = 57^\circ\ 16'\ 22'' \text{ (Appr.)}$$

- If an arc of length 'ℓ' makes an angle 'θ' radian at the centre of a circle of radius 'r', we have $\theta = \frac{\ell}{r}$.

-

Quadrant →	I	II	III	IV
t-functions which are postive	All	sinx cosecx	tanx cotx	cosx secx

-

Function	Domain	Range
sinx	R	[–1, 1]
cosx	R	[–1, 1]
tanx	$R - \left\{(2n+1)\frac{\pi}{2}\right\};\ n \in z$	R
cosecx	$R - \{n\pi\}$; $n \in z$	R – (–1, 1)
secx	$R - \left\{(2n+1)\frac{\pi}{2}\right\};\ n \in z$	R – (–1, 1)
cotx	$R - \{n\pi\}$; $n \in z$	R

- **Allied or related angles:** The angles $\frac{n\pi}{2} \pm \theta$ are called allied or related angles and $\theta \pm n \times 360°$ are called conterminal angles. For general reduction we have the following rules. The value of any trigonometric functions for $\left(\frac{n\pi}{2} \pm \theta\right)$ is numerically equal to :-

  (a) The value of the same function if n is even integer with algebraic sign of the function as per the quadrant in which angles pie (π).

  (b) Corresponding co-function of 'θ' if n is an odd integer algebraic sign of the function for the quadrant in which it lies. Here sine and cosine; tan and cot; sec and cosec, are co-functions of each other.

- **Trigonometric Identities:**

(i) sin (x + y) = sinx cosy + cosx siny

(ii) sin (x – y) = sinx cosy – cosx siny

(iii) cos (x + y) = cosx cosy – sinx siny

(iv) cos (x – y) = cosx cosy + sinx siny

(v) $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x . \tan y}$

(vi) $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x . \tan y}$

(vii) $\cot(x + y) = \frac{\cot x . \cot y - 1}{\cot y + \cot x}$

(viii) $\cot(x - y) = \frac{\cot x . \cot y + 1}{\cot y - \cot x}$

(ix) $\sin 2x = 2\sin x \cos x = \dfrac{2\tan x}{1+\tan^2 x}$

(x) $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

(xi) $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

(xii) $\sin 3x = 3\sin x - 4\sin^3 x$

(xiii) $\cos 3x = 4\cos^3 x - 3\cos x$

(xiv) $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

(xv) $\cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

(xvi) $\cos x - \cos y = 2\sin\dfrac{x+y}{2}\sin\dfrac{y-x}{2}$

(xvii) $\sin x + \sin y = 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

(xviii) $\sin x - \sin y = 2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

(xix) $2\sin x \cos y = \sin(x + y) + \sin(x - y)$

(xx) $2\cos x \sin y = \sin(x + y) - \sin(x - y)$

(xxi) $2\cos x \cos y = \cos(x + y) + \cos(x - y)$

(xxii) $2\sin x \sin y = \cos(x - y) - \cos(x + y)$

(xxiii) $\sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}$

(xxiv) $\cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}$

(xxv) $\tan\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}}$

sign '+' or '–' will be decided according to the quadrant in which angle $\dfrac{A}{2}$ lies.

- **Solution of trigonometric equations:**

  (a) **Principal Solutions:** The solutions of a trigonometric equation for which $0 \le \theta \le 2\pi$ are called principal solutions.

  (b) **General Solutions:** The expression involving integer 'n' which gives all solutions of a trigonometric equation is called the general solution.

- **General solution of trigonometric equations:**

  (i) If $\sin\theta = 0 \quad \Rightarrow \quad \theta = n\pi, \quad n \in z$

  (ii) If $\cos\theta = 0 \quad \Rightarrow \quad \theta = (2n+1)\dfrac{\pi}{2}, \quad n \in z$

  (iii) If $\tan\theta = 0 \quad \Rightarrow \quad \theta = n\pi, \quad n \in z$

  (iv) If $\sin\theta = \sin\alpha \quad \Rightarrow \quad \theta = n\pi + (-1)^n\alpha, \quad n \in z$

  (v) If $\cos\theta = \cos\alpha \quad \Rightarrow \quad \theta = 2n\pi \pm \alpha, \quad n \in z$

  (vi) If $\tan\theta = \tan\alpha \quad \Rightarrow \quad \theta = n\pi + \alpha, \quad n \in z$

  (vii) If $\sin^2\theta = \sin^2\alpha$, $\quad \Rightarrow \quad \theta = n\pi \pm \alpha, \quad n \in z$
  $\cos^2\theta = \cos^2\alpha$,
  $\tan^2\theta = \tan^2\alpha$

- Maximum and minimum values of the expression $A\cos\theta + B\sin\theta$ are $\sqrt{A^2 + B^2}$ and $-\sqrt{A^2 + B^2}$ respectively, where A and B are constants.

- $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$ $\qquad$ $\cos 18^\circ = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$

  $\sin 36^\circ = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$ $\qquad$ $\cos 36^\circ = \dfrac{\sqrt{5}+1}{4}$

VERY SHORT ANSWER TYPE QUESTIONS

1. Write the radian measure of 5° 37' 30".

2. Write the degree measure of $\frac{11}{16}$ radian.

3. Write the value of $\tan\left(\frac{19\pi}{3}\right)$.

4. What is the value of sin (–1125°).

5. Write the general solution of $\sin\left(x+\frac{\pi}{12}\right)=0$.

6. Write the value of 2sin75° sin15°.

7. What is the maximum value of 3 – 7 cos5x.

8. Express $\sin 12\theta + \sin 4\theta$ as the product of sines and cosines.

9. Express 2cos4x sin2x as an algebraic sum of sines and cosines.

10. Write the maximum value of cos (cosx) and also write its minimum value.

11. Write is the value of $\tan\frac{\pi}{12}$.

Choose the correct answer from the given four options in exercise 12 to 30.

12. If $\tan\theta = \frac{-4}{3}$, then $\sin\theta$ is -

(a) $\frac{-4}{5}$ but not $\frac{4}{5}$ (b) $\frac{-4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $\frac{-4}{5}$ (d) None of these.

13. The greatest value of sinx cosx is -

(a) 1 (b) 2

(c) $\sqrt{2}$ (d) $\frac{1}{2}$.

14. If $\sin\theta + \text{cosec}\theta = 2$, then $\sin^2\theta + \text{cosec}^2\theta$ is equal to -

(a) 1 (b) 4

(c) 2 (d) None of these.

15. If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$ then the value of $\theta + \phi$ is -

(a) $\frac{\pi}{6}$ (b) π

(c) 0 (d) $\frac{\pi}{4}$.

16. Which of the following is not correct -

(a) $\sin\theta = \frac{-1}{5}$ (b) $\cos\theta = 1$

(c) $\sec\theta = \frac{1}{2}$ (d) $\tan\theta = 20$.

17. The value of $\tan 1° \times \tan 2° \times \tan 3° \ldots\ldots\ldots \tan 89°$ is -

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) Not defined.

18. The value of $\cos 1° \times \cos 2° \times \cos 3° \ldots\ldots\ldots \cos 179°$ is -

(a) $\frac{1}{\sqrt{2}}$ (b) 0

(c) 1 (d) –1.

19. The value of $\frac{1-\tan^2 15°}{1+\tan^2 15°}$ is -

(a) 1 (b) $\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) 2.

20. The minimum value of 3cosx + 4sinx + 8 is -

(a) 5 (b) 9

(c) 7 (d) 3.

21. The value of sin50° – sin70° + sin10° is equal to -

(a) 1 (b) 0

(c) $\frac{1}{2}$ (d) 2.

22. If $\sin\theta + \cos\theta = 1$, then the value of $\sin 2\theta$ is equal to -

(a) 1 (b) $\frac{1}{2}$

(c) 0 (d) 2.

23. If $\alpha + \beta = \frac{\pi}{4}$, then value of $(1 + \tan\alpha) . (1 + \tan\beta)$ is -

(a) 1 (b) 2

(c) –2 (d) Not defined.

24. The value of $\cos^2 48° - \sin^2 12°$ is -

(a) $\frac{\sqrt{5}+1}{8}$ (b) $\frac{\sqrt{5}-1}{8}$

(c) $\frac{\sqrt{5}+1}{5}$ (d) $\frac{\sqrt{5}+1}{2\sqrt{2}}$.

25. The number of solutions of the equation $4\sin x - 3\cos x = 7$ are -

(a) 0 (b) 1

(c) 2 (d) 3.

26. If $\cos x = \frac{1}{2}\left(a+\frac{1}{a}\right)$, then $\cos 3x$ is -

(a) $\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$ (b) $\frac{3}{2}\left(a^3+\frac{1}{a^3}\right)$

(c) $\frac{1}{2}\left(a^3-\frac{1}{a^3}\right)$ (d) $\frac{3}{2}\left(a^3-\frac{1}{a^3}\right)$.

27. If $\cos x = \frac{-1}{2}$ and $0 < x < 2\pi$, then solutions are -

(a) $x=\frac{\pi}{3}, \frac{4\pi}{3}$ (b) $x=\frac{2\pi}{3}, \frac{4\pi}{3}$

(c) $x=\frac{2\pi}{3}, \frac{7\pi}{6}$ (d) $x=\frac{2\pi}{3}, \frac{\pi}{3}$.

28. If $P = 2\sin^2 x - \cos^2 x$, then P lies in the interval -

(a) [1, 3] (b) [1, 2]

(c) [–1, 2] (d) None of these.

29. If $\frac{\pi}{4} < x < \frac{\pi}{2}$, then write the value of $\sqrt{1-\sin 2x}$ is -

(a) $\cos x - \sin x$ (b) $\cos x + \sin x$

(c) $\sin x - \cos x$ (d) 2.

30. If $\sin x + \cos x = a$, then the value of $|\sin x - \cos x|$ is -

(a) $\sqrt{2-a^2}$ (b) $\sqrt{a^2-2}$

(c) $\sqrt{a^2+2}$ (d) 1.

Fill in the blanks (Exercise 31 to 35] :-

31. The value of $\frac{\sin 50°}{\sin 130°}$ is _________.

32. If $\tan A = \frac{1-\cos B}{\sin B}$, $\tan 2A$ = _________.

33. If $3\sin x + 4\cos x = 5$, then $4\sin x - 3\cos x$ is _________.

34. If $\cos(A - B) = \frac{3}{5}$ and $\tan A \tan B = 2$, then the value of $\cos A \cos B$ is _________.

35. If $A + B = \frac{\pi}{3}$ and $\cos A + \cos B = 1$, then the value of $\cos\left(\frac{A-B}{2}\right)$ is ________.

36. If the following match each item given under column C_1 to its correct answer given under column C_2 :-

C_1	C_2
(a) $\sin(x + y)\sin(x - y)$	(i) $\cos^2 x - \sin^2 y$
(b) $\cos(x + y)\cos(x - y)$	(ii) $\frac{1-\tan x}{1+\tan x}$
(c) $\cot\left(\frac{\pi}{4}+x\right)$	(iii) $\frac{1+\tan x}{1-\tan x}$
(d) $\tan\left(\frac{\pi}{4}+x\right)$	(iv) $\sin^2 x - \sin^2 y$

37. Match each item given under column C_1 to its correct answer given under column C_2 :-

	C_1		C_2
(a)	$\frac{1-\cos x}{\sin x}$	(i)	$\cot^2 \frac{x}{2}$
(b)	$\frac{1+\cos x}{1-\cos x}$	(ii)	$\cot \frac{x}{2}$
(c)	$\frac{1+\cos x}{\sin x}$	(iii)	$\lvert\cos x + \sin x\rvert$
(d)	$\sqrt{1+\sin 2x}$	(iv)	$\sin^2 x - \sin^2 y$

The statements given are true or false (Exercise 38 to 45] :-

38. If $0 \le x \le \pi$ then $\cos 0 \le \cos x \le \cos\pi$.

39. If $0 \le x \le \frac{\pi}{2}$ then $\sin 0 \le \sin x \le \sin\frac{\pi}{2}$.

40. If $\pi \le x \le \frac{3\pi}{2}$ then $\sin\pi \le \sin x \le \sin\frac{3\pi}{2}$.

41. If $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$ then tanx is an increasing function.

42. The period of sinx function is '2π'

43. The period of cosx function is '2π'

44. The period of tanx function is '2π'

45. The range of $f(x) = \sec x$ is $R - [-1, 1]$.

SHORT ANSWER TYPE QUESTIONS

46. Find the length of an arc of a circle of radius 5cm subtending a central angle measuring 15°.

47. If $\sin A = \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$ Find $\cos A$, $\sin 2A$.

48. What is the sign of cos x/2 – sin x/2 when

    (i) $0 < x < \pi/4$ (ii) $\frac{\pi}{2} < x < \pi$

49. Prove that cos 510° cos 330° + sin 390° cos 120° = –1.

50. Find the maximum and minimum value of $7 \cos x + 24 \sin x$.

51. Evaluate $\sin(\pi + x) \sin(\pi - x) \operatorname{cosec}^2 x$.

52. Find the angle in radians between the hands sof a clock at 7 : 20 PM.

53. If $\cot\alpha = \frac{1}{2}$ $\sec\beta = \frac{-5}{3}$ where $\pi < \alpha < 3\pi/2$ and $\frac{\pi}{2} < \beta < \pi$. Find the value of $\tan(\alpha + \beta)$.

54. If $\cos x = \frac{-1}{3}$ and $\pi < x < \frac{3\pi}{2}$. Find the value of $\cos x/2$, $\tan x/2$

55. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$ then find the value of $A + B$

SHORT ANSWER TYPE QUESTIONS

56. A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces 72° at the centre, find the length of the rope.

57. Find the minimum and maximum value of $\sin^4 x + \cos^2 x$; $x \in R$

58. Solve $\sec x . \cos 5x + 1 = 0$

59. Solve $2\tan^2 x + \sec^2 x = 2$ for $0 < x^2\pi$.

60. Solve $\sqrt{3}\cos x - \sin x = 1$.

61. Solve $\sqrt{2}\sec\theta - \tan\theta = \sqrt{3}$.

62. Solve 3 tan x + cot x = 5 cosec x.

63. Find x if 3 tan (x – 15°) = tan (x + 15°)

64. Solve $\tan x + \tan 2x + \sqrt{3}\tan x.\tan 2x = \sqrt{3}$.

65. Solve $\tan x + \sec x = \sqrt{3}$.

66. If $\sec x = \sqrt{2}$ and $\frac{3\pi}{2} < x < 2\pi$, find the value of $\frac{1 - \tan x - \text{cosec } x}{1 - \cot x - \text{cosec } x}$

67. Prove that sin 10° sin 30° sin 50° sin 70° = $\frac{1}{16}$.

68. If $f(x) = \frac{\cot x}{1 + \cot x}$ and $\alpha + \beta = \frac{5\pi}{4}$ then find $f(\alpha).f(\beta)$.

69. Prove that tan 70° = tan 20° + 2 tan 50°

70. Prove that tan 13x = tan 4x + tan 9x + tan 4x tan 9x tan 13x.

Prove the following Identities

71. $\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta.\cos 4\theta$.

72. $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$.

73. $\frac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x.\sin x + \cos 6x.\cos x} = \tan 2x$.

74. $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=\tan\dfrac{\theta}{2}$.

75. $\tan\alpha.\ \tan(60°-\alpha).\ \tan(60°+\alpha)=\tan 3\alpha$.

76. $\sqrt{2+\sqrt{2+2\cos 4\theta}}=2\cos\theta$.

77. $\dfrac{\cos x}{1-\sin x}=\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$.

78. $\cos 10° + \cos 110° + \cos 130° = 0$.

79. $\dfrac{\sin(x+y)-2\sin x+\sin(x-y)}{\cos(x+y)-2\cos x+\cos(x-y)}=\tan x$

80. $\sin x+\sin 2x+\sin 4x+\sin 5x=4\cos\dfrac{x}{2}.\cos\dfrac{3x}{2}.\sin 3x$

81. $\dfrac{\sec 8\theta-1}{\sec 4\theta-1}=\dfrac{\tan 8\theta}{\tan 2\theta}$

82. Find the value of $\sqrt{3}$ cosec $20° - \sec 20°$

83. $\cos\dfrac{\pi}{5}.\cos\dfrac{2\pi}{5}.\cos\dfrac{4\pi}{5}.\cos\dfrac{8\pi}{5}=\dfrac{1}{16}$

84. $\cos 20°.\cos 40°.\cos 80°=\dfrac{1}{8}$

Find the general solution of the following equations (Q.No. 85 to Q.No. 87)

85. $\sin 7x = \sin 3x$.

86. $\cos 3x - \sin 2x = 0$.

87. $\sin x-3\sin 2x+\sin 3x=\cos x-3\cos 2x+\cos 3x$.

88. Draw the graph of cos$x$, sin$x$ and tan$x$ in $[0, 2\pi]$.

89. Draw sin$x$, sin$2x$ and sin$3x$ on same graph and with same scale.

90. Evaluate:

(i) $\cos 36°$ (ii) $\tan\left(\frac{13\pi}{12}\right)$

91. Evaluate:

$$\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\left(\frac{5\pi}{8}\right)+\cos^4\left(\frac{7\pi}{8}\right)$$

92. If $\tan A - \tan B = x$, $\cot B - \cot A = y$ prove that $\cot(A - B) = \frac{1}{x}+\frac{1}{y}$

93. If $\frac{\sin(x+y)}{\sin(x-y)}=\frac{a+b}{a-b}$ then prove that $\frac{\tan x}{\tan y}=\frac{a}{b}$.

94. If $\cos x=\cos\alpha.\cos\beta$ then prove that $\tan\left(\frac{x+\alpha}{2}\right).\tan\left(\frac{x-\alpha}{2}\right)=\tan^2\frac{\beta}{2}$

95. If $\tan(\pi\cos\theta)=\cot(\pi\sin\theta)$ then prove that $\cos\left(\theta-\frac{\pi}{4}\right)=\pm\frac{1}{2\sqrt{2}}$.

96. If $\sin(\theta+\alpha)=a$ and $\sin(\theta+\beta)=b$ then prove that

$$\cos 2(\alpha-\beta)-4ab\cos(\alpha-\beta)=1-2a^2-2b^2$$

97. Find the range of $5\sin x - 12\cos x + 7$.

98. If α and β are the solution of the equation, $a\tan\theta+b\sec\theta=c$ then show that $\tan(\alpha+\beta)=\frac{2ac}{a^2-c^2}$.

99. Prove that

$$\cos^2 x+\cos^2 y-2\cos x.\cos y.\cos(x+y)=\sin^2(x+y)$$

100. Prove that :

$$2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) = \cos 2\alpha$$

101. Solve : $81^{\sin^2 x} + 81^{\cos^2 x} = 30 \quad 0 < x < \pi$

102. Find the minimum value of p for which $\cos(p\sin x) = \sin(p\cos x)$ has a solution in $[0,\ 2\pi]$.

103. Prove that : $\cos A \cos 2A \cos 4A \cos 8A = \dfrac{\sin 16A}{16.\sin A}$.

104. Solve: $4\sin x.\ \sin 2x.\ \sin 4x = \sin 3x$

105. Solve: $\cos\theta \cos 2\theta \cos 3\theta = \dfrac{1}{4}$

106. Evaluate: $\left(1+\cos\dfrac{\pi}{8}\right)\left(1+\cos\dfrac{3\pi}{8}\right)\left(1+\cos\dfrac{5\pi}{8}\right)\left(1+\cos\dfrac{7\pi}{8}\right)$

107. Prove that : $4\sin\alpha.\sin\left(\alpha+\dfrac{\pi}{3}\right).\sin\left(\alpha+\dfrac{2\pi}{3}\right) = \sin 3\alpha$.

CASE STUDY TYPE QUESTIONS

108. After retirement, Mr. D. N. Sharma purchased a farm house in shape of quadrilateral ABCD with $\angle A = 90°$, $\angle B = 72°$, $\angle C = 108°$ and $\angle D = 90°$. He also purchased a horse and cow. One day, he tied the horse with a rope at vertex B and oserved that it describes an arc of length 88 m when it moves along a circular path keeping the rope tight.

Based on above information answer the following :-

i. What is radian measure of $\angle B$?

(a) $2\pi/5$ (b) $3\pi/5$ (c) $\pi/5$ (d) $3\pi/10$

ii. What is length of rope?

(a) 50 m (b) 60 m (c) 70 m (d) 80 m

iii. What will be the length of arc described by horse if he doubles the rope length?

(a) 44 m (b) 176 m (c) 132 m (d) 156 m

iv. What will be the length of arc described by cow if it is tied at vertex c with the rope of same length as horse?

(a) 156 m (b) 132 m (c) 144 m (d) 176 m

v. What is the ratio of area that horse can cover to that of cow with same length of rope?

(a) 1 : 1 (b) 3 : 2 (c) 2 : 3 (d) 2 : 5

109. While playing with this nephew Shashank, Mr. V. S. Malik observes a vertical pole in park. A wire is tied from top of pole to a point on ground level. Mr. Malik asks Shashank some mathematics related questions. Mr. Shashank is Class-XI student and very intelligent in Maths. Using some tools he measure the distance of point at ground where wire is tied as 10 m. and angle between wire and ground level as 75°.

Based on above information answer the following :-

i. What is the value of tan75° ?

(a) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (c) $\frac{\sqrt{3}}{\sqrt{3}+1}$ (d) $\frac{\sqrt{3}}{\sqrt{3}-1}$

ii. What is the height of pole?

(a) $10\left(\sqrt{3}+1\right)$ (b) $10\left(\sqrt{3}-1\right)$

(c) $10\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (d) $10\frac{\sqrt{3}-1}{\sqrt{3}+1}$

iii. What is the value of sin75° ?

(a) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

iv. What is the length of wire?

(a) $10\sqrt{2}\left(\sqrt{3}+1\right)$ (b) $10\left(\sqrt{3}+1\right)$

(c) $10\sqrt{2}\left(\sqrt{3}-1\right)$ (d) $10\left(\sqrt{3}-1\right)$

iii. What is the value of sin105° ?

(a) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

ANSWERS

1. $\frac{\pi}{32}$
2. 39° 22’ 30”
3. $\sqrt{3}$
4. $-\frac{1}{\sqrt{2}}$
5. $n\pi-\frac{\pi}{12}, \quad n \in z.$
6. $\frac{1}{2}$
7. 10
8. 2 sin8θ cos4θ
9. $\sin 6x-\sin 2x$
10. 1 and –1
11. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$
12. (b)
13. (d)
14. (c)
15. (d)
16. (c)
17. (b)

18. (b)

19. (c)

20. (d)

21. (b)

22. (c)

23. (b)

24. (a)

25. (a)

26. (a)

27. (b)

28. (c)

29. (c)

30. (a)

31. 1

32. tanB

33. 0

34. $\frac{1}{5}$

35. $\frac{1}{\sqrt{3}}$

36. (a) $\rightarrow$ (iv)
    (b) $\rightarrow$ (i)
    (c) $\rightarrow$ (ii)
    (d) $\rightarrow$ (iii)

37. (a) $\rightarrow$ (iv)
    (b) $\rightarrow$ (i)
    (c) $\rightarrow$ (ii)
    (d) $\rightarrow$ (ii)

38. False

39. True

40. False

41. True

42. True

43. True

44. False

45. False

46. 70m

47. $\frac{-4}{5}, \frac{-24}{25}$

48. (i) +ve (ii) –ve

50. Max value 25;
    Min value –25

51. –1

52. $\frac{5\pi}{9}$

53. $\frac{2}{11}$

54. $-1/\sqrt{3}, -2$

55. $\pi/4$

56. 70 m

57. $\min = \frac{3}{4}, \ \max = 1$

58. $x = (2n+1)\frac{\pi}{6}$, or $x = (2n+1)\frac{\pi}{4}, n \in z$

59. $x = \frac{\pi}{6}, \ \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

60. $2n\pi \pm \frac{\pi}{3} - \frac{\pi}{6}, \ n \in z$

61. $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}, \ n \in z$

62. $2n\pi \pm \frac{\pi}{3}, \ n \in z$

63. $x = \frac{n\pi}{2} + (-1)^4 \frac{\pi}{4}, \quad n \in z$

64. $\frac{n\pi}{3} + \frac{\pi}{9}$

65. $2n\pi \pm \frac{2\pi}{3} - \frac{2\pi}{6} - \frac{\pi}{6}, n \in z$

66. 1

68. $\frac{1}{2}$

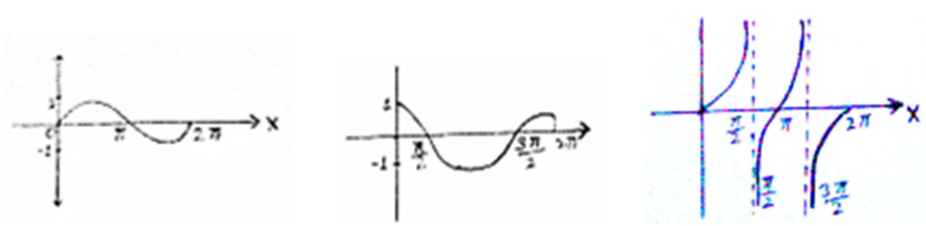
82. 4

85. $(2n+1)\frac{\pi}{10}, \frac{n\pi}{2}, \ n \in z$

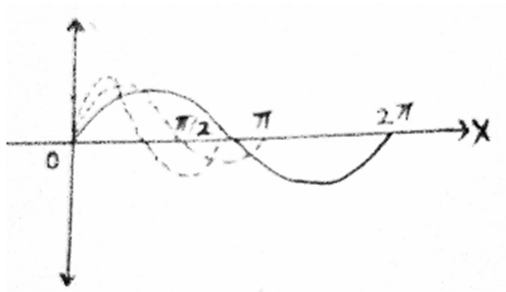
86. $\frac{1}{5}(2n\pi + \frac{\pi}{2}), 2n\pi - \frac{\pi}{2}, \quad n \in z$

87. $x = \frac{n\pi}{2}, \quad n \in z$

88.



89.



90. (i) $\frac{1+\sqrt{5}}{4}$ (ii) $2-\sqrt{3}$

91. $\frac{3}{2}$

97. [–6, 20]

101. $x=\frac{\pi}{6}, \frac{5\pi}{6}$

102. $\frac{\pi}{\sqrt{8}}, \frac{5\sqrt{2}\pi}{4}$

104. $n\pi,\ n\pi \pm \frac{\pi}{3},\ n \in z$

105. $(2n+1)\frac{\pi}{8},\ n\pi \pm \frac{\pi}{3},\ n \in z$

106. $\frac{1}{8}$

108. i. (a) ii. (c) iii. (b) iv. (c) v. (c)

109. i. (b) ii. (c) iii. (c) iv. (a) v. (c)

CHAPTER - 4

PRINCIPLE OF MATHEMATICAL INDUCTION

KEY POINTS

- A meaningful sentence which can be judged to be either true or false is called a statement.
- A statement involving mathematical relations is called as mathematical statement.
- Induction and deduction are two basic processes of reasoning.
- Deduction is the application of a general case to a particular case. In contrast to deduction, induction is process of reasoning from particular to general.
- Induction being with observations. From observations we arrive at tentative conclusions called conjectures. The processes of induction help in proving the conjectures which may be true.
- Statements like

  (i) $1 + 2 + 3 + \ldots\ldots\ldots + n = \dfrac{n(n+1)}{2} \quad \forall\ n \in N.$

  (ii) $2^n \leq 2 \ \forall\ n \in N.$

  (iii) If n(A) = n then number of all subsets of A = $2^n \ \forall\ n \in N.$

(iv) $Sn = \frac{a(r^n - 1)}{r - 1}$ where Sn is sum of n terms of G.P, a = 1st term and r = common ratio. Are all concerned with $n \in N$ which takes values 1, 2, 3, …….. Such statements are denoted by P(n). By giving particular values to 'n', we get particular statement as P(1), P(2), …….. P(k) for some $k \in N$.

Principle of mathematical Induction:

Let P(n) be any statement involving natural number n such that

(i) P(1) is true, and

(ii) If P(k) is true $\Rightarrow$ P(k + 1) is true for some $k \in N$. that is P(K + 1) is true whenever P(K) is true for some $k \in N$ then P(n) is true $\forall\, n \in N$.

VERY SHORT ANSWER TYPE QUESTIONS

1. Let P(n): $n^2 + n$ is even. Is P(1) true?

2. Let P(n): n(n+1) (n+2) is divisible by 3. What is P(3)?

3. Let P(n): $n^2 > 9$. Is P(2) true?

4. If $10^n + 3.4^{n+2} + K$ is divisible by 9 for all $n \in N$, then the least positive integral value of K is –

   (a) 5 (b) 3 (c) 7 (d) 1

5. For all $n \in N$, $3.5^{2n+1} + 3^{2n+1}$ is divisible by –

   (a) 19 (b) 17 (c) 23 (d) 25

6. If $x^n - 1$ is divisible by x – k, then least positive integral value of K is –

   (a) 1 (b) 2 (c) 3 (d) 4

7. State the following statement is true or false –

"Let P(n) be a statement and let P(k) $\Rightarrow$ P(k + 1), for some natural K, then P(n) is true for all n $\in$ N".

SHORT ANSWER TYPE QUESTIONS

8. Give an example of a statement such that P(3) is true but P(4) is not true.

9. If P(n) : 1 + 4 + 7+ ……... + (3n – 2) = $\frac{1}{2}$n (3n – 1). Verify P(n) for n = 1, 2.

10. If P(n) is the statement "$n^2 - n + 41$ is Prime" Prove that P(1) and P(2) are the but P(41) is not true.

SHORT ANSWER TYPE QUESTIONS

Prove the following by using the principle of mathematical induction $\forall$ n $\in$N. (For Q.11 – Q.32)

Type-1

11. 3.6 + 6.9 + 9.12 + ... + 3n (3n + 3) = 3n(n + 1)(n + 2)

12. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\ldots\left(1-\frac{1}{n+1}\right)=\frac{1}{n+1}$

13. a + (a + d) + (a + 2d) + ... + [a +(n –1) d] = $\frac{n}{2}[2a+(n-1)d]$

HOTS

14. 7 + 77 + 777 + ... + to *n* terms = $\frac{7}{81}(10^{n+1}-9n-10)$

15. $\sin x + \sin 2x + \sin 3x + ...\sin nx = \dfrac{\sin\left(\dfrac{n+1}{2}x\right)\sin\dfrac{nx}{2}}{\sin\dfrac{x}{2}}$

16. $\sin x + \sin 3x + ...\sin(2n-1)x = \dfrac{\sin^2 nx}{\sin x}$

17. $\cos\alpha.\cos 2\alpha.\cos 4\alpha..........\cos\left(2^{n-1}\alpha\right) = \dfrac{\sin 2^n\alpha}{2^n \sin\alpha}$

18. $1^2 + 2^2 + 3^2........ n^2 = \dfrac{n(n+1)(2n+1)}{6}$

Type II

19. $2^{3n} - 1$ is divisible by 7.

20. 3^{2n} when divided by 8 leaves the remainder 1.

21. $4^n + 15n - 1$ is divisible by 9.

HOTS

22. $n^3 + (n + 1)^3 + (n + 2)^3$ is a multiple of 9

23. $11^{n+2} + 12^{2n+1}$ is divisible by 133

24. $x^n - y^n$ is divisible by (x–y) if x and y are any two distinct integers.

25. Given that $5^n - 5$ is divisible by 4 $\forall n \in N$. Prove that $2.7^n + 3.5^n - 5$ is a multiple of 24.

26. $7^{2n} + 2^{3n-3}.3^{n-1}$ is divisible by 25.

Type III

27. $2^{n+1} > 2n+1$

28. $3^n > 2^n$

29. $n < 2^n$

HOTS

30. $1+\frac{1}{4}+\frac{1}{9}+.......+\frac{1}{n^2} < 2-\frac{1}{n}$.

31. $(1+x)^n \geq 1+nx \;\; where \; x > -1$.

32. $2^{n+3} \leq (n+3)!$

CASE STUDY TYPE QUESTIONS

33. A student was asked to prove a statement P(n) by induction. He proved that P(k + 1) is true, whenever P(k) is true for all k > 5 where k∈N and also prove that P(5) is true. On the basis of this-

    i. He conclude that P(n) is true

    (a) for all n∈N (b) for all n > 5

    (c) for all n $\geq$ 5 (d) for all n < 5

    ii. The word induction means –

    (a) Normalization from particular cases or facts

    (b) Generalization from particular cases or facts

    (c) Both (a) and (b)

    (d) None of these

    iii. Application of a general case to a particular case is called

    (a) induction (b) deduction

    (c) formation (d) nothing can be said

iv. If he prove the statement for P(3) instead of P(5). Then P(n) is true for

(a) for all $n \in N$ (b) for all $n \geq 3$

(c) for all $n \leq 3$ (d) for all $n > 3$

v. If P(n) is a statement ($n \in N$) such that P(k) is true $\Rightarrow$ P(k+ 1) is true for $k \in N$, then P(n) is true for

(a) for all $n \in N$ (b) for all $n > 1$

(c) for all $n > 2$ (d) nothing can be said

34. A person trying to prove the statement P(n) : $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \in N$ using induction. Help him to find.

i. P(1) + P(2)

(a) 704 (b) 720

(c) 768 (d) 776

ii. The statement P(n) is true

(a) for all $n \in N$ (b) for all $n > 1$

(c) for all $n > 2$ (d) nothing can be said

iii. To prove the statement the person

(a) prove P(k+1) and assume P(k) is true

(b) prove P(k) and assume P(k+ 1) is true

(c) prove the statement for n = 1 to 10

(d) prove P(k+1) and assume P(k–1) is true

iv. If the person encounters another statement $10^n + 3.4^{n+2} + K$ is divisible by 9 for all $n \in N$ then the positive integral value of k is

(a) 5 (b) 3

(c) 7 (d) 1

v. Also find out for all $n \in N$, $3.5^{2n+1} + 2^{3n+1}$ is divisible by

(a) 19 (b) 17

(c) 23 (d) 25

ANSWER

1. True
2. P(3) : 3 (3 + 1) (3 + 2) is divisible by 3
3. NO.
4. (a)
5. (b)
6. (a)
7. True
8. P(n) : $3n^2 + n$ is divisible by 3 and soon
9. P(1) and P(2) are true.

33. i. (c) ii. (b) iii. (b) iv. (b) v. (d)

34. i. (c) ii. (a) iii. (a) iv. (a) v. (b)

CHAPTER - 5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

KEY POINTS

- The imaginary number $\sqrt{-1} = i$, is called iota
- For any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$
- $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$ if both a and b are negative real numbers
- A number of the form z = a + ib, where a, b $\in$ R is called a complex number.

  a is called the real part of z, denoted by Re(z) and b is called the imaginary part of z, denoted by Im(z)
- a + ib = c + id if a = c, and b = d
- $z_1 = a + ib$, $z_2 = c + id$.

  In general, we cannot compare and say that $z_1 > z_2$ or $z_1 < z_2$ but if b, d = 0 and a > c then $z_1 > z_2$

  i.e. we can compare two complex numbers only if they are purely real.
- 0 + *i* 0 is additive identity of a complex number.
- –z = –a –ib is called the Additive Inverse or negative of z = a + ib
- 1 + i 0 is multiplicative identity of complex number.

- $\bar{z} = a - ib$ is called the conjugate of z = a + ib

- $i^0 = 1$

- $z^{-1} = \frac{1}{z} = \frac{a - ib}{a^2 + b^2} = \frac{\bar{z}}{|z|^2}$ is called the multiplicative Inverse of

  z = a + ib (a $\neq$ 0, b $\neq$ 0)

- The coordinate plane that represents the complex numbers is called the complex plane or the Argand plane

- Polar form of z = a + ib is,

  z = r (cosθ + i sinθ) where $r = \sqrt{a^2 + b^2} = |z|$ is called the modulus of z,θ is called the argument or amplitude of z.

- The value of θ such that, $-\pi < \theta \leq \pi$ is called the principle argument of z.

- Z = x + iy, x > 0 and y > 0 the argument of z is acute angle given by $\tan\alpha = \frac{y}{x}$

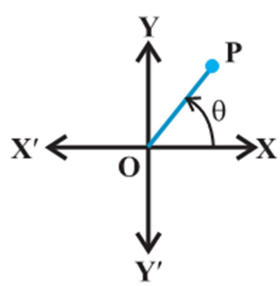
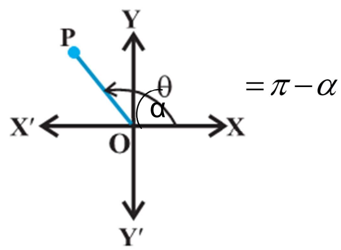


figure (i)

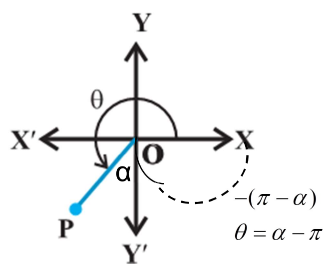
- $Z = x + iy$, $x < 0$ and $y > 0$ the argument of z is $\pi - \alpha$, where α is acute angle given by $\tan\alpha = \left|\frac{y}{x}\right|$



$\theta = \pi - \alpha$

figure (ii)

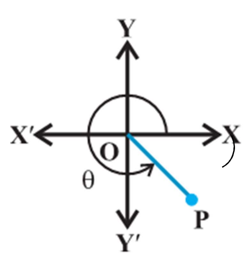
- $Z = x + iy$, $x < 0$ and $y < 0$ the argument of z is $\alpha - \pi$, where π is acute angle given by $\tan\alpha = \left|\frac{y}{x}\right|$



$\theta = \alpha - \pi$

figure (iii)

- $Z = x + iy$, $x > 0$ and $y < 0$ the argument of z is $-\alpha$, where α is acute angle given by $\tan\alpha = \left|\frac{y}{x}\right|$



$\theta = -\alpha$

figure (iv)

- $|z_1 + z_2| \le |z_1| + |z_2|$
- $|z_1 z_2| = |z_1| . |z_2|$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$; $|z^n| = |z|^n$; $|z| = |\bar{z}| = |-z| = |-\bar{z}|$; $z\bar{z} = |z|^2$
- $|z_1 - z_2| \ge |z_1| - |z_2|$
- If $z_1 = r_1 (\cos \theta_1 + i\sin \theta_1)$

  $z_2 = r_2 (\cos \theta_2 + i\sin \theta_2)$

  then $z_1 z_2 = r_1 r_2[\cos (\theta_1 + \theta_2) + i\sin (\theta_1 + \theta_2)]$

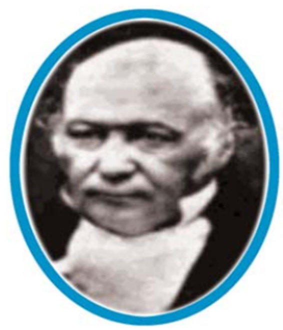
  $$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

- For the quadratic equation $ax^2 + bx + c = 0$,

  $a, b, c \in R$, $a \ne 0$, if $b^2 - 4ac < 0$

  then it will have complex roots given by,

  $$x = \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$



W. R. Hamilton
(1805-1865)

- $\sqrt{a + ib}$ is called square root of $z = a + ib$, $\therefore \sqrt{a + ib} = x + iy$

  squaring both sides we get $a + ib = x^2 - y^2 + 2i(xy)$

  $x^2 - y^2 = a$, $2xy = b$. Solving these we get x and y.

VERY SHORT ANSWER TYPE QUESTIONS

1. Write the value of $i + i^{10} + i^{20} + i^{30}$

2. Write the additive Inverse of $6i - i\sqrt{-49}$

3. Write the multiplicative Inverse of $1 + 4\sqrt{3}\ i$

4. Write the conjugate of $\dfrac{2-i}{(1-2i)^2}$

5. Write the amplitude of $\dfrac{1}{i}$

6. Write the Argument of $(1+\sqrt{3}i)(\cos\theta + i\sin\theta)$

7. Write in the form of $a+ib$: $\dfrac{1}{-2+\sqrt{-3}}$

8. Write the argument of $-i$

9. Write the value of arg(z) + arg$\left(\overline{z}\right)$

10. Multiply $2-3\,i$ by its conjugate.

11. If $\sqrt{7-24i} = x+iy$ and $x = \pm 4,\ y = \pm 3$ then $\sqrt{7-24i}$ = ?

12. What is the least integral value of K which makes the roots of the equation $x^2 + 5x + k = 0$ imaginary?

Fill in the blanks (Exercise 13 to 17) :-

13. The real value of 'a' for which $3i^3 - 2ai^2 + (1-a)i$ is real is ____.

14. If |z| = 2 and arg(z) = $\dfrac{\pi}{4}$, then z = ______.

15. The value of $\left(-\sqrt{-1}\right)^{4n-3}$, when n $\in$ N, is _____.

16. If a complex number lies in the third quadrant, then its conjugate lies in the _______ quadrant.

17. The value of $\sqrt{-25} \times \sqrt{-9}$ is ______.

State true or false for the following statements (Exercise 18 to 22) :-

18. The order relation is defined on the set of complex number

19. Multiplication of a non-zero complex number by $-i$ rotates the point about origin through a right angle n anti-clock wise direction.

20. z is not a complex number.

21. The complex number $\cos\theta + i\sin\theta$ can be zero for some 'θ'.

22. The argument of the complex number $z = (1+i\sqrt{3})(1+i)$ $(\cos\theta + i\sin\theta)$ is $\dfrac{7\pi}{12} + \theta$.

23. Match the following statements of column A and B

	A		B
(a)	The polar form of $i + \sqrt{3}$ is	(i)	Purely real complex number
(b)	The amplitude of $-1 + \sqrt{-3}$ is	(ii)	Forth quadrant
(c)	Reciprocal of $1 - i$ lies in	(iii)	First quadrant
(d)	Conjugate of $1 + i$ lies in	(iv)	$2\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$
(e)	The value of $1 + i^2 + i^4 + i^6 + ... + i^{20}$ is	(v)	$\dfrac{2\pi}{3}$

VERY SHORT ANSWER TYPE QUESTIONS

24. Evaluate :

(i) $\sqrt{-16}+3\sqrt{-25}+\sqrt{-36}-\sqrt{-625}$

(ii) $i\sqrt{-16}+i\sqrt{-25}+\sqrt{49}-i\sqrt{-49}+14$

(iii) $\left(i^{77}+i^{70}+i^{87}+i^{414}\right)^{3}$

(iv) $\dfrac{(3+\sqrt{5i})(3-\sqrt{5}i)}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)}$

25. Find x and y if (x + iy) (2 – 3i) = 4 + i.

26. If n is any positive integer, write value of $\dfrac{i^{4n+1}-i^{4n-1}}{2}$

27. If $z_1 = \sqrt{2}(\cos 30° + i \sin 30°)$, $z_2 = \sqrt{3}(\cos 60 + i \sin 30°)$

Find $R_e\ (z_1z_2)$

28. If $\left|z+4\right|\leq 3$ then find the greatest and least values of $\left|z+1\right|$.

29. Find the real value of a for which $3i^3-2ai^2+(1-a)i+5$ is real.

30. If $\arg(z-1)=\arg(z+3i)$ where $z=x+iy$ find $x-1:y$.

31. If z = x + iy and the amplitude of $(z-2-3i)$ is $\dfrac{\pi}{4}$. Find the relation between x and y.

SHORT ANSWER TYPE QUESTIONS

32. If x + iy = $\sqrt{\dfrac{1+i}{1-i}}$ prove that $x^2 + y^2 = 1$

33. Find real value of θ such that, $\dfrac{1+i\cos\theta}{1-2i\cos\theta}$ is a real number.

34. If $\left|\frac{z-5i}{z+5i}\right|=1$ show that z is a real number.

35. If $x_n = \cos\frac{\pi}{2^n} + i\sin\frac{\pi}{2^n}$ Prove that $x_1\, x_2 \ldots\ldots\, x_\infty = -1$

36. Find real value of x and y if $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$.

37. If $(1+i)(1+2i)(1+3i) \ldots\ldots (1+ni) = x+iy$.

Show, $2.5.10 \ldots\ldots\ldots (1+n^2) = x^2 + y^2$

38. If z = 2 – 3i show that $z^2 - 4z + 13 = 0$, hence find the value of $4z^3 - 3z^2 + 169$.

39. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = a+ib$, find a and b.

40. For complex numbers $z_1 = 6 + 3i$, $z_2 = 3 - i$ find $\frac{z_1}{z_2}$.

41. If $\left(\frac{2+2i}{2-2i}\right)^n = 1$, find the least positive integral value of n

42. If $(x+iy)^{\frac{1}{3}} = a+ib$ prove $\left(\frac{x}{a}+\frac{y}{b}\right) = 4\left(a^2-b^2\right)$.

43. Convert the following in polar form:

(i) $-3\sqrt{2}+3\sqrt{2}i$

(ii) $\frac{\left(\sqrt{3}-1\right)-\left(\sqrt{3}+1\right)i}{2\sqrt{2}}$

(iii) $i(1+i)$

(iv) $\frac{5-i}{2-3i}$

44. Solve

(i) $x^2-\left(3\sqrt{2}-2i\right)x-6\sqrt{2}i=0$

(ii) $x^2-(7-i)x+(18-i)=0$

45. Find the square root of $7-30\sqrt{-2}$.

46. Prove that $x^2 + 4 = (x + 1 + i)(x + 1 - i)(x - 1 + i)(x - 1 - i)$.

47. Show that $\left|\frac{z-2}{z-3}\right|=2$ represent a circle find its centre and radius.

48. Find all non-zero complex number z satisfying $\bar{z}=iz^2$.

49. If $iz^3 + z^2 - z + i = 0$ then show that $|z| = 1$.

50. If z_1, z_2 are complex numbers such that, $\frac{2z_1}{3z_2}$ is purely imaginary number then find $\left|\frac{z_1-z_2}{z_1+z_2}\right|$.

51. If z_1 and z_2 are complex numbers such that,

$\left|1-\bar{z}_1 z_2\right|^2-\left|z_1-z_{2^2}\right|^2=k\left(1-\left|z_1\right|^2\right)\left(1-\left|z_2\right|^2\right)$. Find value of k.

LONG ANSWER TYPE QUESTIONS

52. Find number of solutions of $z^2+|z|^2=0$.

53. If z_1, z_2 are complex numbers such that $\left|\frac{z_1-3z_2}{3-z_1.\bar{z}_2}\right|=1$ and $|z_2|\neq 1$ then find $|z_1|$.

54. Evaluate $x^4 - 4x^3 + 4x^2 + 8x + 44$, When $x = 3 + 2i$

55. If z_1, z_2 are complex numbers, both satisfy $z+\bar{z}=2|z-1|$

$\arg|z_1-z_2|=\frac{\pi}{4}$, then find $\operatorname{Im}(z_1+z_2)$.

56. Solve $2x^2 - (3 + 7i)x - (3 - 9i) = 0$

57. What is the locus of z if amplitude of $z - 2 - 3i$ is $\frac{\pi}{4}$.

58. If $z = x + iy$ and $w = \frac{1 - iz}{z - i}$ show that if $|w| = 1$ then z is purely real.

59. Express the complex number in the form $r(\cos\theta + i\sin\theta)$

(i) $1 + i\tan\alpha$

(ii) $1 - \sin\alpha + i\cos\alpha$

60. If $\left(\frac{1+i}{1+2^2 i}\right)\times\left(\frac{1+3^2 i}{1+4^2 i}\right)\times........\times\left(\frac{1+(2n-1)^2 i}{1+(2n)^2 i}\right) = \frac{a+ib}{c+id}$ then show that $\frac{2}{17}\times\frac{82}{257}\times.......\times\frac{1+(2n-1)^4}{1+(2n)^4} = \frac{a^2+b^2}{c^2+d^2}$.

61. Find the values of x and y for which complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate to each other.

62. Show that the complex number z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a equilateral triangle.

63. If $f(z) = \frac{7 - z}{1 - z^2}$ where $z = 1 + 2i$ then show that $|f(z)| = \frac{|z|}{2}$.

64. If z_1, z_2, z_3 are complex numbers such that

$|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$ then find the value of $|z_1 + z_2 + z_3|$

CASE STUDY TYPE QUESTIONS

65. Suresh and Ramesh try to find the square root of $-15-8i$ by assuming $x+iy=\sqrt{-15-8i}$. Using this information answer the following

i. The value of x^2-y^2 and xy respectively is

(a) –15, 8 (b) 15, –8 (c) –15, –8 (d) 15, 8

ii. The value of x^2+y^2 is

(a) –17 (b) 17 (c) 16 (d) –16

iii. The value of x and y respectively is

(a) $\pm 1, \pm 4$ (b) $\mp 1, \mp 4$ (c) $\mp 1, \pm 4$ (d) $\pm 1, \mp 4$

iv. The required square roots of $-15-8i$ is

(a) $\pm 1, \pm 4i$ (b) $\mp 1, \mp 4i$ (c) $\mp 1, \pm 4i$ (d) $\pm 1, \mp 4i$

v. The square roots of $-15-8i$ are

(a) Complex conjugate of each other
(b) Reciprocal of each other
(c) Multiplicative inverse of each other
(d) None of these

66. While solving a typical equation a person finds that one of the root of the equation is a complex number $z=\frac{1+2i}{1-3i}$, help him to find

i. The standard form of z

(a) $-\frac{1}{2}+\frac{i}{2}$ (b) $\frac{1}{2}-\frac{i}{2}$ (c) $-\frac{1}{2}-\frac{i}{2}$ (d) $\frac{1}{2}+\frac{i}{2}$

ii. If $z = 2x + (4 - y)i,$ then

(a) $x = \frac{1}{4},\ y = \frac{7}{2}$ (b) $x = -\frac{1}{4},\ y = \frac{7}{2}$

(c) $x = \frac{1}{4},\ y = -\frac{7}{2}$ (d) $x = -\frac{1}{4},\ y = -\frac{7}{2}$

iii. The argument of z is

(a) $7\pi/4$ (b) $5\pi/4$ (c) $\pi/4$ (d) $3\pi/4$

iv. The modulus of z is

(a) 1/3 (b) 1/2 (c) $1/\sqrt{2}$ (d) $1/\sqrt{3}$

v. The square roots of $-15 - 8i$ are

(a) $\cos 3\pi/4 + i\sin 3\pi/4$

(b) $\sin 3\pi/4 + i\cos 3\pi/4$

(c) $1/\sqrt{2}\left(\cos 3\pi/4 + i\sin 3\pi/4\right)$

(d) $1/\sqrt{2}\left(\cos 3\pi/4 + i\sin 3\pi/4\right)$

ANSWERS

1. $-1 + i$
2. $-7 - 6i$
3. $\frac{1}{49} - \frac{4\sqrt{3}i}{49}$
4. $\frac{-2}{25} + \frac{11i}{25}$
5. $\frac{-\pi}{2}$
6. $\theta + \frac{\pi}{3}$
7. $\frac{-2}{7} - \frac{i\sqrt{3}}{7}$
8. $\frac{-\pi}{2}$
9. 0
10. 13

11. $-4 + 3i$ and $4 + 3i$

12. 7

13. –2

14. $z = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

15. $-i$

16. First

17. –15

18. False

19. False

20. False

21. False

22. True

23. (a) $\rightarrow$ (iv)
    (b) $\rightarrow$ (v)
    (c) $\rightarrow$ (ii)
    (d) $\rightarrow$ (iii)
    (e) $\rightarrow$ (i)

24. (i) 0
    (ii) 19
    (iii) –8
    (iv) $\frac{-7}{\sqrt{2}}i$

25. $x = \frac{5}{13}, \quad y = \frac{14}{13}$

26. i

27. 0 (zero)

28. 6 and zero

29. a = –2

30. 1 : 3

31. Locus of z is straight line i.e., x – y + 1 = 0

33. $\theta = (2n+1)\frac{\pi}{2}$

36. x = 3, y = –1

38. zero

39. a = 0, b = –2

40. $\frac{z_1}{z_2} = \frac{3(1+i)}{2}$

41. n = 4

43. (i) $6\left(\cos\frac{3\pi}{4}+i\sin\frac{\pi}{4}\right)$

(ii) $1\left[\cos\left(\frac{-5\pi}{12}\right)+i\sin\left(\frac{-5\pi}{12}\right)\right]$

(iii) $\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

(iv) $\sqrt{2}\left[\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right]$

44. (i) $3\sqrt{2}$ and $-2i$

(ii) $4-3i$ and $3+2i$

45. $\pm\left(5-3\sqrt{2}\,i\right)$

47. Centre $\left(\frac{10}{3},0\right)$ and radius = $\frac{2}{3}$

48. $z=0,\ i,\ \frac{\sqrt{3}}{2}-\frac{1}{2}i,\ -\frac{\sqrt{3}}{2}-\frac{1}{2}i$

50. 1

51. K = 1

52. Infinitely many solutions of the form $z=0\ \pm\ iy;\ y\in R$

53. $|z_1|=\sqrt{x^2+y^2}$

54. 5

55. 2

56. $\frac{3}{2}+\frac{1}{2}i$ and $3i$

57. x – y + 1 = 0 straight line

59. (i) $\sec\alpha\left(\cos\alpha+i\sin\alpha\right)\ ,\ 0\le\alpha<\dfrac{\pi}{2}$

$-\sec\alpha\left[\cos\left(\alpha-\pi\right)+i\sin\left(\alpha-\pi\right)\right],\ \dfrac{\pi}{2}<\alpha\le\pi$

(ii) $\sqrt{2}\left(\cos\dfrac{\alpha}{2}-\sin\dfrac{\alpha}{2}\right)\left[\cos\left(\dfrac{\pi}{4}+\dfrac{\alpha}{2}\right)+i\sin\left(\dfrac{\pi}{4}+\dfrac{\alpha}{2}\right)\right]$ if $0\le\alpha<\dfrac{\pi}{2}$

$-\sqrt{2}\left(\cos\dfrac{\alpha}{2}-\sin\dfrac{\alpha}{2}\right)\left[\cos\left(\dfrac{\alpha}{2}-\dfrac{3\pi}{4}\right)+i\sin\left(\dfrac{\alpha}{2}-\dfrac{3\pi}{4}\right)\right]$ if $\dfrac{\pi}{2}<\alpha<\dfrac{3\pi}{2}$

$-\sqrt{2}\left(\cos\dfrac{\alpha}{2}-\sin\dfrac{\alpha}{2}\right)\left[\cos\left(\dfrac{\alpha}{2}-\dfrac{3\pi}{4}\right)+i\sin\left(\dfrac{\alpha}{2}-\dfrac{3\pi}{4}\right)\right]$ if $\dfrac{3\pi}{2}<\alpha<2\pi$

60. When x = 1, y = –4 or x = –1, y = –4

61. 1 (one)

65. i. (c) ii. (b) iii. (d) iv. (d) v. (a)

66. i. (a) ii. (b) iii. (d) iv. (c) v. (c)

CHAPTER - 6

LINEAR INEQUALITIES

KEY POINTS

- **Inequalities:** A statement involving '<', '>', '≥' or '≤' is called inequality. Eg., $7 > 5$, $5x - 3 \leq 4$

  - Inequalities which do not involve variables are called numerical inequalities.
    Eg., $5 > 9$ and $13 > -2$

  - Inequalities which involve variables are called literal inequalities.
    Eg., $3x - 4 \leq 15$ and $4x - 3y \geq 5$

  - Inequalities involving the symbols '>' or '<' are called strict inequalities.

  - Inequalities involving the symbols '≥' or '≤' are called slack inequalities.

- **Linear inequalities in one variable:** The inequalities of form $ax + b + 0$, $ax + b < 0$, $ax + b \geq 0$ or $ax + b \leq 0$; $a \neq 0$ are called linear inequalities in one variable.

  Eg., $4x - 5 \geq 20$ and $-3x - 2 < 5x + 4$

- **Algebraic solutions of linear inequalities in one variables:**

  - **Rule-1**
    Equal numbers may be added (or subtracted from) to both sides without affecting sign of inequalities.

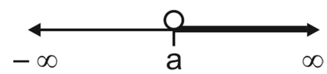
- **Rule-2**
  - (i) If both sides of inequality are multiplied (or divided) by same positive number, then sign of inequality remains unchanged.
  - (ii) If both sides are multiplied (or divided) by any negative number, then sign of inequality is reversed.

  Eg., (i) $4x \geq 8 \quad \Rightarrow \frac{4x}{4} \geq \frac{8}{4} \quad \Rightarrow x \geq 2$

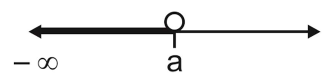
  (ii) $-4x \geq 8 \Rightarrow \frac{-4x}{-4} \geq \frac{8}{-4} \Rightarrow x \leq -2$

▶ **Graphical representation of solutions on number line:**

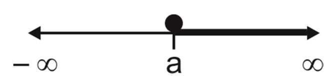
(i) $x > a \Leftrightarrow a < x < \infty \Leftrightarrow x \in (a, \infty) \Leftrightarrow$



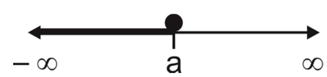
(ii) $x < a \Leftrightarrow -\infty < x < a \Leftrightarrow x \in (-\infty, a) \Leftrightarrow$



(iii) $x \geq a \Leftrightarrow a \leq x < \infty \Leftrightarrow x \in [a, \infty) \Leftrightarrow$



(iv) $x \leq a \Leftrightarrow -\infty < x \leq a \Leftrightarrow x \in (-\infty, a] \Leftrightarrow$



(v) $a < x < b \Leftrightarrow x \in (a, b) \Leftrightarrow$

(vi) $a \leq x \leq b \Leftrightarrow x \in [a, b] \Leftrightarrow$

▶ **Linear inequalities in two variables:** The inequalities of form $ax + by + c > 0$, $ax + by + c < 0$, $ax + by + c \geq 0$ or $ax + by + c \leq 0$ are linear inequalities in two variables. $(a, b \neq 0)$

Eg., $4x - 3y < 15$ and $-4x + 15y + 3 \geq 4$

▶ Graphical solution of linear inequalities in two variables

- A line divides the Cartesian plane into two parts. Each part is known as a half plane.
- The region containing all the solutions of the inequality is called solution region.
- In order to identify the half plane represented by an inequality (solution region), it is just sufficient to take any point (a, b) not on the line and check whether it satisfy the inequality or not.
- If it satisfies, then the regions containing that point (a, b) is solution region.
- If it does not satisfy, then the other region is solution region.
- If inequality contains '$\geq$' or '$\leq$', then points on line $ax + by = c$ are also included in solution region. In this case we draw dark line while sketching graph of $ax + by = c$.
- If inequality contains '$>$' or '$<$', then points on line $ax + by = c$ are not included in solution region. In this case we draw dotted line while sketching graph of $ax + by = c$.

Note: While solving system of linear inequalities in two variables, the common of solution regions of each inequality is solution region of system.

VERY SHORT ANSWER TYPE QUESTIONS

1. Solve $5x < 24$ when $x \in N$
2. Solve $3 - 2x < 9$ when $x \in R$. Express the solution in the form of interval.
3. Show the graph of the solution of $2x - 3 > x - 5$ on number line.

4. Solve $\frac{1}{x-2} \le 0$, $x \in R$.

5. Solve $0 < \frac{-x}{3} < 1$, $x \in R$

6. Solve $-3 \le -3x + 2 < 4$, $x \in R$.

7. Draw the graph of the solution set of $x + y \ge 4$.

8. Draw the graph of the solution set of $x < y$.

9. Fill in the blanks

(a) If $3x + 17 \le 2(1 - x)$, then $x \in$ _______.

(b) If $\frac{x^2}{x-2} > 0$, then $x \in$ _______.

(c) If $x^2 \le 4$, then $x \in$ _______.

(d) Statement $4x - 3 \ge 10$ is ___________.

(e) If $|x| > 5$, then then $x \in$ _______.

10. If $-4x > 20$ and $x \in z^+$ then x belongs to -

(a) $\{-6, -7, -8, \ldots\ldots\}$ (b) ϕ

(c) $\{-4, -3, -2, -1\}$ (d) $\{1, 2, 3, 4, \ldots\ldots\}$.

11. If $\frac{x-3}{x-2} > 0$ then x belongs to -

(a) $(-\infty, 3) \cup (5, \infty)$ (b) $(-\infty, -3) \cup (-5, \infty)$

(c) $(-\infty, 3] \cup [5, \infty)$ (d) $(3, 5)$

12. Solution set for inequality $|x - 1| \leq 5$ is -

(a) $[-6, 4]$ (b) $[-4, 0]$

(c) $[-4, 6]$ (d) $[0, 6]$.

13. Solution set for inequality $\frac{1}{x-2} < 0$ is -

(a) $(2, \infty)$ (b) ϕ

(c) $(0, 2)$ (d) $(-\infty, 2)$.

14. Solution set for inequality $5x - 3 < 3x + 1$, $x \in N$ is -

(a) $(-\infty, 2)$ (b) $\{0, 1, 2\}$

(c) $\{1\}$ (d) ϕ.

15. Which of the following point lies in solution region of inequality $3x - y \leq 5$?

(a) $(5, 1)$ (b) $(1, 5)$

(c) $(2, 0)$ (d) $(2, -1)$.

16. If $x > 0$ and $y < 0$ then (x, y) lies in -

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant.

17. If $x^2 > 9$ then x belongs to -

(a) $(-3, 3)$ (b) $(0, 3)$

(c) $(3, \infty)$ (d) $(-\infty, -3) \cup (3, \infty)$.

18. Solution set for inequality $-8x \leq 5x - 3 < 7$ is -

(a) $(-1, 2)$ (b) $(2, 3)$

(c) $[-1, 2)$ (d) $[2, 3]$.

19. True / False

(a) Solution set for inequality $2x - 6 \leq 0$ is (0, 3].

(b) Solution set for inequality $-8 \leq 5x - 3 < 7$ is [–1, 2).

(c) Inequality $4x - 7 \geq 3x + 4$ is slack inequality.

(d) Inequality $4x - 7 < 8$ is numerical inequality.

VERY SHORT ANSWER TYPE QUESTIONS

20. Solve $\dfrac{(x-1)(x-2)}{(x-3)(x-4)} \geq 0, \quad x \in R.$

21. Solve $\dfrac{x+3}{x-1} > 0, \quad x \in R.$

Solve the inequalities for real x and represent solution on number line

22. $\dfrac{2x-3}{4} + 9 \geq 3 + \dfrac{4x}{3}, \quad x \in R.$

23. $\dfrac{2x+3}{4} - 3 < \dfrac{x-4}{3} - 2, \quad x \in R.$

24. $-5 \leq \dfrac{2-3x}{4} \leq 9, \quad x \in R.$

25. $\dfrac{x+3}{x-2} > 0, \quad x \in R$

26. $\dfrac{x-3}{x-5} > 2$

27. $\dfrac{2x-1}{3} \geq \dfrac{3x-2}{4} - \dfrac{2-x}{5}$

28. $\frac{2x+3}{x-3} \le 4$

29. Find the pair of consecutive even positive integers which are greater than 5 and are such that their sum is less than 20.

SHORT ANSWER TYPE QUESTIONS

30. A company manufactures cassettes and its cost and revenue functions are C(x) = 26000 + 30x and R(x) = 43x respectively, where x is number of cassettes produced and sold in a week. How many cassettes must be sold per week to realise some profit.

31. While drilling a hole in the earth, it was found that the temperature (T°C) at x km below the surface of the earth was given by T = 30 + 25(x – 3), when $3 \le x \le 15$.

    Between which depths will the temperature be between 200°C and 300°C?

32. The water acidity in a pool is considered normal when the average PH reading of their daily measurements is between 7.2 and 7.8. If the first two PH reading are 7.48 and 7.85. Find the range of PH value for the 3rd reading that will result in acidity level being normal.

Solve the following systems of inequalities for all x $\in$ R

33. $2(2x+3)-10<6(x-2), \quad \frac{2x-3}{4}+6 \ge 4+\frac{4x}{3}$

34. $|2x-3| \le 11, \quad |x-2| \ge 3$

35. $\frac{4x}{3}-\frac{9}{4}<x+\frac{3}{4}, \quad \frac{7x-1}{3}-\frac{7x+2}{6}>x$

36. Solve $\frac{|x|-1}{|x|-2} \geq 0 \quad x \in R, \quad x \neq \pm 2$

37. Solve for real x, $|x+1|+|x|>3$

38. In the first four papers each of 100 marks, Rishi got 95, 72, 73, 83 marks. If he wants an average of greater than or equal to 75 marks be should score in fifth paper.

39. A milkman has 80% milk in this stock of 800 litres of adultered milk. How much 100% pure milk is to be added to it so that purity is between 90% and 95%?

40. $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}, \quad \frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$

41. $\frac{x}{2x+1} \geq \frac{1}{4}, \quad \frac{6x}{4x-1} < \frac{1}{2}$

42. $5(2x - 7) - 3(2x + 3) \leq 0$ and $2x + 19 \leq 6x + 45$.

LONG ANSWER TYPE QUESTIONS

Solve to the following system of inequalities and represent solution on number line:

43. $2x + y \leq 24$, $x + y < 11$, $2x + 5y \leq 40$, $x \geq 0$, $y \geq 0$

44. $3x + 2y \geq 24$, $3x + y \leq 15$, $x \geq 4$

45. $x - 2y \leq 3$

$3x + 4y > 12$

$x \geq 0$, $y \geq 1$

CASE STUDY TYPE QUESTIONS

46. A company produced cassettes, one cassette Cost Company Rs. 30 and also an additional fixed cost 26000 per week. The company sold each Cassette at Rs. 43. If x is number of cassettes produced and sold by the company in a week. From the following information find

i. The cost function of the company

(a) 26000 + 30x (b) 26000 + 43x

(c) 30 + 26000x (d) 43 + 26000x

ii. The revenue function of the company

(a) 30x (b) 26000x

(c) 43x (d) 13x

iii. The profit function of the company

(a) –26000 + 73x (b) –26000 + 13x

(c) 26000 + 43x (d) 26000 + 30x

iii. How many cassettes must be produced by the company in a week to realize some profit?

(a) more than 2000 (b) less than 2000

(c) more than 5000 (d) less than 5000

iv. If company incurred an additional cost of Rs. 3 on each cassette per week. How many cassettes must be produced by the company in a week so that there is no profit no loss?

(a) 2000 (b) 5000

(c) 2600 (d) 1000

47. A and B try to find the solution of the inequality $|x - 1| + |x - 2| \geq 4$. Help them to find the solution of the inequality

i. When $x < 1$

(a) $(-\infty, -1/2)$ (b) $(-\infty, -1)$

(c) $(-\infty, -1/2]$ (d) $(-\infty, 1/2)$

ii. When $1 \leq x < 2$

(a) $(-\infty, \infty)$ (b) $(-\infty, -1)$

(c) Infinite solution (d) no solution

iii. When $2 \leq x < \infty$

(a) $(-\infty, \infty)$ (b) $(7/2, \infty)$

(c) $[7/2, \infty)$ (d) no solution

iv. When $x \in R$

(a) $(-\infty, -1/2] \cup [7/2, \infty)$ (b) $(-\infty, -1/2) \cup [7/2, \infty)$

(c) $(-\infty, -1/2] \cup (7/2, \infty)$ (d) $(-\infty, -7/2] \cup [1/2, \infty)$

v. When $x > 4$

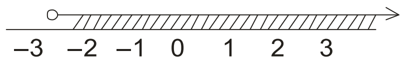
(a) $(-\infty, 4)$ (b) $(-\infty, 4]$

(c) $(4, \infty)$ (d) $[4, \infty)$

ANSWERS

1. {1, 2, 3, 4}

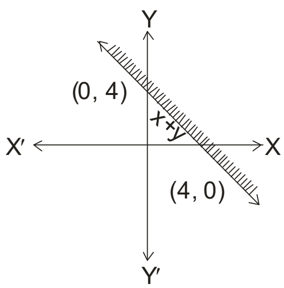
2. (–3, ∞)

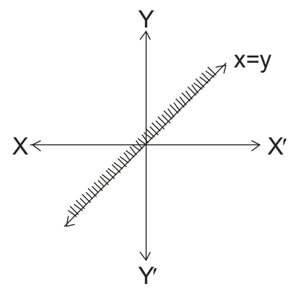
3. 

4. (–∞, 2) or x < 2

5. –3 < x < 0

6. $\left(\frac{-2}{3} \quad \frac{5}{3}\right]$

7. 

8. 

9. (a) [–∞, –3]
   (b) [2, ∞]
   (c) [–2, 2]
   (d) Slack
   (e) (–∞, –5) ∪ (5, ∞)

10. (b)

11. (a)

12. (c)

13. (d)

14. (c)

15. (b)

16. (d)

17. (d)

18. (c)

19. (a) False
    (b) True
    (c) True
    (d) False

20. X > 4

21. $(-\infty, -3) \cup (2, \infty)$

22. $\left(-\infty, \dfrac{63}{10}\right]$

23. $\left(-\infty, \dfrac{-13}{2}\right)$

24. $\left[\dfrac{-34}{3}, \dfrac{22}{3}\right]$

25. $(-\infty, -3) \cup (2, \infty)$

26. $(-12, -5)$

27. $(-\infty, 2]$

28. $(-\infty, -3) \cup (2, \infty)$

29. (6, 8) and (8, 10)

30. More than 2000 cassettes

31. Between 9.8 m and 13.8 m

32. Between 6.27 and 8.07.

33. Solution set = ϕ

34. $[-4, -1] \cup [5, 7]$

35. (4, 9)

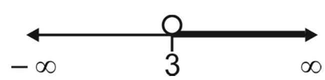
36. $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$

37. $(-\infty, -2) \cup (1, \infty)$

38. He must score greater than or equal to 52 and less than 77.

39. Between 100 litre and 150 litre

40. $(3, \infty)$



41. Number solution

42. $[-7, 11]$

46. i. (a) ii. (c) iii. (b) iv. (a) v. (c)

47. i. (c) ii. (d) iii. (c) iv. (a) v. (c)

CHAPTER - 7

PERMUTATIONS AND COMBINATIONS

KEY POINTS

- **Fundamental principal of counting**

  - **Multiplication Principle:** If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of different ways of occurrence of the two events in order is m × n.

  - **Fundamental Principle of Addition:** If there are two events such that they can occur independently in m and n different ways respectively, then either of the two events can occur in (m + n) ways.

- **Factorial:** Factorial of a natural number n, denoted by n! or n is the continued product of first n natural numbers.

$$n! = n \times (n-1) \times (n-2) \times \ldots\ldots\ldots \times 3 \times 2 \times 1$$
$$= n \times ((n-1)!)$$
$$= n \times (n-1) \times ((n-2)!)$$

- **Permutation:** A permutation is an arrangement of a number of objects in a definite order taken some or all at a time.

  - The number of permutation of n different objects taken r at a time where $0 \le r \le n$ and the objects do not repeat is denoted by ${}^nP_r$ or P(n, r) where,

$${}^nP_r = \frac{n!}{(n-r)!}$$

- The number of permutations of n objects, taken r at a time, when repetition of objects is allowed is n^r.
- The number of permutations of n objects of which p_1 are of one kind, p_2 are of second kind, …….. p_k are of k^{th} kind and the rest if any, are of different kinds, is $\dfrac{n!}{p_1!\,p_2!\,.......\,p_k!}$

▶ **Combination:** Each of the different selections made by choosing some or all of a number of objects, without considering their order is called a combination. The number of combination of n objects taken r at a time where,

$0 \le r \le n$, is denoted by ${}^nC_r$ or C(n, r) or $\binom{n}{r}$ where ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

▶ **Some important result:**

(i) $0! = 1$

(ii) ${}^nC_0 = {}^nC_n = 1$

(iii) ${}^nC_r = {}^nC_{n-r}$ where $0 \le r \le n$, and r are positive integers

(iv) ${}^nP_r = \underline{|n}\ {}^nC_r$ where $0 \le r \le n$, r and n are positive integers.

(v) ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$ where $0 \le r \le n$ and r and N are positive integers.

(vi) If ${}^nC_a = {}^nC_b$ if either $a = b$ or $a + b = n$

VERY SHORT ANSWER TYPE QUESTIONS

1. How many ways are there to arrange the letters of the word "GARDEN" with the vowels in alphabetical order?

2. In how many ways 7 pictures can be hanged on 9 pegs?

3. Ten buses are plying between two places A and B. In how many ways a person can travel from A to B and come back?

4. There are 10 points on a circle. By joining them how many chords can be drawn?

5. There are 10 non collinear points in a plane. By joining them how many triangles can be made?

6. If ${}^nP_4 : {}^nP_2 = 12$, find n.

7. How many different words (with or without meaning) can be made using all the vowels at a time?

8. In how many ways 4 boys can be chosen from 7 boys to make a committee?

9. How many different words can be formed by using all the letters of word "SCHOOL"?

10. In an examination there are three multiple choice questions and each question has 4 choices. Find the number of ways in which a student can fail to get all answer correct.

11. A gentleman has 6 friends to invite. In how many ways can he send invitation cards to them if he has three servants to carry the cards?

12. If there are 12 persons in a party, and if each two of them Shake hands with each other, how many handshakes happen in the party?

13. Fill in the blanks

(a) If ${}^{12}C_5 = {}^{12}C_r$ then r = _______.

(b) $\frac{6!}{3!}$ _______.

(c) If ${}^{12}C_5 + {}^{12}C_6 = {}^{x}C_6$ then x = _______.

(d) If ${}^{n-1}P_3 : {}^{n}P_4 = 1 : 9$ then n = ______.

(e) If ${}^{20}C_r = {}^{20}C_{r-10}$ then ${}^{18}C_r$ = _______.

(f) Number of diagonal of an n-sided polygon is ________.

14. What is the number of ways of arrangement of letters of word 'BANANA' so that no two N's are together -

(a) 40 (b) 60

(c) 80 (d) 100.

15. What is the value of n, if P(15, n – 1) : P(16, n – 2) = 3 : 4 ?

(a) 10 (b) 12

(c) 14 (d) 15.

16. The number of words which can be formed from the letters of the word MAXIMUM, if two consonants can't occur together is -

(a) 4! (b) 3! × 4!

(c) 7! (d) None of these.

17. If 7 points out of 12 are in the same straight line, then what is the number of triangles formed?

(a) 84 (b) 175

(c) 185 (d) 201.

18. In how many ways can be bowler take four wickets in a single 6 balls over?

(a) 6 (b) 15

(c) 20 (d) 30.

19. What is the number of signals that can be sent by 6 flags of different colours taking one or more at a time?

(a) 45 (b) 63

(c) 720 (d) 1956.

20. There are 6 letters and 3 post boxes. The number of wages in which these letters can be posted is -

(a) 6^3 (b) 3^6

(c) ${}^6P_3$ (d) ${}^6C_3$.

21. If ${}^mC_1 = {}^nC_2$, then -

(a) $2m = n$ (b) $2m = n(n + 1)$

(c) $2m = n(n - 1)$ (d) $2n = m(m - 1)$.

22. ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_x$, then x = ?

(a) r (b) r – 1

(c) n (d) r + 1.

23. ${}^{43}C_{r-6} = {}^{43}C_{3r+1}$, then value of r is -

(a) 12 (b) 8

(c) 6 (d) 10.

24. True / False

(a) $0! = 0$.

(b) $\frac{n!}{(n-r)!} = n$.

(c) ${}^{n}P_{n} = 1$.

(d) ${}^{n}C_{r} = {}^{n}C_{n-r}$.

(e) Total number of two letter word, when repetition of letter is not allowed is ${}^{26}P_{2}$.

SHORT ANSWER TYPE QUESTIONS

25. Find n, ${}^{n-1}P_{3}$: ${}^{n}P_{4}$ = 1 : 9.

26. If ${}^{22}P_{r+1}$: ${}^{20}P_{r+2}$ = 11 : 52, find r.

27. In how many ways a picture can be hung from 6 picture nails on a wall.

28. If ${}^{n}P_{r} = 336$, ${}^{n}C_{r} = 56$, find n and r. Hence find ${}^{n-1}C_{r-1}$.

29. A convex polygon has 65 diagonals. Find number of sides of polygon.

30. In how many ways can a cricket team of 11 players be selected out of 16 players, if two particular players are always to be selected?

31. From a class of 40 students, in how many ways can five students be chosen

(i) For an excursion party.

(ii) As subject monitor (one from each subject)

32. In how many ways can the letters of the word "ABACUS" be arranged such that the vowels always appear together?

33. If $^nC_{12} = {^nC_{13}}$ then find the value of the $^{25}C_n$.

34. In how many ways can the letters of the word "PENCIL" be arranged so that I is always next to L.

SHORT ANSWER TYPE QUESTIONS

35. In how many ways 12 boys can be seated on 10 chairs in a row so that two particular boys always take seats of their choice.

36. In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together?

37. From a group of 7 boys and 5 girls, a team consisting of 4 boys and 2 girls is to be made. In how many different ways it can be done?

38. In how many ways can one select a cricket team of eleven players from 17 players in which only 6 players can bowl and exactly 5 bowlers are to be included in the team?

39. A student has to answer 10 questions, choosing at least 4 from each of part A and B. If there are 6 questions in part A and 7 in part B. In how many ways can the student choose 10 questions?

40. Using the digits 0, 1, 2, 2, 3 how many numbers greater than 20000 can be made?

41. If the letters of the word 'PRANAV' are arranged as in dictionary in all possible ways, then what will be 182^{nd} word.

42. From a class of 15 students, 10 are to chosen for a picnic. There are two students who decide that either both will join or none of them will join. In how many ways can the picnic be organized?

43. Using the letters of the word, 'ARRANGEMENT' how many different words (using all letters at a time) can be made such that both A, both E, both R and both N occur together.

44. A polygon has 35 diagonal. Find the number of its sides.

45. How many different products can be obtained by multiplying two or more of the numbers 2, 5, 6, 7, 9?

46. Determine the number of 5 cards combinations out of a pack of 52 cards if at least 3 out of 5 cards are ace cards?

47. How many words can be formed from the letters of the word 'ORDINATE' so that vowels occupy odd places?

48. Find the number of all possible arrangements of the letters of the word "MATHEMATICS" taken four at a time.

49. Prove that 33! in divisible by 2^{15} what is the largest integer n such that 33! is divisible by 2^n?

50. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if a team has

    (i) no girl

    (ii) at least 3 girls

    (iii) at least one girl and one boy?

51. Find n

    if $16\,{}^{n+2}C_8 = 57\,{}^{n-2}P_4$

52. In an election, these are ten candidates and four are to be elected. A voter may vote for any number of candidates, not greater than the number to be elected. If a voter vote for at least one candidate, then find the number of ways in which he can vote.

53. Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

LONG ANSWER TYPE QUESTIONS

54. Using the digits 0, 1, 2, 3, 4, 5, 6 how many 4 digit even numbers can be made, no digit being repeated?

55. There are 15 points in a plane out of which only 6 are in a straight line, then

    (i) How many different straight lines can be made?

    (ii) How many triangles can be made?

56. If there are 7 boys and 5 girls in a class, then in how many ways they can be seated in a row such that

    (i) No two girls sit together?

    (ii) All the girls never sit together?

57. Using the letters of the word 'EDUCATION' how many words using 6 letters can be made so that every word contains atleast 4 vowels?

58. What is the number of ways of choosing 4 cards from a deck of 52 cards? In how many of these,

    (i) 3 are red and 1 is black.

    (ii) All 4 cards are from different suits.

(iii) At least 3 are face cards.

(iv) All 4 cards are of the same colour.

59. How many 3 letter words can be formed using the letters of the word INEFFECTIVE?

60. How many different four letter words can be formed (with or without meaning) using the letters of the word "MEDITERRANEAN" such that the first letter is E and the last letter is R.

61. If all letters of word 'MOTHER' are written in all possible orders and the word so formed are arranged in a dictionary order, then find the rank of word 'MOTHER'?

62. In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats?

63. From 6 different novels and 3 different dictionaries, 4 novels and a dictionary is to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then find the number of such arrangements.

64. The set S = {1, 2, 3, …….12} is to be partitioned into three sets A, B, and C of equal sizes. $A \cup B \cup C = S$, $A \cap B = B \cap C = C \cap A = \phi$. Find the number of ways to partition S.

65. Find the value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$.

66. There are two parallel lines l_1 and l_2 in a plane l_1 contains m different points $A_1, A_2 \ldots\ldots\ldots A_m$ and l_2 contains n different points $B_1, B_2 \ldots\ldots\ldots B_n$. How many triangles are possible with these vertices?

CASE STUDY TYPE QUESTIONS

67. Anita is doing an experiment in which she has to arrange the alphabets of the word "HARYANA" in all possible orders and notes the observations. Help her to find the answers of the following:-

i. Number of words starting with A

(a) 360 (b) 720 (c) 1440 (d) 2880

ii. Number of words having H at end

(a) 72 (b) 120 (c) 240 (d) 480

iii. Number of words having H and N together

(a) 120 (b) 60 (c) 280 (d) 240

iv. Number of words having begin with H and end with N

(a) 20 (b) 24 (c) 60 (d) 48

v. Number of words having vowels together

(a) 240 (b) 120 (c) 240 (d) 720

68. A Company wants to appoint 5 persons, 3 for post A and 2 for post B for its upcoming office in Delhi. They have invited the applications for the same. 14 candidates have applied for the post A and 13 have applied for the post B

i. Find the total number of ways in which the company can make a selection for all the posts.

(a) 5! (b) C(14, 3).C(13,2)

(c) P(13, 2)P(14, 3) (d) none of these

ii. Find the number of ways of selecting one woman for each post, if 3 women have applied for post A and 7 women have applied for post B

(a) 6 (b) 21

(c) 6930 (d) 182

iii. On the day of interview, the candidates were seated in a hall having two chambers. The chairs in both the chambers are placed in line. If the candidates for the two posts are to be seated in two different chambers. Find the total number of ways in which all the candidate can be seated.

(a) 3!2! (b) 11!11!

(c) 14!13! (d) 14! × 13! × 2

iv. During appointment procedure they came to know about a candidate whose resume is excellent and should be selected for the post B. In how many ways can the total selections now be made?

(a) 12 × C(14, 3) (b) 4

(c) 168 (d) 13 × C(13, 3)

v. While checking the applications the management observed that one candidate each who have applied for post A and B are not fit for the job so they cannot be appointed in how many ways can now the post is filled?

(a) 2184 (b) 24024

(c) 18876 (d) 1716

ANSWERS

1. $\frac{6!}{2} = 360$
2. $\frac{9!}{2!}$
3. 100
4. 45
5. 120
6. n = 6
7. 120
8. 35
9. 360
10. 63
11. $3^6 = 729$
12. 66
13. (a) 5 or 7
    (b) 5 or 7
    (c) 13
    (d) 9
    (e) 816
    (f) $\frac{n(n-3)}{2}$
14. (a)
15. (c)
16. (a)
17. (c)
18. (b)
19. (b)
20. (b)
21. (c)
22. (d)
23. (a)
24. (a) True
    (b) True
    (c) False
    (d) True
    (e) True
25. n = 9
26. r = 7
27. 60480
28. n = 8, r = 3 and 21
29. 13
30. 2002

31. (i) 40_{C_5} (ii) 40_{P_5}

32. $\frac{3!}{2!} \times 4!$

31. 1

34. 120

35. $90 \times {}^{10}P_8$

36 56

37. 350

38. 2772

39. 266

40. 36

41. PAANVR

42. $13C_{10} + 13C_8$

43. 5040

44. 10

45. ${}^{n}C_2 - n$

46. 4560

47. 576

48. 2454

49. 31

50. (i) 21;
    (ii) 91;
    (iii) 44133.19

52. ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

53. 48, 144

54. 420

55. (i) 91
    (ii) 435

56. (i) $7! \times {}^{8}P_5$
    (ii) $12! - 8! \times 5!$

57. 24480

58. 52C4

(i) $^{26}C_1 \times {}^{26}C_3$

(ii) $(13)^4$

(iii) 9295 (Hint : Face cards : 4J + 4K + 4Q)

(iv) $2 \times {}^{26}C_4$

59. 265 (*Hint* : make 3 cases i.e.

(i) All 3 letters are different

(ii) 2 are identical 1 different

(iii) All are identical, then form the words.)

60. 59

61. 309

62. $^{14}P_{12}\, 2(2\times 3!)\, ^{11}P_9$

63. $4!\, ^6C_4\, ^3C_1$

64. $^{12}C_4\, ^8C_4\, ^4C_4$

65. $^{56}C_4$

66. $^{m+n}C_3 - {}^mC_3 - {}^nC_3$ or $^mC_2\, ^nC_1 + {}^mC_1\, ^nC_2$

67. i. (a) ii. (b) iii. (c) iv. (a) v. (b)

68. i. (b) ii. (c) iii. (d) iv. (a) v. (c)

CHAPTER - 8

BINOMIAL THEOREM

KEY POINTS

▶ **Binomial Theorem for Positive Integers :**

- $(x + y)^n = {}^nC_0\, y^0\, x^n + {}^nC_1\, x^{n-1}\, y^1 + {}^nC_2\, x^{n-2}\, y^2 + \ldots\ldots\ldots\ldots$

  $\ldots\ldots\ldots\ldots + {}^nC_r\, x^{n-r}\, y^r + \ldots\ldots\ldots\ldots {}^nC_2\, x^0\, y^n,$

  Where n is any positive integer.

- General Term $= T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$, where $0 \le r \le n$.

- Total number of terms in expansion $(x + y)^n$ is $n + 1$.

▶ **Middle Term :**

- If n is even, then there is only are middle term

  M.T. = $\left(\frac{n}{2}+1\right)^{th}$ term

- If n is odd, then there is two middle terms

  (i) M.T. = $\left(\frac{n+1}{2}\right)^{th}$ term

  (ii) M.T. = $\left(\frac{n+1}{2}+1\right)^{th}$ term

▶ **Some important observations :**

- In expansion $(x + y)^n$

  T_{r+1} [$(r + 1)^{th}$ term from beginning] = ${}^nC_r\, x^{n-r}\, y^r$

  T'_{r+1} [$(r + 1)^{th}$ term from end] = ${}^nC_{n-r}\, x^r\, y^{n-r}$

VERY SHORT ANSWER TYPE QUESTIONS

1. Write number of terms in the expansion of $\left\{\left(2x + y^3\right)^4\right\}^7$.

2. Expand $\left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)^6$ using binomial theorem.

3. Write value of

   ${}^{2n-1}c_5 + {}^{2n-1}c_6 + {}^{2n}c_7$ use $\left[{}^nc_r + {}^nc_{r-1} = {}^{n+1}c_r\right]$

4. Which term is greater $(1.2)^{4000}$ or 800?

5. Find the coefficient of x^{-17}, in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$.

6. Find the sum of the coefficients in $(x + y)^8$

   [**Hint :** Put x = 1, y = 1]

7. If ${}^nC_{n-3} = 720$, find n.

8. **Fill in the blanks**

(a) Number of terms in expansion (–2x + 3y) is _______.

(b) Term independent of x in expansion of is $\left(x-\frac{1}{3x^2}\right)^9$ _____.

(c) The middle term in the expansion of $\left(x+\frac{1}{x}\right)^{10}$ is ______.

(d) If the coefficient of x in $\left(x^2+\frac{\lambda}{x}\right)^5$ is 270, then λ = _______.

(e) The coefficient of x^5 in $(x + 3)^8$ is ________.

(f) 4^{th} term from the end in expansion of $\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^a$ is ______.

9. **True / False**

(a) For expansion of $(x + y)^n$, $T_r = {}^nC_r\, x^{n-r}\, y^r$.

(b) Middle term for expansion of $(2x - 3)^8$ is 5th term.

(c) For expansion of $(1 + x)^n$, coefficient of 5th term is ___ ${}^nC_4$.

(d) $6^n - 5n$ is divisible by 5, where $n \in N$.

(e) Number of terms in $(x + y)^5 + (x - y)^5$ is 10.

(f) Coefficient of x^5 in $(1 + x)^{10}$ is ${}^{10}C_5$.

10. The middle term of $\left[2x - \frac{1}{3x}\right]^{10}$ is -

(a) ${}^{10}C_4\, \frac{2^4}{3^4}$

(b) $-{}^{10}C_5\, \frac{2^5}{3^5}$

(c) $-{}^{10}C_4\, \frac{2^4}{3^5}$

(d) ${}^{10}C_5\, \frac{2^5}{3^5}$.

11. For all $n \in N$, $2^{4m} - 15n - 1$ is divisible by -

(a) 125 (b) 225

(c) 450 (d) 625.

12. What is the coefficient of x^n in $(x^2 + 2x)^{n-1}$?

(a) $(n - 1)\, 2^{(n-1)}$ (b) $(n - 1) \times 2^{(n-1)}$

(c) $(n - 1)\, 2^n$ (d) $n \,.\, 2^{(n-1)}$.

13. The coefficient of x^{-3} in the expansion of $\left[x - \frac{m}{x}\right]^{11}$ is -

(a) $-924\, m^7$ (b) $-792\, m^5$

(c) $-792\, m^6$ (d) $-330\, m^7$.

14. In the expansion of $\left[x^2 - \frac{1}{3x}\right]^9$, the term without x is equal to -

(a) $\frac{28}{81}$ (b) $\frac{-28}{243}$

(c) $\frac{28}{243}$ (d) None of these.

15. If in the expansion of $(1 + x)^{20}$, the coefficients of r^{th} and $(r + 4)^{th}$ term are equal, then x is equal to -

(a) 7 (b) 8

(c) 9 (d) 10.

16. If in the expansion of $(1 + x)^5$, the coefficients of $(r - 1)^{th}$ and $(2r + 3)^{th}$ terms are equal, then the value of x -

(a) 5 (b) 6

(c) 4 (d) 3.

17. The total number of terms in expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is -

(a) 202 (b) 51

(c) 50 (d) None of these.

18. The middle term in the expansion of $\left[\frac{2x}{3} - \frac{3}{2x^2}\right]^{2n}$ is -

(a) ${}^{2n}C_n$ (b) $(-1)^n \, {}^{2n}C_n \, x^{-n}$

(c) ${}^{2n}C_n \, x^{-n}$ (d) None of these.

19. If the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are the some, then the value of a is -

(a) $\frac{-7}{9}$ (b) $\frac{-9}{7}$

(c) $\frac{7}{9}$ (d) $\frac{9}{7}$.

SHORT ANSWER TYPE QUESTIONS

20. How many term are free from radical signs in the expansion of $\left(x^{\frac{1}{5}} + y^{\frac{1}{10}}\right)^{55}$.

21. Find the constant term in expansion of $\left(x - \frac{1}{x}\right)^{10}$.

22. Find the value of

$$\frac{{}^8C_0}{6} - {}^8C_1 + {}^8C_2 \times 6 - {}^8C_3 \times 6^2 + \text{........} + {}^8C_8 6^7$$

23. Find 4th term from end in the expansion of find the value of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$.

24. Find middle term in the expansion of $(x - 2y)^8$.

25. Which term is independent of x in the expansion of $\left(3x^3 - \frac{1}{2x^3}\right)^{10}$.

26. Find the 11th term from end in expansion of $\left(2x - \frac{1}{x^2}\right)^{25}$.

SHORT ANSWER TYPE QUESTIONS

27. If the first three terms in the expansion of $(a + b)^n$ are 27, 54 and 36 respectively, then find a, b and n.

28. In $\left(3x^2 - \frac{1}{x}\right)^{18}$ which term contains x^{12}.

29. In $\left(\frac{\sqrt{x}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}x^2}\right)^{10}$ find the term independent of x.

30. Evaluate $\left(\sqrt{2}+1\right)^5 - \left(\sqrt{2}-1\right)^5$ using binomial theorem.

31. In the expansion of $(1 + x^2)^8$, find the difference between the coefficients of x^6 and x^4.

32. Find the coefficients of x^4 in $(1 - x)^2 (2 + x)^5$ using binomial theorem.

33. Show that $3^{2n + 2} - 8n - 9$ is divisible by 8.

34. If the term free from x in the expansion of $\left(\sqrt{x} + \frac{k}{x^2}\right)^{10}$ is 405. Find the value of k.

35. Find the number of integral terms in the expansion of $\left(5^{\frac{1}{2}}+7^{\frac{1}{8}}\right)^{1024}$.

36. If for positive integers r > 1, n > 2 the coefficients of the $(3r)^{th}$ term and $(r + 2)^{th}$ powers of x in the expansion of $(1 +)^{2n}$ are equal, then prove that n = 2r + 1.

37. If a, b, c and d in any binomial expansion be the 6^{th}, 7^{th}, 8^{th} and 9^{th} terms respectively, then prove that $\frac{b^2 - ac}{c^2 - bd} = \frac{4a}{3c}$.

38. If in the expansion of $(1 + x)^n$, the coefficients of three consecutive of three consecutive terms are 56, 70 and 56. Then find n and the position of terms of these coefficients.

39. Show that $2^{4n+4} - 15n - 16$ where $n \in N$ is divisible by 225.

40. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 3 : 5, then show that n = 7.

41. Show that the coefficient of middle term in the expansion of $(1 + x)^{20}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{19}$.

42. Find the value of r, if the coefficient of $(2r + 4)^{th}$ term and $(r - 2)^{th}$ term in the expansion of $(1 + x)^{18}$ are equal.

43. Prove that there is no term involving x^6 in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{11}$.

44. The coefficient of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 6 : 30. Find n.

LONG ANSWER TYPE QUESTIONS

45. Show that the coefficient of x^5 in the expansion of product $(1 + 2x)^6 (1 - x)^7$ is 171.

46. If the 3^{rd}, 4^{th} and 5^{th} terms in the expansion of $(x + a)^n$ are 84, 280 and 560 respectively then find the values of a, x and n.

47. If the coefficients of x^7 in $\left[ax^2 + \frac{1}{bx}\right]^{11}$ and x^{-7} in $\left[ax - \frac{1}{bx^2}\right]^{11}$ are equal, then show that ab = 1.

48. In the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, the ratio of 7^{th} term from the beginning to the 7^{th} term from the end is 1:6 , find n.

49. If a_1, a_2, a_3 and a_4 are the coefficients of any four consecutive terms in the expansion of $(1 + x)^n$

    Prove that $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}$.

50. Using binomial theorem, find the remainder when 5^{103} is divided by 13.

51. Find the remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9.

52. Find the coefficient of x^n in expansions of $(1 + x)(1 - x)^n$.

53. Find the value of $\left(\sqrt{2}+1\right)^6 - \left(\sqrt{2}-1\right)^6$ and show that $\left(\sqrt{2}+1\right)^6$ lies between 197 and 198.

54. Find the term independent of x in the expansion of $(1 + x + 2x^3)$ $\left(\frac{3}{2}x^2 - \frac{1}{3}x\right)^9$.

55. If the coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the expansion of $(1 + x)^4$ are in A.P find the value of r.

56. If the expansion of $(1 - x)^{2n-1}$, the coefficients of x^r is denoted by a_r, then prove $a_{(r-1)} + a_{(2n-r)} = 0$.

57. If the coefficient of 5^{th}, 6^{th} and 7^{th} terms in the expansion of $(1 + x)^n$ are in A.P., then find the value of n.

58. Find the coefficient of x^7 in $\left[ax^2 + \frac{1}{bx}\right]^{11}$ and x^{-7} in $\left[ax - \frac{1}{bx^2}\right]^{11}$ and find the relation between a and b so that these coefficients are equal.

59. The coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(1 + x)^{2n}$ are in A.P. Prove that $2n^2 - 9n + 7 = 0$.

60. Show that the middle term in the expansion of $\left[x - \frac{1}{x}\right]^{2n}$ is $\frac{1 \cdot 3 \cdot 5 \ldots\ldots (2n-1)}{n!}(-2n)^n$.

ANSWERS

1. 29

2. $\frac{x^3}{a^3} - \frac{6x^2}{a^2} + 15\frac{x}{a} - 20 + 15\frac{a}{x} - \frac{6a^2}{x^2} + \frac{a^3}{x^3}$

3. ${}^{2n+1}C_7$

4. $(1.2)^{4000}$

5. –1365

6. 256

7. n = 10

8. (a) 18
   (b) ${}^{9}C_3 \times \left(\frac{-1}{3}\right)^3$
   (c) ${}^{10}C_5$
   (d) 3
   (e) 152
   (f) $\frac{672}{x^3}$

9. (a) False
   (b) True
   (c) True
   (d) False
   (e) False
   (f) True

10. (b)

11. (b)

12. (a)

13. (d)

14. (c)

15. (c)

16. (a)

17. (b)

18. (b)

19. (d)

20. 6 terms (0, 10, 20, 30, 40, 40, 50)

21. $-252 = -{}^{10}C_5$

22. ${}^{31}C_6 - {}^{21}C_6 \ / \ \dfrac{5^8}{6}$

23. $\dfrac{672}{x^3}$

24. $1120\, x^4\, y^4$

25. $\dfrac{-15309}{8}$

26. ${}^{25}C_{15} \times \dfrac{2^{10}}{x^{20}}$

27. a = 3, b = 2, n = 3

28. 9th term

29. $T_3 = \dfrac{5}{6}$

30. 82

31. 28

32. 10

34. k = ±3

35. 129 integral terms

36. $x = \dfrac{1}{\sqrt{10}}$ or 100

38. n = 8, 4th, 5th and 6th

42. r = 6

43. $\left(2x^2 - \dfrac{3}{x}\right)^{11}$

44. n = 41

46. a = 2, x = 1, n = 7

48. 9

50. 8

51. 2

52. $(-1)^n\,[1 - n]$

53. Zero

54. $\dfrac{17}{54}$

55. 5

57. n = 7 or 14

58. ab = 1

CHAPTER - 9

SEQUENCES AND SERIES

KEY POINTS

- A sequence is a function whose domain is the set N of natural numbers or some subset of it.
- A sequence is said to be a progression if the term of the sequence can be expressed by some formula
- **Arithmetic Progression:** A sequence is called an arithmetic progression if the difference of a term and previous term is always same, i.e., $a_{n+1} - a_n$ = constant (=d) for all $n \in N$.
- General A.P. is a, a + d and a + 2d, ...
- $a_n = a + (n-1)d = n^{th}$ term of A.P. $= l$
- S_n = Sum of first n terms of A.P. $= \frac{n}{2}[a + l]$, where l = last term N.

$$= \frac{n}{2}[2a + (n-1)d]$$

- If a, b, c are in A.P. then $a \pm k$, $b \pm k$, $c \pm k$ are in A.P. = ak, bk, ck also in A.P., $k \neq 0$, $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P. where $k \neq 0$.
- Arithmetic mean between a and b is $\frac{a+b}{2}$.
- If $A_1, A_2, A_3, \ldots\ldots A_n$ are n numbers inserted between a and b, such that the resulting sequence is A.P.

then, $A_n = a + n \cdot \dfrac{b-a}{n+1}$

- $S_k - S_{k-1} = a_k$
- $a_m = n,\ a_n = m \Rightarrow a_r = m + n - r$
- $S_m = S_n \Rightarrow S_{m+n} = 0$
- $S_p = q$ and $S_q = p \Rightarrow S_{p+q} = -p - q$
- In an A.P., the sum of the terms equidistant from the beginning and from the end is always same, and equal to the sum of the first and the last term.
- If a, b, c are in A.P. then 2b = a + c.
- If four terms of A.P. are to be taken then we choose then as a – 3d, a – d, a + d, a + 3d.
- If five terms of A.P are to be taken, then we choose then as a – 2d, a – d, a, a + d, a + 2d.
- **G.P. (Geometrical Progression)**

  (i) a, ar, ar^2, ...(General G.P.)

  (ii) $a = ar^{n-1}$

  (iii) $S_n = \dfrac{a\left(r^n - 1\right)}{r-1}$, $r \neq 1$

- If a, b, c are in G.P., then $b^2 = ac$.
- Geometric mean between a and b is $\sqrt{ab}$.
- Reciprocals of terms in GP always form a G.P.

- If $G_1, G_2, G_3, \ldots\ldots\ldots G_n$ are n numbers inserted between a and b so that the resulting sequence is G.P., then

  $$G_k = a\left(\frac{b}{a}\right)^{\frac{k}{n+1}} \quad 1 \le k \le n$$

- If three terms of G.P. are to be taken, then we those then as $\frac{a}{r}$, a, ar.

- If four terms of G.P. are to be taken, then we choose then as $\frac{a}{r^3}, \frac{a}{r}$, a, ar.

- If a, b, c are in G.P. then ak, bk, ck are also in G.P., where $k \neq 0$ and $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ also in G.P. where $k \neq 0$.

- In a G.P., the product of the terms equidistant from the beginning and from the end is always same and equal to the product of the first and the last term.

- If each term of a G.P. be raised to some power then the resulting terms are also in G.P.

- Sum of infinite G.P. is possible if $|r| < 1$ and sum is given by $\frac{a}{1-r}$.

- **Special Series:**

  (i) $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

  (ii) $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

(iii) $\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$

- Let $a_1, a_2, a_3, \ldots\ldots\ldots..$ be a sequence, then the expression $a_1 + a_2 + a_3 + \ldots\ldots\ldots..$ is called series associated with given sequence?

VERY SHORT ANSWER TYPE QUESTIONS

1. If n^{th} term of an A.P. is $6n - 7$ then write its 50^{th} term.
2. If $S_n = 3n^2 + 2n$, then write a_2
3. Which term of the sequence 3, 10, 17, ... is 136?
4. If in an A.P. 7^{th} term is 9 and 9^{th} term is 7, then find 16^{th} term.
5. If sum of first n terms of an A.P is $2n^2 + 7n$, write its n^{th} term.
6. Which term of the G.P. 2, 1, $\frac{1}{2}$, $\frac{1}{4}$ .... is $\frac{1}{1024}$?
7. If in a G.P., $a_3 + a_5 = 90$ and if $r = 2$ find the first term of the G.P.
8. In G.P. $2\sqrt{2}, 4, \ldots\ldots.., 128\sqrt{2}$, find the 4^{th} term from the end.
9. If the product of 3 consecutive terms of G.P. is 27, find the middle term.
10. Find the sum of first 8 terms of the G.P. 10, 5, $\frac{5}{2}$, ……….
11. Find the value of $5^{1/2} \times 5^{1/4} \times 5^{1/8}$ .... upto infinity.
12. Write the value of $0.\overline{3}$
13. The first term of a G.P. is 2 and sum to infinity is 6, find common ratio.

14. **Fill in the blanks**

(a) If 7^{th} and 13^{th} terms of an A.P. be 34 and 64 respectively, then 18^{th} term is _______.

(b) Geometric mean of 4 and 9 is ________.

(c) If the sum of p terms of an A.P. is q and sum of q terms is p, then the sum of p + q terms will be ________.

(d) Sum of infinity of sequence $5, \frac{5}{3}, \frac{5}{9}, \ldots\ldots\ldots$ _______.

(e) If a, b, c are in A.P. and x, y, z are in G.P., then the value of $x^{b-c} \times y^{c-a} \times z^{c-a}$ is ________.

(f) The two geometric means between numbers 1 and 64 are ____________.

15. **True / False**

(a) Common difference of an A.P. is always positive.

(b) n^{th} term of a G.P. is a + (n – 1)d.

(c) $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots + n^n = \frac{n(n-1)(2n-1)}{6}$.

(d) 2 + 4 + 6 + ………… + 2n = n(n + 1).

(e) 0.9, 0.99, 0.999, …………. from G.P.

(f) In a G.P. 5_{∞} is always not defined.

16. The interior angles of a polygon are in A.P. If the smallest angle be 120° and the common difference be 5, then the number of side is -

(a) 8 (b) 10

(c) 9 (d) 6.

17. α and β are the roots of the equation $x^2 - 3x + a = 0$ and γ and δ are the roots of the equation $x^2 - 12x + b = 0$. If α, β, γ and δ form an increasing G.P., then (a, b) -

(a) (3, 12) (b) (12, 3)

(c) (2, 32) (d) (4, 16).

18. If A be the arithmetic mean between two numbers and S be the sum of n arithmetic means between the same numbers, then -

(a) S = nA (b) A = nS

(c) A = S (d) None of these.

19. In an A.P., the m^{th} term is 1/n and n^{th} term is 1/m. What is its $(mn)^{th}$ term?

(a) 1/(mn) (b) m/n

(c) n/m (d) 1.

20. If n geometric means be inserted between a and b, then the n^{th} geometric mean will be -

(a) $a\left[\frac{b}{a}\right]^{\frac{n}{n-1}}$ (b) $a\left[\frac{b}{a}\right]^{\frac{n-1}{n}}$

(c) $a\left[\frac{b}{a}\right]^{\frac{n}{n+1}}$ (d) $a\left[\frac{b}{a}\right]^{\frac{1}{n}}$.

21. What is the 15^{th} term of the series 3, 7, 13, 21, 31, 43 -

(a) 205 (b) 225

(c) 238 (d) 241.

22. If a, b and c are in G.P., then $\frac{1}{a^2-b^2}+\frac{1}{b^2}$ is -

(a) $\frac{1}{c^2-b^2}$ (b) $\frac{1}{b^2-c^2}$

(c) $\frac{1}{c^2-a^2}$ (d) $\frac{1}{b^2-a^2}$.

23. If the 10^{th} term of a G.P. is 9^{th} and 4^{th} term is 4, then what is its 7^{th} term -

(a) 6 (b) 14

(c) 27/14 (d) 56/15.

24. If the arithmetic and geometric means of two numbers are 10 and 8 respectively, then one number exceeds the other number by -

(a) 8 (b) 10

(c) 12 (d) 16.

25. What is the sum of numbers lying between 107 and 253, which are divisible by 5 -

(a) 5220 (b) 5210

(c) 5200 (d) 5000.

26. Sum of all two digit numbers which when divided by 4 yield unity as remainder is -

(a) 1200 (b) 1210

(c) 1250 (d) None of these.

27. The first and last terms of A.P. are 1 and 11. If the sum of its term is 36, then the number of terms will be -

(a) 5 (b) 6

(c) 7 (d) 8.

28. If four numbers in A.P. are such that their sum is 50 and the greatest number is 4 times the least, then the numbers are -

(a) 5, 10, 15, 20 (b) 4, 10, 16, 22

(c) 3, 7, 11, 15 (d) None of these.

29. If the first, second and last term of an A.P. are a, b and 2a respectively, then its sum is -

(a) $\frac{ab}{2(b-a)}$ (b) $\frac{ab}{b-a}$

(c) $\frac{3ab}{2(b-a)}$ (d) None of these.

30. If p^{th}, q^{th} and r^{th} terms of an A.P. are in G.P., then the common ratio of this G.P. is -

(a) $\frac{p-q}{q-r}$ (b) $\frac{q-r}{p-q}$

(c) pqr (d) None of these.

31. The n^{th} term of a G.P. is 128 and the sum of its n term is 225. If its common ratio is 2, then the first term is -

(a) 1 (b) 3

(c) 8 (d) None of these.

32. If A be one A.M. and p, q be two GM's between two numbers, then 2A is equal to -

(a) $\dfrac{p^3+q^3}{pq}$

(b) $\dfrac{p^3-q^3}{pq}$

(c) $\dfrac{p^2+q^2}{2}$

(d) $\dfrac{pq}{2}$.

33. In a G.P. if the $(m + n)^{th}$ term is p and $(m - n)^{th}$ term is q, then its m^{th} term is -

(a) O

(b) pq

(c) $\sqrt{pq}$

(d) $\frac{1}{2}(p+q)$.

34. If $\sum n = 210$, then $\sum n^2 =$

(a) 2870

(b) 2160

(c) 2970

(d) None of these.

35. The sum of 10 terms of the series $\sqrt{2}+\sqrt{6}+\sqrt{18}+......$ is -

(a) $121\left(\sqrt{6}+\sqrt{2}\right)$

(b) $243\left(\sqrt{3}+1\right)$

(c) $\dfrac{1}{\sqrt{3}-1}$

(d) $243\left(\sqrt{3}-1\right)$.

SHORT ANSWER TYPE QUESTIONS

36. Write the n^{th} term of the series, $\dfrac{3}{7.11^2}+\dfrac{5}{8.12^2}+\dfrac{7}{9.13^2}+.........$

37. Find the number of terms in the A.P. 7, 10, 13, ..., 31.

38. In an A.P.,

8, 11, 14, ... find $S_n - S_{n-1}$

39. Find the number of squares that can be formed on chess board?

40. Find the sum of given terms:-

(a) $81 + 82 + 83 \ldots\ldots\ldots\ldots + 89 + 90$

(b) $251 + 252 + 253 + \ldots\ldots\ldots\ldots + 259 + 260$

41. (a) If a, b, c are in A.P. then show that $2b = a + c$.

(b) If a, b, c are in G.P. then show that $b^2 = a \cdot c$.

42. If a, b, c are in G.P. then show that $a^2 + b^2$, $ab + bc$, $b^2 + c^2$ are also in G.P.

SHORT ANSWER TYPE QUESTIONS

43. Find the least value of n for which

$1 + 3 + 3^2 + \ldots + 3^{n-1} > 1000$

44. Find the sum of the series

$(1 + x) + (1 + x + x^2) + (1 + x + x^2 + x^3) + \ldots\ldots\ldots\ldots$

45. Write the first negative term of the sequence $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots\ldots\ldots$

46. Determine the number of terms in A.P. 3, 7, 11, ... 407. Also, find its 11^{th} term from the end.

47. How many numbers are there between 200 and 500, which leave remainder 7 when divided by 9.

48. Find the sum of all the natural numbers between 1 and 200 which are neither divisible by 2 nor by 5.

47. Find the sum of the sequence,

$72 + 70 + 68 + \dots\dots\dots + 40$

48. If in an A.P $\frac{a_7}{a_{10}} = \frac{5}{7}$, find $\frac{a_4}{a_7}$.

49. In an A.P. sum of first 4 terms is 56 and the sum of last 4 terms is 112. If the first term is 11 then find the number of terms.

50. Solve : $1 + 6 + 11 + 16 + \dots\dots\dots + x = 148$

51. The ratio of the sum of n terms of two A.P.'s is $(7n - 1) : (3n + 11)$, find the ratio of their 10^{th} terms.

52. If the I^{st}, 2^{nd} and last terms of an A.P are a, b and c respectively, then find the sum of all terms of the A.P.

53. If $\frac{b+c-2a}{a}, \frac{c+a-2b}{b}, \frac{a+b-2c}{a}$ are in A.P. then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P. [**Hint.** : Add 3 to each term] abc.

54. The product of first three terms of a G.P. is 1000. If 6 is added to its second term and 7 is added to its third term, the terms become in A.P. Find the G.P.

55. If the continued product of three numbers in G.P. is 216 and the sum of their products in pairs is 156, find the numbers.

56. Find the sum to infinity of the series :

$$1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + \dots\dots \infty$$

57. If $A = 1 + r^a + r^{2a} + \dots\dots\dots$ up to infinity, then express r in terms of 'a' and 'A'.

58. Find the sum of first terms of the series 0.7 + 0.77 + 0.777 + …..

59. If $x = a + \frac{a}{r} + \frac{a}{r^2} + \ldots\ldots\ldots\ldots + \infty$; $y = b - \frac{b}{r} + \frac{b}{r^2} - \ldots\ldots\ldots\ldots \infty$ and

$z = c + \frac{c}{r^2} + \frac{c}{r^4} + \ldots\ldots\ldots \infty$ Prove that $\frac{xy}{z} = \frac{ab}{c}$.

60. The sum of first three terms of a G.P. is 15 and sum of next three terms is 120. Find the sum of first n terms.

61. Prove that $0.003\overline{1} = \frac{7}{225}$.

[**Hint:** 0.031 = 0.03 + 0.001 + 0.0001 +..... Now use infinite G.P.]

62. If a, b, c are in G.P. that the following are also in G.P.

(i) a^2, b^2, c^2

(ii) a^3, b^3, c^3

(iii) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in G.P.

63. If a, b, c are in A.P. that the following are also in A.P:

(i) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$

(ii) b + c, c + a, a + b

(iii) $\frac{1}{a}\left(\frac{1}{b}+\frac{1}{c}\right), \frac{1}{b}\left(\frac{1}{c}+\frac{1}{a}\right), \frac{1}{c}\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

64. If the numbers a^2, b^2 and c^2 are given to be in A.P., show that $\frac{1}{b+c}, \frac{1}{c+a}$ and $\frac{1}{a+b}$ are in A.P.

65. Show that : $0.3\overline{56} = \dfrac{353}{990}$

66. Find the sum of n terms of series :

$3 + 5 + 9 + 15 + 23 + \ldots\ldots\ldots\ldots$ n terms

67. Find the sum of n terms of series :

$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 \ldots\ldots\ldots\ldots$ n terms

68. The fourth term of a G.P. is 4. Find product of its first seven terms.

69. If A_1, A_2, A_3, A_4 are four A.M's between $\dfrac{1}{2}$ and 3, then prove $A_1 + A_2 + A_3 + A_4 = 7$.

70. If S_n denotes the sum of first n terms of an A.P. If $S_{2n} = 5S_n$, then prove $\dfrac{S_{6n}}{S_{3n}} = \dfrac{17}{4}$.

LONG ANSWER TYPE QUESTIONS

71. Prove that the sum of n numbers between a and b such that the resulting series becomes A.P. is $\dfrac{n(a+b)}{2}$.

72. A square is drawn by joining the mid points of the sides of a square. A third square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of the first square is 15 cm, then find the sum of the areas of all the squares so formed.

73. If a, b, c are in G.P., then prove that $\dfrac{1}{a^2 - b^2} - \dfrac{1}{b^2 - c^2} = -\dfrac{1}{b^2}$.

[**Hint :** Put $b = ar$, $c = ar^2$]

74. Find two positive numbers whose difference is 12 and whose arithmetic mean exceeds the geometric mean by 2.

75. If a is A.M. of b and c and c, G_1, G_2, b are in G.P., then prove that $G_1^3 + G_2^3 = 2abc$

76. Find the sum of the series,

1.3.4 + 5.7.8 + 9.11.12 + ... upto n terms.

77. Evaluate: $\sum_{r=1}^{10}(2r-1)^2$

78. The sum of an infinite G.P. is 57 and the sum of the cubes of its term is 9747, find the G.P.

79. If $10^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots\ldots + 10(11)^9 = k.(10)^9$, then find the value of k such that $k \in N$.

80. Find the sum of first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....n$ terms.

81. Three positive numbers form an increasing G.P. If the middle term in the G.P. is doubled, then new numbers are in A.P. then find the common ratio of the G.P.

82. Show that if the positive number a, b, c are in A.P. so are the numbers $\frac{1}{\sqrt{a}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P.

83. Find the sum of the series: $1-\frac{3}{2}+\frac{5}{4}-\frac{7}{8}+\frac{9}{16}.........\infty$

84. If the sum of first 'n' terms of an A.P. is $c \cdot n^2$ then prove that the sum of squares of these 'n' terms is $\frac{nc^2\left(4n^2-1\right)}{3}$.

85. Let 'p' and 'q' be the roots of the equation $x^2 - 2x + A = 0$ and let 'r' and 's' be the roots of the equation $x^2 - 18x + B = 0$ if $p < q < r < s$ are in A.P., then prove that A= –3 and B = 77.

86. If S_1, S_2, S_3 …… S_n are the sums of infinite geometric series whose first terms are 1, 2, 3, …… n and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ ...... $\frac{1}{n+1}$ respectively, then show that the value of $S_1^2 + S_2^2 + S_3^2 + ...S_{2n-1}^2 = \frac{1}{6}(2n)(2n+1)(4n+1)-1.$

87. If p^{th}, q^{th} and r^{th} terms of a G.P. are equal and are x, y and z respectively, then prove that $x^{y-z} \cdot y^{z-x} \cdot z^{x-y} = 1$.

88. The sum of infinite G.P. is 57 and sum of their cubes is 9747, find the G.P.

89. Find three numbers in G.P. whose sum is 13 and the sum of whose squares is 9.

CASE STUDY TYPE QUESTIONS

92. Abhishek buys Kisan Vikas Patra (KVP) from post office every year. Each year he exceeds the value of KVP by ₹1000 from last year's purchase. After 5 years he finds that the total value of KVP purchased by him is ₹40,000.00.

Based on the above information answer the following :-

i. The sequence of amount of KVP forms a/an

(a) Anithmetic Progression (b) Geometric Progression

(c) Harmonic Progression (d) None of these

ii. Find the amount of KVP purchased by him initially.

(a) ₹7000 (b) ₹8000

(c) ₹6000 (d) ₹7500

iii. What will be the total amount of KVP purchased by him after 10 years?

(a) ₹1,20,000 (b) ₹1,05,000

(c) ₹1,40,000 (d) ₹1,35,000

iv. What is the amount of KVP purchased by him in the 8^{th} year?

(a) ₹14,000 (b) ₹15,000

(c) ₹13,000 (d) ₹12,000

v. If he buys KVP every year for 10 years, how much will he spend in the purchase of last 4 KVP?

(a) ₹65,000 (b) ₹54,000

(c) ₹75,000 (d) None of these

93. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail it to four different persons with the instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paisa to mail one letter, anwer the following questions.

i. The sequence of letters mailed in each set forms a/an.

(a) Anithmetic Progression (b) Geometric Progression

(c) Harmonic Progression (d) None of these

ii. Find the number of letters mailed in the 4^{th} set.

(a) 64 (b) 16

(c) 256 (d) 1024

iii. Find the total number of letters mailed in the first 5 sets.

(a) 1364 (b) 1650 (c) 1236 (d) 1368

iv. Find the amount spent on the postage when 8^{th} set of letters is mailed?

(a) ₹46,930 (b) ₹54,930 (c) ₹87,380 (d) ₹43,690

v. Find the amount spent on the mailing of 9^{th} set?

(a) ₹1,74,762 (b) ₹1,31,072

(c) ₹1,54,536 (d) None of these

ANSWERS

1. 293
2. 11
3. 20^{th}
4. 0
5. $4n + 5$
6. 12th
7. $\frac{9}{2}$
8. 64
9. 3
10. $20\left(1-\frac{1}{2^8}\right)$
11. 5
12. $\frac{1}{3}$
13. $\frac{2}{3}$
14. (a) 89
    (b) 6
    (c) $-(p + q)$
    (d) 15/2
    (e) 1
    (f) 4 and 16
15. (a) False
    (b) False
    (c) True
    (d) True
    (e) False
    (f) False

16.	(c)	17.	(c)
18.	(a)	19.	(d)
20.	(c)	21.	(d)
22.	(b)	23.	(a)
24.	(c)	25.	(a)
26.	(b)	27.	(b)
28.	(a)	29.	(c)
30.	(b)	31.	(a)
32.	(a)	33.	(c)
34.	(a)	35.	(a)
36.	$\frac{2n+1}{(n+6)(n+10)^2}$	37.	9
38.	3n + 5	39.	204
40.	(a) 855 (b) 2555	43.	n = 7
44.	$\frac{n}{1-x}-\frac{x^2(1-x^n)}{(1-x)^2}$	45.	$-\frac{1}{4}$
46.	102, 367	47.	33
48.	7999	49.	952
50.	$\frac{3}{5}$	51.	11
52.	36	53.	33 : 17

54. $\frac{(b+c-2a)(a+c)}{2(b-a)}$

56. 5, 10, 20, ...; or 20, 10, 5,

57. 18, 6, 2; or 2, 6, 18

58. 6

59. $\left(\frac{A-1}{A}\right)^{\frac{1}{a}}$

60. $\frac{7}{81}\left[9n-1+10^{-n}\right]$

62. $\frac{15}{7}\left(2^{n}-1\right)$

68. $\frac{n(n^2+8)}{3}$

69. $\frac{n(n+1)}{2}$

70. 16384

74. 450 cm^2

76. 16, 4

78. $\frac{n(n+1)}{3}(48n^2-16n-14)$

79. 1330

80. 19, $\frac{38}{3}$, $\frac{76}{9}$, ...

81. k = 100

82. n + 2^{-n} – 1

83. $r=2+\sqrt{3}$

85. $\frac{2}{9}$

90. 19, 38/3, 76/9, ……

91. 1, 3, 9

92. i. (a) ii. (c) iii. (b) iv. (c) v. (b)

93. i. (b) ii. (c) iii. (a) iv. (d) v. (b)

CHAPTER - 10

STRAIGHT LINES

KEY POINTS

- Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

  $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

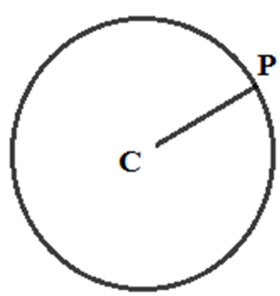
- Let the vertices of a triangle ABC are $A(x_1, y_1)$ $B(x_2, y_2)$ and $C(x_3, y_3)$.Then area of triangle

  $$ABC = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

  Note: Area of a triangle is always positive. If the above expression is zero, then a triangle is not possible. Thus the points are collinear.

- **LOCUS:** When a variable point P(x, y) moves under certain condition then the path traced out by the point P is called the locus of the point.

  For example: Locus of a point P, which moves such that its distance from a fixed point C is always constant, is a circle.



CP = constant

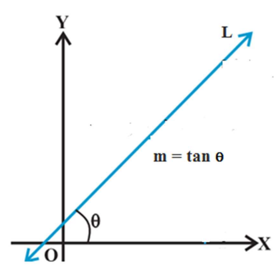
- Locus of an equation: In the coordinate plane, locus of an equation is the pictorial representation of the set of all those points which satisfy the given equation.

- Equation of a locus: is the equation in x and y that is satisfied by the coordinates of every point on the locus.

- A line is also defined as the locus of a point satisfying the condition $ax + by + c = 0$ where a, b, c are constants.

- **Slope of a straight line:**

  If θ is the inclination of a line then $\tan\theta$ is defined as slope of the straight line L and denoted by m



  $m = \tan\theta,\ \theta \neq 90°$
  If $0° < \theta < 90°$ then $m > 0$ and
  $90° < \theta < 180°$ then $m < 0$

  Note-1: The slope of a line whose inclination is 90° is not defined. Slope of x-axis is zero and slope of y-axis is not defined

  Note-2: Slope of any horizontal line i.e. || to x-axis is zero. Slope of a vertical line i.e. || to y-axis is not zero.

- Three points A, B and C lying in a plane are collinear, if slope of AB = Slope of BC.

- Slope of a line through given points (x_1, y_1) and (x_2, y_2) is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

- Two lines are parallel to each other if and only if their slopes are equal.

  i.e., $l_1 \parallel l_2 \Leftrightarrow m_1 = m_2$.

  Note: If slopes of lines l_1 and l_2 are not defined then they must be $\perp$ to x-axis, so they are ||. Thus $l_1 \parallel l_2 \Leftrightarrow$ they have same slope or both of them have not define slopes.

- Two non- vertical lines are perpendicular to each other if and only if their slopes are negative reciprocal of each other.

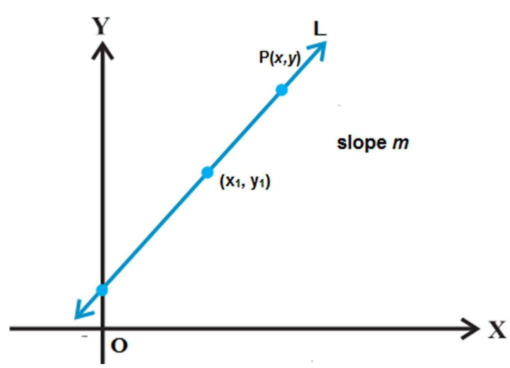
  i.e., $l_1 \perp l_2 \Leftrightarrow m_1\, m_2 = -1 \Leftrightarrow m_2 = \frac{-1}{m}$.

  Note: The above condition holds when the lines have non-zero slopes i.e none of them $\perp$ to any axis.

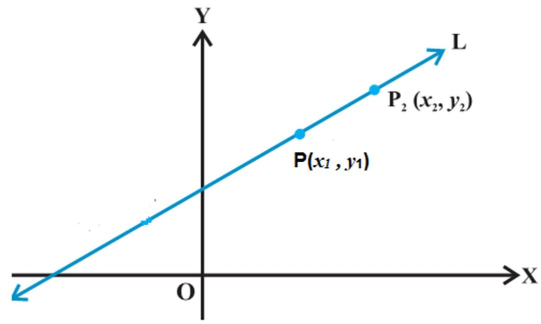
- Acute angle α between two lines, whose slopes are m_1 and m_2 is given by $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$, $1 + m_1 m_2 \neq 0$ and obtuse angle is $\phi = 180 - \alpha$.

- **Point slope form:**

  Equation of a line passing through given point (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$



- **Two Point Form:**



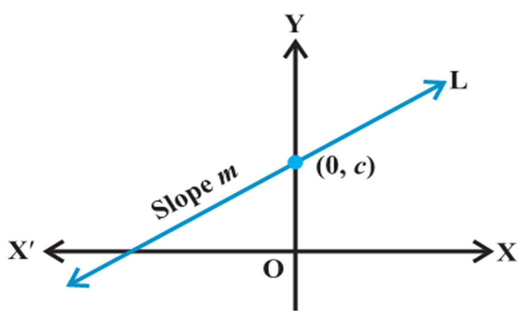
Equation of a line passing through given points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

- **Slope intercept form (y-intercept):**

Equation of a line having slope m and y-intercept 'c' is given by

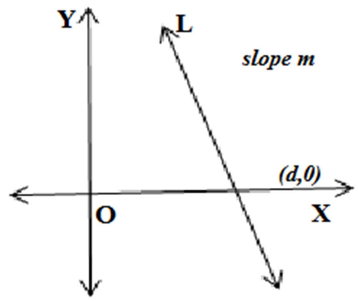
$y = mx + c$



- **Slope intercept form (x-intercept):**

Equation of a line having slope m and y-intercept c is given by

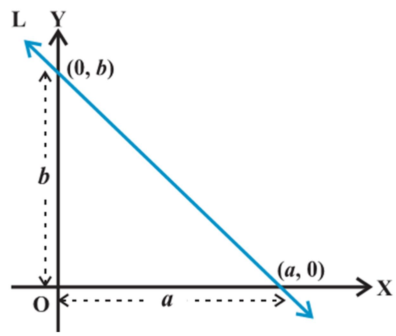
$y = m (x – d)$



- **Intercept Form:**

  Equation of line having intercepts a and b on x-axis and y-axis respectively is given by

  $$\frac{x}{a}+\frac{y}{b}=1$$

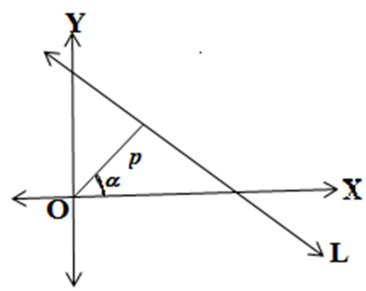


- **Normal Form:**

  Equation of line in normal form is given by x cosα + y sinα = p,

  p = Length of perpendicular segment from origin to the line

  α = Angle which the perpendicular segment makes with positive direction of x-axis



- **General Equation of a line:**

  Equation of line in general form is given by Ax + By + C = 0, A, B and C are real numbers and at least one of A or B is non-zero.

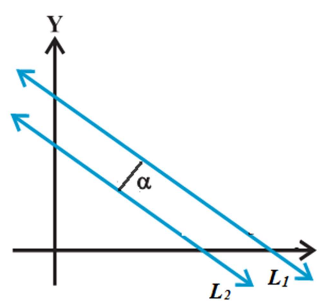
  Slope = $\frac{-A}{B}$ and y-intercept = $\frac{-C}{B}$ x-intercept = $\frac{-C}{A}$.

- Distance of a point (x_1, y_1) from line Ax + By + C = 0 is given by

  $$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

- Distance between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

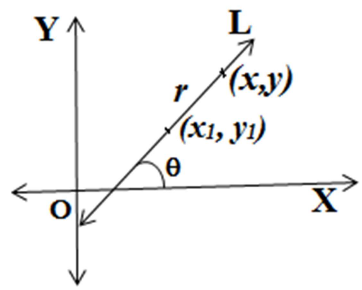
  $$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$



- **Symmetrical (or distance) Form:**

  A straight line passing through the point (x_1, y_1) and inclination θ with x-axis is given by

  $$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

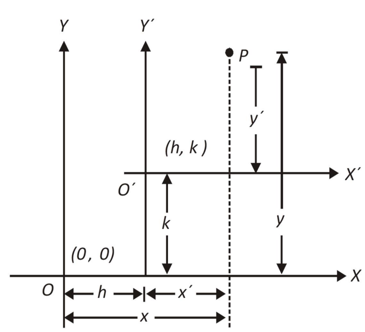


  Where r is the directed distance of any point (x, y) from the point (x_1, y_1).

- **Shifting of Origin:**

  Shifting of origin to a new point without changing the direction of the axes is known as translation of axes.

  Let OX, OY be the original axes and O' be the new origin. Let coordinates of O' referred to original axes be (h, k). Let P(x, y) be point in plane



If the origin is shifted to the point (h, k), then new coordinates (x^1, y^1) and the original coordinates (x, y) of a point are related to each other by the relation

$x' = x - h, y' = y - k$

- Equation of family of lines parallel to $Ax + By + C = 0$ is given by $Ax + By + k = 0$, for different real values of k
- Equation of family of lines perpendicular to $Ax + By + C = 0$ is given by $Bx - Ay + k = 0$, for different real values of k.
- Equation of family of lines through the intersection of lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ is given by $(A_1x + B_1y + C_1) + k(A_2x + B_2y + C_2) = 0$, for different real values of k.

VERY SHORT ANSWER TYPE QUESTIONS

1. Three consecutive vertices of a parallelogram are (–2, –1), (1, 0) and (4, 3), find the fourth vertex.
2. For what value of k are the points (8, 1), (k, –4) and (2, –5) collinear?
3. Coordinates of centroid of $\triangle ABC$ are (1, –1). Vertices of $\triangle ABC$ are A(–5, 3), B(p, –1) and C(6, q). Find p and q.
4. In what ratio y-axis divides the line segment joining the points (3,4) and (–2, 1)?
5. Show that the points (a, 0), (0, b) and (3a,– 2b) are collinear.
6. Find the equation of straight line cutting off an intercept –1 from *y* axis and being equally inclined to the axes.
7. Write the equation of a line which cuts off equal intercepts on coordinate axes and passes through (2, 5).

8. Find k so that the line 2x + ky – 9 = 0 may be perpendicular to 2x + 3y – 1 = 0

9. Find the acute angle between lines x + y = 0 and y = 0

10. Find the angle which $\sqrt{3}x + y + 5 = 0$ makes with positive direction of x-axis.

11. If origin is shifted to (2, 3), then what will be the new coordinates of (–1, 2)?

12. **Fill in the blanks**

(a) The equation of a line with slope 1/2 and making an intercept 5 on y-axis is _________.

(b) Equation of line which is parallel to y-axis and at distance 5 units from y-axis is _________.

(c) The length of perpendicular from a point (1, 2) to a line 3x + 4y + 5 = 0 is _________.

(d) The distance between the lines 3x + 4y = 9 and 6x + 8y = 15 is ________.

(e) Angle between lines 5x + y = 7 and –x + 5y = 9 is _______.

(f) Line 5x – 3y = 12 cuts y-axis at _________.

13. **True / False**

(a) Lines 3x + 2y = 12 and 6x = 4y + 8 are parallel.

(b) Acute angle between two lines with slopes m and m_2 is given by tanθ = $\left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$.

(c) Distance of Point P(–3, 4) from y-axis is 3 units.

(d) Line y = 5 is parallel to y-axis.

(e) x-intercept of line 3x – 4y + 12 = 0 is –4.

(f) $\frac{x}{a}+\frac{y}{b}=1$ is intercept from of line.

14. The angle between the straight lines $x - y\sqrt{3} = 5$ and $\sqrt{3}x + y = 7$ is -

(a) 90° (b) 60°

(c) 75° (d) 30°.

15. If p is the length of the perpendicular drawn from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$, then which one of the following is correct?

(a) $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$ (b) $\frac{1}{p^2}=\frac{1}{a^2}-\frac{1}{b^2}$

(c) $\frac{1}{p}=\frac{1}{a}+\frac{1}{b}$ (d) $\frac{1}{p}=\frac{1}{a}-\frac{1}{b}$.

16. What is the equation of the line passing through (2, –3) and parallel to y-axis?

(a) y = –3 (b) y = 2

(c) x = 2 (d) x = –3.

17. If the lines 3x + 4y + 1 = 0, 5x + λy + 3 = 0 and 2x + y – 1 = 0 are concurrent, then λ is equal to -

(a) –8 (b) 8

(c) 4 (d) –4.

18. If $x \cos\theta + y \sin\theta = 2$ is perpendicular to the line $x - y = 3$, then what is one of the value of θ ?

(a) $\pi / 16$ (b) $\pi / 4$

(c) $\pi / 2$ (d) $\pi / 3$.

19. The x-intercept and the y-intercept of the line $5x - 7 = 6y$, respectively are -

(a) $\frac{7}{5}$ and $\frac{7}{6}$ (b) $\frac{7}{5}$ and $\frac{-7}{6}$

(c) $\frac{5}{7}$ and $\frac{6}{7}$ (d) $\frac{-5}{7}$ and $\frac{6}{7}$.

20. If p be the length of the perpendicular from the origin on the straight line $x + 2y = 2p$, then what is the value of b?

(a) 1/p (b) p

(c) 1/2 (d) $\sqrt{3}/2$.

21. If we reduce $3x + 3y + 7 = 0$ to the form $x \cos\alpha + y \sin\alpha = 9$, then the value of p is -

(a) $\frac{7}{2\sqrt{3}}$ (b) $\frac{7}{3}$

(c) $\frac{3\sqrt{7}}{2}$ (d) $\frac{7}{3\sqrt{2}}$.

22. A straight line through P(1, 2) is such that its intercept between the axes is bisected at P. Its equation is -

(a) $x + y = -1$ (b) $x + y = 3$

(c) $x + 2y = 5$ (d) $2x + y = 4$.

23. If the lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent, then what is the value of b?

(a) 1 (b) 3

(c) 6 (d) 0.

24. The triangle formed by the lines $x + y = 0$, $3x + y = 4$ and $x + 3y = 4$ is -

(a) Isosceles (b) Equilateral

(c) Right angled (d) None of these.

25. What is the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$?

(a) (1, 10) (b) (5, 6)

(c) (6, 5) (d) (7, 4).

SHORT ANSWER TYPE QUESTIONS

26. On shifting the origin to (p, q), the coordinates of point (2, –1) changes to (5, 2). Find p and q.

27. Determine the equation of line through a point (–4, –3) and parallel to *x*-axis.

28. Check whether the points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are the vertices a triangle or not?

29. If a vertex of a triangle is (1, 1) and the midpoints of two sides through this vertex are (–1, 2) and (3, 2). Then find the centroid of the triangle.

30. If the medians through A and B of the triangle with vertices A(0, b), B(0, 0) and C(a, 0) are mutually perpendicular. Then show that $a^2 = 2b^2$.

SHORT ANSWER TYPE QUESTIONS

31. If the image of the point (3, 8) in the line $px + 3y - 7 = 0$ is the point (–1, –4), then find the value of p.

32. Find the distance of the point (3,2) from the straight line whose slope is 5 and is passing through the point of intersection of lines $x + 2y = 5$ and $x - 3y + 5 = 0$

33. The line $2x - 3y = 4$ is the perpendicular bisector of the line segment AB. If coordinates of A are (–3, 1) find coordinates of B.

34. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line $y = 2x + c$. Find c and remaining two vertices.

35. If two sides of a square are along $5x - 12y + 26 = 0$ and $5x - 12y - 65 = 0$ then find its area.

36. Find the equation of a line with slope –1 and whose perpendicular distance from the origin is equal to 5.

37. If a vertex of a square is at (1, –1) and one of its side lie along the line $3x - 4y - 17 = 0$ then find the area of the square.

38. What is the value of y so that line through (3, y) and (2, 7) is parallel to the line through (–1, 4) and (0, 6)?

39. In what ratio, the line joining (–1, 1) and (5, 7) is divided by the line $x + y = 4$?

40. Find the equation of the lines which cut-off intercepts on the axes whose sum and product are 1 and –6 respectively.

41. Find the area of the triangle formed by the lines $y = x$, $y = 2x$, $y = 3x + 4$.

42. Find the coordinates of the orthocentre of a triangle whose vertices are (–1, 3) (2, –1) and (0, 0). [Orthocentre is the point of concurrency of three altitudes].

43. Find the equation of a straight line which passes through the point of intersection of $3x + 4y - 1 = 0$ and $2x - 5y + 7 = 0$ and which is perpendicular to $4x - 2y + 7 = 0$.

44. If the image of the point (2, 1) in a line is (4, 3) then find the equation of line.

45. The vertices of a triangle are (6, 0), (0, 6) and (6, 6). Find the distance between its circumcenter and centroid.

LONG ANSWER TYPE QUESTIONS

46. Find the equation of a straight line which makes acute angle with positive direction of x–axis, passes through point (–5, 0) and is at a perpendicular distance of 3 units from origin.

47. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1, 1). Find the equation of other three sides.

48. If (1, 2) and (3, 8) are a pair of opposite vertices of a square, find the equation of the sides and diagonals of the square.

49. Find the equations of the straight lines which cut off intercepts on x-axis twice that on y-axis and are at a unit distance from origin.

50. Two adjacent sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals is $11x + 7y = 4$, find the equation of the other diagonal.

51. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain its equation.

52. If one diagonal of a square is along the line $8x - 15y = 0$ and one of its vertex is at (1, 2), then find the equation of sides of the square passing through this vertex.

53. If the slope of a line passing through to point A(3, 2) is 3/4 then find points on the line which are 5 units away from the point A.

54. Find the equation of straight line which passes through the intersection of the straight line $3x + 2y + 4 = 0$ and $x - y - 2 = 0$ and forms a triangle with the axis whose area is 8 sq. unit.

55. Find points on the line $x + y + 3 = 0$ that are at a distance of 5 units from the line $x + 2y + 2 = 0$

56. Show that the locus of the midpoint of the distance between the axes of the variable line $x \cos\alpha + y \sin\alpha = p$ is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$, where p is a constant.

57. The line $\frac{x}{a} + \frac{y}{b} = 1$ moves in such a way that $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ where c is constant. Show that the locus of the foot of perpendicular from the origin to the given line is $x^2 + y^2 = c^2$.

58. A point p is such that the sum of squares of its distance from the axes of coordinates is equal to the square of its distance from the line $x - y = 1$. Find the locus of P.

59. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L and the coordinate axes is 5. Find the equation of the line L.

60. The vertices of a triangle are $[at_1t_2, a(t_1 + t_3)]$, $[at_2t_3, a(t_2 + t_3)]$. Find the orthocentre of the triangle.

61. Two equal sides of an isosceles triangle are given by the equation $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side pass through the point $(1, -10)$. Determine the equation of the third side.

62. Let A(2, –3) and B(–2, 1) be the vertices of a ΔABC. If the centroid of this triangle moves on the line $2x + 3y = 1$. Then find the locus of the vertex C.

63. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2 AC. If the coordinates of D and M are (1, 1) and (2, –1) respectively. Then find the coordinates of A.

64. Find the area enclosed within the curve $|x| + |y| = 1$.

65. If the area of the triangle formed by a line with coordinates axes is $54\sqrt{3}$ square units and the perpendicular drawn from the origin to the line makes an angle 60° with the x-axis, find the equation of the line.

66. Find the coordinators of the circumcentre of the triangle whose vertices are (5,7), (6,6) and (2, -2).

67. Find the equation of a straight line, which passes through the point (a, 0) and whose $\perp$ distance from the point (2a, 2a) is a.

68. Line L has intercepts a and b on the coordinate axis when the axis are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then prove that $a^{-2} + b^{-2} = p^{-2} + q^{-2}$.

CASE STUDY TYPE QUESTIONS

69. A person is standing at a point A of a triangular park ABC whose vertices are A(2, 0), B(3, 4) and C(5, 6).

Based on the above information answer the following :-

i. He wants to reach BC in least time. Find the equation of the path he should follow.

(a) $2x + y = 3$ (b) $2x + 3y = 4$

(c) $x + y = 2$ (d) $x + 4y = 7$

ii. Find the shortest distance travelled by him to reach BC -

(a) $\frac{5}{2}\sqrt{2}$ units (b) $\frac{3}{2}\sqrt{2}$ units

(c) $\frac{4}{3}\sqrt{2}$ units (d) $\frac{7}{3}\sqrt{2}$ units

iii. Suppose he meets BC at a point D. Find the coordiantors of the point D.

(a) $\left(\frac{5}{2}, \frac{7}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (c) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{7}{2}, \frac{5}{2}\right)$

iv. Find the area of the triangular park ABC.

(a) 5 sq units (b) 10 sq units

(c) 3 sq units (d) None of these

v. Find the coordinator of the centroid of the triangular park ABC?

(a) $\left(\frac{5}{3}, \frac{7}{3}\right)$ (b) $\left(\frac{10}{3}, \frac{10}{3}\right)$ (c) $\left(\frac{7}{3}, \frac{8}{3}\right)$ (d) $\left(\frac{2}{3}, \frac{8}{3}\right)$

ANSWERS

1.	(1, 2)	2.	k = 3
3.	p = 2, q = –5	4.	3 : 2 (internally)
6.	y = x – 1 and y = – x – 1.	7.	$x + y = 7$
8.	$\frac{4}{3}$	9.	$\frac{\pi}{4}$
10.	$\frac{2\pi}{3}$	11.	(–3, –1)
12.	(a) $y = \frac{x}{2} + 5$	13.	(a) False
	(b) x = 5		(b) True
	(c) 16/5		(c) True
	(d) 3/10		(d) False
	(e) 90°		(e) True
	(f) (0, –4)		(f) True
14.	(a)	15.	(a)
16.	(c)	17.	(b)
18.	(b)	19.	(b)
20.	(d)	21.	(d)
22.	(d)	23.	(c)
24.	(a)	25.	(b)
26.	p = –3, q = –3	27.	y + 3 = 0

28. No

29. $\left(1, \frac{7}{3}\right)$

31. 1

32. $\frac{10}{\sqrt{26}}$

33. (1, –5)

34. c = –4, (2, 0), (4, 4)

35. 49 square units

36. $x + y + 5\sqrt{2} = 0, \; x + y - 5\sqrt{2} = 0$

37. 4 square units

38. y = 9

39. 1 : 2

40. $2x - 3y - 6 = 0$ and $-3x + 2y - 6 = 0$

41. 4 square units

42. (–4, –3)

43. $x + 2y = 1$

44. $x + y - 5 = 0$

45. $3\sqrt{2}$

46. $3x - 4y + 15 = 0$

47. $4x + 7y - 11 = 0, 7x - 4y + 25 = 0$
    $7x - 4y - 3 = 0$

48. $x - 2y + 3 = 0, 2x + y - 14 = 0,$
    $x - 2y + 13 = 0, 2x + y - 4 = 0$
    $3x - y - 1 = 0, x + 3y - 17 = 0$

49. $x + 2y + \sqrt{5} = 0, \; x + 2y - \sqrt{5} = 0$

50. x = y

51. $107x - 3y - 92 = 0$

52. $23x - 7y - 9 = 0$ and $7x + 23y - 53 = 0$

53. $(-1, -1)$ or $(7, 5)$

54. $x - 4y - 8 = 0$ or $x + 4y + 8 = 0$

55. $(1, -4), (-9, 6)$

58. $x^2 + y^2 + 2xy + 2x - 2y - 1 = 0$

59. $x + 5y = \pm 5\sqrt{2}$

60. $\left[-a, a(t_1 + t_2 + t_3 + t_1 t_2 t_3)\right]$

61. $x - 3y - 31 = 0$, $3x + y + 7 = 0$

62. $\frac{x}{-2} + \frac{y}{1} = 1$

63. $\left(1, \frac{-3}{2}\right)$ or $\left(3, \frac{-1}{2}\right)$

64. $\sqrt{3}$

65. $x + \sqrt{3}y = 18$

66. $(2, 3)$

67. $3x - 4y - 3a = 0$ and $x - a = 0$

69. i. (c) ii. (b) iii. (b) iv. (c) v. (b)

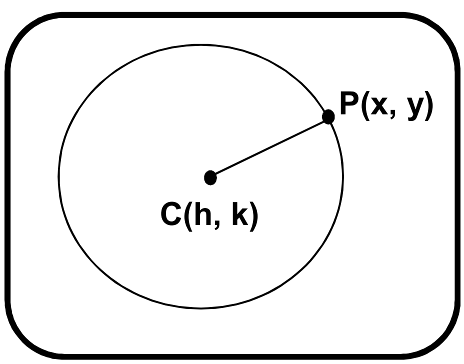
CHAPTER - 11

CONIC SECTIONS

KEY POINTS

- The curves obtained by slicing the cone with a plane not passing through the vertex are called conic sections or simply conics.
- Circle, ellipse, parabola and hyperbola are curves which are obtained by intersection of a plane and cone in different positions.
- A conic is the locus of a point which moves in a plane, so that its distance from a fixed point bears a constant ratio to its distance from a fixed straight line.
- The fixed point is called focus, the fixed straight line is called directrix, and the constant ratio is called eccentricity, which is denoted by '*e*'.
- **Circle:** It is the set of all points in a plane that are equidistant from a fixed point in that plane

  Equation of circle: $(x - h)^2 + (y - k)^2 = r^2$ where Centre (h, k), radius = r



C (h, k)

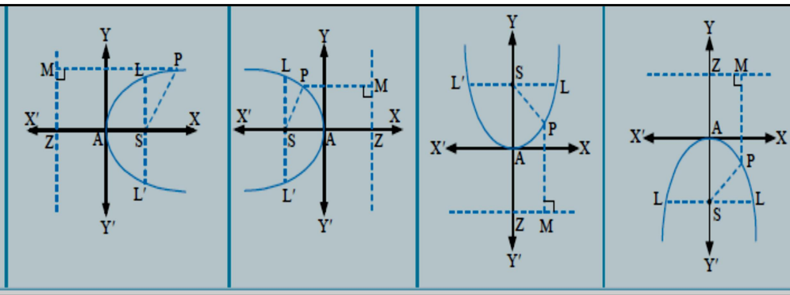
CP = CONSTANT = r

❖ **Parabola:** It is the set of all points in a plane which are equidistant from a fixed point (focus) and a fixed line (directrix) in the plane. Fixed point does not lie on the line

	$y^2 = 4ax$ *Parabola towards right*	$y^2 = -4ax$ *Parabola towards left*	$x^2 = 4ay$ *Parabola opening upwards*	$x^2 = -4ay$ *Parabola opening downwards*
Vertex	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Equation of axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Equation of directrix	$x + a = 0$	$x - a = 0$	$y + a = 0$	$y - a = 0$
Length of latus rectum	$4a$	$4a$	$4a$	$4a$

Note: In the standard equation of parabola, a > 0.

Figure

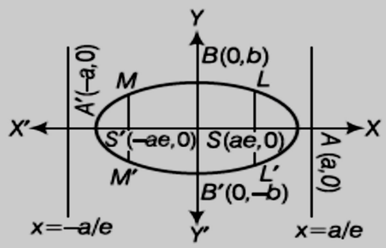
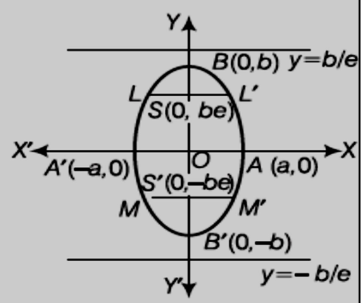


$y^2 = 4ax$ $y^2 = -4ax$ $x^2 = 4ay$ $x^2 = -4ay$

Note: In the figure above, A represents the vertex, S represents the Focus, LL' represents the Latus Rectum and Line MZ represents the Directrix to the parabola.

- **Latus Rectum:** A chord through focus perpendicular to axis of parabola is called its latus rectum.

- **Ellipse:** It is the set of points in a plane the sum of whose distances from two fixed points in the plane is a constant and is always greater than the distances between the fixed points.

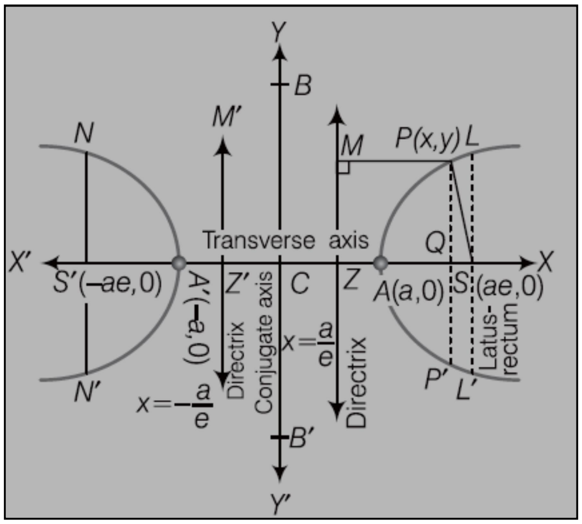
Standard equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\,(a>b)$ (Horizontal form of an ellipse)	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\,(a<b)$ (Vertical form of an ellipse)
Shape of the ellipse		
Centre	(0, 0)	(0, 0)
Equation of major axis	$y = 0$	$x = 0$
Equation of minor axis	$x = 0$	$y = 0$
Length of major axis	$2a$	$2b$
Length of minor axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Equation of directrices	$x = \pm\frac{a}{e}$	$y = \pm\frac{b}{e}$
Eccentricity	$e = \sqrt{\frac{a^2-b^2}{a^2}}$	$e = \sqrt{\frac{b^2-a^2}{b^2}}$
Length of latusrectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

Note: If e = 0 for an ellipse then b = a and equation of ellipse will be converted in equation of the circle. Its eq. will be $x^2 + y^2 = a^2$. It is called auxiliary circle. For auxiliary circle, diameter is equal to length of major axis and e = 0.

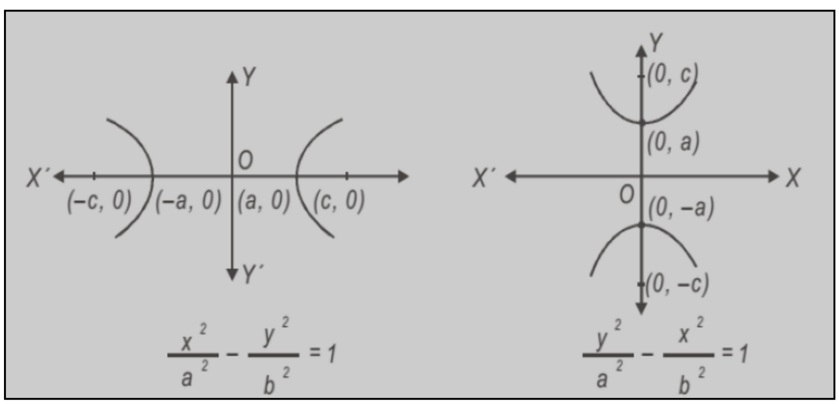
- **Latus rectum:** Chord through foci perpendicular to major axis called latus rectum.
- **Hyperbola:** It is the set of all points in a plane, the differences of whose distance from two fixed points in the plane is a constant.

	Hyperbola	**Conjugate hyperbola**
Standard equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$
Centre	(0, 0)	(0, 0)
Equation of transverse axis	$y = 0$	$x = 0$
Equation of conjugate axis	$x = 0$	$y = 0$
Length of transverse axis	$2a$	$2b$
Length of conjugate axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{b}{e}$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Eccentricity	$e = \sqrt{\frac{a^2 + b^2}{a^2}}$	$e = \sqrt{\frac{a^2 + b^2}{b^2}}$
Length of latusrectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

- **STANDARD HYPERBOLA:**



- **STANDARD HYPERBOLA CONJUGATE HYPERBOLA:**



- **Latus Rectum:** Chord through foci perpendicular to transverse axis is called latus rectum.

  If $e = \sqrt{2}$ for hyperbola, then hyperbola is called rectangular hyperbola.

  For $e = \sqrt{2}$ then b = a and eq. of its hyperbola will be $x^2 - y^2 = a^2$ or $y^2 - x^2 = a^2$.

VERY SHORT ANSWER TYPE PROBLEMS

1. **Fill up in each of the following:**

(a) The centre of the circle $3x^2 + 3y^2 + 6x - 12y - 6 = 0$ is ___.

(b) The radius of the circle $3x^2 + 3y^2 + 6x - 12y - 15 = 0$ is ___.

(c) The equation of circle whose end points of one of its diameter are $(-2, 3)$ and $(0, -1)$ is _______.

(d) If parabola $y^2 = px$ passes through point $(2, -3)$, then the length of latus rectum is _______.

(e) The coordinates of focus of parabola $3y^2 = 8x$ is _____.

(f) The equation of the circle which passes through the point $(4, 6)$ and has its centre at $(1, 2)$ is _______.

(g) The equation of the ellipse having foci $(0, 3)$, $(0, -3)$ and minor axis of length 8 is ___________.

(h) The length of the latus rectum of the ellipse $3x^2 + y^2 = 12$ is _____________.

(i) The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half of the distance between the foci is _______________.

2. **State whether the following are true or false.**

(a) $2x^2 + 2y^2 + 3y + 10 = 0$ represent the equation of a circle.

(b) Latus rectum is the smallest focal chord of any parabola.

(c) The length of latus rectum of parabola $3y^2 = 8x$ is 8.

(d) The point $(-1, 5)$ lies inside the circle $x^2 + y^2 - 2x + 6y + 1 = 0$.

(e) The point $(2, 3)$ lies outside the circle $x^2 + y^2 - 2x + 6y + 1 = 0$.

Note: Q.3 – Q.10 are Multiple Choice Questions (MCQ), select the correct alternatives out of given four alternatives in each.

3. The equation of the circle which passes through the points of intersection of the circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 6y = 0$ and has its centre at (3/2, 3/2) is -

   (a) $x^2 + y^2 + 3x + 3y + 9 = 0$

   (b) $x^2 + y^2 + 3x + 3y = 0$

   (c) $x^2 + y^2 - 3x - 3y = 0$

   (d) $x^2 + y^2 - 3x - 3y + 9 = 0$.

4. The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ -

   (a) (4, 9) (b) (9, 4)

   (c) (7, 4) (d) (4, 7).

5. Value of p, for which the equation $x^2 + y^2 - 2px + 4y - 12 = 0$ represent a circle of radius 5 units is -

   (a) 3 (b) – 3

   (c) both (a) & (b) (d) Neither (a) nor (b).

6. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is 'e' then the value of '5e' is -

   (a) 3 (b) 4

   (c) 2 (d) 1.

7. The centre of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is (a, b) then (2a + 3 b) is -

   (a) 0 (b) 2

   (c) 3 (d) 5.

8. The radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is -

(a) 1 (b) 2

(c) 3 (d) 5.

9. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 8y$ to the ends of its latus rectum is -

(a) 4 sq. units (b) 8 sq. units

(c) 12 sq. units (d) 16 sq. units.

10. Match the following:

	COLUMN 1 Conic		**COLUMN 2 Eccentricity**
A	CIRCLE	P	$e < 1$
B	PARABOLA	Q	$e > 1$
C	ELLIPSE	R	$e = 0$
D	HYPERBOLA	S	$e = 1$

Which one of the following is true?

(a) $A \rightarrow P, B \rightarrow Q, C \rightarrow R, D \rightarrow S$

(b) $A \rightarrow S, B \rightarrow Q, C \rightarrow R, D \rightarrow P$

(c) $A \rightarrow Q, B \rightarrow S, C \rightarrow R, D \rightarrow P$

(d) $A \rightarrow R, B \rightarrow S, C \rightarrow P, D \rightarrow Q$

VERY SHORT ANSWER TYPE QUESTIONS

11. If the lines $5x + 12y = 3$ and $10x + 24y - 58 = 0$ are tangents to a circle, then find the radius of the circle.

12. Find the length of major and minor axis of the following ellipse, $16x^2 + 25y^2 = 400$.

13. Find the eqn. of hyperbola satisfying given conditions foci $(\pm 5, 0)$ and transverse axis is of length 8.

14. Find the coordinates of points on parabola $y^2 = 8x$ whose focal distance is 4.

15. Find the distance between the directrices to the ellipse $\frac{x^2}{36}+\frac{y^2}{20}=1$.

16. If the eccentricity of the ellipse is zero. Then show that ellipse will be a circle.

17. If the eccentricity of the hyperbola is $\sqrt{2}$. Then find the general equation of hyperbola.

18. A circle is circumscribed on an equilateral Triangle ABC where AB = 6 cm. The area of the Circumcircle is $K\pi$ cm^2. Find the value of K.

SHORT ANSWER TYPE QUESTIONS

19. Find equation of an ellipse having vertices (0, ±5) and foci (0, ±4).

20. If the distance between the foci of a hyperbola is 16 and its eccentricity is 2, then obtain the equation of a hyperbola.

21. Find the equation for the ellipse that satisfies the given condition Major axis on the *x*-axis and passes through the points (4, 3) and (6, 2).

22. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the coordinates of the other end of diameter.

23. Find the equation of the ellipse with foci at (± 5, 0) and x = 1.8 as one of the directrices.

24. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, find the equation of the hyperbola if its eccentricity is 2.

25. Find the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ which passes through the points (3, 0) and $(3\sqrt{2}, 2)$.

26. If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentricity.

27. Find equation of circle concentric with circle $4x^2 + 4y^2 - 12x - 16y - 21 = 0$ and of half its area.

28. Find the equation of a circle whose centre is at (4, –2) and $3x - 4y + 5 = 0$ is tangent to circle.

29. If equation of the circle is in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$ then prove that its centre and radius will be $(-g, -f)$ and $\sqrt{g^2+f^2-c}$ respectively.

30. If the end points of a diameter of circle are (x_1, y_1) and (x_2, y_2) then show that equation of circle will be $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

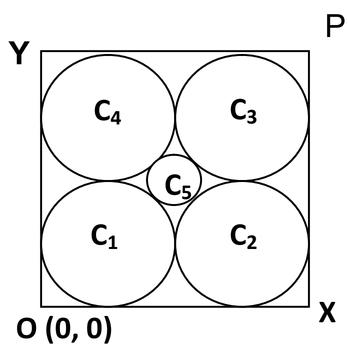
31. Find the equation of the circle which touches the lines $x = 0$, $y = 0$ and $x = 2c$ and $c > 0$.

32. Find the equation of parabola if its focus at (–1, –2) and equation of directrix is $x - 2y + 3 = 0$.

33. Find the equation of the set of all points the sum of whose distance from A(3, 0) and B(9, 0) is 12 unit. Identify the curve thus obtained.

34. Find the equation of the set of all points such that the difference of their distance from (4, 0) and (–4, 0) is always equal of 2 unit. Identify the curve thus obtained.

35. If OXPY is a square of Side 4 cm in First Quadrant, where O is the origin. (OY and OX are lies y-axis and x-axis respectively). Find the equation of the circle C_1, C_2, C_3, C_4 and C_5.



LONG ANSWER TYPE QUESTIONS

36. Prove that the points (1, 2), (3, – 4), (5, – 6) and (11, – 8) are concyclic.

37. A circle has radius 3 units and its centre lies on the line y = x – 1. If it is passes through the point (7, 3) then find the equations of the circle.

38. Find the equation of the circle which passes through the points (20, 3), (19, 8) and (2, – 9). Find its centre and radius.

39. Find the equation of circle having centre (1, – 2) and passing through the point of intersection of the lines 3x + y = 14 and 2x + 5y = 18.

40. Prove that the equation $y^2 + 2Ax + 2By + c = 0$ is represent a parabola and whose axis is parallel to x axis.

41. Show that the points A(5,5), B(6,4), C(–2,4) and D(7,1) all lies on the circle. Find the centre, radius and equation of circle.

42. Find the equation of the ellipse in which length of minor axis is equal to distance between foci. If length of latus rectum is 10 unit and major axis is along the x axis.

43. Find the equation of the hyperbolas whose axes (transverse and conjugate axis) are parallel to x axis and y axis and centre is origin such that Length of latus rectum length is 18 unit and distance between foci is 12 unit.

44. Prove that the line $3x + 4y + 7 = 0$ touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$. Also find the point of contact.

45. Find the equation of ellipse whose focus is (1, 0) and the directrix $x + y + 1 = 0$ and eccentricity is equal to $\frac{1}{\sqrt{2}}$.

46. If y_1, y_2, y_3 be the ordinates of a vertices of the triangle inscribed in a parabola $y^2 = 4ax$, then show that the area of the triangle is $\frac{1}{8a}\left|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)\right|$.

47. Find the equations of tangents to the circle

(a) $x^2 + y^2 - 2x - 4y - 4 = 0$ which are parallel to $3x - 4y - 1 = 0$

(b) $x^2 + y^2 - 4x - 6y - 12 = 0$ which are perpendicular to $4x + 3y = 7$

48. Find the equation of parabola whose focus is (1, –1) and whose vertex is (2, 1). Also, find its axis and latus rectum.

49. Find the equation of hyperbola whose focus is (1, 2), the directrix $2x + y = 1$ and eccentricity is equal to $\sqrt{3}$.

50. Find the equation of Circle in each of the following cases:

(a) Touches both the coordinate axes in first quadrant and having radius = 1 unit

(b) Touches both the coordinate axes in second quadrant and having radius = 2 units

(c) Touches both the coordinate axes in third quadrant and having radius = 3 units

(d) Touches both the coordinate axes in fourth quadrant and having radius = 4 units

(e) Touches the x-axis at origin and having radius = 5 units

(f) Touches the y-axis at origin and having radius = 6 units

CASE STUDY TYPE QUESTIONS

51. A beam is supported at its ends by supports which are 12m apart. Since the load is concentrated at its centre, there is a deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola.

Based on the above information answer the following :-

i. How for from the centre is deflection of 1cm?

(a) $2\sqrt{6}$ m (b) $3\sqrt{6}$ m

(c) $2\sqrt{3}$ m (d) $4\sqrt{3}$ m

ii. What will be the equation of parabola?

(a) $x^2 = 2400y$ (b) $x^2 = 1200y$

(c) $x^2 = 1600y$ (d) $x^2 = 1000y$

iii. At a distance of 2m from the centre, what will be the deflection of the beam?

(a) $\frac{2}{300}$ (b) $\frac{8}{300}$ (c) $\frac{3}{400}$ (d) $\frac{1}{500}$

iv. What is the length of latus rectum of the parabola?

(a) 1000 (b) 1200 (c) 1300 (d) 1400

v. What is the difference of deflection of beam at a distance of 1m and 2m from the centre?

(a) $\frac{1}{300}$ (b) $\frac{1}{500}$ (c) $\frac{1}{400}$ (d) $\frac{3}{700}$

52. A window is in the shape of parabola with a triangle inscribed in it. The triangle is formed in such a way that the verticles of triangle coincides with vertex of parabola and end points of latus rectum. The equation of parabola is given by $x^2 - 24y$.

Based on the above information answer the following :-

i. Which type of parabola is represented by the given equation?

(a) Parabola towards right

(b) Parabola towards left

(c) Parabola opening upwards

(d) Parabola opening downwards

ii. Find the length of altitude of the triangle -

(a) 12 units (b) 6 units

(c) 18 units (d) 3 units

iii. Find the area of the triangle?

(a) 60 sq. units (b) 96 sq. units

(c) 72 sq. units (d) 110 sq. units

iv. Find the length of the longest side of the triangle?

(a) $6\sqrt{5}$ units (b) 24 units

(c) 12 units (d) 48 units

v. Find the length of latus rectum of the parabola?

(a) 24 units (b) 12 units

(c) 6 units (d) 48 units

ANSWERS

1.	(a)	$(-2, 4)$	2.	(a)	False
	(b)	5 Units		(b)	True
	(c)	$x^2 + y + 2x - 2y - 3 = 0$		(c)	False
	(d)	4.5 units		(d)	True
	(e)	$\left(\frac{2}{3}, 0\right)$		(e)	False
	(f)	$(x - 4)^2 + (y - 2)^2 = 25$		(f)	False
3.	(c)		4.	(d)	
5.	(c)		6.	(b)	
7.	(a)		8.	(d)	
9.	(b)		10.	(d)	

11. 2 units

12. Length of Major Axis = 10

Length of Major Axis = 8

13. $\frac{x^2}{16}-\frac{y^2}{9}=1$

14. (2, ±4)

15. 18

17. $x^2 - y^2 = a^2$ or $y^2 - x^2 = a^2$

18. K = 12

19. $\frac{x^2}{9}+\frac{y^2}{25}=1$

20. $x^2 - y^2 = 32$

21. $\frac{x^2}{52}+\frac{y^2}{13}=1$

22. (1, 2)

23. $\frac{x^2}{36}+\frac{y^2}{11}=1$

24. $\frac{x^2}{4}-\frac{y^2}{12}=1$

25. $e=\frac{\sqrt{13}}{3}$

26. $e=\frac{\sqrt{3}}{2}$

27. $2x^2 + 2y^2 - 6x + 8y + 1 = 0$

28. $x^2 + y^2 - 8x + 4y - 5 = 0$

31. $x^2 + y^2 - 2cx \pm 2cy + c^2 = 0$

32. $4x^2 + 4xy + y^2 + 4x + 32y + 16 = 0$

33. $3x^2 + 4y^2 = 36$, Ellipse

34. $15x^2 - y^2 = 15$, Hyperbola

35. $C_1 : (x-1)^2 + (y-1)^2 = 1$

$C_2 : (x-3)^2 + (y-1)^2 = 1$

$C_3 : (x-3)^2 + (y-3)^2 = 1$

$C_4 : (x-1)^2 + (y-3)^2 = 1$

$C_5 : (x-2)^2 + (y-2)^2 = \left(\sqrt{2}-1\right)^2$

37. $x^2 + y^2 - 8x - 6y + 16 = 0$ or

$x^2 + y^2 - 14x - 12y + 76 = 0$

38. $x^2 + y^2 - 14x - 6y - 111 = 0$

Centre (7, 3), Radius = 13 units

39. $(x - 1)^2 + (y + 2)^2 = 25$

41. $x^2 + y^2 - 4x - 2y - 20 = 0$

Centre (2, 1), Radius = 5 units

42. $x^2 + 2y^2 = 100$

43. $3x^2 - y^2 = 27$

44. Point of contact = (–1, –1)

45. $3x^2 - 2xy + 3y^2 - 10x - 2y + 3 = 0$

47. (a) $3x - 4y - 10 = 0$ or $3x - 4y + 20 = 0$

(b) $3x - 4y + 31 = 0$ or $3x - 4y - 19 = 0$

48. $4x^2 - 4xy + y^2 + 8x + 46y - 71 = 0$;

Eq. of axis: $2x - y - 3 = 0$,

length of L.R. = $4\sqrt{5}$

49. $7x2 + 12xy - 2y2 - 2x + 14y - 22 = 0$

50. (a) $(x - 1)^2 + (y - 1)^2 = 1$ (b) $(x + 2)^2 + (y - 2)^2 = 4$

(c) $(x + 3)^2 + (y + 3)^2 = 9$ (d) $(x - 4)^2 + (y + 4)^2 = 16$

(e) $x^2 + (y \pm 5)^2 = 25$ (f) $(x \pm 6)^2 + y^2 = 36$

51. i. (a) ii. (b) iii. (b) iv. (b) v. (c)

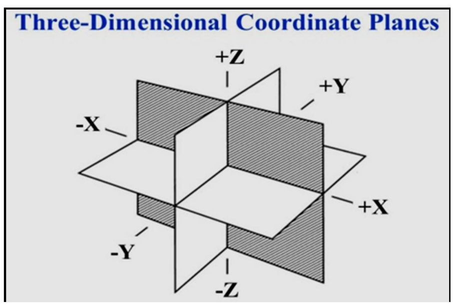
52. i. (d) ii. (b) iii. (c) iv. (b) v. (a)

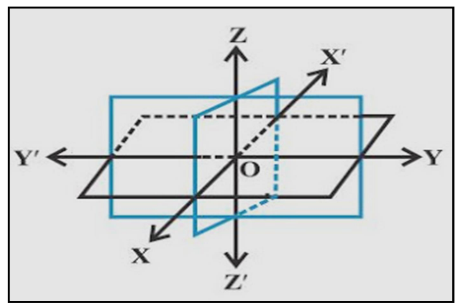
CHAPTER - 12

INTRODUCTION TO THREE-DIMENSIONAL COORDINATE GEOMETRY

KEY POINTS

- Three mutually perpendicular lines in space define three mutually perpendicular planes, called Coordinate planes, which in turn divide the space into eight parts known as octants and the lines are known as Coordinate axes.





❖ **Coordinate axes:** XOX', YOY', ZOZ'

❖ **Coordinate planes:** XOY, YOZ, ZOX or XY, YX, ZX planes

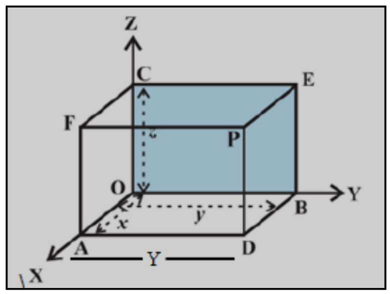
❖ **Octants:** OXYZ, OX'YZ, OXY'Z, OXYZ', OX' Y'Z, OXY'Z', OX'YZ', OX'Y'Z'

❖ Coordinates of a points lying on x-axis, y-axis and z-axis are of the form (x, 0, 0), (0, y, 0), (0, 0, z) respectively.

❖ Coordinates of a points lying on xy-plane, yz-plane and xz-plane are of the form (x, y, 0), (0, y, z), (x, 0, z) respectively.

- The reflection of the point (x, y, z) in xy-plane, yz-plane and xz-plane is (x, y, –z), (–x, y, z) and (x, –y, z) respectively.

- Absolute value of the Coordinates of a point P (x, y, z) represents the perpendicular distances of point P from three coordinate planes YZ, ZX and XY respectively.



- The distance between the point A(x_1, y_1, z_1) and B(x_2, y_2, z_2) is given by

  $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- Let P(x_1, y_1, z_1) and Q (x_2, y_2, z_2) be two points in space and let R be a point on line segment PQ such that it divides PQ in the ratio m : n

  (a) Internally, then the coordinates of R are

  $$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right).$$

  (b) Externally, then the coordinates of R are

  $$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right).$$

- Coordinates of Centroid of a triangle whose vertices are A(x_1, y_1, z_1), B(x_2, y_2, z_2) and C(x_3, y_3, z_3) are

  $$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right).$$

VERY SHORT ANSWER TYPE QUESTIONS

1. **Fill up in each of the following:**

   (a) The coordinate of the image of (–1, 2, –3) in XZ plane is ______________.

   (b) The coordinate of the image of (–1, 2, –3) in YZ plane is _______________.

   (c) The coordinate of the image of (–1, 2, –3) in XY plane is _____________.

   (d) The Point P (–5, 4, –3) lies in the octant ____________.

   (e) If $a < 0$, $b > 0$ & $c < 0$, then The Point P $(a, b, -c)$ lies in the octant____________.

   (f) The perpendicular distance of the point P(- 6, 7, –8) from xy-plane is ____________.

   (g) The perpendicular distance of the point P(- 3, 5, –12) from x-axis is ____________.

   (h) The perpendicular distance of the point P(- 3, 4, –5) from z-axis is ____________.

   (i) The coordinates of foot of perpendicular from (3, 7, 9) on y-axis is _____________.

2. **State whether the following statements are true or false.**

   (a) If the distance between the points (a, 2, 1) and (1, –1, 1) is 5, then the sum of all possible value of a is 2.

   (b) The x-axis and z-axis, together determine a plane known as yz-plane.

   (c) The point P (1, 2, –3) lies in the 7^{th} octant.

(d) The y-axis is the intersection of two planes xy-plane and yz-plane.

(e) Distance of the point (3, 4, 5) from the origin (0, 0, 0) is 10.

(f) The distance of point P(–3, –4, –5) from the yz-plane is –3.

(g) The distance of point P(–3, –4, –5) from the y-axis is –4.

(h) If $(c - 1) > 0$, $(a + 2) < 0$ and $b > 0$ then the point P(a, –b, c) lies in the 4th octant.

(i) The length of the foot of perpendicular drawn from the point P(5, 12, 10) on z-axis is 10.

SHORT ANSWER TYPE QUESTIONS

11. What are the coordinates of the vertices of a cube whose edge is 2 unit, one of whose vertices coincides with the origin and the three edges passing through the origin?

    Coincides with the positive direction of the axes through the origin?

12. Let A, B, C be the feet of perpendiculars from point P(1, –2, –3) on the xy-plane, yz-plane and xz-plane respectively. Find the coordinates of A, B, C.

13. If a parallelepiped is formed by planes drawn through the point (5, 8, 10) and (3, 6, 8) parallel to the coordinates planes, then find the length of the gonal of the parallelepiped.

14. Find the length of the longest piece of a string that can be stretched straight in a rectangular room whose dimensions are 13, 10 and 8 unit.

15. Find the coordinate of the point P which is three-fourth of the way from A(–1, 0, 2) to B (5, –7, –10).

SHORT ANSWER TYPE QUESTIONS

16. Show that points (4, –3, –1), (5, –7, 6) and (3, 1, –8) are collinear.

17. Find the point on y–axis which is equidistant from the point (3, 1, 2) and (5, 5, 2).

18. Determine the point in yz plane which is equidistant from three points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

19. The centroid of ΔABC is at (1, 1, 1). If coordinates of A and B are (3,–5, 7) and (–1, 7, –6) respectively, find coordinates of point C.

20. Find the length of the medians of the triangle with vertices A(0, 0, 3), B(0, 4, 0) and C (5, 0, 0).

21. If the extremities (end points) of a diagonal of a square are (1,– 2, 3) and (2,–3, 5) then find the length of the side of square.

22. Three consecutive vertices of a parallelogram ABCD are A(6, –2, 4) B(2, 4, –8), C(–2, 2, 4). Find the coordinates of the fourth vertex.

23. If the points A(1, 0, –6), B(3, p, q) and C(5, 9, 6) are collinear, find the value of p and q.

24. Show that the point A(1, 3, 0), B(–5, 5,, 2), C(–9, –1, 2) and D(–3, –3, 0) are the vertices of a parallelogram ABCD, but it is not a rectangle.

25. The mid-points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, –1). Find its vertices and hence find centroid.

26. Find the coordinate of the points which divides the line segment AB in four equal parts where A(–2, 0, 6) and B(10, –6, –12).

27. Prove that the points (0, –1, –7), (2, 1, –9) and (6, 5, –13) are collinear. Find the ratio in which first point divides the join of the other two.

28. Let A(3, 2, 0), B(5, 3, 2) C(–9, 6, –3) be three points forming a triangle. AD, the bisector of <BAC, meets BC in D. Find the coordinates of the point D.

29. Describe the vertices and edges of the rectangular parallelepiped with one vertex (3, 4, 5) placed in the first octant with one vertex at origin and edges of parallelepiped lie along x, y and z-axis.

30. Find the coordinates of the point which is equidistant from the point (3, 2, 2), (–1, 2, 2), (0, 5, 6) and (2, 1, 2).

CASE STUDY TYPE QUESTIONS

31. Consider a ΔABC with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$. AD, BE and CF are medians of ΔABC.

Based on the above information, answer the following questions:-

i. Coordinates of Point D are ?

(a) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$ (b) $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right)$

(c) $\left(\frac{x_3+x_1}{2}, \frac{y_3+y_1}{2}, \frac{z_3+z_1}{2}\right)$ (d) None of these

ii. A point G divides AD in 2 : 1, the coordinates of G are

(a) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

(b) $\left(\frac{x_1+2x_2}{3}, \frac{y_1+2y_2}{3}, \frac{z_1+2z_2}{3}\right)$

(c) $\left(\frac{x_2+2x_1}{3}, \frac{y_2+2y_1}{3}, \frac{z_2+2z_1}{3}\right)$

(d) None of these

iii. For ΔABC, G is

(a) Incentre (b) Circumcentre

(c) Centroid (d) Orthocentre

iv. G divides BE in ratio

(a) 1 : 2 (b) 2 : 1

(c) 3 : 1 (d) 1 : 3

v. If ΔABC is equilateral, then coordinates of circumcentre are

(a) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

(b) $\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}, \frac{z_1+z_2+z_3}{2}\right)$

(c) $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}, \frac{z_1+z_3}{2}\right)$

(d) None of these

32. ABCD is a field in shape of parallelogram coordinate of A, B andn C are (3, –1, 2), (1, 2, –4) and (–1, 1, 2) resp.

Based on the above information answer the following :-

i. Coordinates of mid point of AC be

(a) (–1, 1, 2) (b) (–1, 0, –2)

(c) (1, 0, 2) (d) (1, 0, –2)

ii. Coordinates of D be

(a) (1, –2, 8) (b) (–1, –2, 8)

(c) (0, –2, 8) (d) (1, 2, 8)

iii. Length of side BC is

(a) 7 (b) $\sqrt{41}$

(c) $\sqrt{47}$ (d) $\sqrt{43}$

iv. Coordinates of centroid G of ΔABC be

(a) $\left(\frac{5}{3}, \frac{1}{3}, \frac{2}{3}\right)$ (b) $\left(\frac{-5}{3}, \frac{-1}{3}, \frac{2}{3}\right)$

(c) $\left(\frac{5}{3}, -1, \frac{2}{3}\right)$ (d) $\left(\frac{5}{3}, \frac{-1}{3}, 2\right)$

v. Ratio in which AC is divided by centroid G is

(a) 2 : 1 (b) 1 : 2

(c) 3 : 1 (d) 1 : 3

ANSWERS

1. $(-1, -2, -3)$
2. $(1, 2, -3)$
3. $OX'YZ'$
4. $OX'YZ$
5. 8 units
6. 13 units
7. 5 units
8. $(0, 7, 9)$
9. 5 or -3
10. $5\sqrt{2}$
11. $(2, 0, 0)$, $(2, 2, 0)$, $(0, 2, 0)$, $(0, 2, 2)$, $(2, 0, 2)$, $(0, 0, 0)$, $(2, 2, 2)$
12. $(4, -3, 0)$, $(0, -3, -5)$, $(4, 0, -5)$
13. $2\sqrt{3}$
14. $\sqrt{333}$
15. 2 : 3 (externally)
17. $(0, 5, 0)$
18. $(0, 1, 3)$
19. $(1, 1, 2)$
20. $7, \sqrt{34}, 7$
21. $\sqrt{3}$
22. $(2, -4, 16)$
23. $p = 6, q = 2$
25. Vertices : $(-3, 4, -7)$, $(7, 2, 5)$, $(3, 21, 17)$
    Centroid : $\left(\frac{7}{3}, 6, 5\right)$
26. $(8, -5, -9)$
27. 1 : 3 externally

28. $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

29. (0, 0, 0), (3, 0, 0), (3, 5, 0), (0, 5, 0), (0, 5, 6)

(0, 0, 6), (3, 0, 6), (3, 5, 6), $\sqrt{61}$, $\sqrt{45}$, $\sqrt{34}$

30. (1, 3, 5)

31. i. (b) ii. (a) iii. (c) iv. (b) v. (a)

32. i. (c) ii. (a) iii. (b) iv. (d) v. (b)

CHAPTER - 13

LIMITS AND DERIVATIES

KEY POINTS

- To check whether limit of f(x) as x approaches to exists i.e., $\lim_{x \to c} f(x)$ exists, we proceed as follows.

  (i) Find L.H.L at x = a using L.H.L. = $\lim_{h \to 0} f(a - h)$.

  (ii) Find R.H.L at x = a using R.H.L. = $\lim_{h \to 0} f(a + h)$.

  (iii) If both L.H.L. and R.H.L. are finite and equal, then limit at x = a i.e., $\lim_{x \to a} f(x)$ exists and equals to the value obtained from L.H.L or R.H.L else we say "limit does not exist".

- $\lim_{x \to c} f(x) = l$, if and only if $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = l$

- $\lim_{x \to c} a = a$, where a is a fixed real number.

- $\lim_{x \to c} x^n = c^n$, for all $n \varepsilon N$

▶ **ALGEBRA OF LIMITS:** Let f, g be two functions such that $\lim_{x \to c} f(x) = l$, and $\lim_{x \to c} g(x) = m$.

- $\lim_{x \to c} [\alpha f(x)] = \alpha \lim_{x \to c} f(x) = \alpha l$, for all $\alpha \varepsilon R$

- $\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = l \pm m$

- $\lim_{x \to c} [f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x) = l \cdot m$

- $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}=\dfrac{\lim\limits_{x\to c}f(x)}{\lim\limits_{x\to c}g(x)}=\dfrac{l}{m},\ m\neq 0,\ g(x)\neq 0$

- $\lim\limits_{x\to c}\dfrac{1}{f(x)}=\dfrac{1}{\lim\limits_{x\to c}f(x)}=\dfrac{1}{l},\ l\neq 0, f(x)\neq 0$

- $\lim\limits_{x\to c}[f(x)]^n=[(\lim\limits_{x\to c}f(x)]^n=l^n$, for all $n\ \varepsilon\ N$

▶ SOME IMPORTANT RESULTS ON LIMITS:

- $\lim\limits_{x\to a}\dfrac{x^n-a^n}{x-a}=n\cdot a^{n-1}$

- $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$

- $\lim\limits_{x\to 0}\dfrac{\tan x}{x}=1$

- $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x}=0$

- $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}=1$

- $\lim\limits_{x\to 0}\dfrac{a^x-1}{x}=\log_e a$

- $\lim\limits_{x\to 0}\dfrac{\log(1+x)}{x}=1$

- $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=e$

- $\lim\limits_{x\to 0^-}f(x)=\lim\limits_{x\to 0^+}f(-x)$

SOME IMPORTANT RESULTS ON DERIVATIVE:

- $\dfrac{d(\sin x)}{dx} = \cos x$
- $\dfrac{d(\cos x)}{dx} = -\sin x$
- $\dfrac{d(\tan x)}{dx} = \sec^2 x$

- $\dfrac{d(\cot x)}{dx} = -\text{cosec}^2 x$
- $\dfrac{d(\sec x)}{dx} = \sec x \cdot \tan x$
- $\dfrac{d(\operatorname{cosec} x)}{dx} = -\text{cosec}x \cdot \cot x$

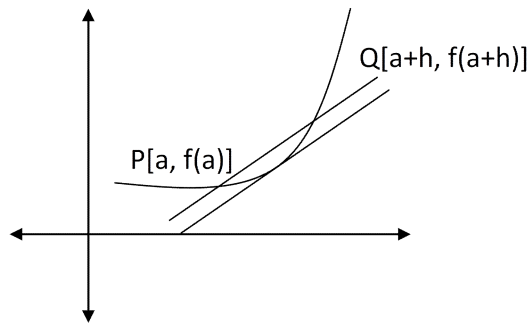
- $\dfrac{d(x^n)}{dx} = n \cdot x^{n-1}$
- $\dfrac{d(\sqrt{x})}{dx} = \dfrac{1}{2\sqrt{x}}$
- $\dfrac{d(a)}{dx} = 0,\ a = \text{constant}$

- $\dfrac{d(e^x)}{dx} = e^x$
- $\dfrac{d(\log x)}{dx} = \dfrac{1}{x}$
- $\dfrac{d(a^x)}{dx} = a^x \cdot \log a$

Logarithm Properties:

- $\log_e(A \cdot B) = \log_e A + \log_e B$
- $\log_e\left(\dfrac{A}{B}\right) = \log_e A - \log_e B$
- $\log_e(A^m) = m \cdot \log_e A$
- $\log_a(1) = 0$
- $\log_B(A) = x,$ then $B^x = A$

- Let y = f(x) be a function defined in some neighbourhood of the point 'a'. Let P[a, f(a)] and Q[a + h, f(a + h)] are two points on the graph of f(x) where h is very small and h ≠ 0.



$$\text{Slope of } PQ = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

- If $\lim_{h\to 0}$ point Q approaches to P and the line PQ becomes a tangent to the curve at point P.

  $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ (if exists) is called derivative of f(x) at the point 'a'.

  It is denoted by f'(a).

▶ ALGEBRA OF DERIVATIVES:

- $\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)],$ where c is a constan t
- $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

Product Rule:

- $\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot \frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$

Quotient Rule:

- $$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\cdot\frac{d}{dx}[f(x)]-f(x)\cdot\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

- If y = f(x) is a given curve then slope of the tangent to the curve at the point (h,k) is given by $\frac{dy}{dx}\Big|_{(h,k)}$ and is denoted by 'm'

VERY SHORT ANSWER TYPE QUESTIONS

1. **Fill in the blanks in each of the followings:**

(a) $\lim\limits_{x\to 2^-}[x]$ = ___________________.

(b) $\lim\limits_{x\to 2^+}[x]$ = ___________________.

(c) $\lim\limits_{x\to 2^-}\frac{|x|}{x}$ = ___________________.

(d) $\lim\limits_{x\to 2^+}\frac{|x|}{x}$ = ___________________.

(e) If f(x) = Sin^2x, then derivative of f(x) is _____________.

2. **State whether the following statements are True or False.**

(a) If $L=\lim\limits_{x\to 0}\frac{\sin 2x}{\tan 3x}$; So the value of L is 1.

(b) If $L=\lim\limits_{x\to 0}\frac{\sin x-\text{tann}\,x}{x^3}$; So the value of 2L is 1.

(c) If f(x) = x^2 – 3x + 1, then derivative of f(x) at x = 2, f '(2) = 1.

(d) $\lim\limits_{x\to 3}[x]$ exists and equal to 3.

(e) $\lim\limits_{x\to 3}|x|$ exists and equal to 3

Note: Q.3 – Q.10 are Multiple Choice Questions (MCQ), select the correct alternatives out of given four alternatives in each.

3. $\lim_{x \to \pi} \frac{\sin x}{x - \pi}$ is -

(a) 1 (b) 2

(c) –1 (d) does not exist.

4. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80,$ then n is -

(a) 2 (b) 3

(c) 4 (d) 5.

5. If $L = \lim_{x \to 1} \frac{x^4 - 1}{x^3 - 1},$ then 3L is -

(a) 2 (b) 3

(c) 4 (d) None of these.

6. $\lim_{x \to 0} \frac{(1+x)^{16} - 1}{(1+x)^4 - 1}$ is -

(a) 0 (b) 4

(c) 8 (d) 16.

7. $\lim_{x \to 1} \frac{x + x^2 + x^3 + x^4 - 4}{x - 1}$ is -

(a) 0 (b) 4

(c) 10 (d) Does not exist.

8. $\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1}$ is -

(a) 0 (b) 1

(c) 2 (d) 4.

9. If $y = \sin^4 x + \cos^4 x$, then $\frac{dy}{dx} =$

(a) $4\sin^3 x + 4\cos^3 x$ (b) $4\sin^3 x - 4\cos^3 x$

(c) $-\sin 4x$ (d) 0.

10. Match the following:

	Column-1		Column-2
A	$\lim_{x\to\infty} \frac{1+2+3+...+x}{x^2} =$	P	$\frac{1}{3}$
B	$\lim_{x\to\infty} \frac{1+4+9+...+x^2}{x^3} =$	Q	$\frac{1}{3}$
C	$\lim_{x\to\infty} \frac{1+8+27+...+x^3}{x^4} =$	R	$\frac{1}{2}$

Which one of the following is true?

(a) A → P, B→ Q, C→ R

(b) A → Q, B→ P, C→ R

(c) A → Q, B→ R, C→ P

(d) A → R, B→ P, C→ Q

11. Evaluate $\lim_{x\to 0} \frac{(1+x)^m - 1}{(1+x)^n - 1}$

12. Evaluate $\lim_{x\to 0} \frac{(\sin 2x) + 3x}{2x + (\tan 3x)}$

13. Evaluate $\lim_{x\to 0} \frac{1-\cos 2x}{1-\cos 4x}$

14. If $y = \sin^2 x.\cos^3 x$, then $\frac{dy}{dx}$.

15. If $y = \sin 2x.\cos 3x$, then $\frac{dy}{dx}$.

SHORT ANSWER TYPE QUESTIONS

16. Differentiate $\frac{\sin x}{x}$ with respect to x.

17. Differentiate $x^3+3^3+3^x$ with respect to x.

18. Differentiate $\sin^2(x^3+x-1)+\frac{1}{\sec^2(x^3+x-1)}$ with respect to x.

19. Differentiate $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a}$ with respect to x.

20. Differentiate $\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}$ w.r.t to x.

21. Find the derivative of x using first principle method.

22. If $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then find the value of k.

23. Evaluate: $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$.

24. Find the derivative of $(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$ with respect to x.

25. Differentiate $\frac{x^8-1}{x^4-1}$ with respect to x.

LONG ANSWER TYPE - I QUESTIONS

26. Differentiate $\text{Sin}^2 x$ with respect to x using First principle method.

27. Differentiate $\text{Sin}(x^2)$ with respect to x using First principle method.

Differentiate the following with respect to x using First principle method. (For Q. 28 – 35)

28. $\cos\sqrt{x}$

29. $\sqrt{\tan x}$

30. $\sec^3 x$

31. $\operatorname{cosec}(2x+3)$

32. $\sin^{\frac{1}{3}} x = \sqrt[3]{\sin x}$

33. $\dfrac{x^2}{x+1}$

34. $\dfrac{2x+3}{x+1}$

35. $\sqrt{x} + \dfrac{1}{\sqrt{x}}$

Evaluate the following Limits: (For Q. 36 – 53)

36. $\lim\limits_{x \to \infty} \dfrac{2x^8 - 3x^2 + 1}{x^8 + 6x^5 - 7}$

37. $\lim\limits_{x \to 1} \dfrac{2x^8 - 3x^2 + 1}{x^8 + 6x^5 - 7}$

38. $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{x \cdot \tan 3x}$

39. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sin x - \cos x}{x - \dfrac{\pi}{4}}$

40. $\lim_{x \to \frac{\pi}{6}} \frac{\sqrt{3}\sin x - \cos x}{\frac{\pi}{6} - x}$

41. $\lim_{x \to 0} \frac{\sin x}{\tan x^0}$ (where x^0 represents x degree)

42. $\lim_{x \to 9} \frac{x^{\frac{3}{2}} - 27}{x^2 - 81}$

43. $\lim_{x \to a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x - a}$

44. $\lim_{x \to 0} \frac{\cos ax - \cos bx}{1 - \cos x}$

45. $\lim_{x \to a} \frac{\cos x - \cos a}{\cot x - \cot a}$

46. $\lim_{x \to \pi} \frac{1 + \sec^3 x}{\tan^2 x}$

47. $\lim_{x \to 1} \frac{x - 1}{\log_e x}$

48. $\lim_{x \to e} \frac{x - e}{(\log_e x) - 1}$

49. $\lim_{x \to 2} \left[\frac{4}{x^3 - 2x^2} + \frac{1}{2 - x} \right]$

50. $\lim_{x \to a} \left[\frac{\sqrt{a + 2x} - \sqrt{3x}}{\sqrt{3a + x} - 2\sqrt{x}} \right]$

51. $\lim\limits_{x\to 0}\dfrac{\sin(2+x)-\sin(2-x)}{x}$

52. $\lim\limits_{x\to 0}\dfrac{1-\cos x\cdot\sqrt{\cos 2x}}{\sin^2 x}$

53. $\lim\limits_{x\to 0}\dfrac{6^x-2^x-3^x+1}{\log(1+x^2)}$

54. Differentiate the following w.r.t.

(a) $\dfrac{(x-1)(x-2)(x-3)}{x^2-5x+6}$

(b) $\left(x-\dfrac{1}{x}\right)\left(x+\dfrac{1}{x}\right)\left(x^2+\dfrac{1}{x^2}\right)\left(x^4+\dfrac{1}{x^4}\right)$

(c) $\dfrac{x\sin x+\cos x}{x\sin x-\cos x}$

(d) $x\cdot\sin x\cdot e^x$

55. Find the values of a and b if $\lim\limits_{x\to 2} f(x)$ and $\lim\limits_{x\to 4} f(x)$ exists where

$$f(x)=\begin{vmatrix} x^2+ax+b, & 0\le x<2 \\ 3x+2, & 2\le x\le 4 \\ 2ax+5b, & 4<x<8 \end{vmatrix}$$

CASE STUDY TYPE QUESTIONS

56. Mr. Pradeep has a rectangular plot, which is used for growing vegetables. Perimeter of plot is 50m. Length and width of plot are x m and y m respectively.

Based on the above information, answer the following questions:-

i. Relation between x and y is

(a) $x + y = 50$ (b) $x + y = 100$

(c) $x + y = 25$ (d) $x = y$

ii. Area function, A(x) =

(a) $x^2 - 5$ (b) $25x - x^2$ (c) $x^2 - 25x$ (d) $25 - x$

iii. Derivative of A(x) w.r.t. x [A'(x)] =

(a) $2x$ (b) $-2x$ (c) $25 - 2x$ (d) $2x - 25$

iv. Value of x for which A'(x) = 0 is

(a) 25 (b) 12.5 (c) 5 (d) 0

v. Value of A(x) at x = 12.5 is

(a) 625 (b) 250 (c) 156.25 (d) 144.25

57. Consider the following functions.

$u(x) = \sqrt{x}, \quad v(x) = \cot x, \quad f(x) = u(x) \times v(x)$

$g(x) = \dfrac{u(x)}{v(x)}$ and $h(x) = \dfrac{v(x)}{u(x)}$

Based on the above information answer the following :-

i. Derivative of $u(x)$ is

(a) $\dfrac{1}{\sqrt{x}}$ (b) $\dfrac{2}{\sqrt{x}}$ (c) $\dfrac{\sqrt{x}}{2}$ (d) $\dfrac{1}{2\sqrt{x}}$

ii. Derivative of $v(x)$ is

(a) $-\text{cosec}\, x \cot x$ (b) $-\text{cosec}^2 x$ (c) $\sec^2 x$ (d) $\tan x$

iii. Derivative of $f(x)$ is

(a) $\dfrac{-2x\text{cosec}^2 x + \cot x}{2\sqrt{x}}$ (b) $\dfrac{-\text{cosec}^2 x}{2\sqrt{x}}$

(c) $\dfrac{-2x\cot^2 x + \text{cosec} x}{2\sqrt{x}}$ (d) $\dfrac{-\cot^2 x}{2\sqrt{x}}$

iv. Derivative of $g(x)$ is

(a) $\dfrac{\cot x - 2x\text{cosec}^2 x}{2\sqrt{x}\cot^2 x}$ (b) $\dfrac{\cot x + 2x\text{cosec}^2 x}{2\sqrt{x}\cot^2 x}$

(c) $\dfrac{\text{cosec}^2 x - 2x\cot^2 x}{2\sqrt{x}\cot^2 x}$ (d) $\dfrac{\text{cosec}^2 x + 2x\cot^2 x}{2\sqrt{x}\cot^2 x}$

v. Derivative of $h(x)$ is

(a) $\dfrac{2x\text{cosec}^2 x + \cot x}{2x^{3/2}}$ (b) $\dfrac{\text{cosec}^2 x + 2x\cot x}{2x^{3/2}}$

(c) $\dfrac{-(2x\text{cosec}^2 x + \cot x)}{2x^{3/2}}$ (d) $\dfrac{-\text{cosec}^2 x}{\sqrt{x}}$

ANSWERS

1. (a) 1
   (b) 2
   (c) –1
   (d) 1
   (e) sin2x

2. (a) False
   (b) False
   (c) True
   (d) False
   (e) True

3. (c)

4. (d)

5. (c)

6. (b)

7. (c)

8. (c)

9. (c)

10. (d)

11. $\frac{m}{n}$

12. 1

13. $\frac{1}{4}$

14. $\cos^2x.\sin x(2\cos^2x - 3\sin^2x)$

15. $2\cos 2x.\cos 3x - 3\sin 2x.\sin 3x$

16. $\frac{x\cos x - \sin x}{x^2}$

17. $3x^2 + 3x.\log 3$

18. 0

19. 0

20. 0

21. 1

22. $\frac{8}{3}$

23. $\frac{1}{2}$

24. $8x^7$

25. $4x^3$

26. sin2x

27. $2x.\cos(x)^2$

28. $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$

29. $\frac{\sec^2x}{2\sqrt{\tan x}}$

30. $3\sec^3x.\tan x$

31. $-2\text{cosex}(2x + 3).\cot(2x + 3)$

32. $\frac{\cos x}{3\sqrt[3]{\sin^2x}}$

33. $\frac{x^2+2x}{(x+1)^2}$

34. $\frac{-1}{(x+1)^2}$

35. $\frac{1}{2\sqrt{x}}-\frac{1}{2x\sqrt{x}}$

36. 2

37. $\frac{5}{17}$

38. $\frac{2}{3}$

39. $\sqrt{2}$

40. 2

41. $\frac{180^0}{\pi}$

42. $\frac{1}{4}$

43. $\frac{5(a+2)^{\frac{3}{2}}}{2}$

44. $b^2 - a^2$

45. $\sin^3 a$

46. $\frac{-3}{2}$

47. 1

48. e

49. –1

50. $\frac{1}{\sqrt{3}}$

51. 2cos2

52. $\frac{3}{2}$

53. (log2)(log3)

54. (a) 1

(b) $8x^7 + 8x^{-9}$

(c) $\frac{-2(x+\sin x\cdot\cos x)}{(x\sin x-\cos x)^2}$

(d) $e^x (x\sin x + x\cos x + \sin x)$

55. a = –1, b = 6

56. i. (c) ii. (b) iii. (c) iv. (b) v. (c)

57. i. (d) ii. (b) iii. (a) iv. (b) v. (c)

CHAPTER - 14

MATHEMATICAL REASONING

CONCEPT MAP

- A sentence is called a statement if it is either true or false but not both simultaneously.

- The denial of a statement p is called its negation and is written as ~p and read as not p.

- Compound statement is made up of two or more simple statements. These simple statements are called component statements.

- 'And', 'or', 'If–then', 'only if' 'If and only if' etc. are connecting words, which are used to form a compound statement.

- Two simple statements p and q connected by the word 'and' namely 'p and q' is called a conjunction of p and q and is written as $p \wedge q$.

- Compound statement with 'And'

    ❖ is true if all its component statements are true

    ❖ false if any of its component statement is false

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- Two simple statements p and q connected by the word 'or' the resulting compound statement 'p or q' is called disjunction of p and q and is written as $p \vee q$.

- Compound statement with 'Or' is
  - ❖ true when at least one component statement is true,
  - ❖ false when both the component statements are false.

p	q	p $\wedge$ q
T	T	T
T	F	T
F	T	T
F	F	F

- The negation of the compound statement 'p or q' is '~p and ~q' $\Rightarrow$ ~(p $\vee$ q) = ~p $\wedge$ ~q.
- The negation of the compound statement 'p and q' is '~p or ~q' $\Rightarrow$ ~(p $\wedge$ q) = ~p $\vee$ ~q.
- A statement with "If p then q" can be rewritten as:-
  - (a) p implies q
  - (b) p is sufficient condition for q
  - (c) q is necessary condition for p
  - (d) p only if q
  - (e) (~q) implies (~p)
- If in a compound statement containing the connective ''or'' all the alternatives cannot occur simultaneously, then the connecting word ''or'' is called as exclusive ''or''.
- If, in a compound statement containing the connective ''or'', all the alternative can occur simultaneously, then the connecting word ''or'' is called as inclusive ''or''.
- Contrapositive of the statement p $\Rightarrow$ q is the statement ~q $\Rightarrow$ ~p
- Converse of the statement p $\Rightarrow$ q is the statement q $\Rightarrow$ p
- "For all", "For every" are called universal quantifiers
- A statement is called valid or invalid according as it is true or false.

VERY SHORT ANSWER TYPE QUESTIONS

1. **State whether the following statements are true or false:**

   (a) Prime factors of 6 are 2 and 3, represents a Statement.

   (b) Students can take French or Spanish as their third language. The "OR" used in the statement here is "INCLUSIVE OR".

   (c) Two lines intersect at a point or are parallel. The "OR" used in the statement here is "EXCLUSIVE OR".

   (d) The Compound statement "$\sqrt{2}$ is a rational number or an irrational number" is True.

   (e) The Compound Statement "All integers are either even or odd" is False.

2. **Fill up blanks in each of the following:**

   (a) The negation of the statement "Zero is a positive number" is _________________.

   (b) The negation of the statement "For every real number x, either $x > 1$ or $x < 1$." is _______________________.

   (c) The Converse of the statement "If a number x is even, then x^2 is also even" is _______________________.

   (d) The Converse of the statement "If n is a prime number, then n is odd." is _______________________.

   (e) The quantifier used in the statement "There exists a number which is equal to its square" is ____________________.

   (f) The Contra positive of the statement "If a triangle is equilateral, it is isosceles" is _______________________.

Note: Q.3 – Q.7 are Multiple Choice Questions (MCQ), select the correct alternatives out of given four alternatives in each.

3. Which of the following is a statement?
   (a) 2 is not a prime number.
   (b) Mind your own business.
   (c) Be punctual.
   (d) Do not tell lies.

4. The negation of the statement "It is raining and weather is cold." is -
   (a) It is not raining and weather is cold.
   (b) It is raining or weather is not cold.
   (c) It is not raining or weather is not cold.
   (d) It is not raining and weather is not cold.

5. Which of the following is the converse of the statement?
   "If Raju secure good marks, then he will get a Pen."
   (a) If Raju will not get Pen, then he will not secure good marks.
   (b) If Raju will get a Pen, then he will secure good marks.
   (c) If Raju will get a Pen, then he will not secure good marks.
   (d) If Raju will not get a Pen, then he will secure good marks.

6. Which of the following is a mathematical statement?
   (a) n is a real number
   (b) Switch off the light
   (c) 5 is a prime number
   (d) Let's go there.

7. The negation of the statement "A circle is an ellipse" is -
   (a) An ellipse is a circle.
   (b) An ellipse is not a circle.
   (c) A circle is not an ellipse.
   (d) A circle is an ellipse.

8. Write negation of the statement: "π is not a rational number".

9. Write negation of the statement: "There exists a complex number which is not a real Number"

10. Write the converse of the statement:
"If $3 \times 7 = 21$ then $3 + 7 = 10$"

11. Write the converse of the statement: "If x is zero, then x is neither positive nor negative"

SHORT ANSWER TYPE QUESTIONS

12. Check whether the compound statement is true or false. Write the component statements.
    (a) A square is a quadrilateral and its four sides are equal.
    (b) "0" is either a positive number or negative number.

13. Identify the quantifiers in the following statements:
    (a) For every integer p, $\sqrt{p}$ is a real number.
    (b) There exists a capital for every country in the world.

14. Write the negation of the following compound statements.
    (a) It is daylight and all the people have arisen.
    (b) Square of an integer is positive or negative.

15. Identify the type 'Or' (Inclusive or Exclusive) used in the following statements
    (a) To enter in a country, you need a visa or citizenship card.
    (b) $\sqrt{2}$ is a rational number or an irrational number.

16. Write the contra positive of the following statements:
    (a) If $5 > 7$ then $6 > 7$.
    (b) x is even number implies that x^2 is divisible by 4.

ANSWERS

1. (a) True (b) False (c) True
   (d) True (e) True

2. (a) Zero is not a positive number.
   (b) There exists a real number x such that $x \leq 1$ and $x \geq 1$.
   (c) If x^2 is even, then x is also even.
   (d) If n is odd then n is Prime number.
   (e) There Exists.
   (f) If triangle is not Isosceles then it is not equilateral.

3. (a) 4. (c) 5. (b)
6. (c) 7. (c)

8. π is a rational number or π is not an irrational number

9. For all complex number x, x is a real number.

10. If $3 + 7 = 10$ then $3 \times 7 = 21$

11. If x is neither positive nor negative then x is zero.

12. (a) True;
   p : A square is a quadrilateral,
   q : All the four sides of a square are equal.
   (b) False;
   p : 0 is a positive number.,
   q : 0 is a negative number

13. (a) For every
   (b) There exists, For every

14. (a) It is not daylight or it is false that all the people have arisen.
   (b) There exists an integer whose square is neither positive nor negative.

15 (a) INCLUSIVE (b) EXCLUSIVE

16. (a) If $6 \leq 7$ then $5 \leq 7$
   (b) If x^2 is not divisible by 4 then x is not even.

CHAPTER - 15

STATISTICS

KEY CONCEPT

- Range of Ungrouped Data and Discrete Frequency Distribution.
- RANGE = Largest observation – smallest observation.
- Range of Continuous Frequency Distribution.
- RANGE = Upper Limit of Highest Class – Lower Limit of Lowest Class.
- **Mean deviation for ungrouped data or raw data:**

$$M.D.\ (\text{about mean}) = \frac{\sum |x_i - \bar{x}|}{n}, \text{ where } \bar{x} \text{ is the Mean.}$$

$$M.D.\ (\text{about mean}) = \frac{\sum |x_i - M|}{n}, \text{ where } M \text{ is the Median.}$$

- **Mean deviation for grouped data (Discrete frequency distribution and Continuous frequency distribution):**

$$M.D.\ (\text{about mean}) = \frac{\sum |f_i - \bar{x}|}{N}, \text{ where } \bar{x} \text{ is the Mean.}$$

$$M.D.\ (\text{about mean}) = \frac{\sum |f_i - M|}{N}, \text{ where } M \text{ is the Median.}$$

Note: $N = \sum f_i$

- Variance is defined as the mean of the squares of the deviations from mean.

- Standard deviation 'σ' is positive square root of variance.

  $\sigma = \sqrt{\text{Variance}}$

- Variance 'σ^2' and standard deviation (SD) σ for ungrouped data

  $$\sigma^2 = \frac{1}{n}\sum (x_i - \overline{x})^2 \Rightarrow \boxed{S.D. = \sigma = \sqrt{\frac{1}{n}\sum (x_i - \overline{x})^2}}$$

- Standard deviation of a discrete frequency distribution

  $$S.D. = \sigma = \sqrt{\frac{1}{n}\sum f_i(x_i - \overline{x})^2} = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - (\sum f_i x_i)^2}$$

- Short cut method to find variance and standard deviation

  $$\text{Variance} = \sigma^2 = \frac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i^2\right)\right]$$

  $$S.D. = \sigma = \frac{h}{N}\sqrt{N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2}$$

  $$\text{Where}: \ y_i = \frac{x_i - A}{h}$$

- Coefficient Of Variation (C.V.) = c

- If each observation is multiplied by a positive constant k then variance of the resulting observations becomes k^2 times of the original value and standard deviation becomes k times of the original value.

- If each observation is increased by k, where k is positive or negative, then variance and standard deviation remains same.

- Standard deviation is independent of choice of origin but depends on the scale of measurement.
- The series having higher coefficient of variation is called more variable than the other. While the series having lesser coefficient of variation is called more consistent or more stable. For series with equal means the series with lesser standard deviation is more stable.
- The mean of first 'n; natural number is $\frac{n+1}{2}$.
- The mean of first 'n' even natural numbers = (n + 1)

VERY SHORT ANSWER TYPE QUESTIONS

1. **Fill up the blanks in each of the following:**

   (a) The mean of first ten natural number is ______________.

   (b) The mean of first ten even natural number is ___________.

   (c) The mean of first ten odd natural number is ___________.

   (d) Coefficient of Variation (C.V.) = $\frac{\cdots\cdots\cdots}{\text{Mean}} \times 100, \ \bar{x} \neq 0.$

   (e) If the variance of a data is 7225, then the standard deviation of the data is ______________.

2. **State weather the following are True or False.**

   (a) The range of observations 1, 2, 5, 3, 0, 8, 10, 9 is eight (8).

   (b) The mean deviation about Mean for 1, 3, 5, 7, 9 is 2.4

   (c) The mean deviation about Median for 1, 3, 5, 7, 9 is 5.

(d) If the mean of a, b, c, d, e is 10, then mean of (a + 3), (b + 3), (c + 3), (d + 3), (e + 3) is also 10.

(e) If the Variance of a, b, c, d, e is 10, then variance of (a + 3), (b + 3), (c + 3), (d + 3), (e + 3) is also 10.

3. The sum of the squares of deviation for 10 observations taken from their mean 50 is 250. Find Standard Deviation.

4. The sum of the squares of deviation for 10 observations taken from their mean 25 is 500. Find Variance.

5. If the variance of 14, 18, 22, 26, 30 is 'k', then find the variance of 28, 36, 44, 52, 60.

> **Note:** Q.6 – Q.10 are Multiple Choice Questions (MCQ), select the correct alternatives out of given four alternatives in each.

6. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what is the new mean -

(a) 12	(b) 16
(c) 24	(d) 48.

7. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what is the new standard deviation -

(a) 4	(b) 8
(c) 16	(d) 32.

8. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what is the new mean-

(a) 25	(b) 29
(c) 30	(d) 35.

9. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what is the new variance -

(a) 4 (b) 14

(c) 16 (d) 25.

10. Match the following:

If the mean of $x_1, x_2, \ldots\ldots, x_{20}$ is 10.

	Column-1		Column-2
A	mean of $2x_1, 2x_2, \ldots\ldots, 2x_{20}$	P	0
B	mean of $(-3x_1 + 32), (-3x_2 + 32), \ldots\ldots, (3x_{20} + 32)$	Q	2
C	mean of $(x_1 + 2), (x_2 + 2), \ldots\ldots, (x_{20} + 2)$	R	12
D	mean of $(x_1 - 10), (x_2 - 10), \ldots\ldots, (x_{20} - 10)$	S	20

(a) A → P, B → Q, C → R, D → S

(b) A → S, B → Q, C → R, D → P

(c) A → Q, B → S, C → R, D → P

(d) A → S, B → Q, C → P, D → R

VERY SHORT ANSWER TYPE QUESTIONS

11. Find the Variance of First 10 Natural Numbers.

12. Find the Variance of First 5 Multiples of 6.

13. Find the Standard Deviations of First 10 Even Natural numbers.

14. Find the Standard deviation for the following data:

10, 20, 30, 40, 50, 50, 60, 70, 80, 90

15. Find the variance for the following Data:

Class-Interval	Frequency
0 - 10	1
10 - 20	2
20 – 30	3
30 – 40	3
40 – 50	1

LONG ANSWER TYPE – I QUESTIONS

16. In a series of '2p' observations, half of the observations are equal 'a' each and remaining half equal (–a) each. If the standard deviation of the observations is 2, then find the value of |a|.

17. In the following Distribution

x	f
A	2
2A	1
3A	1
4A	1
5A	1
6A	1

Where A is positive integer, has a variance of 160. Determine the value of A.

18. Find the mean deviation from mean of first n terms of an Arithmetic Progression (A.P.) with first term is 'a' and Common difference is 'd'.

19. Find the Variance and Standard Deviation of first n terms of an Arithmetic Progression (A.P.) with first term is 'a' and Common difference is 'd'.

20. Consider the first 10 positive integers. If we multiply each number by –1 and then add 1 to each number, find the variance of the numbers so obtained.

21. Coefficients of variation of two distributions A and B are 60 and 80 respectively while their standard deviations are 21 and 36 respectively. What are their means?

22. The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observations are 1, 2 and 6. Find the other two observations.

23. Calculate the possible values of 'x' if standard deviation of the numbers 2, 3, 2x and 11 is 3.5.

24. Mean and standard deviation of the data having 18 observations were found to be 7 and 4 respectively. Later it was found that 12 was miscopied as 21 in calculation. Find the correct mean and the correct standard deviation.

25. Suppose a population A has 100 observations 101, 102, ..., 200. Another population B has 100 observations 151, 152, ..., 250. If V_A and V_B represent the variances of the two populations respectively then find the ratio of V_A and V_B.

LONG ANSWER TYPE – II QUESTIONS

26. Calculate the mean deviation about mean for the following data.

X	2	4	6	8	10	12	14	16
f	2	2	4	5	3	2	1	1

27. If for a distribution $\sum(x-5)=3,\ \sum(x-5)^2=43$ and the total number of item is 18, find the mean and standard deviation.

28. Calculate the mean deviation about median for the following data:

X	10	15	20	25	30	35	40	45
f	7	3	8	5	6	8	4	4

29. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test :

X	0	1	2	3	4	5
f	$p - 2$	p	p^2	$(p + 1)^2$	$2p$	$2p + 1$

where p is positive integer. Determine the mean and standard deviation of the marks.

30. Calculate the mean deviation about mean

Class Interval	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
f	2	3	8	14	8	3	2

31. Mean and standard deviation of 100 observations were found to be 40 and 10 respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively. Find correct standard deviation.

32. Calculate the mean deviation about mean for the following data:

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	5	8	15	16	6

33. Calculate the mean deviation about median for the following data:

Class Interval	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90
f	8	10	10	16	14	2

34. The mean and standard deviation of some data taken for the time to complete a test are calculated with following results:

Number of observations = 25,
mean = 18.2 seconds
Standard deviation = 3.25 seconds

Further another set of 15 observations x_1, x_2, ..., x_{15}, also in $\sum_{i=1}^{15} x_i^2 = 5524$.

Calculate the standard deviation based on all 40 observations.

35. Find the coefficient of variation of the following data:

Class Interval	**f**
20 - 29	5
30 - 39	12
40 - 49	15
50 - 59	20
60 - 69	18
70 - 79	10
80 - 89	6
90 - 99	4

CASE STUDY TYPE QUESTIONS

36. Following data represents the salaries of 11 employees in a firm

10000, 12000, 15000, 13000, 11000, 12000, 12000, 14000, 10000, 13000, 12000.

i. Find the mean salary.

(a) 11181.82 (b) 12181.82

(c) 13181.82 (d) 10000.82

ii. What is the median salary?

(a) 12000 (b) 11000

(c) 12181.82 (d) 11181.82

iii. When arranged in ascending order, which entry gives the median salary?

(a) 6^{th} (b) 5^{th} (c) 4^{th} (d) 7^{th}

iv. The mean deviation about the median salary is

(a) 1190.99 (b) 1000 (c) 1100 (d) 1090.91

v. What is the range of salaries?

(a) 4500 (b) 4000 (c) 5000 (d) 6000

37. Following are the prices of shares X and Y (of ten days) :

Days	**X**	**Y**
1	35	108
2	54	107
3	52	105
4	53	105
5	56	106
6	58	107
7	52	104
8	50	103
9	51	104
10	49	101

i. What is the mean prince of the share X during these 10 days?

(a) 52 (b) 51 (c) 50.3 (d) 51.5

ii. What is the mean prince of the share Y during these 10 days?

(a) 105 (b) 106 (c) 104 (d) 107

iii. What is the standard deviation of the price of share X?

(a) 5.01 (b) 6.75 (c) 5.92 (d) 7.25

iv. What is the standard deviation of the price of share Y?

(a) 1.75 (b) 2.87 (c) 1.25 (d) 2

v. If a person wants to invest in shares (X or Y) whose price remain more stable. He should invest in

(a) X

(b) Y

(c) Both are equally stable. So, he can invest iin anyone

(d) Insufficient data to decide

ANSWERS

1. (a) 5.5
   (b) 11
   (c) 10
   (d) Standard Deviation
   (e) 85
2. (a) False
   (b) True
   (c) False
   (d) False
   (e) True
3. 5
4. 25
5. 4 k
6. (d)
7. (c)
8. (d)
9. (c)
10. (b)
11. 8.33
12. 72
13. $\sqrt{33}$
14. $10\sqrt{6}$
15. $\sqrt{129}$
16. 2

17. A = 7

18. $\dfrac{(n-1)(d-1)}{2}$

19. Variance = $\dfrac{(n^2-1)}{12}d^2$

Standard Deviation = $d\sqrt{\dfrac{(n^2-1)}{12}}$

20. 8.25

21. 35, 45

22. 4, 9

23. 3, $\dfrac{7}{3}$

24. 6.5, 2.5

25. 1 : 1

26. 2.8

27. Mean = 5.17,
Standard Deviation = 1.53

28. 10.1

29. Mean = 2.8,
Standard deviation = 1.12

30. 10

31. 10.24

32. 9.44

33. 11.44

34. 3.87

35. 31.24

36. i. (b) ii. (a) iii. (a) iv. (d) v. (c)

37. i. (b) ii. (a) iii. (c) iv. (d) v. (b)

CHAPTER - 16

PROBABILITY

KEY CONCEPT

- **Random Experiment:** If an experiment has more than one possible outcome and it is not possible to predict the outcome in advance then experiment is called random experiment.

- **Sample Space:** The collection or set of all possible outcomes of a random experiment is called sample space associated with it. Each element of the sample space (set) is called a sample point.

  Some examples of random experiments and their sample spaces

  (i) A coin is tossed
  S = {H, T}, n(S) = 2 Where n(S) is the number of elements in the sample space S.

  (ii) A die is thrown
  S = { 1, 2, 3, 4, 5, 6], n(S) = 6

  (iii) A card is drawn from a pack of 52 cards n (S) = 52.

  (iv) Two coins are tossed
  S = {HH, HT, TH, TT}, n(S) = 4

  (v) Two dice are thrown

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

- Two cards are drawn from a well shuffled pack of 52 cards
  with replacement $n(S) = 52 \times 52$
  without replacement = ${}^{52}C_2$

- **Event:** A subset of the sample space associated with a random experiment is called an event.

- **Elementary or Simple Event:** An event which has only one Sample point is called a simple event.

  For Example: when an unbiased die is thrown, then getting an even prime number on the die is an example of Simple event. {2}

- **Compound Event:** An event which has more than one Sample point is called a Compound event.

  For Example: when an unbiased die is thrown, then getting an even number on the die is an example of Compound event. {2, 4, 6}

- **Sure Event:** If event is same as the sample space of the experiment, then event is called sure event. In other words an event which is certain to happen is sure event.

  For Example: when an unbiased die is thrown, then getting a number less than 7 on the die is an example of sure event. {1, 2, 3, 4, 5, 6}

- **Impossible Event:** Let S be the sample space of the experiment, $\phi \subset S$, ϕ is called impossible event. In other words an event which is impossible to be happen is the impossible event.
  For Example: when an unbiased die is thrown, then getting a number More than 6 on the die is an example of sure event. $\{\} = \phi$.

- Exhaustive and Mutually Exclusive Events: Events E_1, E_2, E_3……..E_n are such that

  (i) $E_1 \cup E_2 \cup E_3 \cup \ldots\ldots \cup E_n = S$ then Events E_1, E_2, E_3……..E_n are called exhaustive events.

  (ii) $E_i \cap E_j = \phi$ for every $i \neq j$ then Events E_1, E_2, E_3……..E_n are called mutually exclusive.

- **Probability of an Event:** For a finite sample space S with equally likely outcomes, probability of an event A is defined as:

  $$P(A) = \frac{n(A)}{n(S)}$$

  where n(A) is number of elements in A and n(S) is number of elements in set S and $0 \leq P(A) \leq 1$

  (a) If A and B are any two events then

  $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  $= P(A) + P(B) - P(A \text{ and } B)$

  (b) A and B are mutually exclusive events, then

  $P(A \cup B) = P(A) + P(B)$ (since $P(A \cap B) = 0$ for mutually exclusive events)

  (c) $P(A) + P(\bar{A}) = 1$ or $P(A) + P(\text{not } A) = 1$

  (d) P (Sure event) = P(S) = 1

  (e) P (impossible event) = $P(\phi) = 0$

(f) $P(A - B) = P(A) - P(A \cap B) = P(A \cap \bar{B})$

(g) $P(B - A) = P(B) - P(A \cap B) = P(\bar{A} \cap B)$

(h) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

(i) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$

- **Addition theorem for three events**

  Let A, B and C be any three events associated with a random experiment, then

  $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

- **Axiomatic Approach to Probability:**

  Let S be a sample space containing elementary outcomes w_1, w_2, ..., w_n

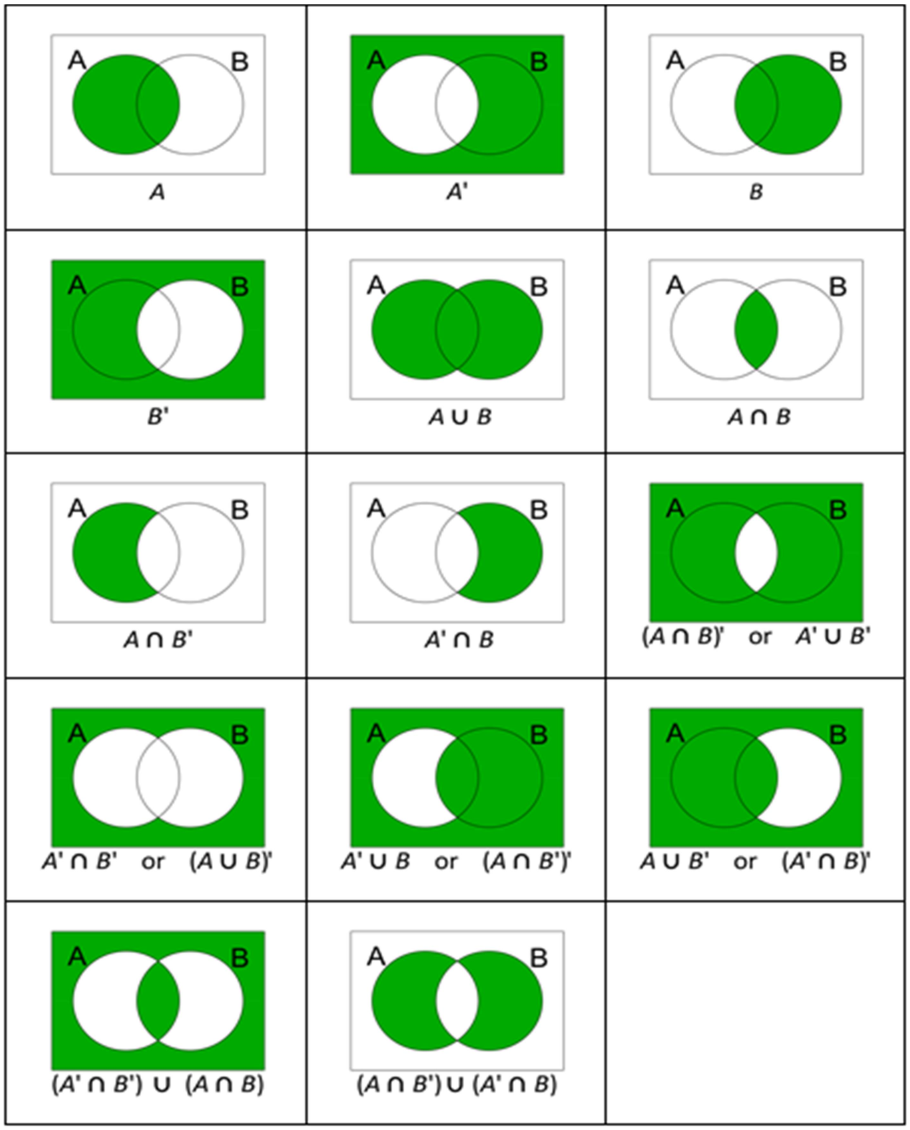
  i.e. $S = \{w_1, w_2, \ldots\ldots, w_n\}$

  (i) $0 \le P(w_i) \le 1, \text{ for all } w_i \in S$

  (ii) $P(w_1) + P(w_2) + P(w_3) + \ldots\ldots + P(w_n) = 1$

  (iii) P (A) = $\sum P(w_i)$, for any event A containing elementary events w_i.

▶ VENN DIAGRAM OF DIFFERENT SETS



- The cards J, Q and K are called face cards. There are 12 face cards in a deck of 52 cards.
- There are 64 squares in a chess board i.e.32 white and 32 Black.

VERY SHORT ANSWER TYPE QUESTIONS

1. **State whether the following statements are True or False.**

When a die is rolled, sample space $S = \{1, 2, 3, 4, 5, 6\}$. Let some of the events are

A = {2, 3}, B = {1, 3, 5}, C = {4, 6}, D = {6} and E = {1, 5}.

(a) Events A and B are Mutually Exclusive Events.

(b) Events A and C are Mutually Exclusive Events.

(c) Events A, B and C are Exhaustive Events.

(d) Event A is Simple Event.

(e) Event D is Compound Event.

2. Fill in the blanks in each of the followings:

(a) Let S = {1, 2, 3, 4, 5, 6} and E = {1, 3, 5}, then $\bar{E}$ is ________________.

(b) The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of either A or B is ____________.

(c) The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of neither A nor B is _____________.

(d) The probability that the Indian team will win the world Cup 2019 is 0.92, the probability that it will shared by two countries is 0.01, and the probability that India will not won the World Cup 2019 is _____________.

(e) Suppose a fair die is rolled. Then the probability of getting a multiple of 2 or 3 or 5 is ________________.

(f) When a pair of fair dice is rolled, then the probability of getting the sum atleast 7 is _____________________.

(g) When a pair of fair dice is rolled, then the probability of getting the sum as multiple of 3 is ________________.

(h) When a pair of fair dice is rolled, then neither the probability of getting the sum neither even nor a multiple of 5 is _________________.

(i) Three letters are written to three different persons and addresses on three envelopes are also written. Without looking at the addresses, then the probability that exactly one letter goes to the right envelopes is _____________.

(j) Three letters are written to three different persons and addresses on three envelopes are also written. Without looking at the addresses, Then the probability that none of the letters go into the right envelopes is _______________.

Note: Q.3 – Q.11 are Multiple Choice Questions (MCQ), select the correct alternatives out of given four alternatives in each.

3. Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is -

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$

(c) $\frac{5}{9}$ (d) $\frac{1}{30}$.

4. Three digit numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers. What is the probability that this number has the same digits?

(a) $\frac{1}{16}$ (b) $\frac{16}{25}$

(c) $\frac{1}{65}$ (d) $\frac{1}{25}$.

5. The probability that a non-leap year selected at random will have 52 Sundays is -

(a) 0 (b) 1

(c) $\frac{1}{7}$ (d) $\frac{2}{7}$.

6. The probability that a non-leap year selected at random will have 53 Sundays is -

(a) 0 (b) 1

(c) $\frac{1}{7}$ (d) $\frac{2}{7}$.

7. The probability that a leap year selected at random will have 54 Sundays is

(a) 0 (b) 1

(c) $\frac{1}{7}$ (d) $\frac{2}{7}$.

8. Three unbiased coins are tossed. If the probability of getting at least 2 tails is p, Then the value of 8p -

(a) 0 (b) 1

(c) 3 (d) 4.

9. Four unbiased coins are tossed. If the probability of getting odd number of tails is p, then the value of 16p -

(a) 1 (b) 2

(c) 4 (d) 8

10. From 4 red balls, 2 white balls and 4 black balls, four balls are selected. The probability of getting 2 red balls is p, then the value of 7p -

(a) 1	(b) 2
(c) 3	(d) 4

11. Describe the Sample Space for the experiment:

    A coin is tossed twice and number of heads is recorded.

12. Describe the Sample Space for the experiment:
    A card is drawn from a deck of playing cards and its colour is noted.

13. Describe the Sample Space for the experiment:
    A coin is tossed repeatedly until a tail comes up.

14. Describe the Sample Space for the experiment:
    A coin is tossed. If it shows head, we draw a ball from a bag consisting of 2 red and 3 black balls. If it shows tail, coin is tossed again.

15. Describe the Sample Space for the experiment:
    Two balls are drawn at random in succession without replacement from a box containing 1 red and 3 identical white balls.

16. A coin is tossed n times. Find the number of element in its sample space.

17. One number is chosen at random from the numbers 1 to 21. What is the probability that it is prime?

18. What is the probability that a given two-digit number is divisible by 15?

19. If $P(A \cup B) = P(A) + P(B)$, then what can be said about the events A and B?

20. If $P(A \cup B) = P(A \cap B)$, then find relation between P(A) and P(B).

SHORT ANSWER TYPE QUESTIONS

21. Let A and B be two events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$, find P(B) if $P(A \cap B) = P(A)\,P(B)$.

22. Three identical dice are rolled. Find the probability that the same number appears on each of them.

23. In an experiment of rolling of a fair die. Let A, B and C be three events defined as under:

    A : a number which is a perfect square

    B : a prime number

    C : a number which is greater than 5.

    Is A, B, and C exhaustive events?

24. Punching time of an employee is given below:

DAY	**MONDAY**	**TUESDAY**	**WEDNESDAY**	**THURSDAY**	**FRIDAY**	**SATURDAY**
TIME (AM)	**10:35**	**10:20**	**10:22**	**10:27**	**10:25**	**10:40**

If the reporting time is 10:30 a.m, then find the probability of his coming late.

25. A game has 18 triangular blocks out of which 8 are blue and rest are red and 19 square blocks out of which 7 are blue and rest are yellow. On piece is lost. Find the probability that it was a square of blue colour.

26. A card is drawn from a pack of 52 cards. Find the probability of getting:
    (i) a jack or a queen
    (ii) a king or a diamond
    (iii) a heart or a club
    (iv) either a red or a face card.
    (v) neither a heart nor a king
    (vi) neither an ace nor a jack
    (vii) a face card

27. In a leap year find the probability of
    (i) 53 Mondays and 53 Tuesdays
    (ii) 53 Mondays and 53 Wednesday
    (iii) 53 Mondays or 53 Tuesdays
    (iv) 53 Mondays or 53 Wednesday

28. In a non-leap year, find the probability of
    (i) 53 Mondays and 53 Tuesdays.
    (ii) 53 Mondays or 53 Tuesdays.

29. Two card are drawn at random from a deck of 52 playing cards. Find the probability of drawing two kings.

30. Three candidates A, B, and C are going to play in a chess competition to win FIDE (World Chess Federation) cup this year. A is thrice as likely to win as B and B is twice as likely as to win C. Find the respective probability of A, B and C to win the cup.

LONG ANSWER QUESTIONS

31. Find the probability that in a random arrangement of the letters of the word UNIVERSITY two I′s come together.

32. An urn contains 5 blue and an unknown number x of red balls. Two balls are drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, find x.

33. Out of 8 points in a plane 5 are collinear. Find the probability that 3 points selected at random form a triangle.

34. Find the probability of at most two tails or at least two heads in a toss of three coins.

35. A, B and C are events associated with a random experiment such that

    $P(A) = 0.3$,

    $P(B) = 0.4$, $P(C) = 0.8$, $P(A \cap B) = 0.08$, $P(A \cap C) = 0.28$ and $P(A \cap B \cap C) = 0.09$. If

    $P(A \cup B \cup C) \geq 0.75$ Then prove that $P(B \cap C)$ lies in the interval $[0.23, 0.48]$.

36. $\frac{1+3p}{3}$, $\frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probability of three mutually exclusive events. Then find the set of all values of p.

37. An urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from urn B and placed in urn A. Now if one ball is drawn at random from urn A then find the probability that it is found to be red.

38. If three distinct numbers are chosen randomly from the first 100 natural numbers, then find the probability that all three of them are divisible by both 2 and 3.

39. S = {1, 2, 3, ………, 30}, A = {x : x is multiple of 7}, B = { x : x is multiple of 5}, C = {x : x is a multiple of 3}.

If x is a member of S chosen at random find the probability that

(i) $x \in A \cup B$

(ii) $x \in B \cap C$

(iii) $x \in A \cap \bar{C}$

40. One number is chosen at random from the number 1 to 100. Find the probability that it is divisible by 4 or 10.

41. The number lock of a suitcase has 4 wheels with 10 digits, i.e. fro 0 to 9. The lock open with a sequence of 4 digits with repeats allowed. What is the probability of a person getting the right sequence to open the suit case?

42. If A and B are any two events having $P(A \cup B) = \frac{1}{2}$ and $P(\bar{A}) = \frac{2}{3}$, then find the $P(\bar{A} \cap B)$.

43. Three of the six vertices of a regular hexagon are chosen at random. What is probability that the triangle with these vertices is equilateral?

44. A typical PIN (Personal identification number) is a sequence of any four symbols chosen from the 26 letters in the alphabet and ten digits. If all PINs are equally likely, what is the probability that a randomly chosen PIN contains a repeated symbol?

45. An urn contains 9 red, 7 white and 4 black balls. If two balls are drawn at random. Find the probability that the balls are of same colour.

46. The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it will get both awards is 0.2. What is the probability, that

(i) it will get at least one of the two awards

(ii) it will get only one of the awards.

47. A girl calculates that the probability of her winning the first prize in a lottery is 0.02. If 6000 tickets were sold, how many tickets has she bought?

48. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9?

49. All the face cards are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 and similar value for other cards. Find the probability of getting a card with value less than 7.

50. If A, B and C are three mutually exclusive and exhaustive events of an experiment such that

$3P(A) = 2P(B) = P(C)$, then find the value of $P(A)$.

CASE STUDY TYPE QUESTIONS

51. To make a healthy routine and to do some physical exercise during lockdown a family decided to roll a dice and based on the outcomes, they will decide activities to be done.

- If the outcome is 2, 4 or 6, they will do 30 minutes walk on the roof.
- If it shows 1 or 3 on the dice, 15 minutes meditation to be done.

- If outcome is 5, then they will toss a coin. If it shows "Head", the family will do 5 minutes of rope skipping. If there is "Tail", family will do 20 minutes of Yoga.

i. How many elements are there in the sample space?

(a) 6 (b) 9 (c) 7 (d) 8

ii. What is the probability of doing walking?

(a) $\frac{5}{6}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{1}{6}$

iii. What is the probability of doing rope skipping?

(a) $\frac{1}{3}$ (b) $\frac{2}{7}$ (c) $\frac{1}{6}$ (d) $\frac{1}{7}$

iv. What is the probability of doing yoga or meditation?

(a) 1 (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{1}{2}$

v. Two activities having the same probability are

(a) Walking and Yoga

(b) Yoga and Rope Skipping

(c) Rope Skipping and Walking

(d) Walking and Meditation

52. In a class of 60 students, hobbies were discussed. 30 liked reaing, 32 liked singing and 24 liked about reading and singing.

i. Find the probability that the student liked reading or singing.

(a) $\frac{17}{30}$ (b) $\frac{19}{30}$ (c) $\frac{23}{30}$ (d) $\frac{29}{30}$

ii. How many students neither like reading nor singing?

(a) 30 (b) 28 (c) 22 (d) 38

iii. Find the probability that the student neither like singing nor reading?

(a) $\frac{11}{30}$ (b) $\frac{13}{30}$ (c) $\frac{7}{30}$ (d) $\frac{1}{30}$

iv. Find the probability that a student like singing but not reading?

(a) $\frac{4}{15}$ (b) $\frac{7}{15}$ (c) $\frac{1}{15}$ (d) $\frac{2}{15}$

v. Find the probability that a student like reading only.

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{7}{10}$ (d) 0

ANSWERS

1.	(a)	False	2.	(a)	{2, 4, 6}	(f)	$\frac{7}{12}$
	(b)	True		(b)	0.8	(g)	$\frac{1}{3}$
	(c)	True		(c)	0.2	(h)	$\frac{7}{18}$
	(d)	False		(d)	0.07	(i)	$\frac{1}{2}$
	(e)	False		(e)	$\frac{5}{6}$	(j)	$\frac{1}{3}$
3.	(c)		4.	(d)			
5.	(b)		6.	(c)			
7.	(a)		8.	(d)			
9.	(d)		10.	(c)			

11. {0, 1, 2}

12. {R, B}

13. {T, HT, HHT, …..}

14. {HR_1, HR_2, HB_1, HB_2, HB_3, TH, TT}

15. {RW, WW, WR}

16. 2n

17. $\frac{8}{21}$

18. $\frac{1}{15}$

19. Mutually Exclusive

20. P(A) = P(B)

21. $\frac{5}{7}$

22. $\frac{1}{36}$

23. Yes, A, B and C are Exhaustive Events

24. $\frac{1}{3}$

25. $\frac{7}{37}$

26. (i) $\frac{2}{13}$ (ii) $\frac{4}{13}$ (iii) $\frac{1}{2}$ (iv) $\frac{8}{13}$

    (v) $\frac{9}{13}$ (vi) $\frac{11}{13}$ (vii) $\frac{3}{13}$

27. (i) $\frac{1}{7}$ (ii) 0 (iii) $\frac{3}{7}$ (iv) $\frac{4}{7}$

28. (i) 0 (ii) $\frac{2}{7}$

29. $\frac{1}{221}$

30. $P(A) = \frac{6}{9} = \frac{2}{3}$, $P(B) = \frac{2}{9}$, $P(C) = \frac{1}{9}$

31. $\frac{1}{5}$

32. 3

33. $\frac{23}{28}$

34. $\frac{7}{8}$

35. $0.23 \le P(B) \le 0.48$

36. $\frac{-1}{3} \le P \le \frac{-1}{3}$

37. $\frac{32}{55}$

38. $\frac{4}{1155}$

39. (i) $\frac{1}{3}$ (ii) $\frac{1}{15}$ (iii) $\frac{1}{10}$

40. $\frac{3}{10}$

41. $\frac{1}{10000}$

42. $\frac{1}{6}$

43. $\frac{1}{10}$

44. $\frac{265896}{1679616}$

45. $\frac{63}{190}$

46. (i) 0.64
(ii) 0.44

47. 120

48. $\frac{5}{12}$

49. $\frac{3}{5}$

50. $\frac{2}{11}$

51. i. (c) ii. (b) iii. (d) iv. (c) v. (d)

52. i. (b) ii. (c) iii. (a) iv. (d) v. (a)

PRACTICE PAPER – 1

Class – XI

MATHEMATICS

Time Allowed : 90 minutes **Maximum Marks: 40**

General Instructions:

(i) This question paper contains three sections – A, B and C. Each part is compulsory.

(ii) Section – A has 20 MCQs, attempt any 16 out of 20.

(iii) Section – B has 20 MCQs, attempt any 16 out of 20.

(iv) Section – C has 10 MCQs, attempt any 8 out of 10.

(v) There is no negative marking.

(vi) All questions carry equal marks.

SECTION-A

In this section, attempt any 16 questions out of questions 1-20. Each question is of 1 mark weightage.

1. Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second. The values of m and n are respectively

(a) 4, 7 (b) 7, 4

(c) 4, 4 (d) 7, 7

2. If A = {1, 2, 3}, B = {1, 4, 6, 9} and R is a relation from A to B defined by ‘x is greater than y’. The range of R is

(a) {1, 4, 6, 9} (b) {1}

(c) {4, 6, 9} (d) {1, 9}

3. $\lim_{x\to 3} \frac{x-3}{1x-31}$, is equal to

(a) –1 (b) 1

(c) 3 (d) does not exit

4. If the set A has p elements, B has q elements, then the number of elemetns in A × B is

(a) p + q (b) p + q – 1

(c) pq (d) q^2

5. Let S = { x : x is a positive multiple of 3 less than 100}, P = { x : x is a prime no. less than 20} Then, n(S) + n(P) is

(a) 34 (b) 31

(c) 33 (d) 30

6. If α and β are the roots of $4x^2 + 3x + 7 = 0$, then the value of $\frac{1}{\alpha}+\frac{1}{\beta}$ is

(a) $\frac{4}{7}$ (b) $\frac{-4}{7}$

(c) $\frac{-3}{7}$ (d) $\frac{3}{7}$

7. The n^{th} term of a G.P. is 128 and the sum of its n terms is 225. If its common ratio is 2, then its first term is

(a) 1 (b) 3

(c) 8 (d) None of these

8. The least value of K which makes the roots of the equation $x^2 + 5x + K = 0$ imaginary is

(a) 4 (b) 5

(c) 6 (d) 7

9. $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-1}{x}$ is equal to

(a) 2 (b) $\frac{1}{2}$

(c) –1 (d) does not exist

10. Let n(A) = m and n(B) = n. then the total number of non-empty relations that can be defined from A to B is

(a) m^n (b) $n^m - 1$

(c) $m^n - 1$ (d) $2^{mn} - 1$

11. If $x + iy = \frac{3+5i}{7-6i}$ then y =

(a) $\frac{9}{85}$ (b) $\frac{-9}{85}$

(c) $\frac{53}{85}$ (d) None of these

12. The value of $\left(i^5 + i^6 + i^7 + i^8 + i^9\right) \big/ (1+i)$ is

(a) $\frac{1}{2}(1+i)$ (b) $\frac{1}{2}(1-i)$

(c) 1 (d) $\frac{1}{2}$

13. A line passes through the point (2, 2) and is perpendicular to the line $3x + y = 3$. Its y intercept is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) 1 (d) $\frac{4}{3}$

14. The distance of the point of intersection of the lines $2x - 3y + 5 = 0$ and $3x + 4y = 0$ from the lines $5x - 2y = 0$, is

(a) $\frac{130}{17\sqrt{29}}$ (b) $\frac{13}{\sqrt{29}}$

(c) $\frac{30}{7}$ (d) None of these

15. Ina town of 840 persons, 450 persons read Hindi, 300 read English and 200 both. Then the number of persons who read neither is

(a) 290 (b) 180

(c) 210 (d) 260

16. Mean of 10 items is 17. If an observation 21 is replaced with 12, then new mean is

(a) 17 (b) 26

(c) 8 (d) 16.1

17. $\lim_{x\to 0} \frac{\sin ax + bx}{ax + \sin bx}$ $a, b, a+b \neq 0$ is equal to

(a) –1 (b) 0

(c) $\frac{a}{b}$ (d) 1

18. Slopes of lines which cuts off intercept of equal lengths on the coordinate axes are

(a) 0 (b) $\pm\frac{1}{\sqrt{3}}$

(c) ± 1 (d) $\pm\sqrt{3}$

19. The coordinates of the image of the point (2, 3) in the line mirror $x + y - 11 = 0$ are

(a) (5, 6) (b) (11, 9)

(c) (8, 9) (d) (–9, –8)

20. Mean deviation of the data 3, 10, 10, 4, 7, 10, 5 from the mean is

(a) 2 (b) 2.57

(c) 3 (d) 3.75

SECTION-B

In this section, attempt any 16 questions out of questions 21-40. Each question is of 1 mark weightage.

21. $\lim\limits_{x \to 0} \dfrac{(1+x)^6 - 1}{(1+x)^2 - 1}$ is equal to

(a) 3 (b) 6

(c) 0 (d) 5

22. If A and B are two sets, then $A \cap (A \cup B)$ equals

(a) A (b) B

(c) ϕ (d) $A \cap B$

23. The domain of the function *f* given by $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - x - 6}$ is

(a) $R - \{-2, 3\}$ (b) $R - \{-3, 2\}$

(c) $R - [-2, 3]$ (d) $R - (-2, 3)$

24. The two geometric means between the numbers 1 and 64 are

(a) 1 and 64 (b) 2 and 16

(c) 4 and 16 (d) 8 and 16

25. Let $A = \{x : x \in R, x > 4\}$ and $B = \{x : x \in R, x < 5\}$ then $A \cap B =$

(a) $(4, 5]$ (b) $(4, 5)$

(c) $[4, 5)$ (d) $[4, 5]$

26. The range of the function $f(x) = \dfrac{x+2}{|x+2|}$, $x \neq -2$ is

(a) $\{-1, 1\}$ (b) $\{-1, 0, 1\}$

(c) $\{1\}$ (d) $(0, \infty)$

27. The equation of the line passing through (1, 5) and perpendicular to the line $3x - 5y + 7 = 0$ is

(a) $5x + 3y - 20 = 0$ (b) $3x - 5y + 7 = 0$

(c) $3x - 5y + 6 = 0$ (d) $3x + 3y + 7 = 0$

28. Solutions of the quaduatic equation $x^2 - 5ix - 6 = 0$ are

(a) $-3i, -2i$ (b) $3i, 2i$

(c) $-3i, 2i$ (d) $3i, -2i$

29. Domain of $f(x) = \sqrt{a^2 - x^2}$, $a > 0$ is

(a) $(-a, a)$ (b) $[-a, a]$

(c) $[0, a]$ (d) $(-a, 0]$

30. The angle between the lines $2x - y + 3 = 0$ and $x + 2y + 3 = 0$ is

(a) 90° (b) 60°

(c) 45° (d) 30°

31. $\lim\limits_{x\to\pi/2} \dfrac{\tan 2x}{x - \pi/2}$ is equal to

(a) 2 (b) $\dfrac{1}{2}$

(c) 1 (d) –1

32. The value of x for which the expression $\dfrac{1 - i\sin x}{1 + 2i\sin x}$ is, purely real, is where n ∈ N

(a) $(n+1)\dfrac{\pi}{2}$ (b) $(2n+1)\dfrac{\pi}{2}$

(c) $n\pi$ (d) None of these

33. The following information relates to a sample of size 60;

$\sum x^2 = 18000$, $\sum x = 960$

The variance of the data is

(a) 6.63 (b) 16

(c) 22 (d) 44

34. The domain of the function f defined by $f(x) = \dfrac{1}{\sqrt{x - |x|}}$ is

(a) R (b) R^+

(c) R^- (d) None of these

35. If the lines $x + q = 0$, $y - 2 = 0$ and $3x + 2y + 5 = 0$ are concurrent, then the value of q will be

(a) 1 (b) 2

(c) 3 (d) 5

36. The inclinations of the line $x - y + 3 = 0$ with positive direction of x-axis is

(a) 45° (b) 135°

(c) –45° (d) –135°

37. When tested, the lives (in hours) of 5 bulbs, were noted as follows : 1357, 1090, 1666, 1494, 1623

The mean deviation (in hours) from the mean is

(a) 179 (b) 178

(c) 220 (d) 356

38. The modulus of the complex number $\dfrac{1+i}{1-i}$ is

(a) 1 (b) –1

(c) $\sqrt{2}$ (d) $\dfrac{1}{\sqrt{2}}$

39. If $\frac{(a^2+1)^2}{2a-i} = x+iy$, then x^2+y^2 is equal to

(a) $\frac{(a^2+1)^4}{4a^2+1}$ (b) $\frac{(a+1)^2}{4a^2+1}$

(c) $\frac{(a^2-1)^4}{(4a^2-1)^2}$ (d) None of these

40. If the variance of the numbers 2, 4, 5, 6, 8, 17 is 23.33, then the variance of 4, 8, 10, 12, 16, 34 will be

(a) 23.33 (b) 46.66

(c) 93.32 (d) None of theses

SECTION-C

In this section, attempt any 8 questions. Each question is of 1 mark weightage. Questions 46-50 are based on case-study.

41. A survey shows that 63% of the people watch a news channel wherease 76% watch another channel if x % of the people watch both channel, then

(a) $x = 35$ (b) $x = 63$

(c) $39 \le x \le 63$ (d) $x = 39$

42. Let A = {1, 2, 3}, B = {2, 3, 4} then which of the following is a function from A to B is

(a) {(1, 2), (1, 3), (2, 3), (3, 3)} (b) {(1, 3), (1, 2)}

(c) {(1, 3), (2, 2), (3, 3)} (d) {(1, 2), (1, 3), (3, 2), (3, 4)}

43. The mean of 100 observations is 50 and their standard deviation is 5. The sum of squares of all the observation is

(a) 50000 (b) 250000

(c) 252500 (d) 255000

44. $\lim_{x \to 0} \frac{1-\cos x}{x^2} =$

(a) $\frac{1}{2}$ (b) 1

(c) 0 (d) None of these

45. If A = {1, 2, 4}, B = {2, 4, 5}, C = {2, 5}, then (A – B) × (B – C) is

(a) {(1, 2), (1, 5), (2, 5)} (b) {(1, 4}

(c) (1, 4) (d) None of these

CASE STUDY

A manufacturer of AC produced 600 units of AC in the third year and 700 units of AC in the seventh year. The production increases uniformly by a fixed number every year. On the basis of above information answer the following questions :

46. The sequence of production of AC forms a/an

(a) Arithmetic Progression (b) Geometric Progress

(c) Harmonic Progression (d) None of these

47. What was his production in the first year?

(a) 650 (b) 600

(c) 550 (d) 400

48. What was the total production in the first seven year?

(a) 5000 (b) 4375

(c) 3490 (d) 6995

49. In which year his production will be 825 ?

(a) 10^{th} year (b) 11^{th} year

(c) 12^{th} year (d) 13^{th} year

50. What is the difference of production in 8^{th} and 5^{th} year ?

(a) 100 (b) 75

(c) 50 (d) 120

ANSWERS

Practice Paper-1

1	b	21	a	41	c
2	b	22	a	42	c
3	d	23	a	43	c
4	c	24	c	44	a
5	b	25	b	45	b
6	c	26	a	46	a
7	a	27	a	47	c
8	d	28	b	48	b
9	b	29	b	49	c
10	d	30	a	50	b
11	c	31	a		
12	a	32	c		
13	d	33	d		
14	a	34	d		
15	a	35	c		
16	d	36	a		
17	d	37	b		
18	c	38	a		
19	a	39	a		
20	b	40	c		

SAMPLE PAPER-2

Term – 1

Time Allowed: 90 minutes **Maximum Marks: 40**

General Instructions:
1. This question paper contains **three sections – A, B and C**. Each part is compulsory.
2. **Section - A** has 20 MCQs, attempt **any 16 out of 20**.
3. **Section - B** has 20 MCQs, attempt **any 16 out of 20**
4. **Section - C** has 10 MCQs, attempt **any 8 out of 10**.
5. There is no negative marking.
6. All questions carry equal marks.

SECTION – A

In this section, attempt any 16 questions out of Questions 1 – 20.
Each Question is of 1 mark weightage.

1.	If $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 6\}$ and $C = \{3, 4, 6, 7\}$, then (a) $A - (B \cap C) = \{1, 3, 4\}$ (b) $A - (B \cap C) = \{1, 2, 4\}$ (c) $A - (B \cup C) = \{2, 3\}$ (d) $A - (B \cup C) = \{\phi\}$
2.	The number of the proper subset of {a, b, c} is: (a) 3 (b) 8 (c) 6 (d) 7
3.	If A and B are finite sets, then which one of the following is the correct equation? (a) $n(A - B) = n(A) - n(B)$ (b) $n(A - B) = n(B - A)$ (c) $n(A - B) = n(A) - n(A \cap B)$ (d) $n(A - B) = n(B) - n(A \cap B)$ [n (A) denotes the number of elements in A]

4.	Which of the following sets is a finite set? (a) A = $\{x : x \in Z \text{ and } x^2 - 5x + 6 = 0\}$ (b) B = $\{x : x \in Z \text{ and } x^2 \text{ is even}\}$ (c) D = $\{x : x \in Z \text{ and } x > -10\}$ (d) All of these
5.	In a group of 500 students, there are 475 students who can speak Hindi and 200 can speak Bengali. What is the number of students who can speak Hindi only ? (a) 275 (b) 300 (c) 325 (d) 350
6.	$1 + i^2 + i^4 + i^6 + \ldots + i^{2n}$ is (a) positive (b) negative (c) 0 (d) cannot be determined
7.	If α, β are roots of the equation $x^2 - 5x + 6 = 0$, then the equation whose roots are $\alpha + 3$ and $\beta + 3$ is (a) $2x^2 - 11x + 30 = 0$ (b) $-x^2 + 11x = 0$ (c) $x^2 - 11x + 30 = 0$ (d) $2x^2 - 5x + 30 = 0$
8.	For a, b, c to be in G.P. What should be the value of $\frac{a-b}{b-c}$? (a) ab (b) bc (c) $\frac{a}{b}$ or $\frac{b}{c}$ (d) None of these
9.	The value of $\lim_{x \to 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{1+x}$ is (a) 2 (b) –2 (c) 1 (d) –1
10.	If $A \times B = \{(5, 5), (5, 6), (5, 7), (8, 6), (8, 7), (8, 5)\}$, then the value A is (a) $\{5\}$ (b) $\{8\}$ (c) $\{5, 8\}$ (d) $\{5, 6, 7, 8\}$

11.	Amplitude of $\frac{1+\sqrt{3}i}{\sqrt{3}+1}$ is : (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$
12.	The real part of $\frac{(1+i)^2}{(3-i)}$ is (a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $-\frac{1}{3}$ (d) None of these
13.	The inclination of the line $x - y + 3 = 0$ with the positive direction of x-axis is (a) 45° (b) 135° (c) –45° (d) –135°
14.	Line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (x, 24). Find the value of x. (a) 2 (b) 3 (c) 4 (d) 5
15.	A straight line makes an angle of 135° with x-axis and cuts y-axis at a distance of – 5 from the origin. The equation of the line is (a) $2x+y+5=0$ (b) $x+2y+3=0$ (c) $x+y+5=0$ (d) $x+y+3=0$
16.	The lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular to each other (a) $a_1b_1 - b_1a_2 = 0$ (b) $a_1^2b_2 + b_1^2a_2 = 0$ (c) $a_1b_1 + a_2b_2 = 0$ (d) $a_1a_2 + b_1b_2 = 0$
17.	If A = {x : x is a multiple of 3} and B = {x : x is a multiple of 5}, then A – B is equal to (a) $\overline{A} \cap B$ (b) $A \cap \overline{B}$ (c) $\overline{A} \cap \overline{B}$ (d) $\overline{A \cap B}$

18.	If $f(x)=\begin{cases} x^2+1, & x\geq 1 \\ 3x-1, & x<1 \end{cases}$, then the value of $\lim\limits_{x\to 1} f(x)$ is (a) 2 (b) –2 (c) 1 (d) –1
19.	Find the mean and variance for the following data 6, 7, 10, 12, 13, 4, 8, 12 (a) mean = 9, variance = 9.25 (b) mean = 3, variance = 7.5 (c) mean = 7, variance = 12 (d) mean = 9, variance = 12.5
20.	The mean deviation from the mean of the following data : Marks: 0-10, 10-20, 20-30, 30-40, 40-50 No. of Students: 5, 8, 15, 16, 6 is (a) 10 (b) 10.22 (c) 9.86 (d) 9.44

SECTION – B

In this section, attempt any 16 questions out of the Questions 21 - 40.
Each Question is of 1 mark weightage.

21.	The value of $\lim\limits_{x\to 0}\dfrac{\cos x}{\pi - x}$ is (a) π (b) $-\pi$ (c) $\dfrac{1}{\pi}$ (d) $-\dfrac{1}{\pi}$
22.	The set {x : x is a positive integer less than 6 and $3^x - 1$ is an even number} in roster form is (a) {1, 2, 3, 4, 5} (b) {1, 2, 3, 4, 5, 6} (c) {2, 4, 6} (d) {1, 3, 5}

23. If $f(x) = x^3 - \frac{1}{x^3}$, then $f(x) + f\left(\frac{1}{x}\right)$ is equal to

(a) $2x^3$ (b) $2\frac{1}{x^3}$

(c) 0 (d) 1

24. The first and eight terms of a G.P. are x^{-4} and x^{52} respectively. If the second term is x^t, then t is equal to:

(a) –13 (b) 4 (c) $\frac{5}{2}$ (d) 3

25. The interval [a, b] is represented on the number line as

(a) 

(b) 

(c) 

(d) 

26. The domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ is equal, i.e. $f(x) = g(x)$, is

(a) $\{0, 2\}$ (b) $\left\{\frac{1}{2}, -2\right\}$

(c) $\left\{-\frac{1}{2}, 2\right\}$ (d) $\left\{\frac{1}{2}, 2\right\}$

27. The line $(3x - y + 5) + \lambda(2x - 3y - 4) = 0$ will be parallel to y-axis, if λ =

(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

28. The roots of equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ is

(a) one (b) two

(c) infinite (d) None of these

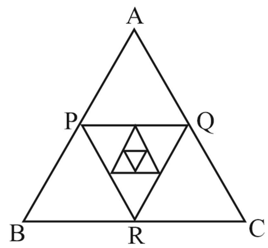
29.	Let $A = \{1, 2, 3, 4\}$, $B = \{1, 5, 9, 11, 15, 16\}$ and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$. Then, (a) f is a relation from A to B (b) f is a function from A to B (c) Both (a) and (b) (d) None of these
30.	If the points (x, y), $(1, 2)$ and $(-3, 4)$ are collinear, then (a) $x+2y-5=0$ (b) $x+y-1=0$ (c) $2x+y-4=0$ (d) $2x-y+10=0$
31.	$\lim\limits_{x\to 0} \dfrac{2\sin^2 3x}{x^2}$ is equal to : (a) 12 (b) 18 (c) 0 (d) 6
32.	If $(x+iy)^{\frac{1}{3}} = a + ib$, where $x, y, a, b \in R$, then $\dfrac{x}{a} - \dfrac{y}{b} =$ (a) $a^2 - b^2$ (b) $-2(a^2 + b^2)$ (c) $2(a^2 - b^2)$ (d) $a^2 + b^2$
33.	The variance of n observations $x_1, x_2, \ldots, x_n$ is given by (a) $\sigma^2 = \dfrac{1}{n}\sum\limits_{i=1}^{n}(x_i - \bar{x})$ (b) $\sigma^2 = \dfrac{1}{n}\sum\limits_{i=1}^{n}(x_i - \bar{x})^2$ (c) $\sigma^2 = \dfrac{1}{n}\sum\limits_{i=1}^{n}(x_i + \bar{x})$ (d) $\sigma^2 = \dfrac{1}{n}\sum\limits_{i=1}^{n}(x_i + \bar{x})^2$
34.	If $A = \{1, 2\}$, $B = \{1, 3\}$, then $(A \times B) \cup (B \times A)$ is equal to (a) $\{(1, 3), (2, 3), (3, 1), (3, 2), (1, 1), (2, 1), (1, 2)\}$ (b) $\{(1, 3), (3, 1), (3, 2), (2, 3)\}$ (c) $\{(1, 3), (2, 3), (3, 1), (3, 2), (1, 1)\}$ (d) None of these

35.	What is the angle between the two straight lines y= $(2-\sqrt{3})x + 5$ and y= $(2+\sqrt{3})x - 7$? (a) 60° (b) 45° (c) 30° (d) 15°
36.	The equation of the straight line that passes through the point (3, 4) and perpendicular to the line $3x + 2y + 5 = 0$ is (a) $2x + 3y + 6 = 0$ (b) $2x - 3y - 6 = 0$ (c) $2x - 3y + 6 = 0$ (d) $2x + 3y - 6 = 0$
37.	The coefficient of variation is computed by: (a) $\frac{\text{mean}}{\text{standard deviation}}$ (b) $\frac{\text{standard deviation}}{\text{mean}}$ (c) $\frac{\text{mean}}{\text{standard deviation}} \times 100$ (d) $\frac{\text{standard deviation}}{\text{mean}} \times 100$
38.	If $z_1 = 2 - i$ and $z_2 = 1 + i$, then value of $\left\lvert \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right\rvert$ is (a) 2 (b) 2i (c) $\sqrt{2}$ (d) $\sqrt{2}i$
39.	If $\frac{(1+i)^3}{(1-i)^3} - \frac{(1-i)^3}{(1+i)^3} = x + iy$ (a) $x = 0, y = -2$ (b) $x = -2, y = 0$ (c) $x = 1, y = 1$ (d) $x = -1, y = 1$
40.	While dividing each entry in a data by a non-zero number *a*, the arithmetic mean of the new data: (a) is multiplied by *a* (b) does not change (c) is divided by *a* (d) is diminished by *a*

SECTION – C

In this section, attempt any 8 questions.
Each question is of 1-mark weightage.
Questions 46-50 are based on a Case-Study.

41.	A market research group conducted a survey of 2000 consumers and reported that 1720 consumers like product P_1 and 1450 consumers like product P_2. What is the least number that must have liked both the products? (a) 1150 (b) 2000 (c) 1170 (d) 2500
42.	Domain of $\sqrt{a^2 - x^2}$, $(a > 0)$ is (a) $(-a, a)$ (b) $[-a, a]$ (c) $[0, a]$ (d) $(-a, 0]$
43.	When tested, the lives (in hours) of 5 bulbs were noted as follows 1357, 1090, 1666, 1494, 1623 The mean deviations (in hours) from their mean is (a) 178 (b) 179 (c) 220 (d) 356
44.	Value of $\lim\limits_{x \to 5} \dfrac{1-\sqrt{x-4}}{x-5}$ is (a) 0 (b) $\dfrac{1}{2}$ (c) $-\dfrac{1}{2}$ (d) does not exist
45.	If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$, then $f\left(\dfrac{1}{2}\right) \times g(14)$ is (a) $\dfrac{1336}{5}$ (b) $\dfrac{1363}{4}$ (c) 1251 (d) 1608

	The side of equilateral triangles is 20 cm. The mid points of its sides are joined to form another triangle. The process is continued as shown in figure  Based on above information, answer the following
46.	The side of 5^{th} triangle is (a) 5 cm (b) 2.5 cm (c) 1.25 cm (d) None of these
47.	The perimeter of sixth triangle is (a) 3 cm (b) $\frac{15}{8}$ cm (c) 15 cm (d) None of these
48.	Sum of perimetres of first five triangle is (a) $\frac{465}{4}$ cm (b) $\frac{465}{2}$ cm (c) 465 cm (d) None of these
49.	The area of third triangle (a) $25\sqrt{3}$ cm^2 (b) $\frac{25\sqrt{3}}{4}$ cm^2 (c) $\frac{75\sqrt{3}}{4}$ cm^2 (d) None of these
50.	The sum of areas of first four triangles (a) $\frac{2125\sqrt{3}}{8}$ cm^2 (b) $\frac{2125\sqrt{3}}{4}$ cm^2 (c) $2125\sqrt{3}$ cm^2 (d) None of these

ANSWERS

Practice Paper-2

1	b
2	d
3	c
4	a
5	b
6	d
7	c
8	c
9	a
10	c
11	c
12	d
13	a
14	c
15	c
16	d
17	b
18	a
19	a
20	d

21	c
22	a
23	c
24	b
25	b
26	b
27	b
28	b
29	a
30	a
31	b
32	b
33	b
34	a
35	a
36	c
37	d
38	c
39	a
40	c

41	c
42	b
43	a
44	c
45	b
46	a
47	b
48	a
49	b
50	d

PRACTICE PAPER – 3

Class – XI

Time Allowed : 3 Hrs. **Maximum Marks: 80**

General Instructions:

(i) This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.

(ii) Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.

(iii) Both Part A and Part B have choices.

Part-A :

(i) It consist of two section I and II

(ii) Section II comprises of 16 very short answer type questions.

(iii) Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part-B :

(i) It consist of two sections III, IV and V

(ii) Section III comprises of 10 questions of 2 marks each.

(iii) Section IV comprises 7 questions of 3 marks each.

(iv) Section V comprises of 3 questions of 5 marks each.

Part – A

SECTION - I

Q. 1 to Q. 16 carries 1 mark each

1. Write the set A = { x : x is an integer, $-1 \le x < 4$} in roster form.

2. Let A, B be any two sets. Using properties of sets prove that $(A - B) \cup B = A \cup B$.

3. Write all the possible subsets of A = {5, 6}.

4. Let A = {1, 2, 3, 4}, B = {1, 4, 9, 16, 25} and R be a relation defined from A to B as, $R = \{(x, y) : x \in A, y \in B \text{ and } y = x^2\}$. Depict this relation using arrow diagram.

OR

How many words can be formed from the letters of the word 'ORDINATE' so that vowels occupy odd places?

5. Write the contrapositive of the following statements: If a triangle is equilateral, it is isosceles.

6. Find the radian measure corresponding to 5° 37' 30".

7. Find the real value of x and y, if $(x + iy)(2 - 3i) = 4 + i$

8. Solve the following system of equation in R. $x + 3 > 0$, $2x < 14$.

9. Using the digits 1, 2, 3, 4, 5 how many 3 digit numbers (without repeating the digits) can be made?

OR

How many words can be formed from the letters of the word 'ORDINATE' so that vowels occupy odd places?

10. Find the term containing x^3 in the expansion of $(x - 2y)^7$.

11. Find 4th term of an A.P. is equal to 3 times the first term and the seventh term exceeds twice the 3rd term by 1. Find the 1st term and the common difference?

12. Find the distance of the point (–1, 1) from the line $12(x + 6) = 5(y - 2)$.

13. Find the centre and radius of the circle $x^2 + y^2 - 2x + 3y = 4$.

14. Find the distance between (–3, 4, –6) and its image in the XY-Plane.

15. If E and F are events such that P(E) = $\frac{1}{4}$, P(F) = $\frac{1}{2}$ and P(E and F) = $\frac{1}{8}$, Find P (not E and not F).

16. Write the converse of the following statements.

    If you do all the exercises in the book, you get A grade in the class.

SECTION - II

Case Study Based Questions. Attempt any 4 subparts each part carries 1 mark each.

17. If P and Q are two differents solution of the equation $a\cos x + b\sin x = c$ then answer the following question.

i. The value of $\tan\left(\frac{P+Q}{2}\right)$ is

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) $\frac{c}{a}$ (d) $\frac{a}{c}$

ii. The value of $\sin\left(\frac{P+Q}{2}\right)$ is

(a) $\frac{b}{\sqrt{a^2+b^2}}$ (b) $\frac{a}{\sqrt{a^2+b^2}}$ (c) $\frac{c}{\sqrt{a^2+b^2}}$ (d) $\frac{\sqrt{a^2+b^2}}{b}$

iii. The value of $\sin(P+Q)$ is

(a) $\frac{2ac}{a^2+b^2}$ (b) $\frac{2bc}{a^2+b^2}$ (c) $\frac{2ab}{a^2+b^2}$ (d) $\frac{2a}{a^2+b^2}$

iv. The value of $\cos(P+Q)$ is

(a) $\frac{a^2-c^2}{a^2+b^2}$ (b) $\frac{c^2-b^2}{a^2+b^2}$ (c) $\frac{a^2+b^2}{a^2-b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

v. The value of $\tan(P+Q)$ is

(a) $\frac{2ac}{a^2-b^2}$ (b) $\frac{2ab}{a^2-b^2}$ (c) $\frac{2ab}{a^2+b^2}$ (d) $\frac{2ac}{a^2+b^2}$

18. Four candidates A, B, C and D are applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B, and B and C are given same chance of being selected, while c is twice as likely to be selected as D, what are the probabilities that

i. A will be selected

(a) $\frac{2}{9}$ (b) $\frac{3}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4}{9}$

ii. B will be selected

(a) $\frac{2}{9}$ (b) $\frac{3}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4}{9}$

iii. D will be selected

(a) $\frac{2}{9}$ (b) $\frac{3}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4}{9}$

iv. Either C or D will be selected

(a) $\frac{2}{9}$ (b) $\frac{3}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4}{9}$

v. Neither C or D will be selected

(a) $\frac{2}{9}$ (b) $\frac{3}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4}{9}$

Part – B

SECTION - III

Q. 19 to Q. 28 carries 2 marks each

19. Draw the graph of the Greatest Integer Function also find its range.

20. Find domain and range of $\frac{x^2}{1} + x^2$.

21. Prove that $\frac{\sin 11x \sin x + \sin 7x \sin 3x}{\cos 11x \sin x + \cos 7x \sin 3x} = \tan 8x$.

22. Find the value of $\tan\left(\frac{\pi}{8}\right)$.

23. If $x+iy+\frac{a+b}{a-ib}$, prove that $x^2+y^2=1$.

24. Prove that the coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficient x^n of in the expansion of $(1+x)^{(2n-1)}$.

25. A committee of 6 is to be formed from 6 boys and 4 girls. In how many ways can this be done if the committee contains : (a) 2 girls? (b) atleast 2 girls?

26. Find the co-ordinates of the foci, vertices, the length of latus rectum & eccentricity of the ellipse $100x^2+25y^2=2500$.

27. A point R with x-coordiante 4 lies on the line segment joining the points P(2, –3, 4) and Q(8, 0, 10). Find the coordinates of the point R.

28. Evaluate

$$\lim_{x\to 0}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3x+x}-2\sqrt{x}}$$

SECTION - IV

Q. 29 to Q. 35 carries 3 marks each

29. Let A and B be two finites sets such that n(A) = m and n(B) = n. If the ratio of number of elements of power sets of A and B is 64 and n(A) + n(B) = 32. Find the value of m and n.

30. Solve the equation $4\sin x\cos x+2\sin x+2\cos x+1=0$.

31. Prove that by the principle of mathematical induction $3^{2n}+7$ is divisible by 8 for all $n\in N$.

32. Convert the complex number $Z=\dfrac{i-1}{\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)}$ in the polar form

OR

If a and b are different complex numbers with r $|\beta| = 1$, find $\left|\frac{\beta - \alpha}{1 - \alpha\beta}\right|$.

33. The sum of two numbers is six times their geometric mean. Show that the numbers are in the ratio $\frac{\left(3+2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)}$.

34. If the image of the point (2, 1) in a line is (4, 3). Find the equation of the line.

35. Differentiate using First Principle $\sin(x-3)$.

SECTION - IV

Q. 36 to Q. 38 carries 5 marks each

36. Solve the following systems of inequations graphically :

$5x + y \geq 10,\ 2x + 2y \geq 12,\ x + 4y \geq 12,\ x \geq 0,\ y \geq 0$

37. Show that $\frac{1 \times 2^2 + 2 \times 3^2 + ... + nx(n+1)^2}{1^2 \times 2 + 2^2 \times 3 + ... + n^2 x(n+1)} \quad \frac{3n+5}{3n+1}$.

38. The diameters of circles (in mm) drawn in a design are given below :

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

ANSWERS
Practice Paper-3

1. $\{-1, 0, 1, 2, 3\}$
3. $\{\phi, \{5\}, \{6\}, \{5,6\}\}$
4. or R
5. If triangle is not isosceles then it is not equilateral.

6.	$\pi/32$	7.	$x = 5/13$ and $y = 14/13$
8.	$(-3, 7)$	9.	60 or 576
10.	$560y^4$	11.	$a = 3, d = 2$
12.	5	13.	$C(1, -3/2),\ r = \sqrt{29}/2$
14.	12	15.	3/8

16. If you get a grade in the class then you do all the exercise in the book.
17. (i) b (ii) a (iii) c (iv) d (v) c
18. (i) d (ii) a (iii) c (iv) b (v) c
19. Range : Z
20. Domain : R, Range : [0, 1)

22. $\sqrt{2}-1$

25. (a) 90 (b) 185
26. $F\left(0,\ \pm\sqrt{75}\right),\ V\left(0,\ \pm 10\right),\ LR = 5,\ \ e = \sqrt{3}/2$
27. $(4, -2, 6)$

28. $2\sqrt{3}/9$

29. $m = 19, n = 13$
30. $x = n\pi + (-1)^n 7\pi/6$ and $x = 2n\pi \pm 2\pi/3$
32. $\sqrt{2}\left(\cos 5\pi/12 + i\sin 5\pi/12\right)$

34.	$x + y - 5 = 0$	35.	$\cos(x-3)$
38.	5.55	43.	5

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