DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
(2021-2022)

Class : IX
MATHEMATICS
(ENGLISH MEDIUM)

Under the Guidance of

Shri H. Rajesh Prasad
Pr. Secretary (Education)

Shri Udit Prakash Rai
Director (Education)

Dr. Rita Sharma
Addl. DE (School & Exam.)

Coordinators

Mr. Sanjay Subhas Kumar	Mrs. Sunita Dua	Mr. Raj Kumar	Mr. Krishan Kumar
DDE (Exam)	OSD (Exam)	OSD (Exam)	OSD (Exam)

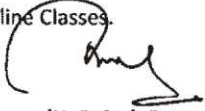Production Team
Anil Kumar Sharma

H. RAJESH PRASAD
IAS

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MESSAGE

I would like to congratulate the members of Core Academic Unit and the subject experts of the Directorate of Education, who inspite of dire situation due to Corona Pandemic, have provided their valuable contributions and support in preparing the Support Material for classes IX to XII.

The Support Material of different subjects, like previous years, have been reviewed/ updated in accordance with the latest changes made by CBSE so that the students of classes IX to XII can update and equip themselves with these changes. I feel that the consistent use of the Support Material will definitely help the students and teachers to enrich their potential and capabilities.

Department of Education has taken initiative to impart education to all its students through online mode, despite the emergency of Corona Pandemic which has led the world to an unprecedented health crises. This initiative has not only helped the students to overcome their stress and anxiety but also assisted them to continue their education in absence of formal education. The support material will ensure an uninterrupted learning while supplementing the Online Classes.

(H. Rajesh Prasad)

UDIT PRAKASH RAI, IAS
Director, Education & Sports

सत्यमेव जयते

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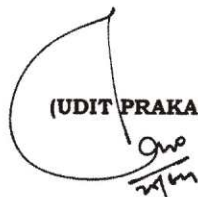E-mail : diredu@nic.in

MESSAGE

The main objective of the Directorate of Education is to provide quality education to all its students. Focusing on this objective, the Directorate is continuously in the endeavor to make available the best education material, for enriching and elevating the educational standard of its students. The expert faculty of various subjects undertook this responsibility and after deep discussions and persistent efforts, came up with Support Material to serve the purpose.

Every year the Support Material is revised/updated to incorporate the latest changes made by CBSE in the syllabus of classes IX to XII. The contents of each lesson/chapter are explained in such a way that the students can easily comprehend the concept and get their doubts solved.

I am sure, that the continuous and conscientious use of this Support Material will lead to enhancement in the educational standard of the students, which would definitely be reflected in their performance.

I would also like to commend the entire team members for their contributions in the preparation of this incomparable material.

I wish all the students a bright future.

(UDIT PRAKASH RAI)

Dr. RITA SHARMA
Additional Director of Education
(School/Exam)

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MESSAGE

It gives me immense pleasure to present the revised edition of the Support Material. This material is the outcome of the tireless efforts of the subject experts, who have prepared it following profound study and extensive deliberations. It has been prepared keeping in mind the diverse educational level of the students and is in accordance with the most recent changes made by the Central Board of Secondary Education.

Each lesson/chapter, in the support material, has been explained in such a manner that students will not only be able to comprehend it on their own but also be able to find solution to their problems. At the end of each lesson/chapter, ample practice exercises have been given. The proper and consistent use of the support material will enable the students to attempt these exercises effectively and confidently. I am sure that students will take full advantage of this support material.

Before concluding my words, I would like to appreciate all the team members for their valuable contributions in preparing this unmatched material and also wish all the students a bright future.

(Rita Sharma)

भारत का संविधान

उद्देशिका

हम, भारत के लोग, भारत को एक [1][संपूर्ण प्रभुत्व-संपन्न समाजवादी पंथनिरपेक्ष लोकतंत्रात्मक गणराज्य] बनाने के लिए, तथा उसके समस्त नागरिकों को :

सामाजिक, आर्थिक और राजनैतिक **न्याय,**

विचार, अभिव्यक्ति, विश्वास, धर्म

और उपासना की **स्वतंत्रता,**

प्रतिष्ठा और अवसर की **समता**

प्राप्त कराने के लिए,

तथा उन सब में

व्यक्ति की गरिमा और [2][राष्ट्र की एकता

और अखंडता] सुनिश्चित करने वाली **बंधुता**

बढ़ाने के लिए

दृढ़संकल्प होकर **अपनी इस संविधान सभा में** आज तारीख 26 नवम्बर, 1949 ई० (मिति मार्गशीर्ष शुक्ला सप्तमी, संवत् दो हजार छह विक्रमी) को एतद्द्वारा इस संविधान को अंगीकृत, अधिनियमित और आत्मार्पित करते हैं ।

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1. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3-1-1977 से) "प्रभुत्व-संपन्न लोकतंत्रात्मक गणराज्य" के स्थान पर प्रतिस्थापित ।
2. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3-1-1977 से) "राष्ट्र की एकता" के स्थान पर प्रतिस्थापित ।

मूल कर्तव्य

51क. मूल कर्तव्य–भारत के प्रत्येक नागरिक का यह कर्तव्य होगा कि वह–

(क) संविधान का पालन करे और उसके आदर्शों, संस्थाओं, राष्ट्र ध्वज और राष्ट्रगान का आदर करे ;

(ख) स्वतंत्रता के लिए हमारे राष्ट्रीय आंदोलन को प्रेरित करने वाले उच्च आदर्शों को हृदय में संजोए रखे और उनका पालन करे ;

(ग) भारत की प्रभुता, एकता और अखंडता की रक्षा करे और उसे अक्षुण्ण रखे ;

(घ) देश की रक्षा करे और आह्वान किए जाने पर राष्ट्र की सेवा करे ;

(ङ) भारत के सभी लोगों में समरसता और समान भातृत्व की भावना का निर्माण करे जो धर्म, भाषा और प्रदेश या वर्ग पर आधारित सभी भेदभाव से परे हो, ऐसी प्रथाओं का त्याग करे जो स्त्रियों के सम्मान के विरुद्ध है ;

(च) हमारी सामासिक संस्कृति की गौरवशाली परंपरा का महत्व समझे और उसका परिरक्षण करे ;

(छ) प्राकृतिक पर्यावरण की, जिसके अंतर्गत वन, झील, नदी और वन्य जीव हैं, रक्षा करे और उसका संवर्धन करे तथा प्राणि मात्र के प्रति दयाभाव रखे ;

(ज) वैज्ञानिक दृष्टिकोण, मानववाद और ज्ञानार्जन तथा सुधार की भावना का विकास करे ;

(झ) सार्वजनिक संपत्ति को सुरक्षित रखे और हिंसा से दूर रहे ;

(ञ) व्यक्तिगत और सामूहिक गतिविधियों के सभी क्षेत्रों में उत्कर्ष की ओर बढ़ने का सतत प्रयास करे जिससे राष्ट्र निरंतर बढ़ते हुए प्रयत्न और उपलब्धि की नई ऊंचाइयों को छू ले ;]

²[(ट) यदि माता-पिता या संरक्षक है, छह वर्ष से चौदह वर्ष तक की आयु वाले अपने, यथास्थिति, बालक या प्रतिपाल्य के लिए शिक्षा के अवसर प्रदान करे ।]

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1 संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 11 द्वारा (3-1-1977 से) अंतःस्थापित ।

2 संविधान (छियासीवां संशोधन) अधिनियम, 2002 की धारा 4 द्वारा (1-4-2010 से) अंतःस्थापित ।

PART IV A

FUNDAMENTAL DUTIES

51A. **Fundamental Duties** – It shall be the duty of every citizen of India.

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem.

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom.

(c) to uphold and protect the sovereignty, unity and integrity of India.

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women.

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers and wild life and to have compassion for living creatures.

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individuals and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

(k) who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

1. Ins. by the Constitution (Forty second Amendment Act, 1976, s 11 (w.e.f. 3-1-1977)
2. Ins. by the Constitution (Eighty Sixth Amendment) Act, 2002, s 4 (w.e.f. 1-4-2010)

DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
(2021-2022)

Class : IX
MATHEMATICS

NOT FOR SALE

LIST OF GROUP LEADER AND SUBJECT-EXPERTS
FOR PREPARATION / REVIEW OF SUPPORT MATERIAL

Class - IX (2021-22)
Subject : Mathematics

1. **Ms. Barkha Dawar**   Vice-Principal
                                  S Co-ed S S School, 2P-Block,
                                  Pitampura, Delhi (1411007)

2. **Ms. Aakanksha**   PGT (Mathematics)
                                  Core Academic Unit
                                  (CAU)

3. **Dr. Preeti Sharma**   TGT (mathematics)
                                  RPVV, Sec. XI, Rohini, Delhi
                                  (1413076)

4. **Mr. Sunil Kumar Tiwari**   TGT (Mathematics)
                                  SBV, Moti Nagar, Delhi
                                  (1516010)

5. **Ms. Namita Tiwari**   TGT (Mathematics)
                                  GSKV, B-3, Paschim Vlhar
                                  New Delhi, (1617011)

6. **Ms. Shalini Bahri**   TGT (Mathematics)
                                  SKV No. 1, Narela, Delhi
                                  (13100368)

7. **Mr. Julfikar Ahmad**   TGT (Mathematics)
                                  Dr. Zakir Hussain Memorial
                                  Sr. Sec. School, Jafrabad
                                  (1105137)

CONTENTS

*   The chapters are not included for assessment purpose for the session 2021-22 as per CBSE guidelines.

MATHEMATICS (IX)

The Syllabus in the subject of Mathematics has undergone changes from time to time in accordance with growth of the subject and emerging needs of the society. The present revised syllabus has been designed in accordance with National Curriculum Framework 2005 and as per guidelines given in the Focus Group of Teaching of Mathematics which is to meet the emerging needs of all categories of students. For motivating the teacher to related the topics to real life problems and other subject areas, greater emphasis has been laid on applications of various concepts.

The curriculum at secondary stage primarily aims at enhancing the capacity of students to employ Mathematics in solving day-to-day life problem adn studying the subject as a separate discipline. IT is expected that students should acquired the ability to solve problem using algebraic methods and apply the knowledge of simple trigonometry to solve problem of height and distances. Carrying out experiments with numbers and forms of geometry, framing hypothesis and verifying these with further observations form inherent part of Mathematics learning at this stage. The proposed curriculum includes the study of number system, algebra, geometry, trigonometry, mensuration, mensuration, statistics, graphs and coordinate geometry, etc.

The teaching of Mathematics should be imparted through activities which may involve the use of concrete materials, models, patterns, charts, pictures, posters, games, puzzles and experiments.

Objectives

The broad objectives of teaching of Mathematics at secondary stage are to help the learners to:

- consolidate the Mathematical knowledge and skills acquired at the upper primary stage; acquire knowledge and understanding, particularly by way of motivation and visualization, of basic concepts, terms, principles and symbols and underlying processes and skills; develop mastery of basic algebraic skills;

- develop drawing skills;

- feel the flow of reason while proving a result or solving a problem;

- apply the knowledge and skills acquired to solve problems and wherever possible, by more than one method;

- to develop ability to think, analyze and articulate logically;

    to develop awareness of the need for national integration, protection of environment, observance of small family norms, removal of social barriers, elimination of gender biases;

- to develop necessary skills to work with modern technological devices and mathematical software's.

- to develop interest in mathematics as a problem-solving tool in various fields for its beautiful structures and patterns, etc.

- to develop reverence and respect towards great Mathematicians for their contributions to the field of Mathematics;

- to develop interest in the subject by participating in related competitions;

- to acquaint students with different aspects of Mathematics used in daily life;

- to develop an interest in students to study Mathematics as a discipline.

Course Structure

Term-I

Units	Unit Name	Marks
I	Number Systems	08
II	Algebra	05
III	Coordinate Geometry	14
IV	Geometry	13
V	Mensuration	4
VI	Statistics & Probability	6
	Total	40
	Internal Assessment	10
	Grand Total	50

Term-II

Units	Unit Name	Marks
II	Algebra (Cont.)	12
IV	Geometry (Cont.)	15
V	Mensuration (Cont.)	9
VI	Statistics & Probability (Cont.)	4
	Total	40
	Internal Assessment	10
	Grand Total	50

Term-I

UNIT I : Number System

Chapter 1 : Number System

Review of representation of natural numbers, integers and rational numbers on the number line.

Rational numbers as recurring/ terminating decimals. Operations on real numbers.

Examples of non-recurring/ non-terminating decimals. Existence of non-rational numbers (irrational numbers) such as $\sqrt{2}$, $\sqrt{3}$ and their representation on the number line.

Rationalization (with precise meaning) of real numbers of the type $\dfrac{1}{a+b\sqrt{x}}$ and $\dfrac{1}{\sqrt{x}+\sqrt{y}}$ (and their combinations) where x and y are natural number and a and b are integers.

Recall of laws of exponents with integral powers. Rational exponents with positive real boses (to be done by particular cases, allowing learner to arrive at the general laws.)

UNIT III : COORDINATE GEOMETRY

Chapter 3 : Coordinate Geometry

The Cartesian plane, coordinates of a point, names and terms associated with the coordinate plane, notations plotting points in the plane.

Chapter 4 : Linear Equations in Two Variables

Recall of linear equations in one variable. Introduction to the equation in two variables. Focus on linear equations of the type $ax + by + c = 0$. Explain that a linear equation in two variables has infinitely many solutions and justify their being written as ordered pairs of real numbers, plotting them and showing that they lie on a line. Graph of linear equations in two variables. Examples, problems fro real life with algebraic and graphical solutions being done simultaneously.

UNIT IV : GEOMETRY

Chapter 6 : Lines and Angles

1. (Motivate) If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and their converse.

2. (Prove) If two lines intersect, vertically opposite angles are equal.

3. (Motivate)

Chapter 7: Triangles

1. (Motivate) Two triangles are congruent if any two sides and the included angle of one triangle is equal to any two sides and the included angle of the other triangle (SAS Congruence).

2. (Motivate) Two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle (ASA Congruence).

3. (Motivate) Two triangles are congruent if the three sides of one triangle are equal to three sides of the other triangle (SSS Congruence).

4. (Motivate) Two right triangles are congruent if the hypotenuse and a side of one triangle are equal (respectively) to the hypotenuse and a side of the other triangle. (RHS Congruence)

5. (Prove) The angles opposite to equal sides of a triangle are equal.

6. (Motivate) The sides opposite to equal angles of a triangle are equal.

UNIT V : MENSURATION

Chapter 12: Heron's Formula

Area of a triangle using Heron's formula (without proof).

UNIT VI : STATISTICS

Chapter 14: Statistics

Introduction to Statistics: Collection of data, presentation of data — tabular form, ungrouped/ grouped, bar graphs, histograms.

➢ Mental Maths

➢ Revision from Support Material

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Term II

UNIT II : ALGEBRA

Chapter 2 : Polynomials

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Definition of a polynomial in one variable with examples and counter examples. Coefficients of a polynomial, terms of a polynomial and zero polynomial. Degree of a polynomial. Constant, linear, quadratic and cubic polynomials. Monomials, binomials, trinomials. Factors and multiples. Zeroes of a polynomial. Factorization of axe + bx + c, a 0 where a, b and c are real numbers, and of cubic polynomials using the Factor Theorem. Recall of algebraic expressions and identities. Verification of identities:

$$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$$

$$(x \pm y)^3 = x^3 \pm y^3 \pm 3xy(x \pm y)$$

$$x^3 \pm y^3 = (x \pm y)(x^2 \mp xy \pm y^2)$$

and their use in factorization of polynomials.

<div align="center">

UNIT IV : GEOMETRY

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Chapter 8: Quadrilaterals

1. (Prove) The diagonal divides a parallelogram into two congruent triangles.

2. (Motivate) In a parallelogram opposite sides are equal, and conversely.

3. (Motivate) In a parallelogram opposite angles are equal, and conversely.

4. (Motivate) A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal.

5. (Motivate) In a parallelogram, the diagonals bisect each other and conversely.

6. (Motivate) In a triangle, the line segment joining the mid points of any two sides is parallel to the third side and is half of it and (motivate) its converse.

Chapter 10: Circles

Through examples, arrive at definition of circle and related concepts-radius, circumference, diameter, chord, arc. secant. sector, segment. subtended angle.

1. (Prove) Equal chords of a circle subtend equal angles at the centre and (motivate) its converse.

2. (Motivate) The perpendicular from the centre of a circle to a chord bisects the

chord and conversely, the line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

3. (Motivate) Equal chords of a circle (or of congruent circles) are equidistant from the centre (or their respective centre) and conversely.

4. (Motivate) The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

5. (Motivate) Angles in the same segment of a circle are equal.

6. (Motivate) The sum of either of the pair of the opposite angles of a cyclic quadrilateral is 180° and its converse.

Chapter 11: Constructions

Construction of bisectors of line segments and angles of measure 60°, 90°, 45° etc., equilateral triangles.

Construction of a triangle given its base, sum/difference of the other two sides and one base angle.

UNIT V : MENSURATION

Chapter 13: Surface Areas and Volumes

Surface areas and volumes of cubes, cuboids, spheres (including hemispheres) and right circular cylinders/cones.

UNIT VI : STATISTICS & PROBABILITY

Chapter 15: Probability

History, repeated experiments and observed frequency approach to probability. Focus is on empirical probability. (A large amount of time to be devoted to group and to individual activities to motivate the concept; the experiments to be drawn from real - life situations, and from examples used in the chapter on statistics)

➢ Mental Maths

➢ Revision from Support Material

Note :

1. The above syllabus is as per CBSE guidelines for the session 2021-22 only.

2. However this support material contains the portion in addition to the above prescribed syllabus.

MATHEMATICS
QUESTION PAPER DESIGN
CLASS–IX (2021-22)

Time : 3 Hrs.   **Maximum Marks : 80**

Sr. No.	Typology of Questions	Total Marks	% Weightege (approx.)
1	**Remembering :** Exhibit memory of previously learned material by recalling facts, terms, basic concepts and answers. **Understanding :** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	43	54
2	**Applying :** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	19	24
3	**Analyzing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations. **Evaluation:** Present and defend opinions by making Judgements about information, validity of ideas, or quality of work based on a set of criteria. **Creating :** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions.	18	22
	Total	80	100

INTERNAL ASSESSMENT	20 Marks
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

CHAPTER-1
NUMBER SYSTEMS
MIND MAP

Natural Numbers (N): 1, 2, 3, ...∝

Whole Numbers (W): 0, 1, 2, 3, ...

Integers (Z): -∝...........-3, -2, -1, 0, 1, 3, ...∝

Rational Number: form $\frac{p}{q}$, p, q are integers q ≠ 0

Irrational Number: cannot be written in $\frac{p}{q}$ form, p, q are integers q ≠ 0

Real Number

NUMBER SYSTEMS

Laws of exponents:

$$a^m \cdot a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$

Rationalisation:

Term	Rationalisation Factor
$\frac{1}{\sqrt{a}}$	$\sqrt{a}$
$\frac{1}{\sqrt{a}-b}$	$\sqrt{a} + b$
$\frac{1}{\sqrt{a}+b}$	$\sqrt{a} - b$
$\frac{1}{\sqrt{a}+\sqrt{b}}$	$\sqrt{a} - \sqrt{b}$
$\frac{1}{\sqrt{a}-\sqrt{b}}$	$\sqrt{a} + \sqrt{b}$

Number line: Every point on the number line represents real numbers

-4 -3 -2 -1 0 1 2 3 4

CHAPTER-1
NUMBER SYSTEMS
KEY POINTS
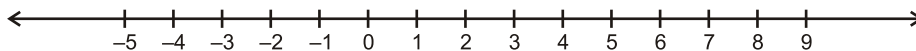
- 1, 2, 3, ... are natural numbers which are represented by N.

- 0, 1, 2, 3, ... are whole numbers which are represented by W.

- .... –3, –2, –1, 0, 1, 2, 3, ... are Integers which are represented by Z or I.

- A number is a rational number if

  (a) it can be represented in the form of p/q, where p and q are integers and $q \neq 0$.

    or

  (b) its decimal expansion is terminating (e.g. $\frac{2}{5}$ = 0.4)

    or

  (c) its decimal expansion is non-terminating recurring (repeating) (e.g. $0.1\overline{234}$ = 0.1234234......)

- A number is irrational number if

  (a) it can not be represented in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

    or

  (b) its decimal expansion is non-terminating non-recurring (e.g. 0.1010010001......)

- All rational and irrational numbers collectively form real numbers.

- There are infinite rational numbers between any two rational numbers.

- There is a unique real number corresponding to every point on the number line. Also, corresponding to each real number, there is a unique point on the number line.

- Rationalisation of a denominator means to change the Irrational denominator to rational form.

- To rationalise the denominator of $\dfrac{1}{\sqrt{a} + b}$, We multiply this by $\dfrac{\sqrt{a} - b}{\sqrt{a} - b}$, where a is a natural number and b is an integer.

- Laws of Exponents : Let $a > 0$ be a real number and m and n are rational numbers, then

1) $a^m \, a^n = a^{m+n}$

2) $a^m \div a^n = a^{m-n}$

3) $(a^m)^n = a^{mn}$

4) $a^m . b^m = (ab)^m$

5) $a^0 = 1$

6) $a^{-m} = \dfrac{1}{a^m}$

- For positive real numbers a and b, the following Identities hold

1) $\sqrt{a} . \sqrt{b} = \sqrt{ab}$

2) $\sqrt{a} \div \sqrt{b} = \sqrt{\dfrac{a}{b}}$

3) $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = a - b$

4) $\left(\sqrt{a} + \sqrt{b}\right)^2 = a + 2\sqrt{ab} + b$

5) $\left(a + \sqrt{b}\right)\left(a - \sqrt{b}\right) = a^2 - b$

All natural numbers, whole numbers and integers are rational

Prime Numbers : All natural numbers that have exactly two factors (i.e., 1 and itself) are called prime numbers. e.g., 2, 3, 5, 7, 11, 13, 17, 19, 23, … etc.

Composite Numbers : Those natural numbers which have more than two factors are known as composite numbers. e.g., 4, 6, 8, 10, 12, …

1 is neither prime nor composite.

Types of Numbers

Real Numbers

Rational Numbers $\left(\dfrac{p}{q}\right)$ — Irrational Numbers

Terminating Decimal Expansion

$q = 2^m \times 5^n$

$\left(\dfrac{2}{5} = 0.4\right)$

Non Terminating Recurring Decimal Expansion

$\left(\dfrac{10}{3} = 3.33... = 3.\overline{3}\right)$

Non-Terminating Non-Recurring Decimal Expansion

$\left(\sqrt{2} = 1.414...\right)$

Real Numbers

Rational Numbers

Eg : $\left(-5, \dfrac{-7}{3}, 0, \dfrac{5}{6}\right)$

Irrational Numbers

Eg. : $\left(\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi, 0.010010001.....\right)$

Integers

(... -3, -2, -1, 0, 1, 2, 3 ...)

Fractions

$\left(\dfrac{1}{2}, \dfrac{5}{3}, \dfrac{7}{5}\right)$

Negative Integers

(... -3, -2, -1)

Whole Numbers (0, 1, 2, 3...)

Zero (0)

Natural Numbers (1, 2, 3, ...)

Even Numbers
(2, 4, 6, 8 ....)

Odd Numbers
(1, 3, 5, 7, 9...)

Prime Number
(2)

Composite Numbers
(4, 6, 8. 10 ...)

(1)
[Neither Prime
nor composite]

Prime Numbers
(3, 5, 7, 11 ....)

Composite
Numbers
(9, 15 ...)

* $\sqrt[n]{a} \;=\; a^{1/n}$

where 'a' is a positive real number and n is a positive integer.

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

where 'a' is a positive real number, m and n are co prime integers, and n > 0.

Very-Very Short Answer Type (1 Mark)

1. If x=2 and y=4, then $\left(\frac{x}{y}\right)^{x-y} + \left(\frac{y}{x}\right)^{y-x}$ = ...

2. $\dfrac{(32)^{0.2} + (81)^{0.25}}{(256)^{0.5} - (121)^{0.5}}$ = ...

3. $\dfrac{3}{7}$ lies between $\dfrac{42}{99}$ and ....

4. Express $0.3\overline{2}$ in the form $\dfrac{p}{q}$, where p and q are integers and $q \neq 0$

5. The number 0.318564318564318564........... is a .... number.

6. Express $0.\overline{7}$ is the form $\dfrac{p}{q}$, where p and q are integers and $q \neq o$.

7. Find the value of $0.\overline{23} + 0.\overline{22}$.

8. Find the Value of $[3-4(3-4)^4]^3$

9. The cube root of 125 divided by square root of 25, is ....

10. If $y^2 = 625$ then y is a .... number.

11. $2\sqrt{(81)^{2.5}}$ = _____.

12. Find the value of x, if $5^{x-3} \cdot 3^{2x-8} = 225$.

13. If $a = 2 + \sqrt{3}$, then the value of $\dfrac{1}{a}$ = ...

14. Which number is the smallest natural number ?

15. Which of the number out of $\sqrt{2}$, $\sqrt{4}$, $\sqrt{9}$ and $\sqrt{25}$ is not a rational number ?

16. _____ is a whole number but not a natural number.

17. The decimal expansion of the number $\sqrt{3}$ is non- ____ and _____.

18. Find a rational number between $\dfrac{-2}{3}$ and $\dfrac{1}{4}$.

19. Which of $\sqrt{\dfrac{4}{9}}$, $\sqrt{81}$, $\dfrac{\sqrt{12}}{\sqrt{3}}$ and $\sqrt{7}$ is an irrational number ?

20. Give example of two irrational number the product of which is a rational number.

21. $\sqrt{6} \times \sqrt{8}$ = _____

22. After rationalising the denominator of $\dfrac{3\sqrt{2}}{3\sqrt{2} - 2\sqrt{2}}$,we get the denominator as _____.

23. $\left[\sqrt{a^3}\right]^{\frac{2}{3}}$ = _____.

---

24. Every rational number is a _____ number.

25. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Fill in the blanks

26. The sum of a rational and an irrational numbers is always _____ number.

27. The difference of a rational and an irrational number is always _____ number.

28. The decimal expansion of every rational number is either _____ or non - terminating _____ .

29. The decimal expansion of every irrational number is always _____ .

30. Every number whose decimal expansion is non-terminating non-recurring is _____ number.

31. Between two distinct rational numbers there lie _____ rational numbers.

32. Between two distinct rational numbers there lie _____ irrational numbers.

33. Between two distinct irrational numbers there lie _____ rational numbers.

34. The reciprocal of every (non-zero) rational number is a _____ number.

35. Write first five whole numbers in $\dfrac{p}{q}$ form, where p and q are integers and $q \neq 0$

36. Find decimal expansion of $\dfrac{17}{8}, \dfrac{3}{15}, \dfrac{2}{7}, \dfrac{50}{3}$.

37. Find four rational numbers between $\dfrac{2}{9}$ and $\dfrac{3}{7}$.

38. Find decimal for of $\sqrt{23}$ and $\sqrt{24}$ upto 3 decimal places.

39. Find two irrational numbers between $\sqrt{23}$ and $\sqrt{24}$.

40. Find one irrational and one rational number between 2 and $\sqrt{5}$.

41. Write two numbers whose decimal expansions are terminating.

42. What can be the maximum number of digits in the repeating block of digits in teh decimal expansion of $\dfrac{5}{7}$?

43. Write two numbers whose decimal expansions are non-terminating non-repeating (non-recurring).

44. Find the value of $(256)^{0.16} \times (256)^{0.09}$

45. Find two irrational numbers between 2016 and 2017.

Case Study Based Questions [46-47]

46. Case Study-1

A building has 13 floors above and 4 floors below the ground (basement). Stairs Run from the lowest ground of the basement to the top of the building. Ramesh is standing on the ground. If that point is considered "0" and after every 4 steps of stairs, either above or below, he counts "1". Also, below the ground he counts negative numbers.

Represent his position by number in each of the following cases-
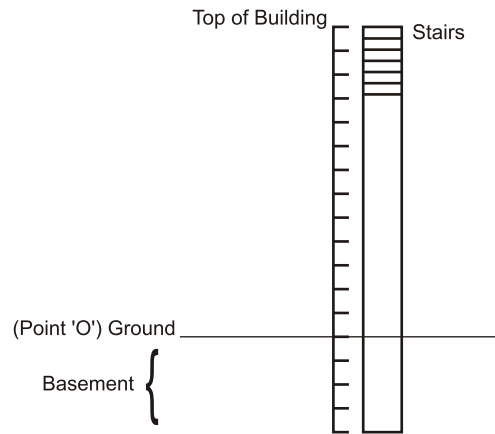
(i) If he has climbed 16 stairs above

    (a) 2   (b) 3   (c) 4   (d) 16

(ii) If he has gone three steps below the ground

    (a) $\dfrac{3}{4}$ (b) $-\dfrac{3}{4}$ (c) -0.75   (d) b & c

(iii) If he has climbed seven steps above the ground

    (a) 7/4   (b) - 7/4   (c) 1.75   (d) b and c   (e) a and c

(iv) If he has gone 15 steps below the ground

    (a) 15/4   (b) -15/4   (c) 3.75   (d) a and c   (e) b and c

(v) Name the numbers used to represent his location position.

    (a) Natural   (b) Whole (c) Integers   (d) Real

47. Case Study 2-

A school wants to plant trees in a row as distance shown in the figure Height of each tree is considered equal and is taken as 1 unit. Distance OA is also 1 unit. First Tree is planted at Point B & second at C.

Consider point 'O' as the zero of the line and OB=OP, OC=OQ, then

[Neglect width of the tree]
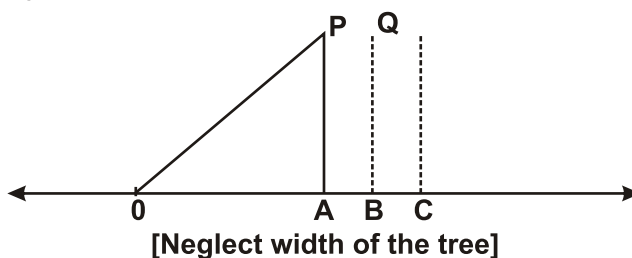
(i) Distance OB will be

    (a) 1 unit   (b) 2 unit   (c) $\sqrt{1}$ unit   (d) $\sqrt{2}$ units

(ii) Distance OC will be

    (a) $\sqrt{3}$ units   (b) $\sqrt{2}$ units   (c) 2 units   (d) 3 units

(iii) Simplifying $\frac{1}{OB}$, we get

    (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\frac{2}{\sqrt{2}}$

(iv) Making denominator of $\frac{OB}{OC}$ rational, we get

    (a) $\frac{\sqrt{6}}{3}$ (b) 2   (c) $\frac{\sqrt{2}}{3}$ (d) $\frac{\sqrt{3}}{3}$

(v) Simplifiying the reciprocal of difference (OC-OB)

    (a) $\frac{1}{\sqrt{3}-\sqrt{2}}$ (b) 1   (c) $\sqrt{3}+\sqrt{2}$ (d) $-(\sqrt{2}+\sqrt{3})$

Very Short answer type Questions (2 Marks)

48. Represent $\frac{-7}{5}$ and $\frac{11}{5}$ on the number line.

49. Represent following on number line

    i) $\sqrt{5}$ ii) $\sqrt{3}$ iii) $\sqrt{2}$

50. Insert two irrational numbers between $\frac{2}{3}$ and $\frac{3}{2}$

51. Simplify : $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{80}+\sqrt{48}-\sqrt{45}-\sqrt{27}}$

52. Find the value of $[1^3+2^3+3^3+8^2]^{-5/2}$

53. Find the value of x if $x^{1/2}=(36)^{0.5}$

54. Find the value of x if $(\sqrt{3})^x=3^7$

55. If $2^{5x} \div 2^x = \sqrt[5]{32}.$ Then find the value of x.

56. Evaluate $a^{x-y}.a^{y-z}. a^{z-x}.$

57. Simplify $12^{\frac{2}{5}} . 5^{\frac{2}{5}}.$

58. Which of the following rational numbers will have a terminating decimal expansion or a non-terminating repeating (recurring) decimal expansion ?

(i) $\dfrac{135}{50}$ (ii) $\dfrac{4}{11}$ (iii) $\dfrac{8}{7}$ (iv) $6\dfrac{3}{8}$

(v) $\dfrac{55}{9}$ (vi) $\dfrac{5^2 \times 3^3}{2 \times 5^3 \times 27}$ (vii) $\dfrac{51}{60}.$

59. Classify the following numbers as terminating decimal or non-terminating recurring decimal or non-terminating non-recurring decimal :

(i) 0.1666...   (ii) 0.250   (iii) 1.01001000100001....

(iv) 0.27696   (v) 2.142857142857.... (vi) $0.\overline{3}$

(vii) 0.2359872785... (viii) 0.484848848....   (ix) 2.502500250002.....

(x) $4.\overline{123456789}$

Also classify these given numbers as Rational and Irrational numbers.

60. Classify the following numbers as rational or Irrational number :

(i) $\sqrt{27}$ (ii) $\sqrt{36}$ (iii) $\sqrt{5} \times \sqrt{125}$ (iv) $2\sqrt{3}$

(v) $\dfrac{7\sqrt{7}}{\sqrt{343}}$ (vi) $2 + \sqrt{21}$ (vii) $5 + 2\sqrt{23} - \left(\sqrt{25} + \sqrt{92}\right)$

(viii) $\dfrac{22}{7}$ (ix) π (x) $\sqrt[3]{27}$

61. Express the following numbers in the form $\dfrac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) 0.0875   (ii) 2.123456789   (iii) 0.181818.....

(iv) $0.4\overline{37}$　　　　(v) $3.65\overline{1}$

62. Do as directed :

(i) Add : $\sqrt{125} + 2\sqrt{27}$ and $-5\sqrt{5} - \sqrt{3}$

(ii) Add : $\sqrt{7} - \sqrt{11}$ and $\sqrt{5} - \sqrt{11} + \sqrt{13}$

(iii) Multiply : $2\sqrt{2}$ by $5\sqrt{2}$.

(iv) Multiply : $\left(-3 + \sqrt{5}\right)$ by 3.

(v) Divide : $7\sqrt{5}$ by $-14\sqrt{125}$

(vi) Divide : $2\sqrt{216} - 3\sqrt{27}$ by 3.

Short answer type questions (3 Marks)

63. Simplify :

(i) $\left(2\sqrt{2} + 3\sqrt{3}\right)\left(2\sqrt{2} - 3\sqrt{3}\right)$　(ii) $\left(2\sqrt{8} - 3\sqrt{2}\right)^2$

(iii) $\left(\sqrt{7} + \sqrt{6}\right)^2$　　　　　(iv) $\left(6 - \sqrt{2}\right)\left(2 + \sqrt{3}\right)$　(v) $\left(9 + \sqrt{2} - \sqrt{3}\right)^2$

64. Evaluate :

(i) $\dfrac{2^{38} + 2^{37} + 2^{36}}{2^{39} + 2^{38} + 2^{37}}$　　　　(ii) $\left[\left(64^{\frac{1}{2}}\right)^{\frac{1}{6}}\right]^2$

65. Find the value of a if $\dfrac{6}{3\sqrt{2} - 2\sqrt{3}} = 3\sqrt{2} - a\sqrt{3}$.

66. Simplify : $\left[5\left(8^{1/3} + 27^{1/3}\right)^3\right]^{\frac{1}{4}}$

67. Simplify : $\dfrac{\left(25\right)^{3/2} \times \left(243\right)^{3/5}}{\left(16\right)^{5/4} \times \left(8\right)^{4/3}}$

68. If $5^{2x-1} - (25)^{x-1} = 2500$, then find the value of *x*.

69. Express $0.6 + 0.\overline{7} + 0.4\overline{7}$ in the form $\dfrac{p}{q}$, where p and q are integers and q $\neq$ 0.

70. Rationalise the denominator of　(i) $\dfrac{1}{\sqrt{3} + \sqrt{5} + \sqrt{7}}$　　(ii) $\dfrac{2}{\sqrt{2} - \sqrt{3} + \sqrt{5}}$

71. Find a and b if $\dfrac{7 + 3\sqrt{5}}{2 + \sqrt{5}} - \dfrac{7 - 3\sqrt{5}}{2 - \sqrt{5}} = a + b\sqrt{5}$

72. If $x = \left(3 - 2\sqrt{2}\right)$, show that $\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right) = \pm\, 2$

73. If $xyz = 1$, then simplify

$$\left(1 + x + y^{-1}\right)^{-1} \times \left(1 + y + z^{-1}\right)^{-1} \times \left(1 + z + x^{-1}\right)^{-1}$$

74. Find the value of x if

    (i) $25^{2x-3} = 5^{2x+3}$ (ii) $(4)^{2x-1} - (16)^{x-1} = 384$

Long Answer Question (5 Marks)

75. Evaluate : $\dfrac{64^{\frac{a}{6}}}{4^a} \times \dfrac{2^{2a+1}}{2^{a-1}}$

76. Simplify : $\dfrac{1}{1 + x^{b-a} + x^{c-a}} + \dfrac{1}{1 + x^{a-b} + x^{c-b}} + \dfrac{1}{1 + x^{a-c} + x^{b-c}}$

77. Simplify : $\left(\dfrac{x^a}{x^{-b}}\right)^{a-b} \times \left(\dfrac{x^b}{x^{-c}}\right)^{b-c} \times \left(\dfrac{x^c}{x^{-a}}\right)^{c-a}$

78. Show that :

$$\dfrac{1}{\left(3 - \sqrt{8}\right)} - \dfrac{1}{\left(\sqrt{8} - \sqrt{7}\right)} + \dfrac{1}{\left(\sqrt{7} - \sqrt{6}\right)} - \dfrac{1}{\left(\sqrt{6} - \sqrt{5}\right)} + \dfrac{1}{\left(\sqrt{5} - 2\right)} = 5$$

79. If $a = \dfrac{\sqrt{7} - \sqrt{6}}{\sqrt{7} + \sqrt{6}}$ and $b = \dfrac{\sqrt{7} + \sqrt{6}}{\sqrt{7} - \sqrt{6}}$, then find the value of $a^2 + b^2 + ab$.

80. Simplify : $\dfrac{2\sqrt{6}}{\sqrt{2} + \sqrt{3}} + \dfrac{6\sqrt{2}}{\sqrt{6} + \sqrt{3}} - \dfrac{8\sqrt{3}}{\sqrt{6} + \sqrt{2}}$

81. If $x = 9 - 4\sqrt{5}$, then find

    (i) $x + \dfrac{1}{x}$ (ii) $x - \dfrac{1}{x}$ (iii) $x^2 + \dfrac{1}{x^2}$ (iv) $x^2 - \dfrac{1}{x^2}$

    (v) $x^3 + \dfrac{1}{x^3}$ (vi) $x^3 - \dfrac{1}{x^3}$ (vii) $\sqrt{x} + \dfrac{1}{\sqrt{x}}$ (viii) $\sqrt{x} - \dfrac{1}{\sqrt{x}}$

(ix) $x^4 + \dfrac{1}{x^4}$ (x) $x^6 + \dfrac{1}{x^6}$ (xi) $x + \dfrac{14}{x}$

82. If $a = 1 + \sqrt{7}$, find the value of $\dfrac{-6}{a}$

83. If $p = 5 - 2\sqrt{6}$, Find $p^2 + \dfrac{1}{p^2}$ and $p^2 - \dfrac{1}{p^2}$

84. Express $0.3\overline{178}$ in the form of p/q where p and q are integers and $q \neq 0$.

85. If $\sqrt{2} = 1.414$, then find the value of $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

86. Find the value of

$$\dfrac{4}{(216)^{\frac{-2}{3}}} + \dfrac{1}{(256)^{\frac{-3}{4}}} + \dfrac{2}{(243)^{\frac{-1}{5}}}$$

CHAPTER-1
NUMBER SYSTEMS
ANSWERS

1. 8

2. 1

3. $\dfrac{42}{99}, \dfrac{4}{9}$

4. 29/90

5. rational

6. $\dfrac{7}{9}$

7. $0.\overline{45}$ or $\dfrac{45}{99}$

8. −1

9. 1

10. rational

11. 243

12. 5

13. $2 - \sqrt{3}$

14. 1

15. $\sqrt{2}$

16. 0

17. Ternimating non-recurring

18. $\dfrac{-5}{24}$ or any other appropriate us.

19. √7

20 Any correct - example

21. 4√3

22. 19

23. a

24. Real

25. 1.5 or any other approriate number

26. an irrational

27. an irrational

28. Terminating, recurring

29. non-terminating non-recurring

30. an irrational

31. infinitely many

32. infinitely many

33. infinitely many

34. rational

35. $\dfrac{0}{1}, \dfrac{1}{1}, \dfrac{2}{1}, \dfrac{3}{1}, \dfrac{4}{1}$

36. $\dfrac{17}{8} = 2.125, \dfrac{3}{15} = 0.2, \dfrac{2}{7} = 0.\overline{285714}, \dfrac{50}{3} = 16.\overline{6}$

37. $\dfrac{15}{63}, \dfrac{16}{63}, \dfrac{17}{63}, \dfrac{18}{63}$ (other answers are also possible).

38. $\sqrt{23} = 4.795, \sqrt{24} = 4.898$

39. 4.8010010001 ...., 4.8020020002 ......, (other answers are also possible)

40. 2.1, 2.010010001 ......, (other answers are possible).

42. 6

44. 4

45. 2016.1010010001 ..... ; 2016.2020020002 ......; (other answers are possible)

46. (i)   (c)   (ii)   (d)

    (iii) (e)   (iv) (b)

    (v)   (c)

47. (i)   (d)   (ii)   (a)

    (iii) (c)   (iv) (a)

    (v)   (c)

50. 0.909009000.........; 1.10100100010000............ (other answers are also possible)

51. 1

52. $\dfrac{1}{10^5}$

53. 36

54. 14

55. $x = \dfrac{1}{4}$

56. 1

57. $(60)^{2/5}$

58. (i) Terminating Decimal   (ii) Non Terminating Repeating Decimal

    (iii) Non-Terminating Repeating Decimal

    (iv) Terminating Decimal   (v) Non-Teminating repeating Decimal

    (vi) Terminating Decimal   (vii) Terminating Decimal

59. (i) Non-Terminating Repeating Decimal (Rational).

(ii) Terminating Decimal (Rational).

(iii) Non-Terminating Non-Repeating Decimal (Irrational).

(iv) Terminating Decimal (Rational)

(v) Non-Terminating Repeating Decimal (Rational)

(vi) Non-Terminating Repeating Decimal (Rational)

(vii) Non-Terminating Non-Repeating Decimal (Irrational)

(viii) Non-Terminating Non-Repeating Decimal (Irrational)

(ix) Non-Terminating Non-Repeating Decimal (Irrational)

(x) Non-Terminating Repeating Decimal (Rational).

60. (i) Irrational (ii) Rational (iii) Rational (iv) Irrational

(v) Rational (vi) Irrational (vii) Rational (viii) Rational

(ix) Irrational (x) Rational

61. (i) $0.0875 = \dfrac{7}{80}$ (ii) $\dfrac{2123456789}{1000000000}$ (iii) $\dfrac{2}{11}$

(iv) $\dfrac{433}{990}$ (v) $\dfrac{1643}{450}$

62. (i) $5\sqrt{3}$ (ii) $\sqrt{5} - 2\sqrt{11} + \sqrt{7} + \sqrt{13}$ (iii) 20

(iv) $-9 + 3\sqrt{5}$ (v) $-\dfrac{1}{10}$ (vi) $4\sqrt{6} - 3\sqrt{3}$

63. (i) -19 (ii) 2 (iii) $13 + 2\sqrt{42}$

(iv) $12 + 6\sqrt{3} - 2\sqrt{2} - \sqrt{6}$ (v) $14 + 18\sqrt{2} - 2\sqrt{6} - 18\sqrt{3}$

64. (i) $\dfrac{1}{2}$ (ii) 2

65. $a = -2$ 66. 5 67. $\dfrac{3375}{512}$

68. $x = 3$ 69. $\dfrac{167}{90}$

70. ^{Hint}

$$\frac{1}{(\sqrt{3}+\sqrt{5})+\sqrt{7}} \times \frac{(\sqrt{3}+\sqrt{5})-\sqrt{7}}{(\sqrt{3}+\sqrt{5})-\sqrt{7}}$$

$$= \frac{\sqrt{3}+\sqrt{5}-\sqrt{7}}{(\sqrt{3}+\sqrt{5})^2-(\sqrt{7})^2} = \frac{\sqrt{3}+\sqrt{5}-\sqrt{7}}{3+5+2\sqrt{15}-7}$$

$$= \frac{\sqrt{3}+\sqrt{5}-\sqrt{7}}{1+2\sqrt{15}} \times \frac{1-2\sqrt{15}}{1-2\sqrt{15}}$$

Ans. (i) $\frac{1}{59}\left(9\sqrt{3}+5\sqrt{5}+\sqrt{7}-2\sqrt{105}\right)$ (ii) $3\sqrt{2}-2\sqrt{3}+\sqrt{30}$

71. a=o, b=2

73. $\dfrac{1}{(1+y+xy)(1+z+yz)(1+x+zx)}$

74. (i) 9/2   (ii) Hint – $4^{2x-1}-16^{x-1}=384$

$$2^{4x-2}-2^{4x-4}=2^7 \times 3$$

$$2^{4x}\left[\frac{1}{2^2}-\frac{1}{2^4}\right]=2^7 \times 3$$

$$= \quad 2^{4x-4} \times 3 = 2^7 \times 3$$

$$x \quad = \frac{11}{4}$$

75. 4

76. 1

77. 1

79 $a^2+b^2+ab=675$

80. 0

81. ^{Hint–}

$$\frac{1}{x}=\frac{1}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}}$$

$$=\frac{9+4\sqrt{5}}{81-80}=9+4\sqrt{5}$$

$$x+\frac{1}{x}=9-4\sqrt{5}+9+4\sqrt{5}=18$$

$$x-\frac{1}{x}=9-4\sqrt{5}-9-4\sqrt{5}=-8\sqrt{5}$$

(vi) $\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3x\frac{1}{x}\left(x-\frac{1}{x}\right)$

$(-8\sqrt{5})^3=\left(x^3-\frac{1}{x^3}\right)-3(-8\sqrt{5})$

$\left(x^3-\frac{1}{x^3}\right)=-2584\sqrt{5}$

(vii) $\left(\sqrt{x}+\dfrac{1}{\sqrt{x}}\right)^2 = \left(x+\dfrac{1}{x}\right)+2$

$\qquad\qquad = 18+2$

$\sqrt{x}+\dfrac{1}{\sqrt{x}} = \sqrt{20} = 2\sqrt{5}$

(ix) $\left(x^2+\dfrac{1}{x^2}\right)^2 = x^4+\dfrac{1}{x^4}+2$

$\left(x^2+\dfrac{1}{x^2}\right)^2-2 = x^4+\dfrac{1}{x^4}$

Ans.　(i) 18　　　　　(ii) $-8\sqrt{5}$　　　　(iii) 322

(iv) $-144\sqrt{5}$　　(v) 5778　　　　(vi) $-2584\sqrt{5}$

(vii) $2\sqrt{5}$　　　(viii) 4　　　　(ix) 103682

(x) 33385282　　(xi) $8\sqrt{3}-14\sqrt{2}$

82.　$1-\sqrt{7}$　　　　83. 98, $-40\sqrt{6}$　　84. $\dfrac{635}{1998}$

85.　28.28　　　　86. Hint-

$\qquad = \dfrac{4}{(6^3)^{-2/3}}+\dfrac{1}{(4^4)^{-3/4}}+\dfrac{2}{(3^5)^{-1/5}}$

$\qquad = \dfrac{4}{6^{-2}}+\dfrac{1}{4^{-3}}+\dfrac{2}{3^{-1}}$

$\qquad = 214$

Practice Test

NUMBER SYSTEMS

Time : 1 Hr.   M.M. 25

1. If $\dfrac{4}{a} = \dfrac{a^2}{16}$, then check whether a is rational or irrational number. (1)

2. Find two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$.   (1)

3. If P = 5 - $2\sqrt{6}$, Find $\dfrac{1}{P^2}$ (1)

4. Simplify:

    $4\sqrt{3} + 3\sqrt{48} \ -\dfrac{5}{2}\sqrt{\dfrac{4}{3}}$ (2)

5. If $\sqrt{3}$ = 1.732, find the value of $\dfrac{2}{\sqrt{3}-1}$ (2)

6. If $(5)^{2x-1} - (25)^{x-1} = 2500$ (2)
    then find the value of 'x'

7. Find the value of x and y   (3)

    $\dfrac{\sqrt{11}\ -\sqrt{7}}{\sqrt{11}\ +\sqrt{7}} = a - b\sqrt{77}$

8. Represent $(2 +\sqrt{3})$ on the number line.   (3)

9. Simplify :   (5)

    $\dfrac{16\ \times\ 2^{a+1}-\ 4\ \times\ 2^a}{16\ \times\ 2^{a+2} - 2\ \times\ 2^{a+2}}$

10. Express the following in the form p/q where p and q are integers and
    $q \neq 0$ (5)
    $0.\overline{4}\ + 0.1\overline{8}$

CHAPTER-2
POLYNOMIALS
MIND MAP

Constant   *Variable*   *Highest Power of Variable*

TERM → Single Term → MONOMIAL

TERM → Two Terms → BINOMIAL

TERM → Three Terms → TRINOMIAL

DEGREE → Degree 1 → LINEAR

DEGREE → Degree 2 → QUADRATIC

DEGREE → Degree 3 → CUBIC

POLYNOMIAL

COEFFICIENTS
Constants are the coefficients of a polynomial

ZERO POLYNOMIAL
When all the constants of a polynomial are zero

CONSTANT POLYNOMIAL
It is the polynomial of degree zero like 3,-8 etc. 3 can be written as $3x^0$

Factor Theorem: If $p(x)$ is a polynomial of degree greater than equal to 0 and a is any real number then
1. $(x-a)$ is a factor of $p(x)$ if $p(a)=0$.
2. $p(a)=0$ if $(x-a)$ is a factor of $p(x)$.

Remainder Theorem: Let $p(x)$ be any polynomial of degree greater than equal to 0 and a is any real number. If $p(x)$ is divided by linear polynomial $(x-a)$ then $p(a)$ is the remainder.

Algebraic Identities :

1. $(x+y)^2=x^2+2xy+y^2$
2. $(x-y)^2=x^2-2xy+y^2$
3. $(x^2-y^2)=(x-y)(x+y)$
4. $(x+a)(x+b) = x^2+(a+b)x+ab$
5. $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$
6. $(x+y)^3=x^3+y^3+3xy(x+y)=x^3+y^3+3x^2y+3xy^2$
7. $(x-y)^3=x^3-y^3-3xy(x-y)=x^3-y^3-3x^2y+3xy^2$
8. $x^3+y^3=(x+y)(x^2-xy+y^2)$
9. $x^3-y^3=(x-y)(x^2+xy+y^2)$
10. $x^3+y^3+z^3=3xyz$ if $x+y+z=0$

POLYNOMIALS

Definition:

A Polynomial $p(x)$ in one variable x of degree n is an algebraic expression in x of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_2x^2 + a_1 x + a_0$, where

(i) $a_0, a_1, a_2 ..... a_n$ are constants and $a_n \neq 0$

(ii) $a_0, a_1, a_2..... a_n$ are respectively the coefficients of $x^0, x^1, x^2, ... , x^n$

(iii) Each of $a_n x^n, a_{n-1}x^{n-1}, a_{n-2}x^{n-2}, ... , a_2x^2, a_1x, a$ are called terms of the polynomial.

(iv) n is called the degree of the polynomial where n is a non-negative integers.

ZEROES OF POLYNOMIAL :

For a polynomial $p(x)$ if $p(a) = 0$, where a is a real number we say that 'a' is a zero of the polynomial.

1. A polynomial having four or more than four terms does not have particular name. These are simply called polynomials.

2. A polynomial of degree five or more than five does not have any particular name. Such a polynomial is usually called a polynomial of degree five or six or ... etc.

3. The degree of zero polynomial is not defined or we can not determine the degree of zero polynomial.

Fact About Polynomial :

4. A polynomial of degree 'n' can have at most n zeroes.

5. A non-zero constant polynomial has no-zero.

6. Every real number is a zero of the zero polynomial.

SECTION I

Very-Very Short Answer (1 Marks)

1. Write all the terms of the polynomial x^3-4x^2-3x+2.

2. Find the value of m if y+2m is a factor of $y^5-4m^2y^3+2y+2m+3$.

3. If side of a square is (x+2y-z) units then the area of the square is_____.

4. Find remainder when the polynomial (p^2-p-29) is divided by (p-6).

5. If p+q+r=9 then find the value of $(3-p)^3+(3-q)^3+(3-r)^3$.

6. The degree of the polynomial $\sqrt{5}$ is _____.

7. Find the value of q(-6) – q(-5) if q(x) = 4x+1.

8. A quadratic polynomial can be written as the product of _____ linear polynomials.

9. A cubic polynomial has _____ zeroes.

10. $(49)^3–(30)^3+$ _____ = 3x49x30x19

11. Check whether x=3 is a zero of the polynomial $x^3-3x+x-3$.

12. Find the zero of the polynomial 7x+1=0.

13. Find the value of k if $x^2+mx-30=(x-5)(x+6)$.

14. When a polynomial q(x) is divided by (x-2) and the remainder q(2)=0 then (x-2) is a _____ of the polynomial.

15. The degree of a zero polynomial is _____.

16. The polynomial (x+a)(x-a) has _____ zeroes.

17. What is the value of polynomial x^3+3x^2+1 at x=-1.

18. A_____ polynomial has one and only one zero.

19. The coefficient of x in x^3+4x^2-2x+1 is _____.

20. x^3-64 is a polynomial of degree _____ having _____terms.

21. Express -7 as a polynomial of degree zero.

22. The constant polynomial has _____ term/s.

23. Write the coefficient of x for the polynomial x^2-9.

24. A quadratic polynomial can have atmost _____ terms.

25. The variable of the polynomial x+b is _____.

26. Find p(0) if p(x) = x^3-2x^2+x+1.

CASE-STUDY QUESTIONS

27. A tree plantation campaign is organised in a government school. Under this campaign, the students of class IX planted total $2x^3+x^2-2x-1$ trees. The total number of students in class IX are $(x+1)$ and each student planted equal number of trees. The trees are arranged in rows and columns. The number of rows are $(2x+1)$.

    (i) The number of trees planted by each student are

    (a) $x+2$ (b) $x+1$ (c) $(2x+1)(x+1)$ (d) $(2x+1)(x-1)$

    (ii) If total 2079 trees are planted then the value of x is

    (a) 100   (b) 10   (c) 1000   (d) 20

    (iii) The number of columns are

    (a) (x^2-1) (b) $(2x+1)$ (c) $(2x-1)$ (d) x^2+1

    (iv) The third factor of the polynomial $2x^3+x^2-2x-1$ is

    (a) $(2x-1)$ (b) $(x+1)$ (c) $(2x+1)$ (d) $(x-1)$

    (v) Which of the following is not a zero of $2x^3+x^2-2x-1$?

    (a) 1   (b) $\dfrac{1}{2}$ (c) -1 (d) $\dfrac{-1}{2}$

28. Mahesh wants to paint a wall of his room. He decides to paint the wall in two colours, pink and white, divided diagonally. The length and breadth of the wall are $(x+4)$ and $(3x+2)$ respectively. The diagonal of the wall is x^2+3x.

    (i) The area of the wall is _____ polynomial.

    (a) linear   (b) quadratic   (c) cubic   (d) biquadratic

    (ii) The area of the wall is

    (a) $(x+4)(3x+2)$ (b) $(x+4)+(3x+2)$

    (c) $(x+4)-(3x-2)$ (d) $(x+4) \div (3x+2)$

    (iii) Mahesh wished to draw flowers in the squared area of the wall. For this the breadth should be reduced by

    (a) $2(x+3)$ (b) $(x-4)$ (c) $(3x+2)$ (d) $2(x-1)$

    (iv) The zero of the polynomial $(x+4)$ is

    (a) 0   (b) 4   (c) -4   (d) $\dfrac{1}{4}$

    (v) If x=2, the area to be painted pink is

    (a) 14   (b) 20   (c) 24   (d) 28

Very short Answer Question (2 Marks)

29. Check whether $q(x)$ is a multiple of $r(x)$ or not.

    Where $q(x) = 2x^3 – 11x^2 – 4x + 5$, $r(x) = 2x+1$

30. Show that $(x–5)$ is a factor of $x^3–3x^2–4x–30$.

31. Evaluate by using suitable identity : $(997)^2$

32. Find the zeroes of the polynomial $p(x) = x\,(x–2)\,(x+3)$

33. Find the quotient when $3x^2–7x–6$ is divided by $(x–3)$

34. Factorize $8x^3 + \sqrt{27}\, y^3$.

35. If $p(x) = x + 9$, then find $p(x) + p(–x)$

36. Find the product without multiplying directly 106×94.

37. Find the value of $(351)^2 – (350)^2$.

38. Find the value of $(a+b)$ if $(x+2)\,(x-5) = x^2+(a+b)x+ab$

39. The factors of $5x^2-18x+9$ are $(ax+b)$ and $(x+b)$. Find the values of a and b.

40. Find $p\,(1) + p\,(–1) + p\,(10)$ if $p(x) = x^2 – 3x + 2$

41. Find $(x-y)^2$ if $\dfrac{x}{y} + \dfrac{y}{x} = 2$.

42. Find the value of k when $(-x^{140}-2x^{151}+k)$ id divided by $(x+1)$.

43. Show that -1 is a zero of $3x^4-x^3+3x-1$.

44. Multiply $(x+1)\,(x-y)$.

45. Write the type of the polynimial x^3+3x^2+1 based on terms and degree.

Short Answer Type Questions   **(3 Marks)**

46. Factorize : $64a^2 + 96ab + 36b^2$

47. Factorize : $x^3 + 6x^2 + 11x + 6$

48. If $x^2+ y^2 = 49$ and $x–y = 3$, then find the value of $x^3–y^3$.

49. Simplify : $(5a–2b)\,(25a^2+10ab+4b^2)–(2a+5b)(4a^2–10ab+25b^2)$

50. Find the sum of remainders when $x^3–3x^2+4x–4$ is divided by $(x–1)$ and $(x+2)$.

51. Find the product $\left(p - \dfrac{1}{p}\right)\left(p + \dfrac{1}{p}\right)\left(p^2 + \dfrac{1}{p^2}\right)\left(p^4 + \dfrac{1}{p^4}\right)$

52. Factorize : $7\sqrt{2}\,k^2 – 10k – 4\sqrt{2}$.

53. Simplify : $(3x–4y)^3 – (3x+4y)^3$

54. Using appropriate identity, expand $(2a)^3 + b^3 + (3c)^3 - 18abc$.

55. Simplify : $(x+y+z)^2 - (x-y-z)^2$.

56. Factorize : $125x^3 + 8y^3 + z^3 - 30xyz$.

57. $x+2$ is a factor of polynomial $ax^3 + bx^2 + x - 2$ and the remainder 4 is obtained on dividing this polynomial by $(x-2)$. Find the value of a and b.

58. If the polynomials $ax^3 + 4x^2 + 3x-4$ & $x^3-4x + a$ leave the same remainder when divided by $(x-3)$, find the value of a.

59. If $\left(\dfrac{9}{10}\right)^3 - \left(\dfrac{2}{5}\right)^3 - \left(\dfrac{1}{2}\right)^3 = \dfrac{x}{50}$, find x

60. If $(x-3)$ and $\left(x - \dfrac{1}{3}\right)$ are factors of the polynomial $px^2 + 3x + r$, show that $p = r$.

Long Answer Type Questions   **(5 Marks)**

61. A literacy campaign was organised by Class IX girl students under NSS. Students made $(x-5)$ rows and $(3x-4)$ columns for the rally. Write the total number of students in the form of a polynomial.

62. i)   Using identity, find the value of $(-7)^3 + (5)^3 + (2)^3$.

    ii)   Find dimensions of cube whose volume is given by the expression $4x^2 + 14x + 6$.

63. Give possible expression for the length and breadth of each of the following rectangles if.

    i)   Area $= (x^2 + 5\sqrt{5}x + 30)$ sq. unit.

    ii)   Area $= (24x^2 - 26x - 8)$ sq. unit.

64. If $a + b + c = 0$, find the value of $\dfrac{(b+c)^2}{bc} + \dfrac{(c+a)^2}{ca} + \dfrac{(a+b)^2}{ab}$

65. Simplify : $\dfrac{(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$

66. Factorize

$(2a-b-c)^3 + (2b-c-a)^3 + (2c-a-b)^3$

67. If the polynomial $4x^3-16x^2+ax+7$ is exactly divisible by $x-1$, then find the value of a. Hence factorize the polynomial.

68. If $p, q, \& r$ are all non zero and $p+q+r = 0$, prove that
$$\frac{p^2}{qr} + \frac{q^2}{rp} + \frac{r^2}{pq} = 3$$

69. Factorize: $9x^3 - 27x^2 - 100x + 300$

70. If $(x+4)$ is a factor of the polynomial $x^3 - x^2 - 14x + 24$, find the other factors.

71. If $\dfrac{x}{y} + \dfrac{y}{x} = -1$ where $x \neq 0, y \neq 0$ then find the value of $x^3 - y^3$.

72. Simplify: $\dfrac{155 \times 155 + 155 \times 55 + 55 \times 55}{155 \times 155 \times 155 - 55 \times 55 \times 55}$

CHAPTER-2
POLYNOMIALS
Hints & Answers

1. $x^3, -4x^2, -3x, 2$
2. $\dfrac{3}{2}$
3. $(x+2y-z)^2$
4. 1
5. $3(3-p)(3-q)(3-r)$
6. 0
7. -4
8. two
9. three
10. $(-19)^3$
11. Not a zero
12. $\dfrac{-1}{7}$
13. m=-11
14. Factor
15. not defined
16. two
17. 3
18. linear
19. -2
20. three, two
21. $-7x°$
22. single
23. zero
24. three
25. -b

26. 1
27. (i) (d) $(2x+1)(x-1)$
    (ii) (b) 10
    (iii) (a) (x^2-1)
    (iv) (d) $(x-1)$
    (v) (d) $\dfrac{-1}{2}$
28. (i) (b) quadratic
    (ii) (a) $(x+4)(3x+2)$
    (iii) (d) $2(x-1)$
    (iv) (c) -4
    (v) (c) 24
29. Not a multiple
30. Put x=5
31. 994009
32. 0, 2, -3
33. 3x+2
34. $(2x+\sqrt{3}y)(4x^2-2\sqrt{3}xy+3y^2)$
35. 18
36. $(100+6)(100-6)=9964$
37. 701
38. -3
39. a=5, b=-3
40. 8
41. 0
42. k=-1
44. $x^2+(1-y)x-y$
45. Three terms - Trinomial
    Degree 3- cubic

46. $(8a + 6b)^2$

47. $(x+1) (x+2) (x+3)$

48. 207

49. $117a^3 - 133b^3$

50. -34

51. $p^8 - \dfrac{1}{p^8}$

52. $(K - \sqrt{2}) (7\sqrt{2}\ K + 4)$

53. $-8y(16y^2 + 27x^2)$ or $-128y^3 - 216\ x^2y$

54. $(2a+b+c) (4a^2 + b^2 + 9c^2 - 4ab - 2bc - 4ac)$

55. $4xy + 4zx$

56. $(5x + 2y + z) (25x^2 + 4y^2 + z^2 - 10xy - 2yz - 5zx)$

57. $a = 0, b = 1$

58. $a = -1$ Hint $p(3) = q (3)$

59. $x = 27$ (Use $a+b+c = 0$, $a^3 + b^3 + c^3 = 3abc$)

61. $3x^2 - 19x + 20$

62. i) -210

ii) 2 ; $(x + 3)$; $(2x + 1)$

63. i) $(x + 2\sqrt{5}) (x + 3\sqrt{5})$

ii) $(4x + 1), (6x - 8)$

64. 3

65. $(a+b)(b+c)(c+b)$

66. $3(2a-b-c)(2b-c-a)(2c-a-b)$

67. $a = 5, (x-1)(2x+1)(2x-7)$

69. $(3x+10)(x-3) (3x-10)$

70. $(x-3)(x-2)$

71. 0

72. $\dfrac{(155)^2+(155 \times 55)+(55)^2}{(155)^3-(55)^3} = \dfrac{(155)^3-(55)^3}{(155-55)[155)^3-(55)^3]} = \dfrac{1}{100} = 0.01$

Practice Test
POLYNOMIALS

Time : 50 Min.   M.M. 25

1.   Is $(x^2)^{½} + 2\sqrt{5}$ a polynomial?   (1)

2.   Show that x = 1 is a zero of the polynomial $3x^3 - 4x^2 + 8x - 7$.   (1)

3.   Find the value of the polynomial 2x+5 at x=-3.   (1)

4.   Find the zeroes of the polynomial $x^2 - 4x + 3$ (2)

5.   Evaluate 38x42 without multiplying directly   (2)

6.   If x + y + z = 6, xy + yz + zx = 11. Find the value of $x^2 + y^2 + z^2$.   (2)

7.   If 3x – 4 is a factor of the polynomial $p(x) = 2x^3 - 11x^2 + kx - 20$,   (3)
   find the value k

8.   Factorise : $a^2 + b^2 + 2(ab + bc + ca)$ (3)

9.   If a + b + c = 0 then find the value of   (5)

   $$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$$

10.   Factorise $x^3 - 23x^2 + 142x - 120$ by using factor theorem.   (5)

CHAPTER-3
COORDINATE GEOMETRY
MIND MAP

Coordinate Geometry

Cartesian System

Plotting a point in Cartesian Plane

Quadrants

Coordinates of a point

Convention of signs of coordeinates in different quadrants

II
(-,+)

I
(+,+)

y

Positive y-axis

Perpendieular distance from - y axis

x coordinate (abscissa)

p (a, b)

y coordinate (ordinate)

Perpendicular distance from x-axis

x ← → x
Negative x-axis

0

← a → Positive x-axis

Negative y-axis

III
(-,-)

IV
(+,-)

y

Key Points

- Co-ordinate Geometry is the branch of Mathematics in which we study the position of any object lying in a plane, with the help of two mutually perpendicular lines in the same plane.

- In cartesian system, there are two perpendicular straight lines xx' and yy' intersect at O (Origin) in a plane (cartesian plane)

- Coordinates of a point are determined by perpendicular distance from x-axis and y-axis respectively.

- The perpendicular distance of a point from y-axis is called abscissa and from x-axis is called ordinate.

- Coordinates of a point on x-axis are of the form (x,0) and on y-axis are of the form (0,y)

- If x-coordinate of two or more points are same, then the line joining these points is parallel to y-axis.

- If y-coordinate of two or more points are same, then the line joining these points is parallel to x-axis.

- Location of a point P(a, b) on graph with sign convention – where a and b are such that –

	Value of Point	Sign of Point	Location of Point
i)	a = 0, b = 0	——	origin
ii)	a > 0, b > 0	(+, +)	Ist Quadrant
iii)	a < 0, b > 0	(−, +)	IInd Quadrant
iv)	a < 0, b < 0	(−, −)	IIIrd Quadrant
v)	a > 0, b < 0	(+, −)	IVth Quadrant

Note: If a point lie on x-axis or y-axis, it does not lie in any quadrant.

- Mirror Image of a point is just a reflection of this point about the mirror.

Point	Mirror Image about x-axis	Mirror Image about y-axis	Mirror Image about origin
(a, b)	(a,-b)	(-a, b)	(-a,-b)
(-a, b)	(-a,-b)	(a, b)	(a,-b)
(-a,-b)	(-a, b)	(a, -b)	(a, b)
(a,-b)	(a, b)	(-a, -b)	(-a, b)

Section-I

Very Very Short Answer (1 Marks)

1. The distance of a point form x-axis is called _____.

2. The distance of a point from y-axis is called _____.

3. In which quadrant will a point lie if its both the coordinates are positive?

4. Find the point at which two coordinate axes meet.

5. Write the coordinates of the point which lies at a distance of x-units from x-axis and y units from y-axis.

6. Find the coordinates of the point which lies on x-axis at a distance of 5 units from y-axis.

7. Find the coordinates of the point which lies on y-axis at a distance of 9 units from x-axis in negative direction.

8. In which quadrant in a cartesion plane the ordinate of a point will be positive and abscissa will be negative?

9. Point (6,-8) will lie in which quadrant ?

10. In which quadrant will a point lie if its ordinate and abscissa both have negative signs?

11. On which axis the point A(-3,0) lie?

12. If the coordinates of the points are P (-2,3) and Q (-3,5) then abscissa of P- abscissa of Q is _____.

13. The measure of the angle between coordinate axes is _____.

14. The perpendicular distance of the point p(-4,-3) from x-axis is _____.

15. The coordinate axes divide the plane info four parts which are called _____ .

16. The point (200, –111) lies in the _____ quadrant.

17. The abscissa of any point on the y-axis is _____ .

18. The ordinate of any point on the x-axis is _____ .

19. The points (0,0), (0,4), and (4,0) form a/an _____ triangle.

20. If (x, y) represents a point and x>0,y>0, then the point will lie in _____ quadrant.

21. The points with coordinates (3, −1) and (−1, 3) are at _____ (same/different) positions of the coordinate plane.

22. If the ordinate of point is 7 and abscissa is −5, then its coordinates are _____ .

23. The coordinates of a point whose abscissa is 5 and which lie on x-axis are _____.

24. Find the image of the point (2,3) under x-axis.

25. Find the mirror image of the point (-5,6) about y-axis.

Case study based questions (4 marks)

26. To make the students aware of personal health and hygiene a race was organized on rectangular playground ABCD of a school. LInes have been drawn with chalk powder at a distance of 1m each. 100 flower ports have been placed at a distance of 1m from each other along AD. Muskan runs $\frac{1}{4}$ of the distance AD on the third line and posts a red flag. Mayank runs $\frac{1}{5}$ of the distance AD on the 7th line and posts a green flag.

   Based on the above information, answer the following questions.

For Reference Only
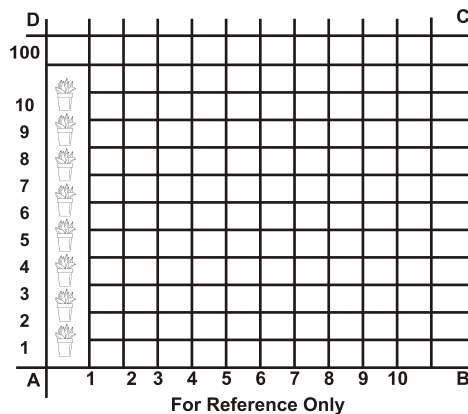
   (i) The coordinates of red flag are

   (a) (25,3)   (b) (100,3)

   (c) (3,25)   (d) (3,100)

   (ii) What are the coordinates of green flag?

   (a) (100,7)   (b) (7,100)

   (c) (20,7)   (d) (7,20)

(iii) How far Muskan had to run to fix the red flag?

(a) 3m   (b) 25m

(c) 100m   (d) Non of these

(iv) How far Mayank had to run to fix green flag?

(a) 7m   (b) 20m

(c) 100m   (d) Non of these

(v) How far is the green flag from the red flag?

(a) $\sqrt{3}$m   (b) $\sqrt{41}$m

(c) $\sqrt{10}$m   (d) None of these

27. The diagram shows a model of pyramid placed on a cartesian plane in a mathematics lab in a school. Based on the diagram, answer the following questions-
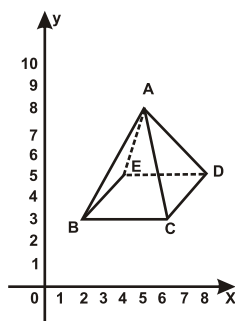
(i) What are the coordinates of the axis of the pyramid ?

(a) (2,3)   (b) (6,3)

(c) (4,5)   (d) (5,8)

(ii) How much is the perpendicular distance between the edges BC and ED?

(a) 3 units   (b) 2 units

(c) 1 unit   (d) 4 units

(iii) If the pyramid is moved 2 units to the right, what will be the coordinates of vertex D?

(a) (10,5)   (b) (8,5)

(c) (8,7)   (d) (10,7)

(iv) If the pyramid is moved 3 units below its actual position, what will be the coordinates of vertex B.

(a) (2,3)   (b) (3,2)

(c) (2,0)   (d) (0,2)

(v) If the vertex B should lie on the origin, what will be the coordinates of vertex E?

(a) (2,2)   (b) (3,3)

(c) (4,4)   (d) None of these

28. MNQ school provide free education to underpreviliged childern. Municipal corporation of a city wants to open an ABC school in the town on a rectangular plot ABCD as shown in figure. But this plot belongs to Amar Singh who has agreed to exchange it with a triangular plot PQR out of the town as shown in figure. Based on the above information, answer the following questions:
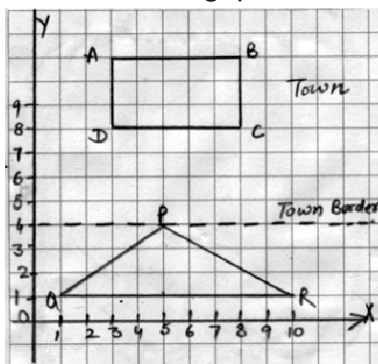
(i) What are the coordinates of vertex C of rectangular plot

(a) (0,0)   (b) (8,8)

(c) (4,4)   (d) (4,8)

(ii) What is the area of rectangular plot ?

(a) 5 sq. units   (b) 3 sq. units

(c) 15 sq. units   (d) 10 sq. units

(iii) What is the area of triangular plot ?

(a) 10 sq. units   (b) 15 sq. units

(c) 5 sq. units   (d) 13.5 sq. units

(iv) Which statement is true in respect of the areas of two plots?

(a) Ar (ABCD) > Ar (PQR)   (b) Ar(ABCD) < Ar (PQR)

(c) Ar (ABCD) = Ar (PQR)   (d) Ar (ABCD) = 2Ar(PQR)

(v) Find the ratio of Ar (ABCD) to Ar (PQR)

(a) 1 : 2   (b) 3 : 4

(c) 10 : 9   (d) 3 : 5

Very Short Answer Type Questions (2 Marks)

29. Find the co-ordinates of two points on x-axis and two points on y-axis which are at equal distance from the origin.

30. Name the quadrant in which the graph of point A(x,y) lies when

    (i) $x > 0$ and $y > 0$ (ii) $x < 0$ and $y < 0$

31. On plotting the points 0(0,0), A(4,0), B(4,3) and c(0,3) and joining OA, AB, BC and CD, which shape is formed?

32. Plot a point P(5,6) on the graph paper. Now plot the reflections of this point on x-axis and y-axes.

33. Find the coordinates of the vertices of a rectangular figure placed in III quadrant in the cartesian plane with laught p units on x-axis and breadth q units on y-axes.

34. Plot the points A(1,-1) and B(4,5) on a cartesian plane and draw a line segment joining these points. Now, write the coordinates of any one point on this line segment which lies between the points A and B.

Short Answer Type Questions   (3 Marks)

35. Draw a line segment on a graph paper whose end points lies in first quadrant and third quadrant. Write the coordinates of its end points and mid point of line segment.

36. Plot the points A (2, 4) & B (2, –5). Is this line AB parallel to any of the axes. If yes, to which axis is it parallel?

37. Plot the points P (2, –3) & Q (–5, –3). To which axis the line P Q is parallel?

38. Plot the points A (7, 6) & B (7, –6) on graph paper. Join them & answer the following :

(i) Write the coordinate of the point where line AB cuts the x-axis.

(ii) To which axis, line AB is parallel ?

39. Draw a triangle ABC on graph paper having the coordinates of its vertices as A (–2, 0), B (4, 0) and C (1, 5). Also find the area of triangle.

40. If we plot the points P(5, 0), Q (5, 5), R(–5, 5) and S (–5, 0), which figure will we get? Name the axis of symmetry of this figure?

41. Find the coordinates of a point which is equidistant from the two points (–4, 0) and (4, 0). How many of such points are possible satisfying this condition?

42. Draw a quadrilateral with vertices A (4, 3), B(–4, 3), C(–4, –3) and D(4, –3). Draw its diagonals and write the coordinates of the point where the diagonals cut each other.

43. A rectangular field is of length 10 units & breadth 8 units. One of its vertex lie on the origin. The longer side is along x-axis and one of its vertices lie in first quadrant. Find all the vertices.

44. Plot the point B (5, 3), E(5, 1), S(0, 1) and T(0, 3) and answer the following :

    i) Join the points and name the figure obtained.

    ii) Find the area of figure.

Hints and Solutions/Answers

1. Ordinate
2. Abscissa
3. I-quadrant
4. (0,0)
5. (y,x)
6. (5,0) or (-5,0)
7. (0,-9)
8. II uadrant
9. IV quadrant
10. III quadrant
11. x-axis
12. 1
13. 90°
14. 3 units
15. quadrants
16. IV quadrant
17. 0
18. 0
19. isosceles
20. I
21. different
22. (-5,7)
23. (5,0)
24. (2,-3)
25. (5,6)
26. (i) (c)
    (ii) (d)
    (iii) (b)
    (iv) (b)
    (v) (b)
27. (i) (d)
    (ii) (b)
    (iii) (a)
    (iv) (c)
    (v) (a)

28. (i) (b)
    (ii) (c)
    (iii) (d)
    (iv) (a)
    (v) (c)
29. $(\pm a, 0)$, $(0, \pm a)$, where a is any real number.
30. (i) I quadrant
    (ii) III quadrant
31. Rectangle
32.

33. (o,o), (-p,o), (-p,-q), (o,-q)
36. Yes, Y-axis
37. X- axis
38. (7,0), Y-axis
39. 15 sq. units
40. Rectangle, Y-axis
41. Any point on Y-axis, Infinite
42. (0,0.)
43. (0,0), (10,0), (10,8) (0,8)
44. (i) Rectangle
    (ii) 10 sq. units.

Practice Test
COORDINATE GEOMETRY

Time : 1 Hr.   M.M. 25

1. In which quadrant, the point (x, y) will lie, where x is
    positive and y is negative number?   (1)

2. Write the y-coordinate of a point which lies on x-axis.   (1)

3. If (x,y)=(y,x), then find the value of (x-y)   (1)

4. Find the value of x and y if: (2)
    (a) (x – 4, 7) = (4, 7)
    (b) (1, 2y – 3) = (1, 7)

5. What is the distance of a point (7, 6) from x-axis and y-axis?(2)

6. Draw the quadrilateral, whose vertices are A(3,2), B(2,3),   (2)
    C(-4,5), D(5,-3)
                                    OR
    In which quadrant, do the following points lie?
    (i) (4,-2) (ii) (-3,7)   (iii) (-1,-2)   (iv) (3,6)

7. Plot the following points in a Cartesian plane and join the   (3)
    points to find the area of the figure so obtained.
    (–3, 5), (–2, 0), (–4, 0)

8. Write the equations of lines *l*   (3)

    and m as shown in the figure.
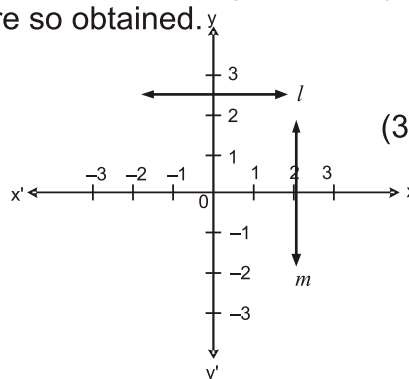
    Also name the line which is

    represented by x = 0.

9. Plot the points O(0, 0), A(4, 0) and C(0, 6). Find the coordinates of
    the fourth point B such that OABC forms a rectangle.   (5)

10. The base AB of two equilateral triangles ABC and ABD with side
    2a, lies along the x-axis such that the mid point of AB is at the
    origin. Find the coordinates of two vertices C and D of the
    triangles.   (5)

CHAPTER-4
Linear Equations In Two Variables

Mind Map

Linear Equation in Two variables

General Form

ax+by+c=0

where → **a,b,c are constants, x,y are variables**

Examples → **2x+5y=10 7x-2y+11=0 y=3x+5 etc.**

Solutions

Values of x and y satisfying the equation

Example

Solutions of x+2y=7 are
(i) x=5, y=1   (iii) x=6, y=$\frac{1}{2}$
(ii) x=3, y=2   (iv) x=1, y=3
etc.

Number of Solutions → **Infinite**

Graph
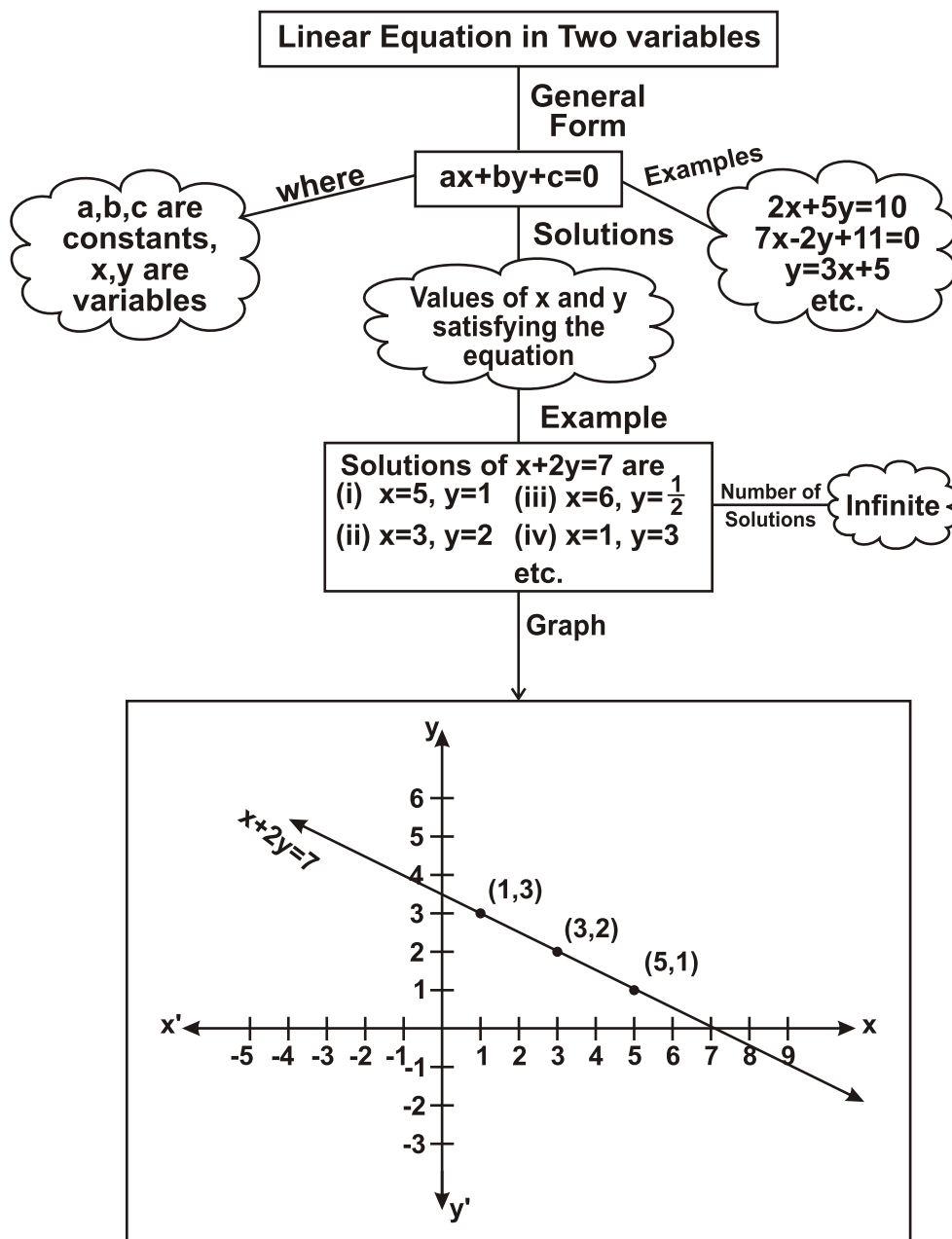
LINEAR EQUATIONS IN TWO VARIABLES

KEY POINTS

- **Linear equation in one variable** – An equation which can be put in the form ax+b=0, $a \neq 0$ and a, b are real numbers is called a linear equation in one variable.

- **Linear equation in two variables** – Any equation which can be put in the form ax+by+c=0, where a, b, and c are real numbers and a, $b \neq 0$, is called a linear equation in two variables.

  Linear equation in one variable has a unique solution

  $$ax + b = 0 \gg x = -\frac{b}{a}$$

- Linear equation in two variables has infinitely many solutions.

- The graph of every linear equation in two variables is a straight line.

- Every point on the line satisfies the equation of the line.

- Every solution of the equation is a point on the line. Thus, a linear equation in two variables is represented geometrically by a line whose points make up the collection of solutions of the equation.

Graph :

* The pair of values of x and y which satisfies the given equation is called solution of the linear equation in two variables.

    Example : x + y = 4

    Solutions of equation

    x+y = 4 are

    (0,4) (1,3) (2,2) (4,0)

    and many more

Class IX - Mathematics
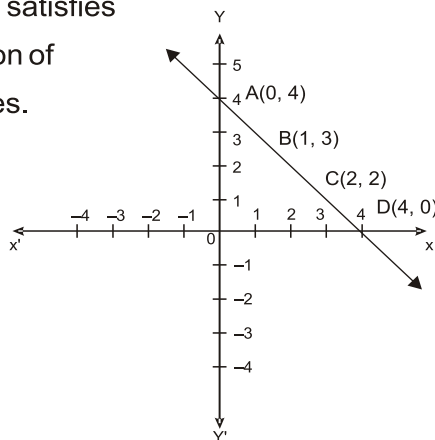
Very Very Short answer type questions (1 mark)

1. At what point, the graph of 3x+2y=9, cuts the y axis?

2. Let y varies directly as x. If y=15 when x=5, then write a linear equation.

3. Write the point of intersection of the lines x=2 and y=-3

4. What is the distance of the point (3,-7) from x-axis?

5. What is the distance of the point (-5,-4) from y-axis?

6. Express the linear equation $\sqrt{2}x-4=5y$ in the form of ax+by+c=0 and thus indicate the values of a, b and c.

7. Express x in terms of y for the equation 3x+4y=7

8. Express y in the terms of x.

   3y+5x = 9

9. Point (9,0) lie on which axis?

10. Find a solution of x + y = 5 which lies on y-axis

11. Express the equation 5y=9 as linear equation in two variables.

12. Write the linear equation which is parallel to x-axis and is at a distance of 2 units from the origin in upward direction.

13. Check whether (1, —2) is a solution of 2x - y = 6.

14. Check whether x = 2 & y = -2 is a solution of 2x - y = 6.

15. How many solutions are there for equation y = 5x + 2.

16. Find the value of K, if x = —1 & y =1 is a solution of equation Kx —2y = 0.

17. If the graph of equation 2x + Ky = 10 K intersects x-axis at point (5,0), find the value of K.

18. The graph of the linear equation 4x=6 is parallel to which axis?

19. At what point the graph of 2x — y = 6, cuts x-axis?

20. On which side of y — axis, x + 3 = 0 lies?

21. On which side of x-axis, 2y-1=0 lies?

Fill in the blanks

22. The equation of a line parallel to x-axis is _____ = a, where a is any non-zero real number.

23. The equation of a line parallel to y-axis is _____ =a, where a is any non-zero real number.

24. The graph of every linear equation in two variables is a_____.

25. An equation of the form ax+b=0, where a, b are real numbers and a ≠ 0, in the variable x, geometrically represents _____.

26. The coefficient of x in the linear equation 2(x+y)-x=7 is _____.

Case study based problem

Case Study - I

27. The eco-club of a school decided to develop a garden of the school and planted three types of plants A, B and C Number of plant A is twice that of plant B and number of plant B is same as number of plant C. If total number of plants is 100.

(A) Correct representation of the above situation in linear equation is.

(a) x+y=100   (b) x+2y=100

(c) x-y=100   (d) x-2y=100

(B) If number of plants of type A is 50, then number of plant B is

(a) 10   (b) 20

(c) 30   (d) None

(C) If number of plants of type A and B together is 75, then number of plant of type C is

(a) 10   (b) 15

(c) 20   (d) 25

(D) The graph of linear equation in two variable for the above situation will not pass through :

(a) (100,0)   (b) (50,50)

(c) (60,20)   (d) (70,15)

(E) The number of plants of type C is always.

(a) more than 25   (b) less than 25

(c) less than or equal to 25   (d) more than or equal to 25.

Case Study -II

28. The RWA of a locality is running a lending library to develop the habit of reading books among society. To run this library they charges a fixed amount of ₹10 for first five days for a book and ₹ 3 for each day there after.
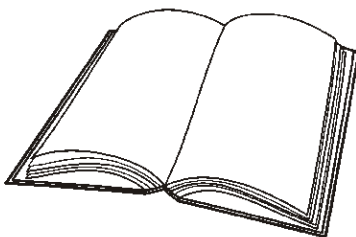
(A) If you borrow the book for 7 days what amount you will have to pay.

(a) ₹ 15   (b) ₹ 16

(c) ₹ 70   (d) ₹ 21

(B)   If you borrow two books and you paid total amount as ₹ 40, then number of days for which book was borrowed.

(a) 10 days   (b) 12 days

(c) 15 days   (d) 30 days

(C) The linear equation to represent the condition by taking x as total number fo days ($x \geq 5$) and total amount to be paid as ₹y.

(a) 3x-y=5   (b) 3x+y=5

(c) y=3x+10   (d) x=3y+10

(D) In the linear equation (correct option from part C) if x=7 then the value of y is.

(a) -1   (b) 31

(c) -16   (d) 16

(E) The correct option to borrow a book for a number of days and amount paid for it is.

(a) 5 days, ₹ 20   (b) 15 days, ₹ 45

(c) 8 days, ₹ 14   (d) 10 days, ₹ 25

Case Study -III

29. During Covid-19 door to door survey, A frontline health worker recorded the temperature of the family of five members. Their name and age is given below.

Name	Age (in years)
Uma	65
Raj Kumar	40
Savita	37
Rohan	14
Jyoti	10

The linear equation that convert temperature from Fahrenheit (°F) to Celsius (°C) is given by

$$C = \frac{5F - 160}{9}$$

(A) If temperature of Uma is 97.7°F, then her temperature is °C is.

    (a) 36   (b) 36.5

    (c) 37   (d) 37.2

(B) If temperature of Raj Kumar is 37°C, then his temperature is °F is

    (a) 98   (b) 98.2

    (c) 98.4   (d) 98.6

(C) Taking C on x-axis and F an y-axis, if graph of given linear equation is drawn , then from which quadrant it will to pass?

    (a) I   (b) II

    (c) III   (d) IV

(D) If normal temperature of a human body lies between 36.5°C and 37.5°C, then in Fahrenheit it lies between:

    (a) 97.7 and 99.5   (b) 98 and 99.2

    (c) 98.5 adn 99.5   (d) 99 and 100

(E) For which numerical value, the Celsius (°C) and Fahrenheit (°F) temperatures are equal ?

    (a) 0   (b) 100

    (c) -40   (d) 40

Very Short Answer Type Question (2 Marks)

30. Find any two solutions of equation

    2x+y=x+5.

31. Find the value of P if x=2, y=3 is a solution of equation 5x+3 Py=4a

32. If the points A (3,5) and B (1,4) lines on the graph of line ax+by=7, find the value of a.

33. Write the coordinates of the point where the graph of the equation 5x+2y=10 intersect both the axes.

34. Write the equations of two lines passing though (3, 10).

35. The cost of coloured paper is 7 more than 1/3 of the cost of white paper. Write this statement in linear equation in two variables.

36. Draw the graph of equation x+y=5.

37. The graph of linear equation 2x-y=6 will pass through which quadrant(s).

38. How many solution of the equation 3x-2=x-3 are there on the

    (i) Number line

    (ii) Cartesian plane....

39. Find the points where the graph of x+y=4 meets line which is

    (i) parallel to x-axis at 3 units from origin in positive direction of y-axis.

    (ii) parallel to y-axis at 2 units on left of origin.

Short Answer Type Questions (3 Marks)

40. If total number of legs in a herd of goats and hens is 40. Represent this situation in the form of a linear equation in two variables.

41. Find the value of a and b, if the line 6ax+by=24 passes through (2,0) and (1,2)

42. Determine the point on the graph of the linear equation 2x+5y=19 whose ordinate is $1\frac{1}{2}$ times its abscissa.

43. Find the points where the graph of the following equation cut the x-axis and y-axis 2x=1-5y

44. Write the equation of the line parallel to x-axis at a distance of 4 units above the origin.

45. If the points A (4,6) and B(1,3) lie on the graph of ax+by=8 then find the value of a and b.

46. Find the value of 'a' if (1, -1) is the solution of the equation 2x+ay=5. Find two more solutions of the equation.

47. Find two solutions of the equation 4x + 5y = 28. Check whether (-2,10) is solution of the given equation.

48. Write the equation of line passing through (3, —3) & (6, -6).

49. If x = 3k — 2, Y=2k is a solution of equation 4x — 7y + 12 =0, then find the value of K.

50. If (m —2, 2m + 1) lies on equation 2x + 3y— 10 = 0, find m.

51. F = (9/5)C + 32.

    (i) If the temperature is 35°C, what is the temperature in Fahrenheit?

    (ii) If the temperature is 30°C, what is the temperature in Fahrenheit?

52. Draw the graph of the linear equation 2x+3y=6. Find out the coordinates of the points where the line intersects at x axis and y-axis.

53. Draw the graph for the linear equations

    3x +4y = 12. If x = 8, find the value of y with the help of graph.

54. Draw the graph of y = x & 2y = —5x on the same graph.

55. Give the geometrical representation of 5x + 7 = 0 as equation.

    (i) in one variable   (ii) in two variables

56. Draw the graph of the linear equations 2y – x = 7. With the help of graph check whether x = 3 and y = 2 is the solution of the equation?

57. Draw the graph of linear equation 3x-y=4. From the graph find the value of p and q if the graph passes through (p,-4) and (3,q)

58. Draw the graph of equations 2x+3y=-5 and x+y=-1 on the same graph. Find the co-ordinate of the point of intersection of two lines.

59. Show that the points A(1,-1) B(2,6) and C(0,-8) lie on the graph of the linear equation 7x-y=8.

Long Answer type questions (5 Marks)

60. Write 3y = 8x in the form of ax+by+c=0. Write x in terms of y. Find any two solutions of the equation. How many solutions you can find out?

61. Rohan and Ramita of Class IX decided to collect ₹ 25 for class cleanliness. Write it in linear equations in two variables. Also draw the graph.

62. Sarika distributes chocolates on the occasion of children's Day. She gives 5 chocolates to each child and 20 chocolates to adults. If number of children is represented by 'x' and total distributed chocolates as 'y'.

   (i) Write it in the form of linear equation in two variables.

   (ii) If she distributed 145 chocolates in total, find number of children?

63. Priyanka and Arti decided to donate ₹1600 for the Army widows. Let Priyanka's share as 'x' and Arti share as 'y'.

   (a) Form a linear equation in two variables.

   (b) If Priyanka donates thrice the amount donated by Arti, then find out the amount donated by both.

64. Riya participates in Diwali Mela with her friends for the charity to centre of handicapped children. They donate ₹3600 to the centre from the amount earned in Mela. If each girl donates ₹150 and each boy donates ₹ 200.

   (a) Form the linear equation in two variables.

   (b) If number of girls are 8, find number of boys.

65. Aftab is driving a car with uniform speed of 60 km/hr. Assuming total distance to be y km & time taken as x hours, form a linear equation. Draw the graph. From the graph read the following:

   (i) distance travelled in 90 minutes.

   (ii) Time taken to cover a distance of 150 km.

66. The parking charges of a car in a private parking is ₹ 20 for the first hour and ₹10 for subsequent hours. Taking total parking charges to be y & total parking time as x hours form a linear equation. Write it in standard form hence find, a, b & c. Draw the graph also.

67. We know that C = 2πr, taking π = 22/7, circumference as y units, radius as x units, form a linear equation. Draw the graph. Check whether the graph passes through (0, 0). From the graph read the circumference when radius is 2.8 units.

LINEAR EQUATIONS IN TWO VARIABLES

Hints and Solutions / Answers

1. (4.5,0)
2. y=3x
3. (2,-3)
4. 7 units
5. 5 units
6. $\sqrt{2}$x-5y-4=0

   a=$\sqrt{2}$, b=-5, C=-4
7. x = $\frac{7-4y}{3}$
8. y = $\frac{9-5x}{3}$
9. x-axis
10. (0,5)
11. ox+5y=9
12. y=2
13. No
14. Yes
15. Infinitely many solutions
16. K(-1)-2(1)=0

    k=-2
17. 2(5)+k(0)=10k

    k=1
18. Parallel to y-axis
19. (3,0)
20. On left side
21. On right side
22. y
23. x
24. Straight line

25. a point on number line
26. 1
27. (A) (b) x+2y=100

    (B) (d) None

    (C) (d) 25

    (D) (b) (50,50)

    (E) (c) less than or equal to 25
28. (A) (b) ₹ 16

    (B) (c) 15 days

    (C) 3x-y = 5

    (D) (b) 31

    (E) (d) 10 days, ₹ 25
29. (A) (b) 36.5

    (B) (d) 98.6

    (C) (d) IV

    (D) (a) 97.7 and 99.5

    (E) (c) -40
30. (1,4) (0,5)

    [or any other possible solutions]
31. p= $\frac{4a-10}{9}$
32. 3a+5b=7; a+4b=7

    3(7-4b) +5b =7

    b=2

    a=-1
33. x axis = (2,0)

    y axis = (0,5)

34. $3x-y+1=0$; $x+y=13$

[or any other possible equation]

35. Let the cost of coloured paper be ₹ x

Let the cost of white paper be ₹ y

$3x-y=21$

36. $x+y=5$

x	0	5	1
y	5	0	4
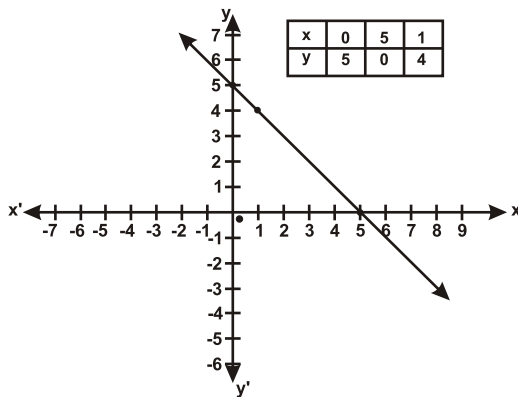
37. I, IV, III

38. (i) one solution

(ii) Infinitely many solution

39. (i) (1,3)

(ii) (-2,6)

40. Let number of goats = x

number of hens = y

$4x+2y=40$

or $2x+y=20$

41. $6a(2) + b(0) = 24$

$12a = 24$

$a = 2$

$6(2)(1) + b(2) = 24$

$12+2b=24$

$2b=12$

$b = 6$

42. abscissa $=2$

ordinate $= 3$

point (2,3)

43. Points cuts x axis $(\frac{1}{2},0)$

Points cut y axis $(0,\frac{1}{5})$

44. $y=4$

45. $4a+6b = 8$

or $2a+3b=4$

$a+3b=8$

$a=-4$ and $b=4$

46. $2(1)+a(-1)=5$

$-a=3$

$a=-3$

$2x-3y=5$, any two solutions

47. (2,4); (7,0)

[or any other possible solution]

No, (-2,10) is not a solution

48. $x+y=0$

49. $4[3k-2]-7[2k]+12=0$

$12k-8-14k+12=0$

$k=2$

50. $2[m-2]+3[2m+1]-10=0$

$2m-4+6m+3-10=0$

$m= \frac{11}{8}$

51. (i) 95°F

(ii) 86°F

52. $2x+3y=6$

x	0	3	-3
y	2	0	4
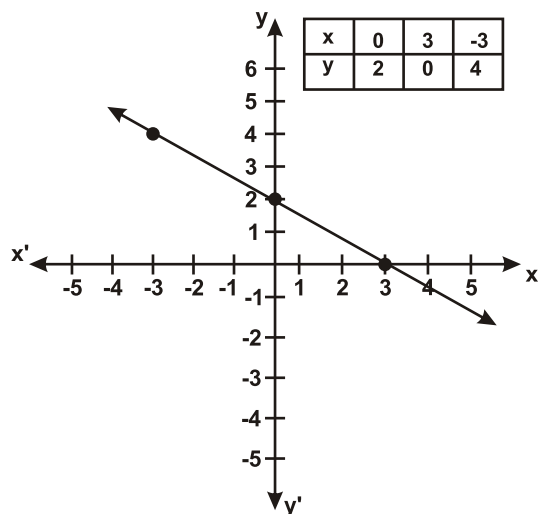
x-axis co-ordinates (3,0)

y-axis co-ordinates (0,2)

53. $y=-3$

55. $x=\dfrac{-7}{5}$ or $x=-1.4$

(i)

(ii)

x	-1.4	-1.4	-1.4
y	0	1	2
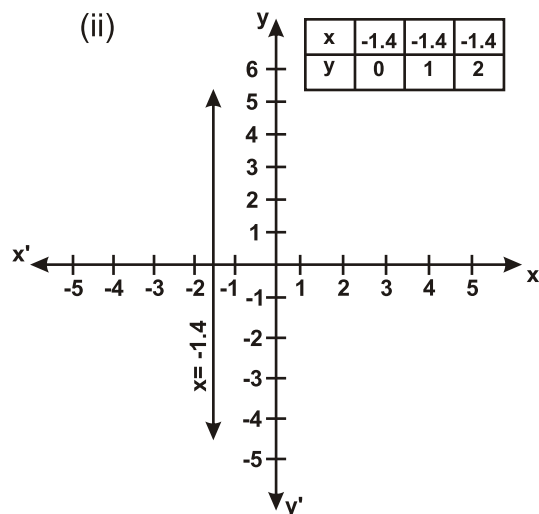
56. No

57. $p=0$; $q=5$

58. Point of intersection (2,-3)

60. $8x-3y+0=0$; $x=\dfrac{3y}{8}$

(0,0) (3x8)

Infinitely many solutions.

61. $x+y=25$

62. (i) $5x+20=y$

(ii) 25

63. (a) $x+y=1600$

(b) Priyanka= ₹ 1200

Arti = ₹400

64. (a) $150x+200y=3600$

(b) Number of boy = 12

65. $y=60x$

(i) 90km

(ii) 2 hours 30 min.

66. $10x+10=y$

$10x-y+10=0$

$a=10$; $b=-1$; $c=10$

67. $y=2\pi x$

yes

when r=2.8 units

c=17.6 units

Practice Test
POLYNOMIALS

Time : 1 hr.   M.M. 25

1. At what point the graph of the linear equation 3x - 2y = 15 cuts the x-axis?   (1)

2. The graph of liner equation 2y=5 is parallel to which axis ?   (1)

3. Write the linear equation the graph of which is parallel to y-axis and is at a distance 3 units on left from the origin   (1)

4. Find the value of a and b. if the line 5bx-3ay=30 passes through (-1,0) and (0,-3)   (2)

5. Write two linear equations passing through the points (2,-3)   (2)

6. If the point (5,2) lies on the graph of the linear equation kx+5y=10k find K?   (2)

7. Write the linear eqution x+ √3y=4 in the form of ax+by+c=0 and hence write the values of a, b and c. Write also x in term of y   (3)

8. Find the solutions of linear equation 2x+y=4 which represents a point on/which   (3)

    (i) x-axis   (ii) y-axis

    (iii) is at 3 unit perpendicular distance from x-axis

9. Give the geometrical representation of 2x+5=0 as a linear equation in(5)

    (a) One variable   (b) two varables

10. A taxi charges ₹ 15 for first kilometer and ₹ 8 each for every subsequent kilometer for a distance of x km, an amount of ₹ y is paid. Write the linear equation representing the above information and draw the graph.   (5)

INTRODUCTION TO EUCLID'S GEOMETRY

KEY POINTS

- **Introduction :** Euclidean geometry, which is taught today is named after Euclid - he is known as "the father of geometry". Euclid also studied and contributed in other areas of mathematics, including number theory and astronomy.

- **Axiom or Postulates** : Axiom or Postulates are the assumptions which are obvious universal truths. They are not proved.

- **Theorems :** Theorems are statements which are proved using definitions, axioms, previously proved statements and deductive reasoning.

SOME OF EUCLID'S AXIOMS

1. Things which are equal to the same thing are equal to one another.

2. If equals are added to equals the whole are equal.

3. If equals are subtracted from equals the remainders are equal.

4. Things which coincide with one another are equal to one another.

5. The whole is greater than the part.

6. Things which are double of the same things are equal to one another.

7. Things which are halves of the same things are equal to one another.

EUCLID'S POSTULATES AND DEFINITIONS

- **Postulates 1 :** A straight line may be drawn from any one point to any other points.

- **Postulate 2 :** A terminated line can be produced indefinitely.

- **Postulate 3 :** A circle can be drawn with any centre and any radius.

- **Postulate 4 :** All right angles are equal to one-another.

- **Postulate 5 :** If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right

DEFINITIONS

1. A <u>Point</u> is that which has no part.

2. A <u>line</u> is breadthless length.

3. The ends of a line are points.

4. A <u>straight</u> line is a line which lies evenly with the points on it self.

5. A <u>surface</u> is that which contain length and breadth only.

6. The <u>edges</u> of a surface are lines.

7. A <u>plane surface</u> is a surface which lies evenly with the straight lines on it self.

8. Two distinct lines can not have more than one point in common.

Very Very Short Answer Type Questions (1 Marks)

1. Write the number of dimensions, that a surface contain.

2. A proof is required for _____ (Postulate, Axioms, Theorem).

3. The number of line segments determined by three collinear points is _____ (Two, three, only one).

4. Euclid stated that if Equals are subtracted from Equal then the remainders are equal in the form of _____ (an axiom, a definition, a postulate).

5. In given figure AD = BC then AC and BD are equal or not?
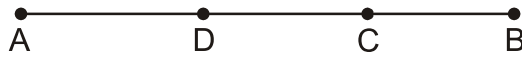
6. How many lines can pass through a single point ?

7. State Euclid's first postulate.

8. Write Euclid's fifth postulate.

9. If a + b = 15 and a + b + c = 15 + c

   which axiom of Euclid does the statement illustrate?

10. If A, B and C are three points on a line and B is between A and C then prove that AC − BC = AB.
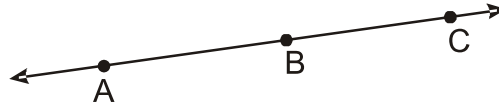
Very Short Answer Type Questions (2 Marks)

11. If x + y = 10 and x = z then show that z + y = 10

12. In given figure AX = AY, AB = AC

    Show that : BX = CY
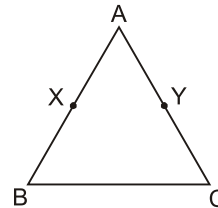
13. In given figure ∠ABC = ∠ACB and ∠3 = ∠4

    Show that ∠1 = ∠2
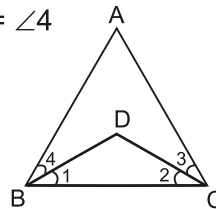
14. In the given figure of AD = CB

then prove that AC = BD
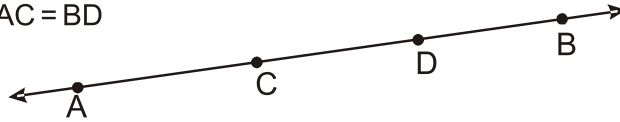
15. Solve the equation x − 10 = 15, State which axiom do you use here.

16. If a point C lies between two points A and B such that AC = BC then prove that

$$AC = \frac{1}{2} AB$$

17. In the given figure

$$AM = \frac{1}{2} AB$$

$$AN = \frac{1}{2} AC$$
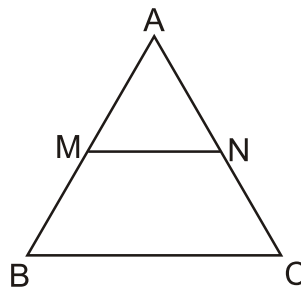
show that AB = AC

18. In the given figure AC = DC, CB = CE
then show that AB = DE
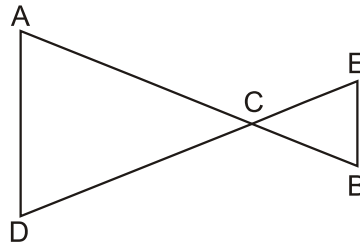
19. Prove that every line segment has one and only one mid point.

20. In fig, A and B are centres of the two intersecting circles, which intersect at C. Prove that AB=AC=BC
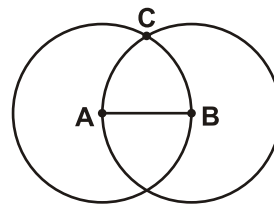
21. Kartik and Himank have the same weight. If they each gain weight by 3kg, how will their new weights be compared? state Euclid's axiom used?

Short Answer Type Questions (3 Marks)

22. In the given figure ∠1 = ∠2 and ∠2 = ∠3

    then show that ∠1 = ∠3
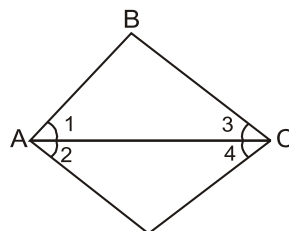
23. In the given figure AB = BC, M is the mid point of AB and N is the mid point of BC. Show that AM = NC
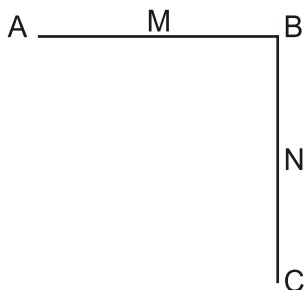
24. In the given figure ∠1 = ∠3 and ∠2 = ∠4

    then show that ∠BAD = ∠BCD
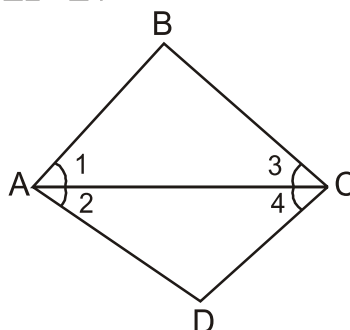
25. An equilateral triangle is a polygon made up of three line segments out of which two line segments are equal to the third one and all the angles are 60° each.

    Can you justify that all sides and all angles are equal in equilateral triangle?

26. RAM and Shyam are two students of Class IX. They give equal donation to a blind school in the month of March. In April each student double their donation.

    (a) compare their donation in April.

    (b) which mathematical concept have been covered in this question?

INTRODUCTION TO EUCLID'S GEOMETRY

ANSWERS

1. Two
2. Theorem
3. Only One
4. Axiom
5. Equal
6. Infinite
9. Second axiom
15. Second Axiom
26. (a) Donation amount is same in April

(b) Euclid's axiom

Introduction to Euclid's Geometry
Practice Paper

Time : 1 hr.   M.M. 25

1. Explain when a system of axioms is called consistent .   (1)

2. How many line segments can be determined by three collinear points ?(1)

3. How many lines can pass through a given point ?   (1)

4. Solve the equation x+3=10 and state Euclid's axiom used.   (2)

5. State Euclid's first postulate   (2)

6. In $\triangle$ ABC, D and E are mid-points of the sides AC and BC respectively, such that AD=CE, show that AC=BC.   (2)

7. If a point C lies between two points A and B such that AC=BC then prove that AC= $\frac{1}{2}$ AB. Explain by drawing the figure.   (3)

8. It is known that if x+y=10, then x+y+z=10+z. State the Euclid's axiom that illustrates the statement.   (3)

9. State Euclid's fifth postulate Explain it and compare it with version of parallal lines.   (5)

10. In fig. PQ=RS, A and B are points on PQ and RS such that AP=$\frac{1}{3}$PQ and RB=$\frac{1}{3}$RS show that AB=RB. State which axiom you use here. Also give two more axioms other then the axiom used in above situation.

LINES & ANGLES

MIND MAP

Obtuse

Acute

Right

Straight

Vertically opposite angles

Reflex

Complementary

Adjacent angles

Corres-ponding angles

Alternate interior angles

Supplementary

Linear Pair

Collinear & Non Collinear points

Pair of Angles

Parallel lines and transversal

Alternate exterior angles

Angles

Line Segment

Interior angles on the same side of transversal

Intersecting & Non Intersecting Lines

Angle sum property of triangle

Basic Terms

LINES AND ANGLES

KEY POINTS

- Line is a collection of points which has only length, neither breadth nor thickness.
- **Line Segment :** A part or portion of a line with two end points.
- **Ray :** A part of a line with one end point.
- **Collinear points :** Three or more points lying on the same line.
- **Non-Collinear Points:** Three or more points which donot lie on same line.
- **Angle :** An angle is formed when two rays originate from the same end point. The rays making an angle are called the arms and the end point is the vertex.
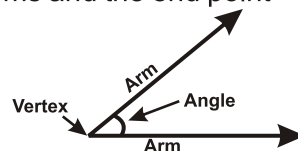
- **Acute angle :** An angle measure between 0° and 90°
- **Right angle :** Angle exactly equal to 90°
- **Obtuse angle :** An angle greater than 90° but less than 180°
- **Straight angle :** An angle exactly equal to 180°
- **Reflex Angle :** An angle greater than 180° but less than 360°
- **Complimentary Angles :** A pair of angles whose sum is 90°
- **Supplementary angle :** A pair of angles whose sum is 180°
- **Complete Angle :** An angle whose measure is 360°.
- **Adjacent angles :** Two angles are adjacent if
  (i) They have a common vertex.
  (ii) a common arm
  (iii) Their non common arms are on opposite sides of common arm.
- **Linear pair of angle :** A pair of adjacent angles whose sum is 180°
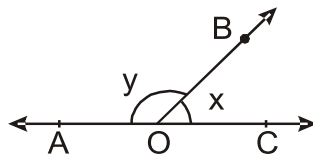
∠AOB & ∠COB are forming linear pair.

- **Vertically opposite angles :** Angles formed by two intersecting lines on opposite side of the point of intersection.

$$\angle x = \angle z$$
$$\angle y = \angle w$$
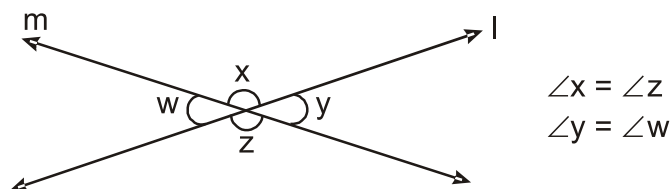
- **Intersecting lines:** Two lines are said to be intersecting when the perpendicular distance between the two lines is not same every where. They intersect at some point.

- **Non Intersecting lines :** Two lines are said to be non-intersecting lines when the perpendicular distance between them is same every where. They do not intersect. If these lines are in the same plane these are known as **Parallel lines.**

- **Transversal line :** In the given figure l || m and t is transversal then

(a) $\left.\begin{array}{l}\angle 1 = \angle 3 \\ \angle 2 = \angle 4 \\ \angle 5 = \angle 7 \\ \angle 6 = \angle 8\end{array}\right\rbrace$ Vertically opposite angle

(b) $\left.\begin{array}{l}\angle 1 = \angle 5 \\ \angle 2 = \angle 6 \\ \angle 3 = \angle 7 \\ \angle 4 = \angle 8\end{array}\right\rbrace$ Corresponding angle

(c) $\left.\begin{array}{l}\angle 3 = \angle 5 \\ \angle 4 = \angle 6\end{array}\right\rbrace$ Alternate Interior angle

(d) $\left.\begin{array}{l}\angle 2 = \angle 8 \\ \angle 1 = \angle 7\end{array}\right\rbrace$ Alternate Exterior Angle

(e) $\left.\begin{array}{l}\angle 3 + \angle 6 = 180° \\ \angle 4 + \angle 5 = 180°\end{array}\right\rbrace$ Angles on the same sides of a transversal are supplementary.

$\angle 3, \angle 6$ and $\angle 4, \angle 5$ are called co-interior angles or allied angles or consecutive interior angles.

- Sum of all interior angles of a triangle is 180°.

- Two lines which are parallel to the third line are also parallel to each other.

Very Very Short Answer Type Questions (1 Marks)

1. In the given figure, identify group of collinear points from (P,Q,R) and (A,B,C,)
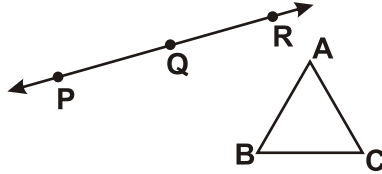
2. In the given figure, write the name of line segment whose one end point is B.
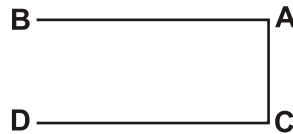
3. In the given figure, name the vertex of the angle.
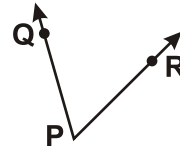
4. In the figure given in Q.3, name the two arms of the angle.

5. Which type of angle is formed in the given figure
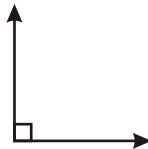
Fill in the blanks-

6. A ray has only _____ end point.

7. A line segment has a _____ length.

8. If two lines are non-intersecting, then they will be _____

9. An angle whose measure is more than 0° but less than 90°, is called an _____ angle.

10. A straight angle has ____ right angles.

11. An angle whose measure is more than 180° but less than 360° is called a _____ angle.

12. If an angle is equal to its complement, then its measure is _____.

13. If two angle are complements to each other, then what is the type of each angle?

14. In the give figure, for what value of x+y, will ABC be a line?
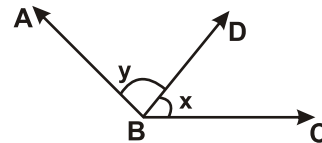
15. If two angles of a triangle are complementary then what type of triangle will be formed?

16. Two lines l and m are perpendicular to the same line n, Now l and m will be intersecting or parallel lines?

17. What is common between the three angles of a triangle and a linear pair?

18. In the given figure, find the value of x where l ||m.
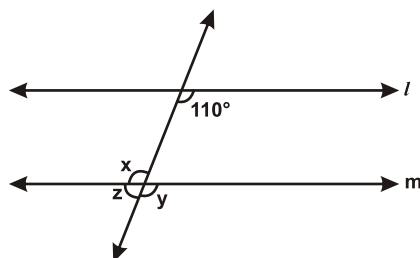
19. In the figure given in Q.18, find the value of y.

20. In the figure given in Q.18, find the value of z.

21. One of the angle of a linear pair is 65°. What will be the measure of the other angle?

22. In the given figure, find the value of x.
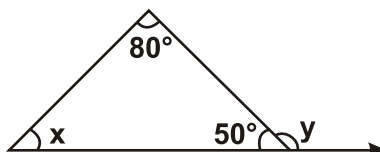
23. In the figure given in Q.22, find the value of y.

24. In the given figure find the value of p.
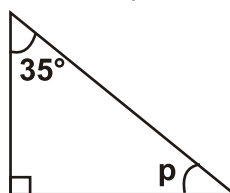
25. In the given figure find the value of m.

Class IX - Mathematics
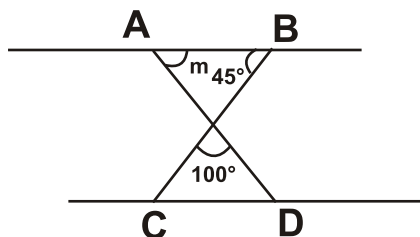
Case Study Based Questions (4 Marks)

26. Rashmi was making a toy butterfly with sticks for her younger sister. She arranged the sticks as shown in figure. AB and CD are two sticks intersecting (Joined) at O and a third stick OP is also joined to hold the toy butterfly.

    Based on the above information, answer the following question-
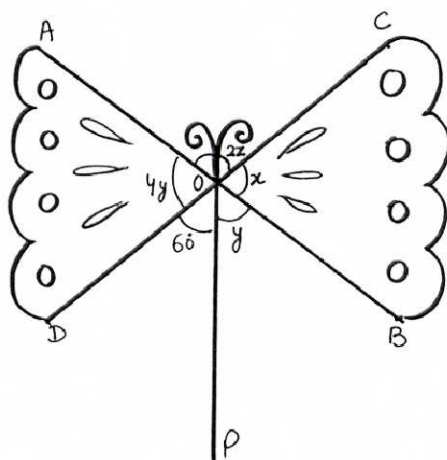
(i) At what angle does Rashmi inclined the two sticks AB and CD?

   (a) 86°   (b) 96°

   (c) 76°   (d) 67°

(ii) Find the value of y in the figure-

   (a) 24°   (b) 42°

   (c) 62°   (d) 26°

(iii) Find the value of Z in the figure-

   (a) 24°   (b) 42°

   (c) 62°   (d) 26°

(iv) If the two sticks of lengths 10cm each are joined at the mid point, then find the value of OA-

   (a) 5cm   (b) 10cm

   (c) 7.5cm   (d) 15cm

(v) What will be the sum of interior angles of both the wings of the butterfly-

   (a) 180°   (b) 270°

   (c) 192°   (d) 520°

27. Once four students of class IX are selected in Eco club of the school for plantation work. They are Shreya, Khushi, Vaibhav and Sushant. Shreya and Vaibhav planted a row of rose plants as shown in figure with line AB. Now Khushi and Sushant want to plant another row of sunflower plants parallel to rose plants row. Also there is a pipeline PQ passing through AB & CD. Based on the above information, answer the following questions-
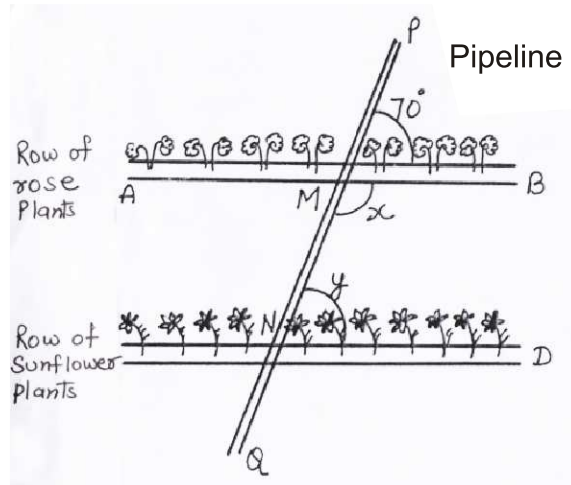
(i)   At what angle with PQ should Khushi and Sushant plant the row CD to make it parallel to row AB.

(a) 60°   (b) 65°

(c) 70°   (d) 75°

(ii) What will be the sum of angles between AB and CD marked as x and y.

(a) 90°   (b) 180°

(c) 70°   (d) 140°

(iii) What will be the value of x?

(a) 110°   (b) 120°

(c) 90°   (d) 70°

(iv) What will be the value of y?

(a) 110°   (b) 120°

(c) 90°   (d) 70°

(v)   What will be the value of 2x+y=?

(a) 290°   (b) 210°

(c) 220°   (d) 100°

28. Two cars are moving on two parallel roads represented as AB and CD respectively in the given figure. First car reaches at point E and takes a turn towards its right at an angle of 50°. At the same time, second car reaches at point F and takes a turn towards its left at an angle of 60°. They both meet at a point G. Based on the above information and given figure, answer the following question (without considering the width of the roads)
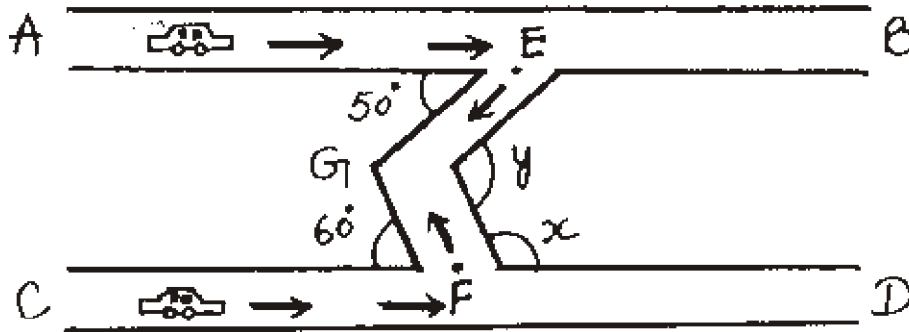
(i) What will be the measure of angle x marked in the figure?

    (a) 60°   (b) 80°

    (c) 100°   (d) 120°

(ii) What will be the measure of $\angle EGF$ marked as y?

    (a) 50°   (b) 100°

    (c) 150°   (d) 250°

(iii) What will be the measure of reflex $\angle EGF$?

    (a) 90°   (b) 100°

    (c) 110°   (d) 120°

(iv) If EF is joined, what type of triangle will EGF made?

    (a) Scalene   (b) Isosceles

    (c) Right angled   (d) None of these.

(v) If ΔEGF would have been an isosceles triangle with EG=GF, then what would be the measure of $\angle GFC$ considering $\angle AEG$ as 50°?

    (a) 50°   (b) 60°

    (c) 70°   (d) 80°

Very short Answer type Questions (2 Marks)

29. In the given figure POQ is a straight line and OM and ON are two rays. The three adjacent angles so formed are consecutive numbers. Find the value of x.
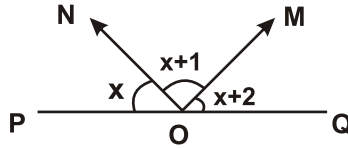
30. If x and y are linear pair and twice of x is 30° less than y, then find the value of x and y.
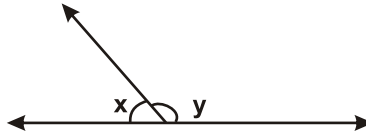
31. One of the angles of a pair of supplementary angles is 2° more than its supplement. Find the angles.

32. In the given figure AB and CD are two straight lines intessecting at O and OP is a ray. What is the measure of $\angle AOD$? Also find the value of x.
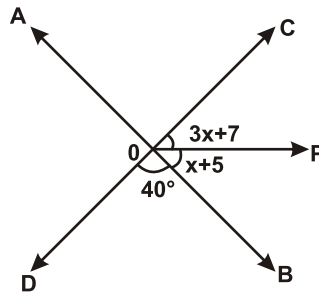
33. If the difference between two supplementary angles is 40°, then find smaller angle.

34. Find the angle which is four times more than its complement.

35. An exterior angle of a triangle is 100° and its two interior opposite angles are equal. Find the measure of these interior angles.

36. Find the value of x in the given figure.
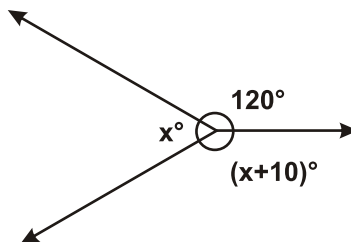
37. In the given figure AB, CD and EF are three straight lines intersecting at O. Find the measure of ∠BOC.
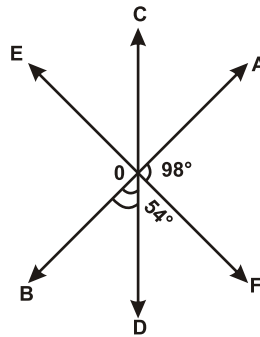
38. ∠x and ∠y forms a linear pair and x-2y=30°. Find the value of x.

39. In the given figure, AB||DC and AD||BC. Prove that ∠DAB=∠DCB.
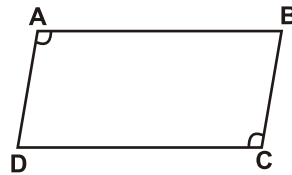
40. In the given figure AB||CD. what is the value of x+y.
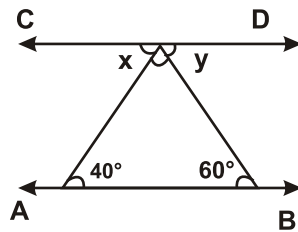
41. In the given figure, find the value of x if AB || CD.
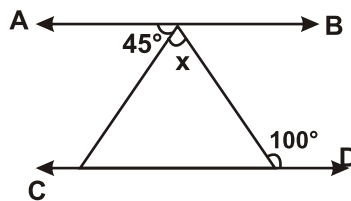
42. In the given figure, if l || m then what is the value of x.
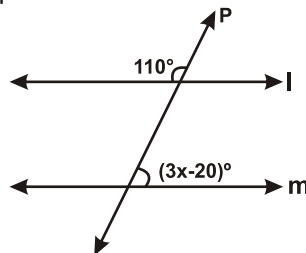
43. In the given figure, SP ⊥ *l* and RQ ⊥ *l*. Find the measure of ∠y.
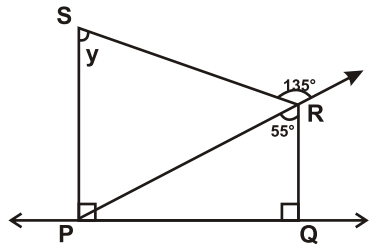
44. In the given figure if AB || ED, then find the value of x.
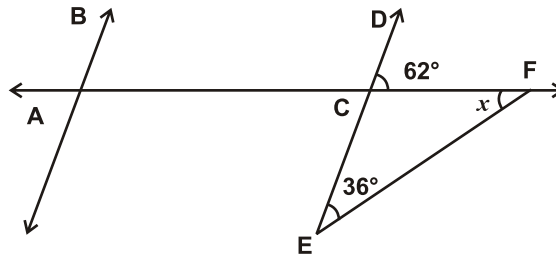
45. If the angles of a triangle are in the ratio 5:3:7 then show that the triangle is an acute angled triangle.

46. Find the value of x.
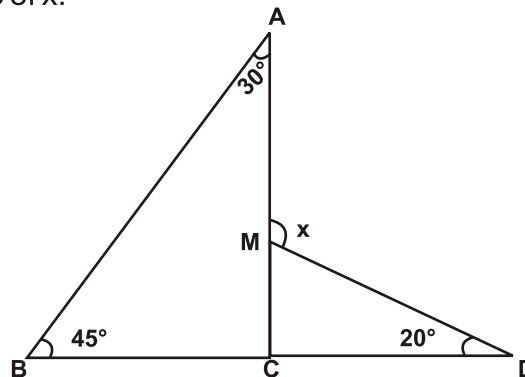
Short Answer Type Question (3 Marks)

47. In the adjoining figure *PQ || RS* find *x and y.*
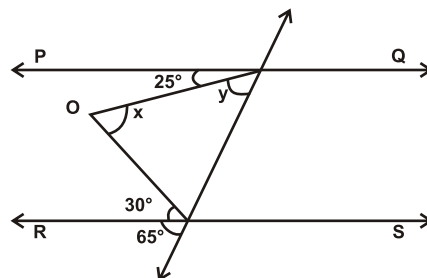
48. By contributing money, 5 friends bought pizza. They want to divide it equally among themselves. But one of them was given double share, as he was very hungry. Find the angle of the piece of pizza each one received.

49. ABC is a triangle in which AB and AC are produced.

   BO and CO are bisectors of exterior $\angle B$ and $\angle C$ intersecting at O. If $\angle A=60°$, $\angle ABC = 70°$, Find $\angle BOC$.

50. In the figure ,

   If $p : q = 11 : 19$, $AB \parallel CE$, then find the values of p, q and r.
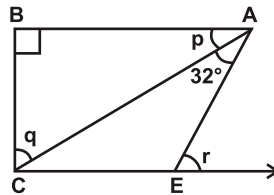
51. Prove that if two lines intersect then vertically opposite angles are equal.

52. In the figure, CD is the angle bisector of $\angle ECB$, $\angle B = \angle ACE$. Prove that $\angle ADC = \angle ACD$.
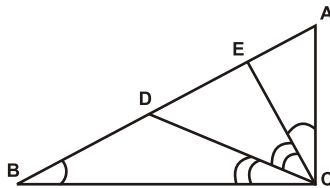
53. In the figure, choose the pair of lines which are parallel. Give reasons also.
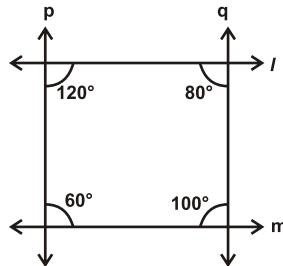
54. In the given figure if AB $\parallel$ DC and $\angle BDC=30°$ $\angle BAD=80°$ find $\angle x$, $\angle y$, $\angle z$.
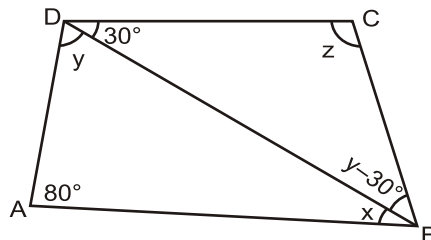
55. If one of the angle of two intersecting lines is right angle then prove that other three angles will also be right angles.

56. AB and CD are intersecting lines. OD is bisector of ∠BOY. Find x.
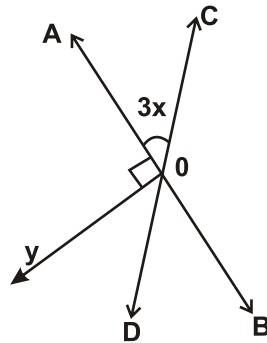
57. If p || q || r, find x, y, z from given figure.
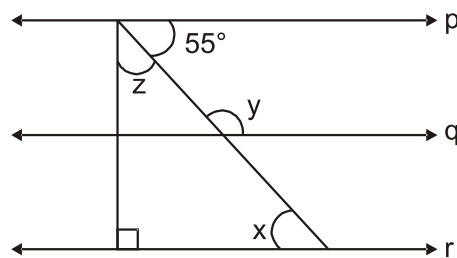
58. In the given figure find ∠DCB if AE || CD
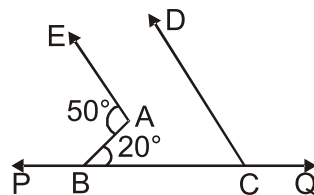
59. In the given figure l || m and n is the transversal, find x.
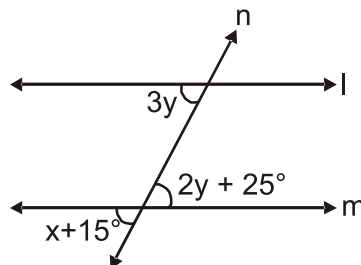
60. Two lines are respectively perpendicular to two parallel lines show that they are parallel to each other.

61. As shown in the figure find x & y if $\angle ACB=100°$, $\angle ADE=120°$.
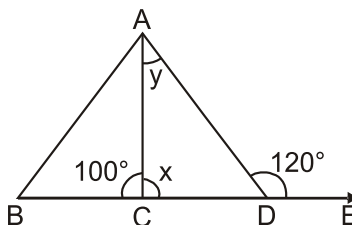
62. Prove that the bisectors of the angles of a linear pair are at right angle.

63. If two complementary angles are such that two times the measure of one is equal to three times the measure of the other. Find the measure of larger angle.

64. Prove that the sum of all exterior angles of a triangle is 360°.

65. If the bisectors of $\angle Q$ and $\angle R$ of a triangle $\triangle PQR$ meet at point S, then prove that

$$\angle QSR = 90° + \frac{1}{2} \angle P$$

66. Show that if sum of the two angles of a triangle is equal to the third angle then the triangle is right angled triangle.

67. If a transversal intersects two parallel lines prove that internal bisectors of the angle on the same side of a transversal meet at right angles.

68. In the figure AE is the bisector of $\angle A$, $AD \perp BC$. Show that

$2(\angle ADE - \angle EAC) = \angle B + \angle C$
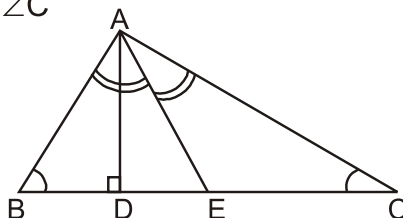
69. Prove that quadrilateral formed by the intersection of bisectors of interior angles made by a transversal on two parallel lines is a rectangle.

70. In right angled $\triangle ABC$ right angled at B, $\angle BCA=2\angle CAB$. Show that hypotenuse AC=2BC.

Long Anser Type Questions (5 Marks)

71. Two lines are respectively perpendicular to two parallel lines. Show that they are parallel to each other.

72. In the given figure, prove that

$\angle ADC = \angle A + \angle B + \angle C$
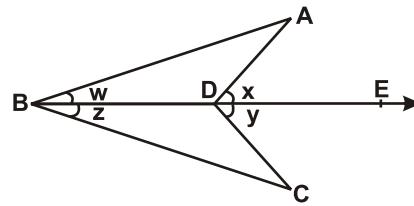
73. In the given figure, $\angle Q > \angle R$, PA is the bisector of $\angle QPR$ and PM $\perp$ QR. Prove that $\angle APM = \frac{1}{2} (\angle Q - \angle R)$
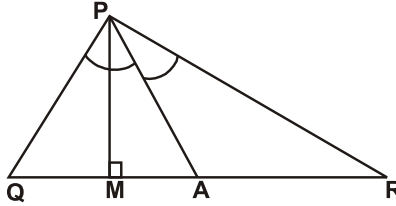
74. If the arms of one angle are respectively parallel to the arms of another angle, then show that the two angels are either equal or supplementary.

75. If the bisectors fo the base angles of a triangle enclose an angle of 135°, then prove that the triangle is a right angled triangle.

LINES & ANGLES
ANSWERS

1. (P,Q,R)
2. $\overline{BA}$
3. P
4. PQ, PR
5. Right Angle
6. One
7. definite
8. parallel
9. acute
10. two
11. reflex
12. 45°
13. acute
14. 180°
15. Right angled
16. Parallel
17. Sum of angles of a triangle and linear pair is 180°
18. 110°
19. 110°
20. 70°
21. 115°
22. 50°
23. 113°
24. 55°
25. 35°
26. (i) (b) 96°
    (ii) (a) 24°
    (iii) (b) 42°
    (iv) (a) 5cm
    (v) (c) 192°

27. (i) (c) 70°
    (ii) (b) 180°
    (iii) (a) 110°
    (iv) (d) 70°
    (v) (a) 290°
28. (i) (d) 120°
    (ii) (d) 250°
    (iii) (c) 110°
    (iv) (a) scalene
    (v) (a) 50°
29. 59°
30. x=50°, y=130°
31. 89°, 91°
32. 140°, x=32°
33. 70°
34. 72°
35. 50°
36. 115°
37. 152°
38. 130°
39. Hint: Use the property that sum of interior angles on the same side of transversal are supplementary.
40. 100°
41. 55°
42. 30°
43. 80°
44. 26°

45. Let the angles are 5x, 3x, 7x then 5x+3x+7x=180°

x=12°

∴ Angles are 60°, 36°, 84°

∴ triangle is acute angled triangle.

46. 95°

47. x=55°, y=40°

48. 4 equal pieces=60°, one double piece=120°

49. 60°

50. 33°, 57°, 65°

52.
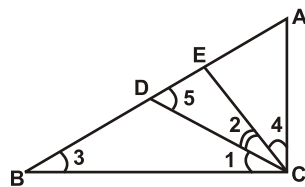
∠1= ∠2

∠3= ∠4

∠1+∠3=∠2+∠4

∠5= ∠2+∠4

∠ADC=∠ACD

53. l || m because sum of interior angles on the same side of transversal is 180°

54. x=30°, y=70°, z=110°

55.
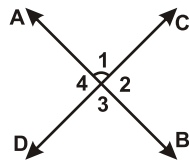
Given :- ∠1=90°

To prove :-∠2=∠3=∠4=90°

∠1+∠2=180° [Linear Pair]

90°+∠2=180°

∠2=90°

∠3=∠1=90° (Vertically Opp. ∠s)

∠4=∠2=90° (Vertically opp. ∠s)

56. x=15°

57. x=55°, y=125°, z=35°

58. 30°

59. 60°

61. x=80°, y=40°

63. 60°

64.
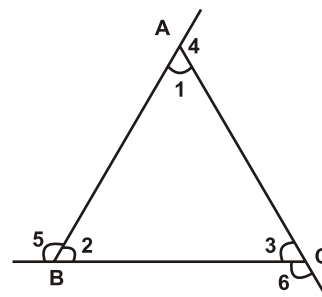
To prove: ∠4+∠5+∠6=360°

∠1+∠2+∠3=180°

∠4=180-∠1

∠5=180-∠2

∠6=180-∠3

∠4+∠5+∠6=3(180)-(∠1+∠2+∠3)

=540-180=360°

65.
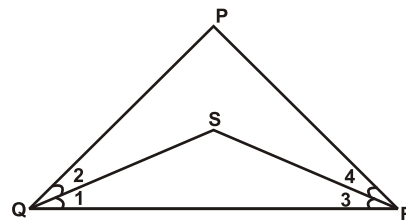
Given:- ∆PQR, QS is bisector of ∠Q, RS is bisector of ∠R.

To prove : $\angle QSR=90+\frac{1}{2}\angle P$

Prof : ∠QSR+∠1+∠3=180°——①

In ∆PQR

$\angle P + \angle PQR + \angle PRQ = 180°$

$\angle P + \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

$\angle P + 2\angle 1 + 2\angle 3 = 180°$

$\frac{1}{2}\angle P + \angle 1 + \angle 3 = 90°$

$\angle 1 + \angle 3 = 90 - \frac{1}{2}\angle P$

Putting the value of $\angle 1 + \angle 3$ in ①

$\angle QSR + 90° - \frac{1}{2}\angle P = 180°$

$\angle QSR = 90 + \frac{1}{2}\angle P$

67.
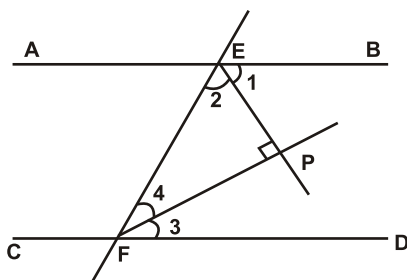
Given: AB || CD, EF is transversal

EP is bisector of $\angle BEF$

FP is bisector of $\angle DFE$

$\therefore \quad \angle 1 = \angle 2, \angle 3 = \angle 4$

To prove: $\angle EPF = 90°$

Proof: $\angle BEF + \angle DFE = 180°$

(Angles on the same side of transversal are supplementary)

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

$2\angle 2 + 2\angle 4 = 180°$

$\angle 2 + \angle 4 = 90°$

Also $\angle 2 + \angle 4 + \angle EPF = 180°$

$\angle EPF = 180 - 90 = 90°$

69. Hint: Take help of solution 67.

71.
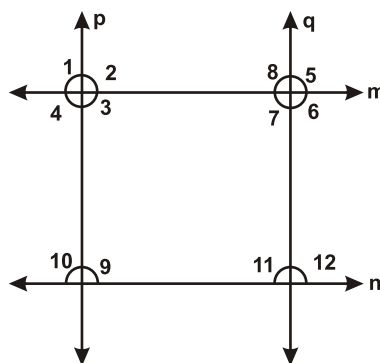
given: n || m, p⊥ m, p⊥n, q⊥m, q⊥n

To prove: p || q

$\angle 1 = \angle 10 = 90°$

$\angle 2 = \angle 9 = 90°$

$\angle 4 = \angle 9 = 90°$

$\angle 3 = \angle 10 = 90°$

$\angle 7 = 90°$ & $\angle 11 = 90°$

$\angle 3 + \angle 7 = 90° + 90° = 180°$

$\therefore$ p ||q $\therefore$ If a transversal intersect two lines such that a pair of interior angles on the same side of transversal is supplementary then two lines are parallel.

72. Hint: use exterior angle property in △ABD & △CBD.

73. In figure

PA is bisector of $\angle QPR$

$\angle QPR = \angle APR$ — (i)

In $\triangle PQM$, $\angle PQM + \angle PMQ + \angle QPM = 180°$

$\angle PQM = 90 - \angle QPM$ —(ii)

In $\triangle PMR$

$\angle PMR + \angle PRM + \angle RPM = 180°$

$90° + \angle PRM + \angle RPM = 180°$

$\angle PRM = 90° - \angle RPM$ —(iii)

Subracting eq (iii) from (ii)

$\angle Q - \angle R = (90° - \angle QPM) - (90° - \angle RPM)$

$\angle Q - \angle R = \angle RPM - \angle QPM$

$\angle Q - \angle R = \angle RPA - \angle APRm - [\angle QPA - \angle APM]$

$\angle Q - \angle R = \angle QPA + \angle APM - \angle QPA + \angle APM$

$\angle Q - \angle R = 2\angle APM$

$\angle APM = \frac{1}{2}(\angle Q - \angle R)$

LINES AND ANGLES

Time : 1 Hr.   M.M. 25

1. If $\angle ABC = 142°$, find reflex $\angle ABC$.   (1)

2. Two angles form a linear pair. If one of the angle is acute, what is the type of other angle?   (1)

3. Find x in the given figure :   (1)
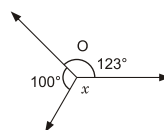
4. An exterior angle of a triangle is 100° and two of its interior opposite angels are equal. Find the angles of the triangle.   (2)

5. In a $\triangle ABC$, $\angle A + \angle B = 125^3$ and $\angle B + \angle C = 150°$. Find all the angle of $\triangle ABC$.   (2)

6. If the difference between two supplementary angles is 40° then find the angles.   (2)

7. l and m are the intersecting lines in the given figure. Find x. y and z.   (3)
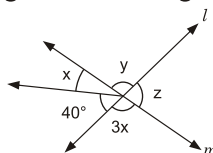
8. If two parallel lines are intersected by a transversal, then prove that the bisectors of the interior angles on both on both sides of transversal form a rectangle.   (3)

9. In $\triangle PQR$, sides PQ and PR are extended to S and T respectively. OQ and OR are bisector of $\angle RQS$ and $\angle QRT$ meeting at O. Show that   (5)
    $2 \angle QOR = \angle PQR + \angle QRP$
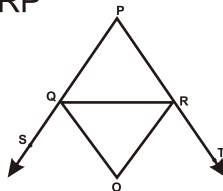
10. If AB || CD, find x
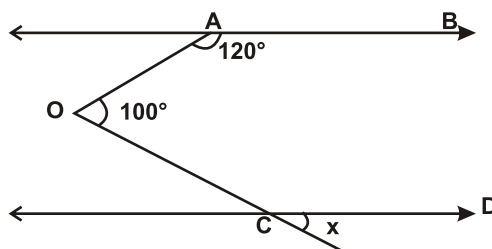
CHAPTER-7
TRIANGLES
MIND MAP

Congruency Criteria in Triangless

ASA

SAS

SSS

RHS

A-Angle

R-Right Angle H-Hypotenus

Inequalities in triangle

Angle Opposite to the longer side is greater

Sum of any Two sides of a triangle is always greater than third side

PROPERTIES OF ISOSCELES TRIANGLE

Angles Opposite to equal sides are equal

If in △ABC
AB = AC
then ∠B = ∠C

Opposite sides to equal angles are equal
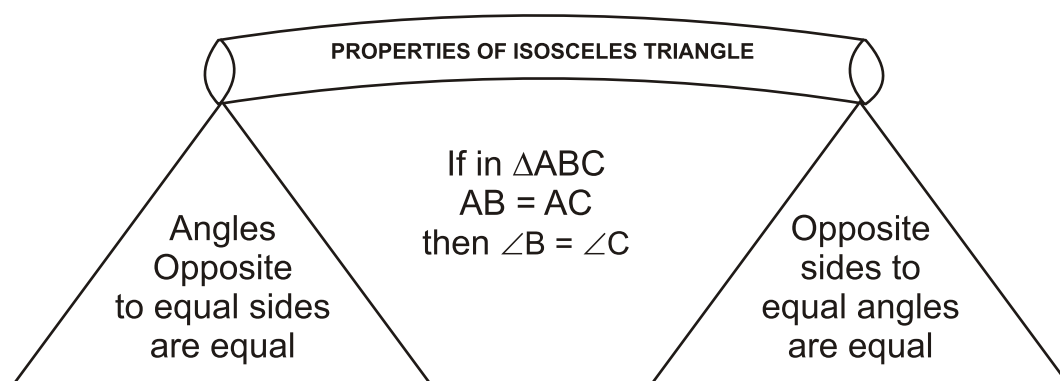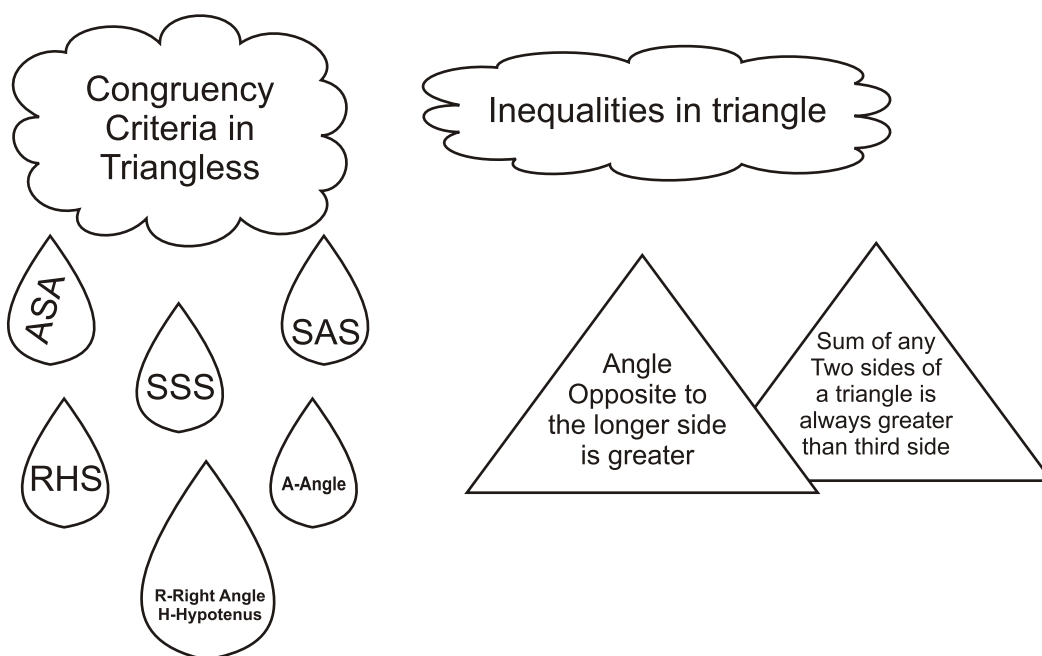
TRIANGLES
KEY POINTS

Congruence in different shapes :

• Two figures having the same shape and size are called congruent figures.

• Two plane figures are congruent, if each one when superimposed on the other, covers the other exactly.

• Two line segments are congruent, if they are of equal lengths.

• Two angles of equal measures are congruent.

• Two circles of the same radii are congruent.

• Two squares of the same sides are congruent.

• Two rectangles are congruent, if they have the same length and breadth.

Congruency Criteria:

• If two triangles ABC and DEF are congruent under the correspondence A $\longleftrightarrow$ D, B $\longleftrightarrow$ E and C $\longleftrightarrow$ F, then symbolically, it is expressed as $\triangle ABC \cong \triangle DEF$.

• There are four congruent conditions for triangles.

(a) **Side-Angle-Side (SAS) congruent rule :** Two triangles are congruent, if two sides and the included angle of the one triangle are respectively equal to the two sides and the included angle of the other triangle.

(b) **Angle-Side-Angle (ASA) congruence rule :** Two triangles are congruent, if two angles and the included side of the one triangle are respectively equal to the two angles and the included side of the other triangle.

(c) **Side-Side-Side (SSS) congruence rule :** Two triangles are congruent, if the three sides of one triangle are respectively equal to the three sides of the other triangle.

(d) **Right angle-Hypotenuse-Side (RHS) congruence rule :** Two right triangles are congruent, if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of another triangle.

INEQUALITIES IN TRIANGLE:

1. In $\triangle ABC$ if $AB > AC$ then $\angle C > \angle B$.

   if $AC > BC$ then $\angle B > \angle A$.

   if $BC > AB$ then $\angle A > \angle C$.

2. In $\triangle ABC$, $AB + BC > AC$ and $AB-BC < AC$

   $BC+AC > AB$ and $BC-AC < AB$

   $AC+AB > BC$ and $AC-AB < BC$

Very-Very Shot Answer questions (1 Marks)

1. In $\triangle ABC$, $AB=AC$ and $\angle B=40°$. Find $\angle C$.

2. Write correct symbolic form of congruency if $AB=QR$, $BC=PR$ and $CA=PQ$.

3. The _____ is the largest side of a right-angled triangle.

4. In the given figure, AC is bisector of $\angle BAD$, $AB=3cm$ and $AC=5cm$. Find AD.
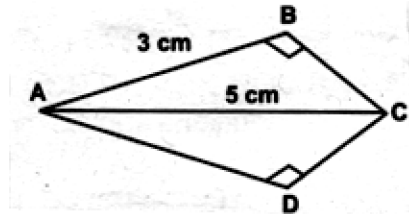
5. Two squares are congruent if they have same _____.

6. Find the diameter of circle O_2 if circle $O_2 \cong$ Circle O_1 and radius of circle O_1 is 6 cm.

7. Write the congruence criteria for triangles $\triangle ABC$ and $\triangle PQR$ where $AB=QP$, $\angle B=\angle P$ and $BC=PR$.

8. For right angled triangle $\triangle ABC$, $AB=BC$, Find $\angle A$.

9. EF is the largest side of $\triangle EFG$. Name the greatest angle.

10. In a triangle, sides opposite to equal angles are _____.

11. Write the congruence criteria for the following triangles.
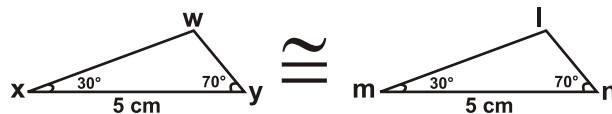
12. If $\triangle ABC$ is obtuse angled at C then AB _____ BC.

13. Name the side equal to side NL if $\triangle PQR \cong \triangle LMN$.

14. Line segment MN=4 cm and TP=4.2cm. Are they congruent?

15. Which is the largest angel in $\triangle QRS$?
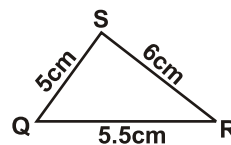
16. The _____ of any two sides of a triangle is always less than the third side.

17. What does it mean if two triangles are congruent by SSS criterion?

18. In $\triangle PQR$, $\angle R = \angle P$, QR=4cm and PR=5cm. Fidn PQ.

19. In the given figure if a=b=c, then name the angle congruent to $\angle AOC$.
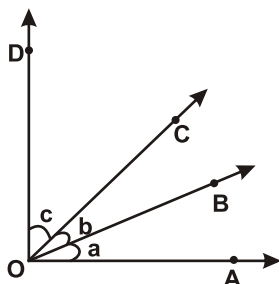
20. If two sides of a triangle are 6 cm and 4cm then the length of the third side should be greater than _____cm.

21. What does 'R' stands for in RHS congruence?

22. In $\triangle BCD$ and $\triangle WXY$, BD=WX and $\angle B = \angle X$. What should be the third possibility to satisfy ASA congruency criterion?

23. Two triangles may not be _____ if all the three angels of one triangle are equal to the angles of other triangle.

Case-Study Question

24. One sunday morning, Amita decided to have bread-pakodas in breakfast. For preparation, she divided a rectangular bread diagonally into two parts. She served bread-pakodas in a plate of diameter 20cm. On arrival of some guests, she cut the bread-pakoda from the middle as shown in the figure. She served these small bread-pakodas in a plate of radius 8cm. Everybody enjoyed bread-pakodas with tea.
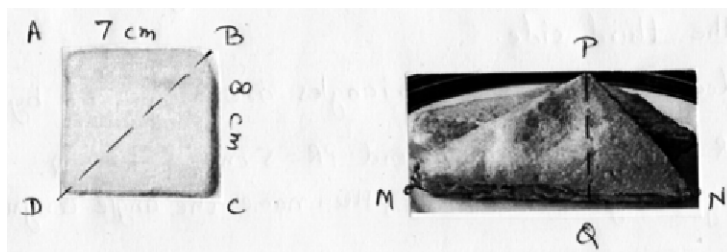
   (i)   After cutting rectangular bread diagonally, which type of triangular bread is obtained?

        (a) Equilateral   (b) Scalene

        (c) Isosceles   (d) Irregular

   (ii)   The line-segment that divided the bread-pakoda from the middle is called.

(a) Diagonal   (b) Altitude

(c) Base   (d) Median

(iii)   How much radius of the plate serving small bread-pakoda is to be increased so as to make it congruent to the other plate?

        (a) 12cm   (b) 4 cm

        (c) 2cm   (d) 6 cm

(iv)   Which of the following is true based on the images?

        (a) MQ=QN   (b) $\triangle ABD \cong \triangle CDB$

        (c) PM+PN=MN   (d) $\triangle PQM \cong \triangle PQN$

(v)   Name the largest angle in the bread-pakoda.

        (a) $\angle P$ (b) $\angle Q$

        (c) $\angle M$ (d) $\angle N$

25. Ritesh opened the door at an angle of 43° to enter the class. In the recess, he came out of the class by opening the door at an angle of 72°. After the recess, he again opened the door at 43° and entered the class. The door length is 80cm.
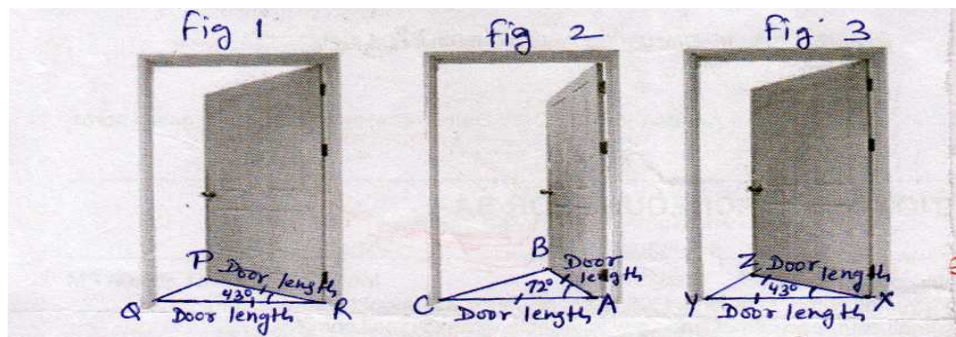
(i)   The type of the triangle formed by opening the door is

        (a) Equilateral   (b) Isosceles

        (c) Scalene   (d) Irregular

(ii)   Which of the following triangles are congruent?

        (a) $\triangle PQR$ and $\triangle ABC$ (b) $\triangle PQR$ and $\triangle XYZ$

        (c) $\triangle ABC$ and $\triangle XYZ$ (d) $\triangle ACB$ and $\triangle XYZ$

(iii)   What is the congruency criteria for these two triangles?

        (a) SSS   (b) ASA

        (c) SAS   (d) RHS

(iv)   If figure 2, which is the largest side?

        (a) BC   (b) AB

        (c) AC   (d) All sides are equal

(v)   What is the measure of $\angle P$?

        (a) $68\frac{1}{2}°$ (b) $69\frac{1}{2}°$

        (c) $21\frac{1}{2}°$ (d) 47°

Very Short Anser Questions (2 Marks)

26. $\triangle ABP \cong \triangle KST$ then

    (a) $\angle P =$ _____   (b) $KT =$ _____

27. In the following figure, which of the two triangles are congruent? Name them in symbolic form.
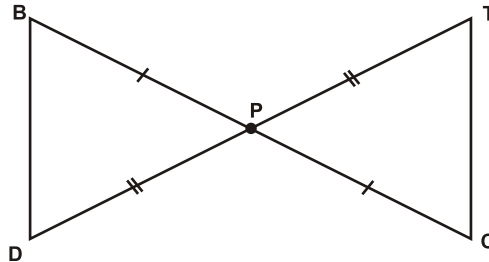
28. Explain why AAA is not a criteria for congruency of two triangles.

29. In the given figure, If $AB = CD$, $AD = BC$ then prove that $\triangle ADC \cong \triangle CBA$
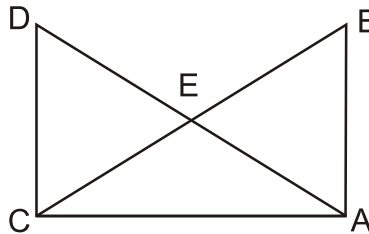
30. If $\triangle ABC$ is an isosceles triangle such that $AB = AC$, then prove that altitude AD from A on BC bisects it.

31. Which criteria of congruence of triangles is satisfied in the given figure.
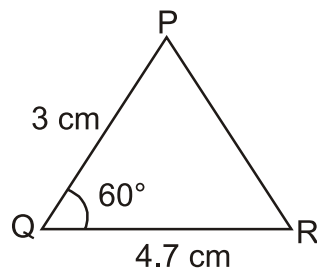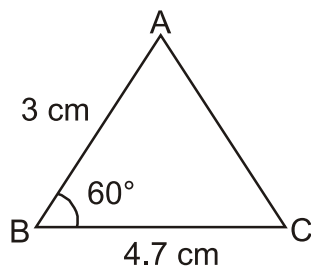
32. In a $\triangle PQR$, $\angle P = 110°$, $PQ = PR$. Find $\angle Q$ and $\angle R$.

33. In the given figure $AB = AC$ and $\angle ACD = 125°$. Find $\angle A$
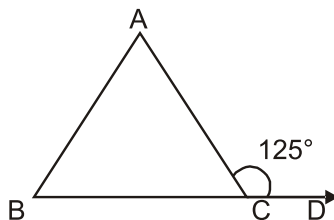
34. In △ABC, if ∠A = 55°, ∠B = 75° then find out the smallest and longest side of the triangle.

35. In the given figure, AC bisects ∠A and ∠C. If AD = 5 cm find AB.
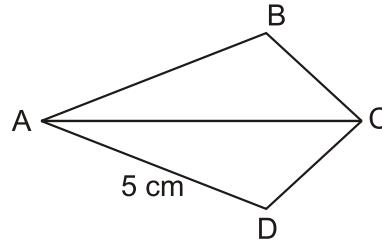
36. The vertex angle of an isosceles triangle is 80°. Find out the meaure of base angles.

Short Anser Questions (3 Marks)

37. In the given figure, Q is a point on the side SR of △PSR such that PQ = PR. Prove that PS > PQ.
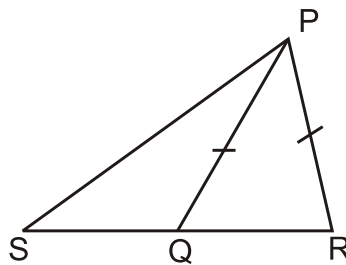
38. ABC is a triangle and D is the mid-point of BC. The perpendicular from D to AB and AC are equal. Prove that triangle is isosceles.

39. Prove that angles opposite to the equal sides of an isosceles triangle are equal.

40. In the given figure, AC > AB and AD bisects ∠BAC

Prove that ∠ADC > ADB.
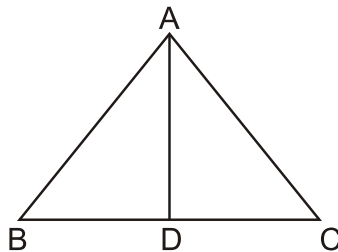
41. S is any point in the interior of a △PQR. Prove that SQ + SR < PQ + PR.

42. In the given figure, if AD = BD = CD, Find ∠BAC
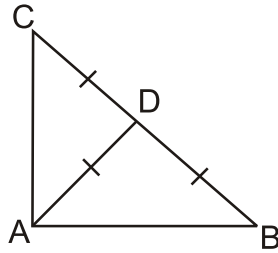
43. In the given figure, if AB = BC and ∠A = ∠C then find the value of x.
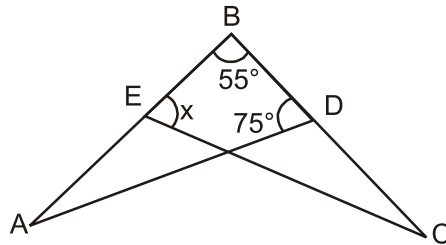
44. In the given figure ∠ABC = ∠BAC, D and E are points on BC and AC respectively such that DB = AE. If AD and BE intersect at O then prove that OA = OB.
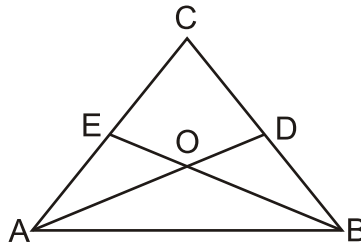
45. In the given figure, if AB = AC, ∠BAD = ∠CAE then prove that ΔADE is an isosceles triangle.
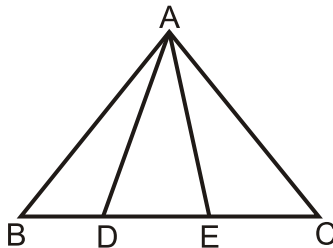
46. In ΔDEF, DM is the angle bisector of ∠EDF that intersects EF at M. If DM = MF, and ∠E=2∠F then prove that ∠EDF = 72°

47. Prove that the angles of an equilateral triangle are 60° each.

48. In the given figure, $\angle a > \angle b$, show that $\angle ATM < \angle AMT$.
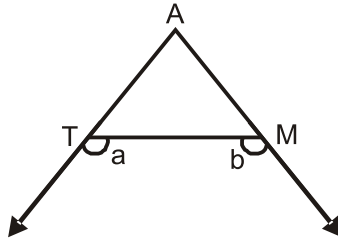
Long Answer Questions (5 Marks)

49. The altitudes AF, BD and CE of $\triangle ABC$ are equal. Prove that ABC is an equilateral triangle.

50. O is any point in the interior of a $\triangle ABC$. Prove that $(OA+OB+OC) > \frac{1}{2}$ (AB+BC+CA)

51. Prove that the perimeter of a triangle is greater than the sum of its three altitudes.

52. Two sides AB, BC and median AM of one $\triangle ABC$ are respectively equal to sides PQ, QR, and median PN of $\triangle PQR$. Show that.

    (i) $\triangle ABM \cong \triangle PQN$

    (ii) $\triangle ABC \cong \triangle PQR$

53. In the given figure, PQR is a triangle in which altitudes QS and RT to sides PR and PQ are equal. Show that.

    (i) $\triangle PQS \cong \triangle PRT$
    (ii) PQR is an isosceles triangle
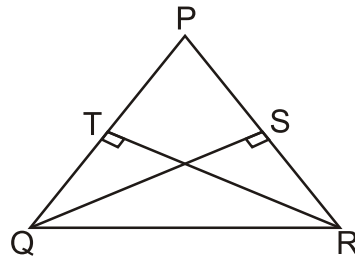
54. In the given figure, AB = AD, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$. Prove that AP=AQ.
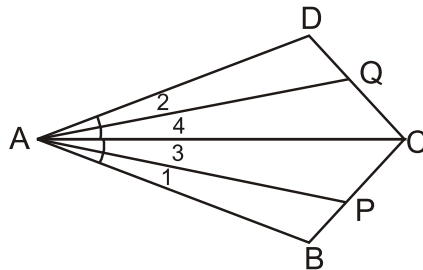
55. In the given figure, ABC is a right angled triangle, right angled at C and M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. D is joined to B. Prove that $CM = \dfrac{1}{2} AB$
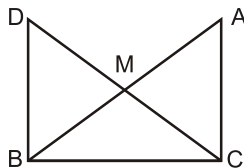
56. Prove that the sum of any two sides of a triangle is greater than its third side.

57. Vandana wishes to literate the poor children of the nearby slum area. She makes flash cards for them as shown in the given figure.

(i)   (ii)   (iii)
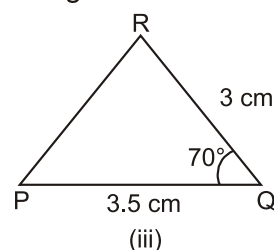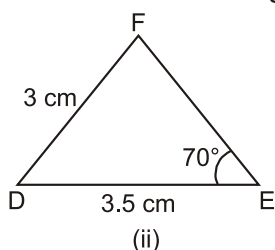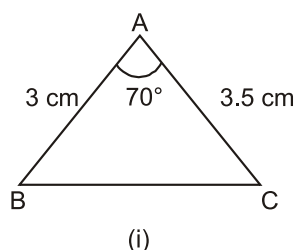
   (a)   Which two flash cards are congruent?
   (b)   Which criteria of congruency is satisfied here?
   (c)   Write the third side of both the triangles which are equal by CPCT.

58. Prove that the sum of any two sides of a triangle is greater than twice the median drawn to the third side.

59. In the given figure, AB = CD, CE = BF and $\angle ACE = \angle DBF$. Prove that

   (i) $\triangle ACE \cong \triangle DBF$

   (ii)   AE = DF
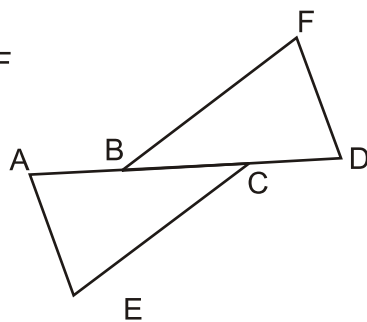
60. Show that the triangles $\triangle ABC$ and $\triangle DEF$ in the given figure are congruent. Hence find the value of x.
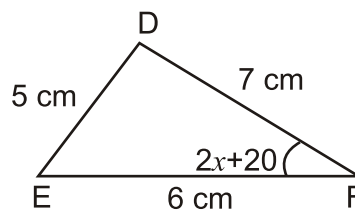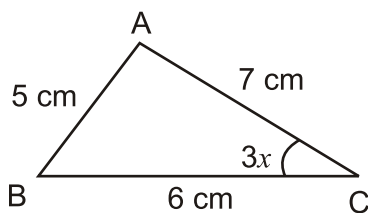
Chapter-7

TRIANGLES
ANSWERS

1. 40°
2. $\triangle ABC \cong \triangle QRP$
3. Hypotenuse
4. AD=3cm
5. Side
6. 12cm
7. $\triangle ABC \cong \triangle QPR$
8. $\angle A = 45°$
9. $\angle G$
10. Equal
11. ASA
12. AB>BC
13. NL=RP
14. NO
15. $\angle Q$
16. Sum
17. It means all the three sides of one triangle are equal to three sides of other triangle.
18. PQ=4cm
19. $\angle BOD$
20. 2cm
21. Right angle
22. $\angle D = \angle W$
23. Congruent
24. (i) b) Scalene
    (ii) b) Altitude
    (iii) c) 2cm
    (iv) b) $\triangle ADB \cong \triangle CDB$
    (v) a) $\angle P$
25. (i) b) Isosceles
    (ii) b) $\triangle PQR$ and $\triangle XYZ$
    (iii) c) SAS
    (iv) a) BC
    (v) a) $68\frac{1}{2}°$

26. (a) $\angle T$ (b) AP
27. $\triangle PBD \cong \triangle PCT$

    or any correct form
28. Because many triangles are possible with given three angles.
31. SAS
32. $\angle Q = \angle R = 35°$
33. $\angle A = 70°$
34. Smallest side =AB

    Longest Side = AC
35. AB=5cm
36. 50°, 50°
37. In $\triangle PQS$, $\angle PQR$ is angle

    outside $\triangle PQS$

    So $\angle PSQ + \angle SPQ = \angle PQR$

    $\Rightarrow \angle PSQ + \angle SPQ = \angle PRQ$ (PQ=PR)

    $\Rightarrow \angle PSQ < \angle PRQ$

    $\Rightarrow PA> PR$

    Since PQ=PR, So PS>PQ
42. $\angle BAC = 90°$
43. 75°
50. For any triangle, the sum of any two sides is always greater than third side. In $\triangle AOB$, $\triangle AOC$ and $\triangle BOC$
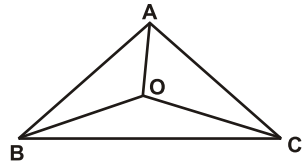
OA+OB>AB——(i)

OA+OC>AC——(ii)

OB+OC>BC——(iii)

Adding OA+OB+OC> $\frac{1}{2}$ (AB+BC+AC)

57. (a) $\triangle ABC \cong \triangle QRP$

    (b) SAS   (c) BC=RP

60. x=20

PRACTICE TEST
TRIANGLES

Time : 1 hour   M.M. 25

1. Find the measure of each exterior angle of an equilateral triangle.   (1)

2. The _____ of an isosceles triangle divides it into two congruent triangles.   (1)

3. Which angle is greater in the scalene triangle?   (1)

4. The vertical angel of an isosceles triangle is thrice the one of its base angle. Find the base angle.   (2)

5. For the given triangle, the third side should be greater than _____ and less than _____.   (2)
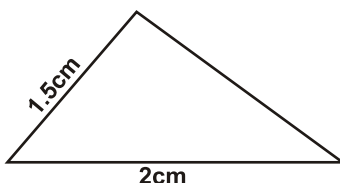
6. Find if △ABC is possible with AB=4.5cm, BC=5cm and AC=9.5cm.   (2)

7. In the given figure, AB=AC and side BA is produced to D such that AB=AD. Prove that ∠BCD=90°   (3)
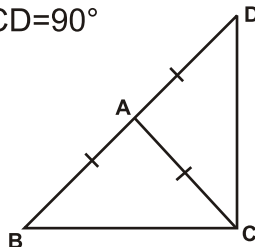
8. AD is a median of △ABC. Show that AB+AC>2AD.   (3)

   [Hint: Produce AD to E such that AD=DE]
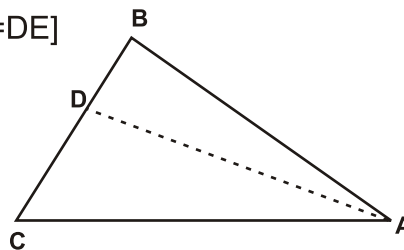
9. In the given figure C is the midpoint of AB, ∠DCA = ∠ECB and ∠DBC = ∠EAC. Prove that DC=EC and BD=AE.   (5)
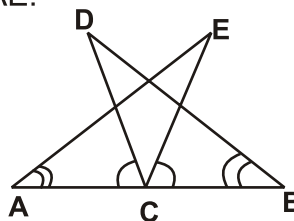
10. In given ΔMRP, ∠R=35°, ∠P=70° and bisector of ∠M meets RP at D. Find the longest and shortest side among MD, RD and DP. Also write them in ascending order.
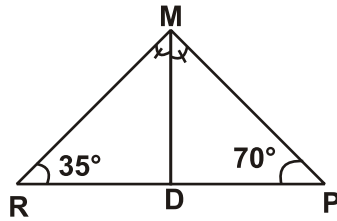
QUADRILATERAL
MIND MAPPING

Quadrilateral : A closed figure in a plane, bounded by four line segments.

Trapezium : A quadrilateral with one pair of opposite sides parallel i.e AB||CD.

Parallelogram: A quadrilateral with both pairs of opposite sides parallel. So we get AB||CD, AB=CD & AD||BC, AD=BC Also AE=CE, BE=DE, $\angle$BAD=$\angle$BCD $\angle$ABC = $\angle$ADC

Rectangle : A parallelogram having one angle 90°. So $\angle$ABC=$\angle$BCD=$\angle$CDA =$\angle$BAD=90° Also AC=BD

Square : A Parallelogram having one angle 90° and one pair of adjacent sides equal. So $\angle$ABC=$\angle$BCD=$\angle$CDA =$\angle$BAD=90° & AB=BC=CD=AD AC=BD & AC $\perp$ BD.

Rhombus: A Parallelogram having one pair of adjacent sides equal. So AB=BC=CD=AD Also AC$\perp$ BD.

QUADRILATERAL

KEY POINTS

1. Quadrilateral : - It is a closed figure bounded by four line segments. In a quadrilateral there are.
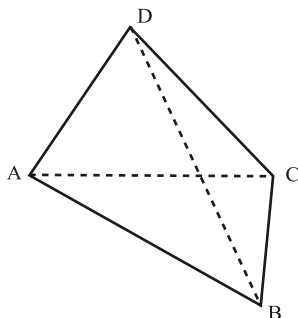
    i) Two pairs of opposite sides (no common point)

        e.g. AB & CD, BC & AD

    ii) Two pairs of opposite angles $\angle A$ & $\angle C$ and $\angle B$ & $\angle D$.

    (iii) Four pairs of adjacent sides AB & BC, BC & CD, CD & AD and AD & AB (one common point)

    (iv) Four pairs of adjacent angles $\angle A$ & $\angle B$, $\angle B$ & $\angle C$, $\angle C$ & $\angle D$, $\angle D$ & $\angle A$.

    (v) Line segment joining opposite vertices called diagonal of quadrilateral. e.g., AC & BD.

    (vi) Sum of the angles of a quadrilateral is 360^0, $\angle A + \angle B + \angle C + \angle D = 360^0$.

2. Parallelogram : A quadrilateral is a parallelogram if.
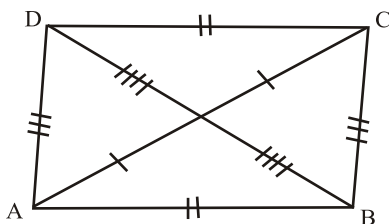
    •   Opposite sides are equal or

    •   Opposite angles are equal or

    •   Diagonals bisects each other or

    •   One pair of opposite sides is equal and parallel

3. A diagonal of a parallelogram divides it into two congruent triangles. other Examples of parallelogram:
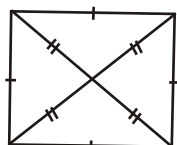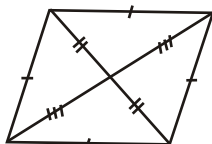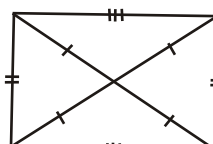
Square   Rhombus   Rectangle

4. Theorem :- A line segment joining the mid points of the two sides of a triangle is parallel to the third side and is half of it. If D & E are mid points then DE II BC and DE = $\frac{1}{2}$ BC.
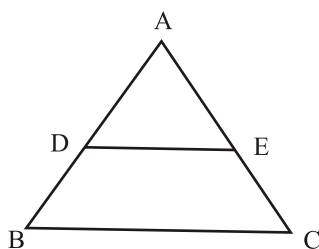
5. Converse of mid point theorem.

The line drawn through the mid point of one side of a triangle, parallel to another side bisects the third side. So, if D is mid point of AB and DE||BC then E will be mid point of AC.

Very Very Shot answer type question (1 Marks)

1. If three angles of a quadrilateral are 75°, 90., and 75° then find the fourth angle.

2. ABCD is a rhombus such that $\angle$ACB = 40°, then find $\angle$ABD.

3. Name the closed figure formed by the bisector of the angles of a parallelogram.

4. Name the figure obtained by joining the midpoints of the sides of the quadrilateral taken in order.

5. The diagonals AC and BD of a parallelogram ABCD intersect each other at point "O" If $\angle$DAC = 32° And $\angle$A0B = 70° then find $\angle$DBC.

6. The angles of a quadrilateral are in the ratio 3: 4: 5:6. Find the respective angles of this quadrilateral.

7. Line segment joining the midpoints of two sides of a triangle is parallel to the third side and is ..... of it.

8. If two consecutive sides of a rhombus are represented by 3x - 6 and x+ 14 then find the perimeter of the rhombus.

9. Points A, B, C and D are midpoints of the sides of square PQRS. If the area of PQRS is 36 sq. cm, then find the area of ABCD.

10. The perimeter of a rhombus is 60cm. If the length of its longer diagonal measures 24cm, find the length of the shorter diagonal.

11. In the given figure ABCD is a rectangle. if m $\angle$ADE = 30° and m $\angle$CFE =150°. What will be the m $\angle$DEF?
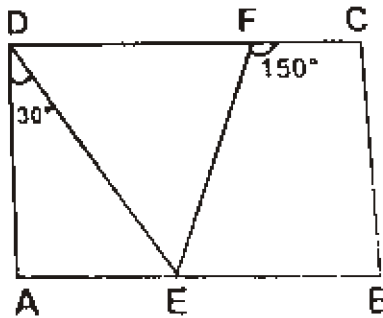
12. Opposite angles of a parallelogram are ....

13. Given four points A, B, C, D such that three points A, B and C are collinear. Name the closed figure obtained by joining these points in order.

14. What is the sum of consecutive angles of parallelogram?

15. In parallelogram ABCD, bisectors of angles A and B intersect each other at "O". Find the value of angle AOB.
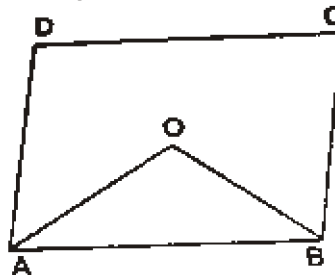
16. If an angle of a parallelogram is two-third of its adjacent angle then find the smallest angle of the parallelogram.

17. Diagonals of a rectangle......... each other and are.....

18. In the given figure PQRS is a rhombus. Find the value of x.

Class IX - Mathematics
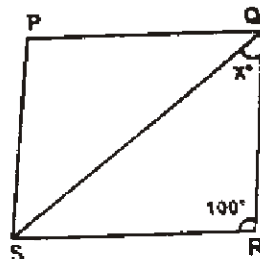
19. If in a rectangle ABCD, diagonal AC bisect $\angle A$ as well as $\angle C$ then ABCD is a ....

20. Two adjacent angles in a parallelogram are in the ratio 2:4. Find the values of these two angles.

21. Diagonals of a parallelogram .... each other.

22. In a rhombus ABCD, if $\angle A = 60^0$ find $\angle B$, $\angle C$ & $\angle D$.

23. The angles of a quadrilateral are in the ratio 1:2:4:5. Find the measure of each angle.

24. If in a rhombus LMNP, $\angle LNM = 40^0$ then what is the measure of $\angle LPM$?

25. In a parallelogram if all the four angles are in the ratio 1:1:1:1 then, what type of parallelogram is this?

26. In the figure, AB II CD, what will be the measure of $\angle ADC$?
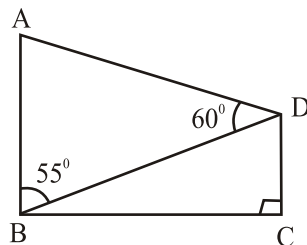
27. In the figure, if D & E are respectively the mid points of AB & AC, what will be the length of ED ?
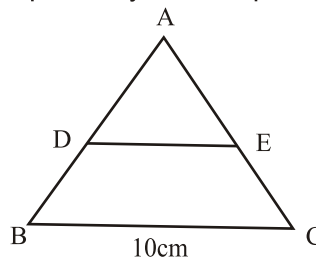
28. PQRS is a rhombus with $\angle QPS = 50^0$. Find $\angle RQS$.

29. In the figure, ABCD is a parallelogram find value of $(x + y)$.

30. In the figure line ℓ II m and p II q, $\angle BCD = 108^0$ find all four angles of quadrilateral ABCD.
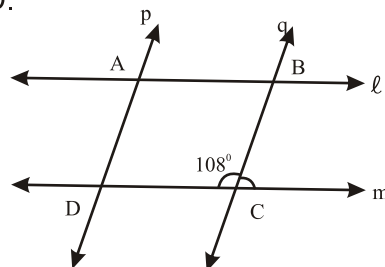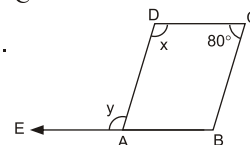
31. If two adjacent angles of a parallelogram ABCD are int the 5:4, find all the angles of the parallelogram.

32. Rohita wants to print her dress with a pattern shown in the figure-
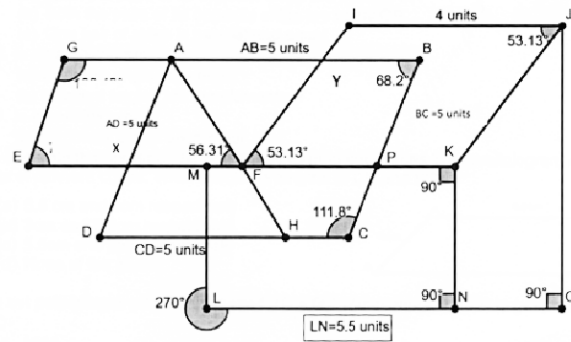
(i) Rohita wants to order a block of shape ABCD. What shape should Rohita mention to the carpenter for a wooden block for printing ABCD -

(a) Parallelogram   (b) Square

(c) Rectangle   (d) Rhombus

(ii) What shape should Rohita mention to the carpenter for a wooden block for printing LMKN -

(a) Parallelogram   (b) Square

(c) Rectangle   (d) Trapezium

(iii) She wants to colour two pairs of parallel lines with same shade. Which of the following pairs should she choose-

(a) GE II AF & LN II KN   (b) IJ II KF & BP II IF

(c) AB II CD & KN II JO   (d) ML II FN & HC II CP

(iv) She wants to colour a trapezium in blue shade. Which of the following should she select —

(a) JKNO   (b) AXF

(c) AFMX   (d) AFYI

(v) Rohita needs to know the measure of $\angle$AFY to construct $\triangle$AFY. What should be $\angle$AFY-

(a) 70.56°   (b) 70°

(c) 80°   (d) 170°

33. Class IXC wants to decorate the display board of their class. They are using following concepts for cutting paper shapes for decoration-

    1. A quadrilateral is called a parallelogram if

      (a) Both the pairs of its opposite sides are parallel OR

      (b) Both the pairs of its opposite sides are equal OR

      (c) One pair of sides is parallel and equal

    2. In a parallelogram

      (a) Opposite angles are equal

      (b) adjacent angles are supplementary

    3. In a parallelogram

      (a) diagonals bisect each other

(i) To decorate the border of the board they want to cut shapes like parallebgram ABCD, using sheets of different colors. What should be lengths of CD and BC -

    (a) 5.5 cm and 5cm respectively

    (b) 3cm and 5.5cm respectively

    (c) 5.5cm and 3cm respectively (d) None of the above
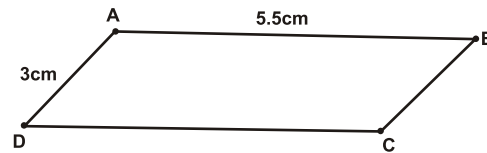
(ii) To cut parallelogram ABCD they fixed ∠D = 60°. What should be ∠B to get parallelogram ABCD-

    (a) 120°   (b) 60°

  (c) 180°   (d) 30°

(iii) What should be ∠A to get parallelogram ABCD-

    (a) 120°   (b) 60°

  (c) 180°   (d) 30°
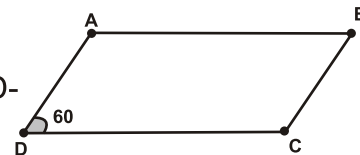
(iv) Some of the parallelograms were cut to get triangles such as AEB, AED CED & BEC to design flowers at the corner of the board. If BD is kept 8cm long and AC is 6cm then what should be length AE?

    (a) 6cm   (b) 3cm

    (c) 4cm   (d) 8cm

(v) What should be length DE?

    (a) 6cm   (b) 3cm

    (c) 4cm   (d) 8cm
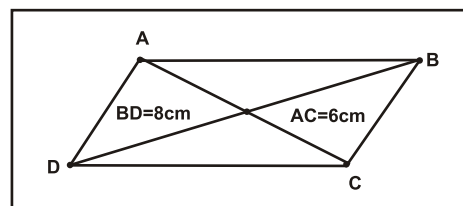
34. 1. A Parallelogram is called a Rectangle if one of its angles is 90°. It makes all the angles of the rectangle 90°. Its diagonals of the rectangle become equal and bisect each other.

2. A Parallelogram is called a square if one of its angles is 90° and one pair of adjacent sides are equal. It makes all the angles of the square as 90° and all the sides equal. The diagonals of a square become equal and bisect each other at 90°.

Sahil is using above learnings to design a wall-hanging. He is using wires to structure a frame and then is going to wrap up wool around the wires.

(i) The first frame he structured from wires is rectangle ABCD, as shown in figure. If the diagonal wire is 7cm and side CD is 6cm then what should be length of BC to get required rectangle-

(a) 13 cm

(b) √13 cm

(c) 10 cm

(d) √10 CM
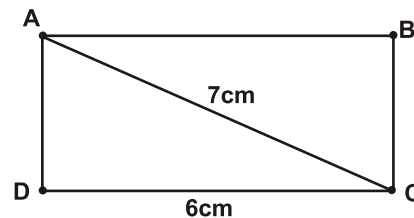
(ii) What should be length of wire BD?

(a) 6cm

(b) 7cm

(c) 13cm

(d) √13 cm

(iii) The second frame is square EFGH, as shown in figure. He has a left out piece of wire having length 8cm. He wants to take this piece as diagonal of EFGH. What should be the length of wire HF?

(a) 8cm

(b) 6cm

(c) 10cm

(d) 4cm

(iv) What should be the length of wire EF?

(a) 64cm

(b) 16cm

(c) 4√2 cm

(d) 2√4 cm

(v) What should be the length of wire OF?

(a) 3cm   (b) 4cm

(c) 4√2 cm   (d) 8cm
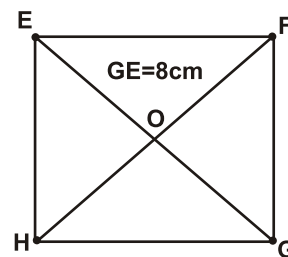
Very short answer type question (3 Marks)

35. Prove that the sum of all the four angles of a quadrilateral is 360:

36. Show that opposite angles of a parallelogram are equal.

37. In a parallelogram ABCD $\angle B=110^0$ determine the measure of $\angle A$ and $\angle D$.

38. In the figure if PQRS is a parallelogram, then find the value of x & y.
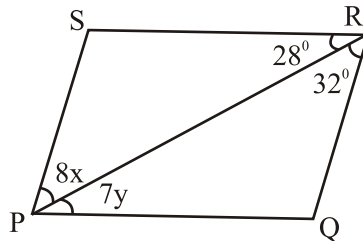
39. The diagonals of a parallelogram ABCD interest at O. A line through O intersects AB at X & DC at Y. Prove that OX=OY.

40. In a parallelogram ABCD diagonals AC and BD intersect at O and AC = 7.4 cm. and BD = 6.2 cm. Find the length of AO and BO.

41. Two opposite angles of a parallelogram are (5x-3) and (4x+12). Find the measure of each angle of the parallelogram.

42. Diagonals of a quadrilateral ABCD bisect each other if $\angle A=35^0$ determine $\angle B$.

43. The perimeter of a parallelogram is 30cm. If longer side is 9.5 cm then find the length of shorter side.

44. In a parallelogram ABCD diagonals AC and BD intersects at O and AC=12.6 cm and BD = 9.4 cm. Find the measures of OC and OD.

45. ABCD is a rhombus in which DO = 3x & AO = 4x, find perimeter of quadrilateral ABCD.

46. The angles of a quadrilateral are (x+20), (x–20), (2x+5), (2x–5). Find the value of x.
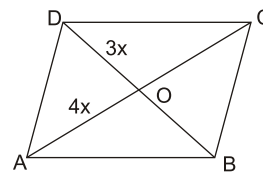
Short answer type questions (3 Marks)

47. In the figure P is the mid point of side BC of a parallelogram ABCD such that $\angle BAP = \angle DAP$ prove that AD = 2CD.
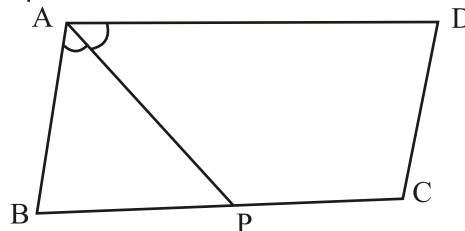
48. In the adjoining figure if PQRS is a parallelogram where $\angle PQR = 100$ and $\angle SPR = 40$. Find $\angle PRQ$ and $\angle SRQ$.
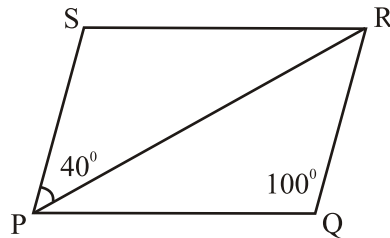
49. Prove that the line segment joining the mid points of two sides of a triangle is parallel to the third side.

50. In the given figure L, M, and N are mid point of the sides PQ, PR and QR respectively of $\triangle PQR$. If PQ = 4.4cm, QR = 5.6 cm and PR = 4.8cm then find the perimeter of $\triangle LMN$.
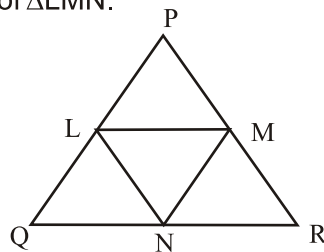
51. A quadrilateral is a parallelogram if one pair of opposite sides are equal and parallel. Prove it.

52. If the diagonals of a quadrilateral bisect each other then quadrilateral is a parallelograms. Prove it.

53. In a parallelograms PQRS, M and N are points on PQ and RS such that PM = RN. Prove that MS II NQ.

54. In a parallelogram ABCD, AP and CQ are drawn perpendiculars from vertices A and C on diagonal BD. Prove that $\triangle APB \cong \triangle CQD$.

55. The diagonals of a rectangle ABCD meet at O. If $\angle BOC = 50°$ then find $\angle ODA$.
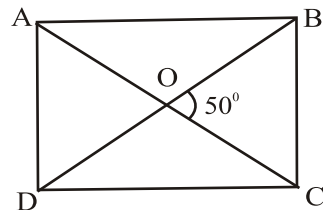
56. In the given figure AD and BE are the medians of $\triangle ABC$ and BE $\parallel$ DF prove that CF = 1/4 AC.
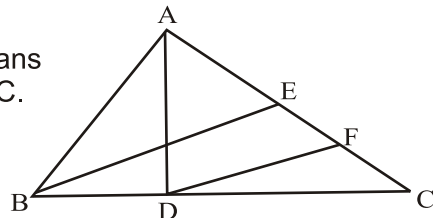
Long Answer type Questions (5 Marks)

57. In the figure LMNO, is a trapezium in which LM is parallel to side ON and P is the mid point of side LO. If Q is a point on the side MN such that segment PQ is parallel to side ON Prove that Q is the mid point of MN and PQ = 1/2 (LM + ON).
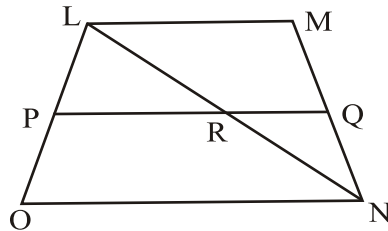
58. In the figure, △ABC is right angled at B. If AB=9 cm AC = 15 cm. and D and E are the mid points of AB & AC respectively calculate.

    (i) The length of BC

    (ii) The area of trapezium BCED
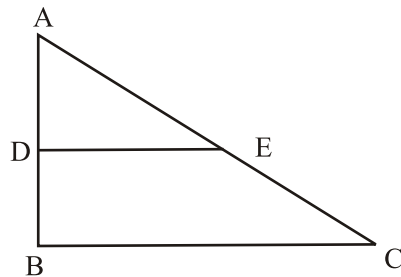
59. A farmer has divided his field into three parts as in the figure. Ist part is used to take care of his cattles. While II and III are used to grow two different crops.

    Answer the following :–

    i) How much area has been used to take care for cattles ?

    ii) Are the two areas part II and part III equal? Justify.

    iii) What is the total area of the field ?
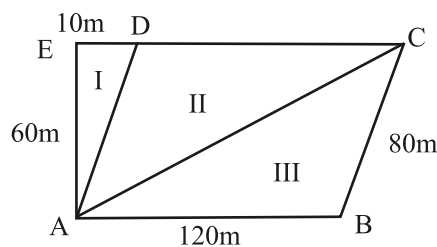
60. ABCD is a parallelogram. Side AB is produced on both sides to E & F as in figure such that BE = BC & AF = AD. Show that EC & FD when produced meets at right angle.
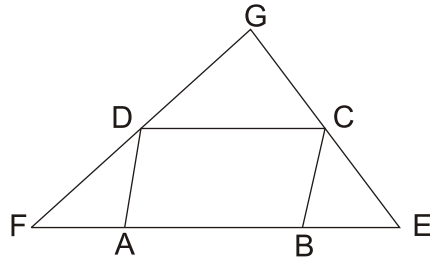
61. P is mid point of side CD of a parallelogram ABCD. A line through C parallel to PA intersects AB at Q & DA produed at R. Prove that DA = AR & CQ = QR.
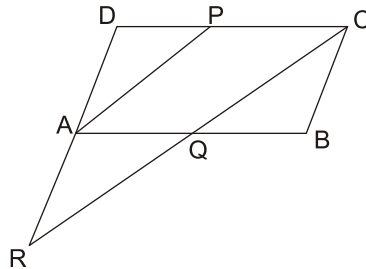
QUADRILATERALS
ANSWERS AND HINTS

1. 120°
2. 50°
3. Rectangle
4. Parallelogram
5. 38°
6. 60°, 80°, 100°, 120°
7. Half
8. 96
9. 18
10. 18
11. 90°
12. Equal
13. A Triangle
14. 180°
15. 90°
16. 72°
17. Bisect, Equal
18. 40°
19. Rhombus
20. 60°, 120°
21. bisect
22. 120°, 60°, 120°
23. 30°, 60°, 120°, 150°
24. 100°
25. Rectangle
26. 115°
27. 5 cm
28. 65°
29. 200°
30. 108°, 72°, 108°, 72°
31. 100°, 80°, 100°, 80°

32. (i) (b)   (ii) (c)
    (iii) (c)   (iv) (a)
    (v) 70.56°
33. (i) (c)   (ii) (b)
    (iii) (a)   (iv) (b)
    (v) (c)
34. (i) (b)   (ii) (b)
    (iii) (a)   (iv) (c)
    (v) (b)
37. 70° 110°
38. x=y=4
39.
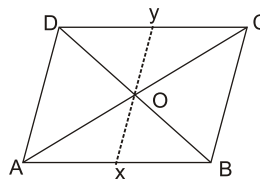
in $\triangle AOX$ & $\triangle COY$
OA=OC
$\angle AOX = \angle COY$ (vertically opposite)
$\angle OAX = \angle OCY$ (Alternate Interior angles)
$\triangle AOX \cong COY$ [ASA]
OX=OY [CPCT]

40. 3.7cm, 3.1cm
41. 72°, 108°, 72°, 108°
42. 145°
43. 43.55 cm
44. 6.3cm 4.7cm
45. Hint : In rt $\triangle OAD$
    $AD^2=(3x)^2+(4x)^2$
    $AD^2=9x^2+16x^2$
    AD=5x
    Perimeter = 20x units

46. x=60.

47. Hint

∠1=∠2

∠2=∠3 [Alt.int .angles]

∠1=∠3

in △ABP

∠1=∠3

BP=AB [why?]

48. 40°, 80°

50. 7.4cm

55. 65°

56. Hint-In △ABC

$EC=\frac{1}{2}AC$ [BE is median]

In △BEC

$CF= \frac{1}{2}EC$

58. 12cm, 40.5cm^2

59. Hint-

(iii) Area of Trapezium ABCE =

$\frac{1}{2}$ [130+120]x60

Ans (i) 300m^2

(ii) Yes

(iii) 7500m^2

60. In parallelogram ABCD
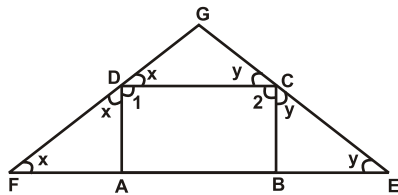
∠1+∠2=180°——(1)

x+x+∠1=180°[Why?]

$x=90-\frac{1}{2}(∠1)$——(2)

Similarly $y=90°-\frac{1}{2}(∠2)$——(3)

In △DGC

∠DGC+x+y=180°

61. APCQ is a parallelogram

Q is mid point of AB

in △AQR & △BQC

∠AQR = ∠BQC

∠QAR = ∠QBC

AQ = BQ

△AQR ≅ △BQC

PRACTICE TEST
Quadrilaterals

Time : 1 Hr.   M.M. 25

1. If the diagonals of a quadrilateral ABCD bisect each other & ∠A=45°, What is m ∠B?   (1)

2. The angles of a Quadrilateral ABCD are in the ratio 2 : 3 : 5 : 8. Find the measure of smallest angle.   (1)

3. Two opposite angles of a Parallelogram are (5x-3)° and (4x+12)°. Find the measure of each angle of the parallelogram.   (1)

4. In a △PQR, median PS is produced to a point T such that PS = ST. Prove that PQTR is a parallelogram.   (2)

5. In the Fig. PQRS is a rhombus in which the diagonal PR is produced to T. If ∠SRT=152°, find x & y.   (2)
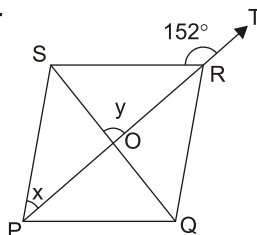
6. In a parallelogram ABCD, AP and CQ are drawn perpendicular from vertices A and C on diagonal BD. Prove that △APB ≅ △CQD   (2)

7. ABCD is a square. A line BM intersects CD at M and the diagonal AC at O such that ∠AOB=70°, find a   (3)
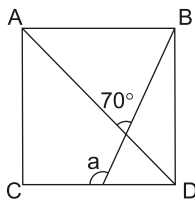
8. AD is median of △ABC & E is the mid point of AD. BE is produced to meet AC in F. Prove that AF=1/3 AC.   (3)

9. Show that the bisectors of angles of a parallelogram form a rectangle. (5)

10. Show that the quadrilateral formed by joining the mid point of the sides of a square is also a square.   (5)

AREAS OF PARALLELOGRAMS TRIANGLES

MIND MAP

Ar.(‖gm ABPQ) = Ar. (‖ gm CDPQ)

‖ ‖ m, AD‖BC, AF‖BE

Ar. (ΔABC) = Ar. (ΔABD)

‖ ‖ AB

On same base AB and between some parallels l and m

Ar. (‖gm ABCD) = Ar. (‖gm ABEF)
and
Ar. (ΔABP) = Ar. (ΔABQ)
also
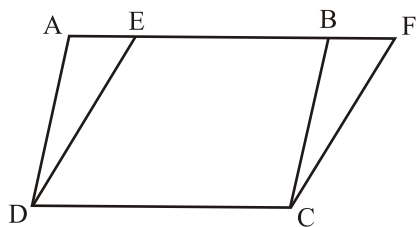Ar. (‖gm ABCD) = 2 Ar. (ΔABD)

Ar.(ΔPCD) = $\frac{1}{2}$ Ar.(‖gm ABCD)
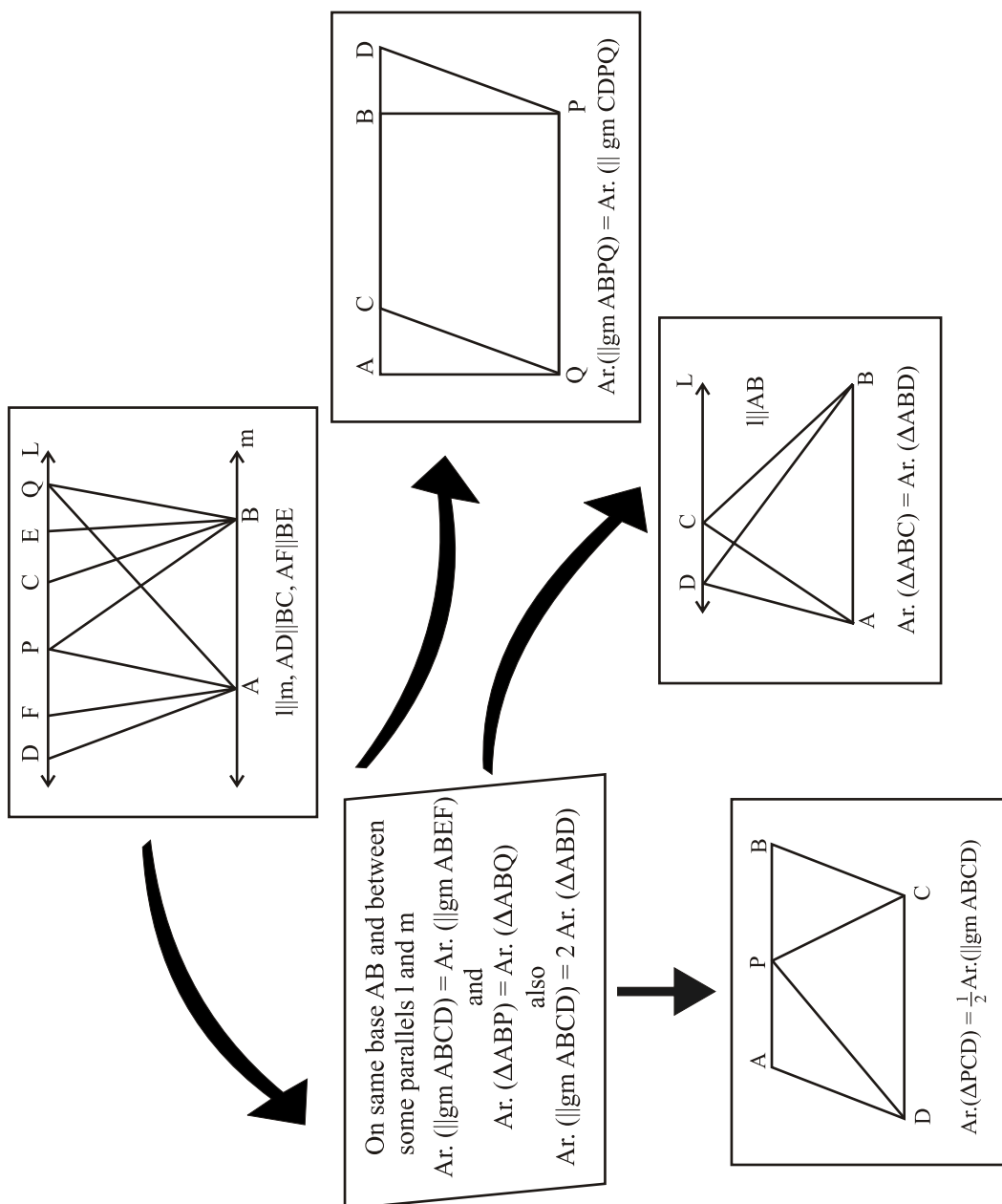
KEY POINTS

1. Parallelograms on the same base and between same parallels are equal in area.

   Two parallelograms ABCD and EFCD on the same base DC and between same parallels AF and DC

   ar (ABCD) = ar (EFCD)

2. Two triangles on the same base and between the same parallels are equal in area.

   Two triangles ABC and PBC on the same base BC and between same Parallel lines BC and AP in the given figure then ar ($\triangle$ABC) = ar ($\triangle$PBC)
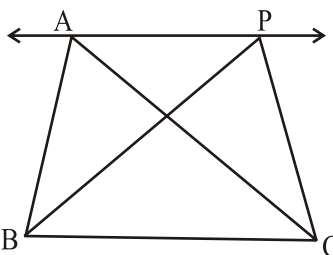
4. If a triangle and a parallelogram are on the same base and between the same parallels then the area of the triangle is half of the area of parallelogram.

   ar ($\triangle$LAB) = $\frac{1}{2}$ ar (ABCD)
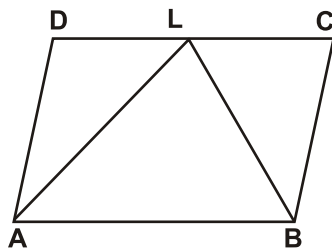
5. The median of a triangle divides it into two triangles of equal area.

Very Short Answer Type (1 Mark)

Fill in blanks :

1. The area of a parallelogram is the product of any of its sides and its corresponding _____

2. The area of parallelogram on the same base and between the same _____ are equal.

3. The diagonal of a parallelogram divides it into triangles having equal _____.

4. Area of trapezium = $\frac{1}{2}$ x height x _____

    State True or False :

5. The median of a triangle divides it into two triangles of .... area.

6. In $\triangle$XYZ, P is the mid-point of side YZ. Find the ratio ar ($\triangle$XYZ) : or ($\triangle$XYP)

7. A square and rhombus are on the same base and between same parallels. What is the ratio of their areas ?

8. If area of Parallelogram ABCD is 80 cm². Find the area of $\triangle$APD.
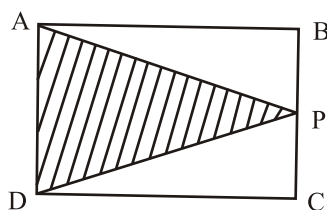
9. If area of Parallelogram PQRS is 88 cm² find K.
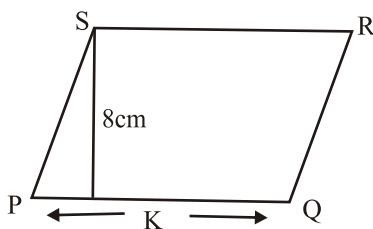
10. PQRS is a Parallelogram and PQM is a triangle. If area of $\triangle$ PQM = 18cm². Find the area of PQRS.
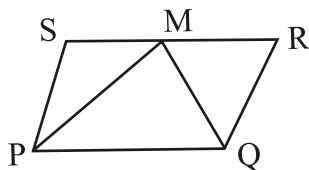
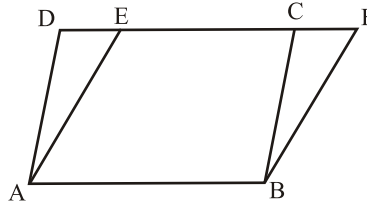11. In $\triangle ABC$, AD is median. If area of $\triangle ABD = 25cm^2$ find the area of $\triangle ABC$.
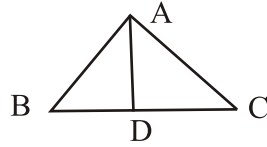
12. In the given figure, area of $\triangle SRN = 21cm^2$ RQ = 6cm find PQ.
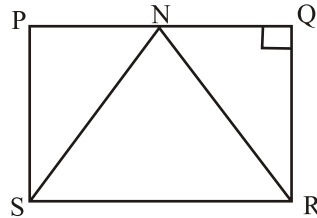
13. In the figure ABCD and ABFE are Parallelograms then find ar $(\triangle BCF)$.

If ar $(ABCE) = 18\,cm^2$

ar $(ABCD) = 25\,cm^2$

14. If two parallelogram are on equal base and between the same parallels, then what is the ratio of their areas?

15. In $\triangle ABC$, D, E, F are respectively the mid points of the sides AB, BC and AC. Find ratio of the area of $\triangle DEF$ and area of $\triangle ABC$.

16. If the base of a parallelogram is 8 cm and its altitude is 5 cm then find its area.

17. If two triangles are on the same base and between the same parallels. Then find the ratio of area of the two triangles.

18. In given figure, if area of parallelogram ABCD is 30 cm^2 then find ar $(\triangle ADE) + ar (\triangle BCE)$
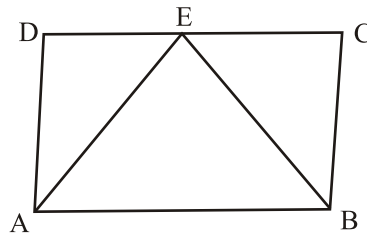
Case study based problem (19-20)

19. In a math lab activity, A student pasted three triangular papers to form of parallelogram. In the figure found from this activity, ABCD is a parallelogram. P lies on DC. If ar ($\triangle$DPA) = 15 cm^2 and or ($\triangle$BPC) = 20 cm^2 then.
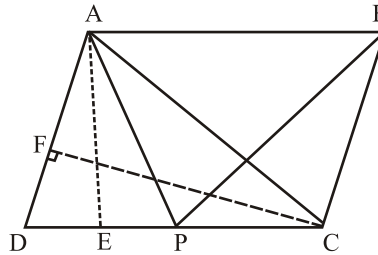
A. Ar ($\triangle$ADC)

  (a) 35cm^2 (b) 70cm^2 (c) 20cm^2 (d) 10cm^2

B. Ar (ABCD)

  (a) 15cm^2 (b) 40cm^2 (c) 70cm^2 (d) 140cm^2

C. Ar ($\triangle$ABP)

  (a) 15cm^2 (b) 35cm^2 (c) 70cm^2 (d) 20cm^2

D. If AE $\perp$ DC and AE = 7cm then AB =

  (a) 7cm   (b) 10cm   (c) 14cm   (d) 15cm

E. If CF $\perp$ AD and AD = 7, then CF=

  (a) 14cm   (b) 7cm   (c)10cm   (d) None

20. Rakesh and Arun are two farmers. Rakesh has a parallelogram shaped field and Arun has triangular shaped field with one common side. Rakesh divides his field in three part to grow three types of vegetables and Arun divides the field in two parts to grow two types of vegetables.

In figure, Rakesh's field is shown by $\triangle$ABCD and Arun's field is shown by $\triangle$CDQ. On measuring, it is found that BC=CQ. If AQ and CD cuts each other at P then area of $\triangle$ABP is 600m^2.
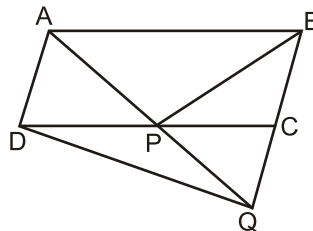
a. The area of Rakesh's field is
   (a) $300m^2$ (b) $600m^2$ (c) $900m^2$ (d) $1200m^2$
b. Area of $\triangle APD$ is
   (a) $300m^2$ (b) $150m^2$ (c) $450m^2$ (d) $600m^2$
c. Area of $\triangle PCQ$ is
   (a) $300m^2$ (b) $450m^2$ (c) $600m^2$ (d) $150m^2$
d. Ar. $(\triangle ABP)$: Ar $(\triangle CDQ)$ is :
   (a) $1:1$ (b) $1:2$ (c) $2:1$ (d) None
E. $\dfrac{Ar\ (\triangle BPC)}{Ar\ (\triangle DPQ)} =$

   (a) 1 (b) 2 (c) $\dfrac{1}{2}$ (d) $\dfrac{3}{2}$

Very Short Answer Type (2 marks)

21. Show that the median of a triangle divides it into two triangles of equal areas.

22. P and Q are any two points lying on the side DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).

23. If the ratio of altitude and area of the parallelogram is 2:11, then find the length of the base of parallelogram.

24. In figure if PQRS is a parallelogram in which PQ=12cm, ST=9cm, QM=6cm, ST $\perp$ PQ, QM $\perp$ SP, then find length of SP.
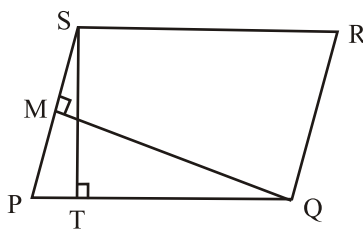
25. In given fig. ABCD is a square whose diagonals are interesting at O. If OD = 2 cm, then find the length of AB.
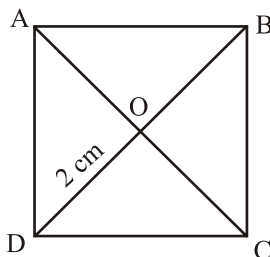
26. Show that the diagonals of a parallelogram divides it into four triangles of equal area.

27. M is any point on the median AD of ΔABC. Show that ar (AMB) = ar (AMC).
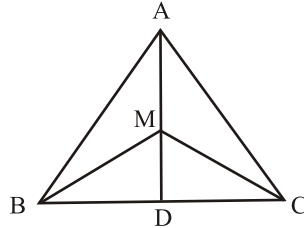
28. If D, E and F are respectively the mid points of sides BC, CA, and AB of ΔABC show that.

    i)   BDEF is a parallelogram.

    ii)   ar (DEF) = 1/4 ar (ΔABC)

29. In the given figure BC = CD = DE and ar (ΔABC) = 60cm^2

    M is the mid point of CD, then find the area of ΔAMC.
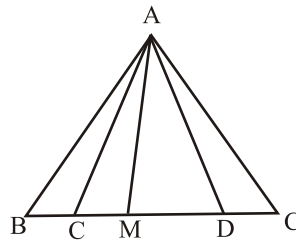
30. In ΔXYZ, A and B are points on sides XZ and YZ respectively. YA and XB interset at O. If AB||XY, then show that or (ΔAOX) = or (ΔBOY)
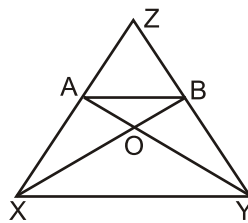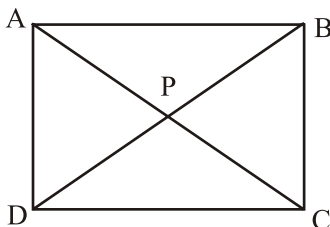
31. In the given figure, the area of parallelogram ABCD is 40 cm². If MN is a median of ΔCDN then find the area of ΔNDM.
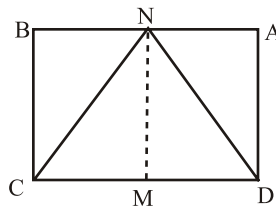
Short Answer type (3 Marks)

32. In the figure, P is the point in the interior of parallelogram ABCD then show that

    (i) ar (APB) + ar (PCD) = 1/2 ar (ABCD)

    (ii) ar (APD) + ar (PBC) = ar (APB) + ar (PCD)

33. ABCD is a trapezium in which AB ‖ DC. If diagonal AC and BD intersect at O then prove that ar (AOD) = ar (BOC).

34. ABCD is a parallelogram whose diagonals AC and BD intersect at O. A line through O intersects AB at P and DC at Q. Prove that ar ($\triangle$POA) = ar ($\triangle$QOC).

35. Diagonal PR and QS of quadrilateral PQRS intersects at T such that

    PT = TR and PS = QR, show that

    ar ($\triangle$PTS) = ar ($\triangle$RTQ).

36. In the figure, ABC and ABD are two triangles on the same base AB. If line segment CD bisects AB at O show that ar ($\triangle$ABC) = ar ($\triangle$ABD).
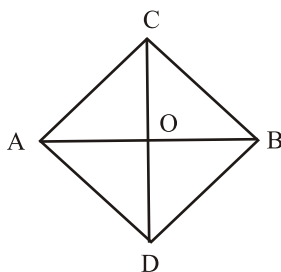
37. In given figure AD is median of $\triangle$ABC. Prove that ar ($\triangle$ABD) = ar ($\triangle$ACD).
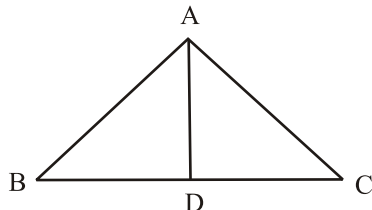
38. Prove that parallelogram on the same base and between same parallels are equal in area.

39. Prove that the two triangles on the same base and between the same parallels are equal in area.

40. If a triangle and parallelogram are on the same base and between the same parallels then prove that the area of triangle is equal to the half the area of parallelogram. Using this find ar ($\triangle$CMD).
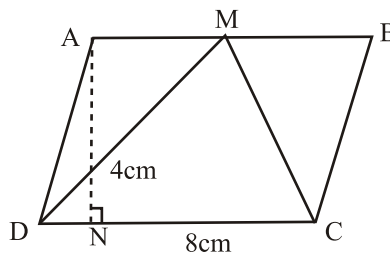

41. XY is a line parallel to side BC of a triangle ABC. If BE $\parallel$AC and CF $\parallel$ AB meet XY at E and F respectively show that ar (ABE) = ar (ACF).

42. If E, F, G and H are respectively the mid points of the sides of a parallelograms ABCD. Show that ar (EFGH) = 1/2 ar (ABCD).

43. There is a plot in a village in the shape of a quadrilateral ABCD. Head of the village wants to get floor cemented so as to use it for panchayat meetings.

    Later he decided to construct playground of shape $\triangle$ABP for children. If AC $\parallel$ DP then

    (a) Prove than ar (ABCD) = ar (ABP)

    (b) area   (ABCD) = 2x _____
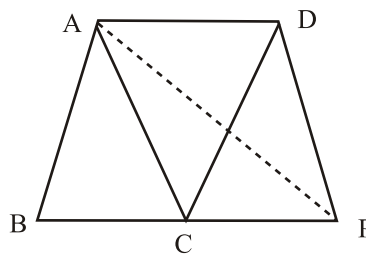
Long Answer type (5 Marks)

44. A farmer has a square plot of land where he wants to grow five different crops at a time. On half of the area in the middle he want to grow different crops.

a) Explain by diagram how he can divide the area to fulfill his purpose.

b) For same base and between the same parallels, write the relation between area of triangle and parallelogram formed.

45. In the adjoining figure, the point D divides the side BC of △ABC in the ratio m:n. Prove that ar (△ABD) : ar (△ADC) = m : n.
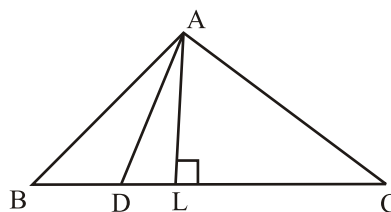
46. ABCD is a parallelogram. E is a point on BA such that BE= 2EA and F is a point on DC such that DF = 2 FC. Prove that AECF is a parallelogram whose area is one third of the area of parallelogram ABCD.
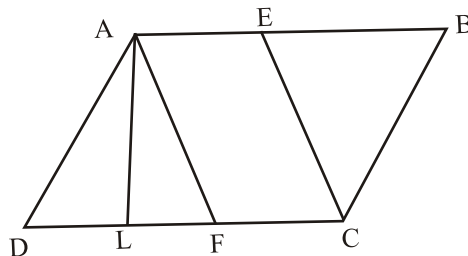
47. In the adjoining figure, two parallelogram ABCD and AEFB are drawn on opposite sides of AB. Prove that

ar(□ABCD) + ar(□ AEFB) = ar (□ EFCD)
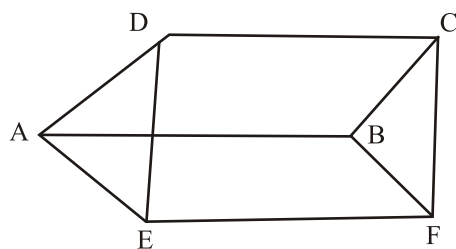
48. In the given figure BC || XY, BX || CA and AB || YC. Prove that ar (△ABX) = ar (△ACY)
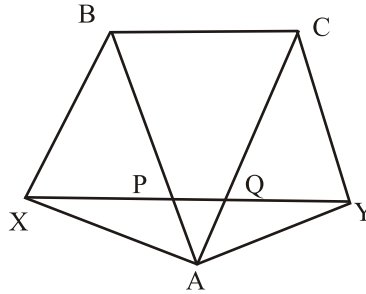
49. In the given figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilateral ABCD and DCPR are trapeziums.
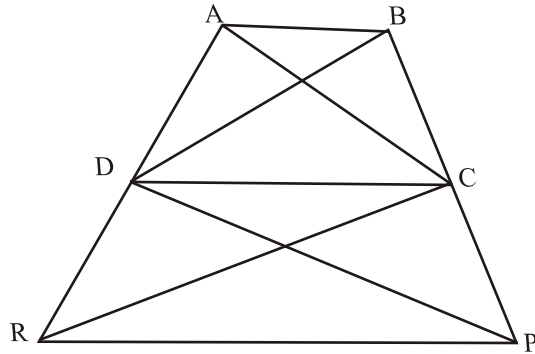
AREAS OF PARALLELOGRAMS TRIANGLES

1. altitude

2. parallels

3. areas

4. Sum of parallel sides

5. equal

6. 2:1

7. 1:1

8. $40cm^2$

9. 11cm

10. $36cm^2$

11. $50cm^2$

12. PQ=7cm

13. $7cm^2$

14.   1:1

15. 1:4

16. $40cm^2$

17. 1:1

18. $15cm^2$

19. (A)   a) $35cm^2$

    (B)   c) $70cm^2$

    (C)   b) $35cm^2$

    (D)   b) 10cm

    (E)   c) 10cm

20. (A)   d) $120cm^2$

    (B)   b) $15cm^2$

    (C)   a) $15cm^2$

    (D)   a) 1:1

    (E)   a) 1

23. $\dfrac{11}{2}$ units

24. ar (PQRS) = PQ x ST

                = $108cm^2$

         108 = SP x 6cm

          SP = 18cm

25. $\sqrt{8}$cm or $2\sqrt{2}$cm

29. $10cm^2$

31. $10cm^2$
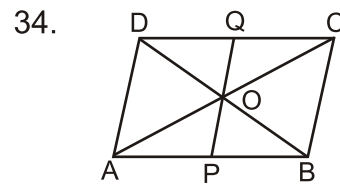
34.

   Δ POA $\cong$ ΔQOC

    → as (ΔPOA) = ar (ΔQOC)

35. Since ΔPTS $\cong$ ΔRTQ

   $\therefore$ ar (ΔPTS) = ar (ΔRTQ)

40. ar (ΔCMD) = $16cm^2$

43. (a)   ar (ΔACP) = ar (ΔACD)

    ar (ΔACP) + ar (ΔABC) = ar (ΔACD) + ar (ABC)

    ar (ΔABP) = ar (quad ABCD)

    (b)   ar (ΔACD)

AREAS OF PARALLELOGRAMS & TRIANGLES

Time : 1hrs.   M.M : 25

1.   What will be ratio of areas of the two parallelogram which lie on the
      same base and between the same parallels ?   (1)

2.   If area of parallelogram ABCD is
      96cm^2, find K.
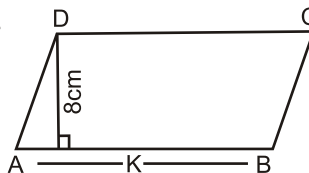
3.   If area of parallelogram ABCD is 60cm^2.   (1)
      Find area of $\triangle$APD

                                                                          (1)
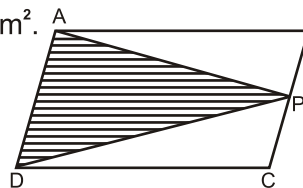
4.   Show that the media of a triangle divides it into two triangles of equal
      area.   (2)

5.   In fig, of PQRS is a parallelogram in which PQ = 12cm, ST=9cm,
      QM=6cm, ST$\perp$PQ, QM$\perp$SP, then find length of SP.   (2)
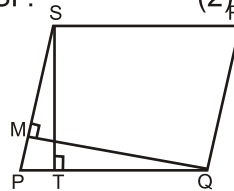
6.   Diagonals AC and BD of a quadrilateral ABCD interest each other at
      P. Show that as ($\triangle$APB) x ar ($\triangle$CPD) = ar ($\triangle$APD) x ar($\triangle$BPC)

7.   ABCD is a parallelogram and O is a point
      in the interior, Prove that
      ar ($\triangle$AOB) + ar ($\triangle$COD) =
      ar ($\triangle$AOD) + ar ($\triangle$BOC)
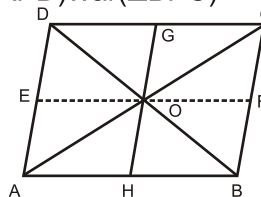
8.   The base BC of $\triangle$ABC is divided at D, Such that BD= $\frac{1}{2}$ DC. Prove
      that ar ($\triangle$ABD) = $\frac{1}{3}$ ar ($\triangle$ABC)

9. Prove that parallelogram on the same base and between same parallels are equal in area.

10. In fig, PQ is a line parallel to the side BC of ABC. If BX || CA and CY||BA meet the line PQ produced in X and Y respectively. Show that ar ($\triangle$ABX) = ar ($\triangle$ACY)
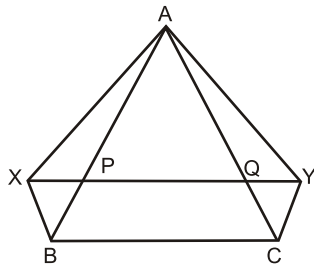
CIRCLES
MIND MAPPING
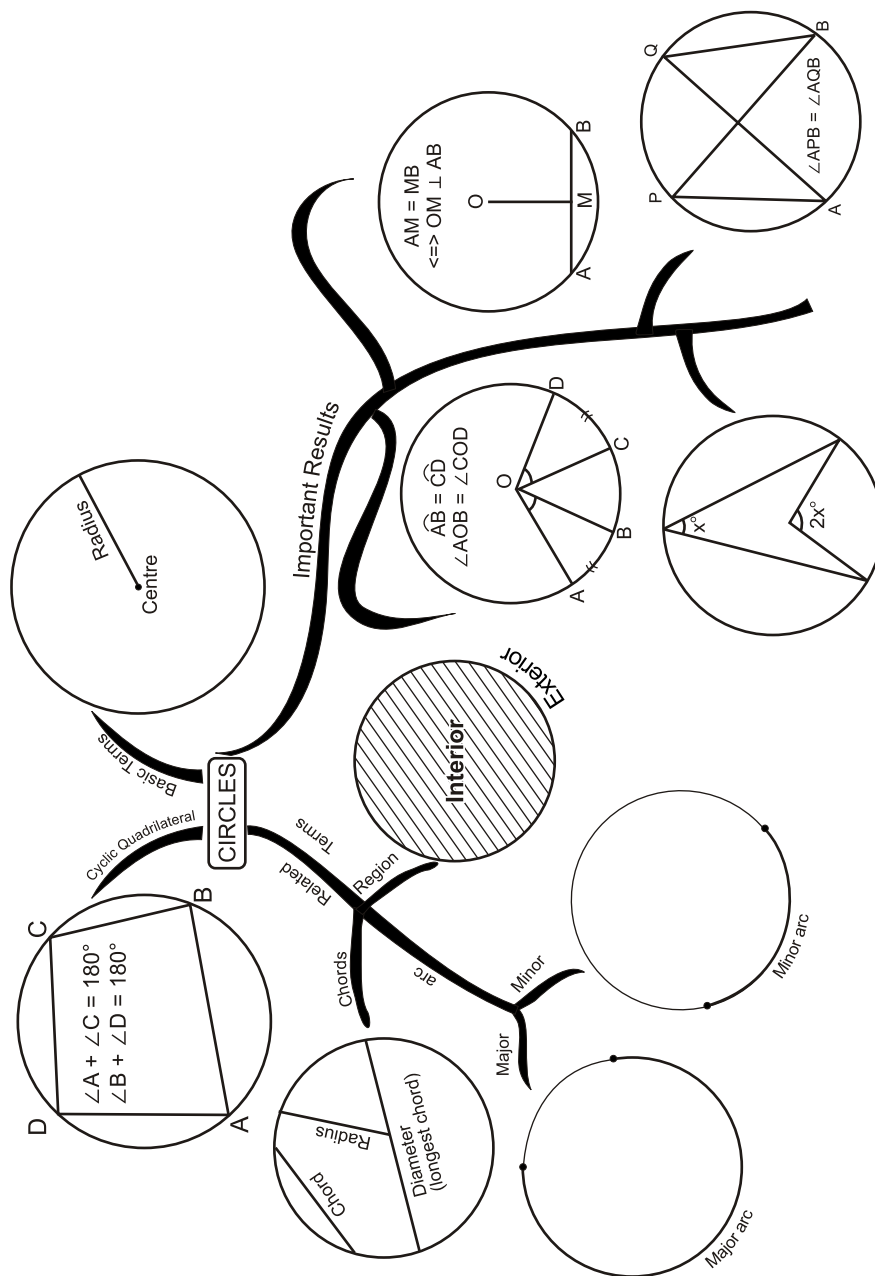
CIRCLES

KEY POINTS

- The collection of those points in a plane which are at a fixed distance from a given fixed point is called a circle. The fixed point is called centre of the circle and the fixed distance is called radius.

Circle and related Terms !
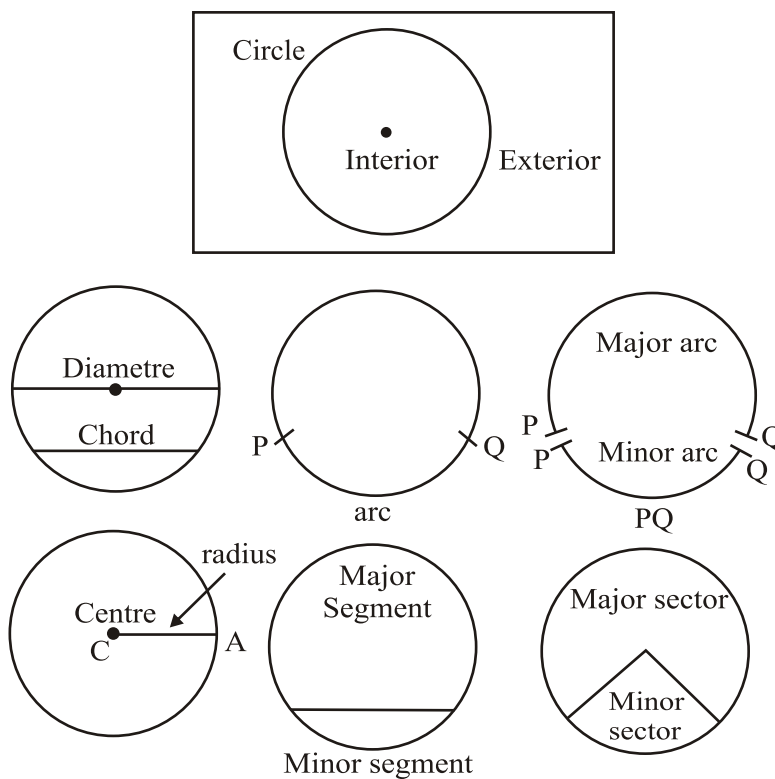
- There is one and only one circle passing through three non-collinear points.

- Equal chords of a circle subtends equal angles at centre.

- If angles subtended by chords at centre are equal then chords are equal.

- The perpendicular from centre to a chord of a circle, bisects the chord.

- The line joining the centre of a circle to the mid point of a chord is perpendicular to the chord.
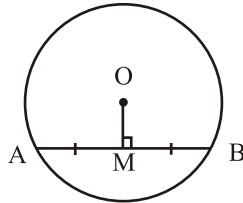
- Equal chords of a circle are equidistant from centre.
- Chords equidistant from centre are equal in length.
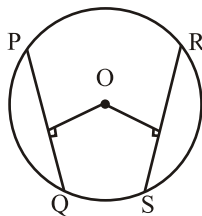
- If two chords of a circle are equal then corresponding arcs are equal.
- If arcs of a circle are equal then corresponding chords are also equal.
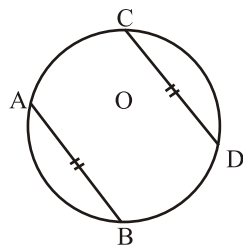
- Congruent arcs (or equal arcs) of a circle subtends equal angles at centre.

$\Rightarrow$ $\boxed{\angle AOB = \angle COD}$
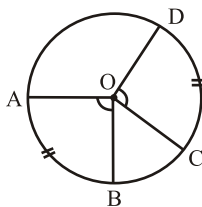
- The angle subtended by an arc at the centre of circle is twice the angle which is subtended at remaining part of the circle.

$\Rightarrow$ $\boxed{\angle AOB = 2\angle APB}$
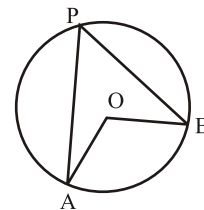
- Any two angles in the same segment of the circle are equal.

  $\Rightarrow$ $\boxed{\angle APB = \angle AQB}$
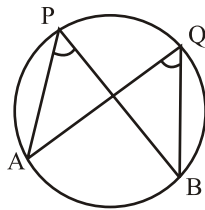
- Angle in semi circle is right angle.

  $\Rightarrow$ $\boxed{\angle APB = 90°}$
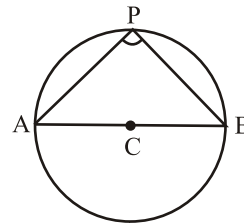
- In a cyclic quadrilateral the sum of opposite angles is 180^0.

  $\Rightarrow$ $\boxed{\angle A + \angle C = 180°}$

  $\Rightarrow$ $\boxed{\angle B + \angle D = 180°}$
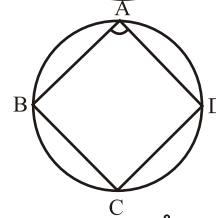
- If sum of opposite angles of a quadrilateral is 180^0 then that quadrilateral is cyclic quadrilateral.
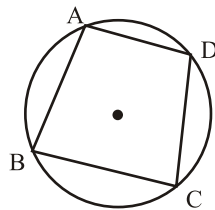
Very-Very Short Answer Type Questions (1 mark)

Fill in the blank :- (Ques 1 to 5)

1. If the sum of a pair of apposite angles of a quadrilateral is 180°, then the quadrilateral is _____

2. Diameter is the _____ chord of a circle.

3. Circles having the same centre are different radii are called _____.

4. Angle in a semicircle is _____

5. If two chords of a circle are equal then corresponding arcs are_____

6. In fig, AOB is a diameter of the circle and AC = BC, find ∠CAB
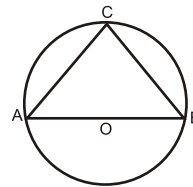
7. In fig, AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chords AB and CD respectively. If ∠POQ = 150°, find ∠APQ
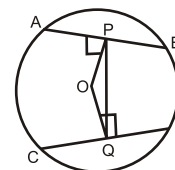
8. In fig, if OA = 5cm, AB = 8cm and OD⊥AB then find CD.
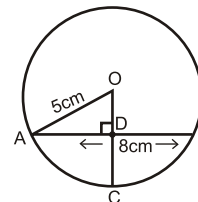
9. The radius of a circle is 13cm and the length of one of its chords is 10cm. Find the perpendicular distance of the chord from the centre.

10. In fig, ∠ABC=20°, find ∠AOC.
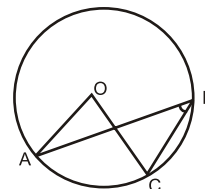
11. If in a △ABC, AB = 12cm, BC=16cm and AB⊥BC, then find the radius of the circle passing through A,B and C

12. In figure, find the values of x and y. Where O is the centre of the circle.
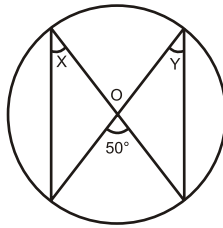
13. A round pizza is cut into 4 equal pieces. What does each piece represent?

14. AD is a diameter of a circle and AB is a chord if AD = 34cm, AB=30 cm then find the distance of AB from the centre of circle.

15. Given two concentric circles with centre O. A line cut the circles at A, B, C and D respectively. If AB = 10cm, then find the length of CD.

16. Find y in given figure
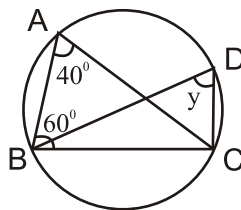
17. Find x
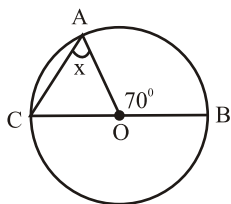
18. Find x
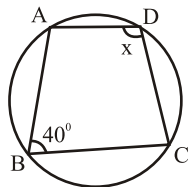
19. In given figure OC is perpendicular segment drawn from centre O on chord AB. If OB = 5cm, and OC = 3cm then find length of AB.
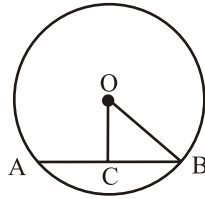
20. In given figure O is centre of circle.

If $\angle AOC = 130^0$ then find $\angle ABC$
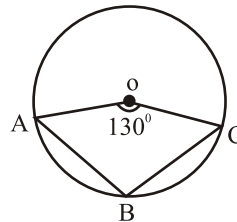
21. In given figure AOB is diameter of circle & P is any point on the circle. Find $\angle APB$.
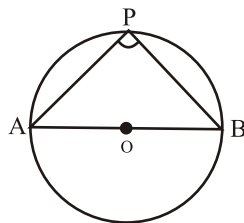
22. Find the value of x in given figure.
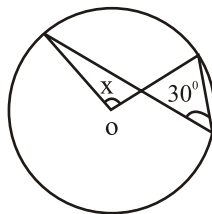
Case Study Based Problems

23. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in hand to talk to each other as shown in figure. AP, BQ and CR are the medians of triangle ABC.
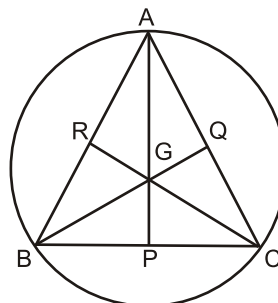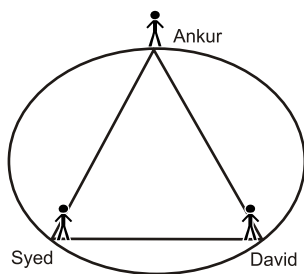
(A) The length of AG is :

a) 20m   b) 10m   c) 22m   d) 25m

(B) The length of AP is :

a) 20m   b) 30m   c) 40m   d) $20\sqrt{2}$m

(C) The measure of angle BGC is :

a) 60°   b) 90°   c) 120°   d) 150°

(D) The measure of angle ABQ is :

a) 20°   b) 30°   c) 45°   d) 60°

(E) The length of AB is :

a) 30m   b) $10\sqrt{3}$m   c) $20\sqrt{3}$m   d) $30\sqrt{3}$m

24.   During a practical activity in mathematics Lab, students were using circular geoboard. The angle subtended by $\overline{BC}$ at the centre is (2a + 50°)
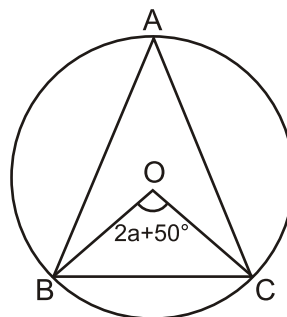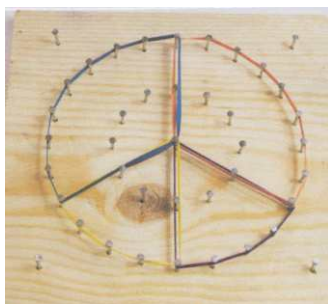
(A) ∠BAC is equals :

a) a   b) 25   c) 2a   d) a + 25°

(B) If a=30°, then ∠BAC is

a) 110°   b) 55°   c) 50°   d) 60°

(C) If a=15° then reflex BOC is

a) 100°   b) 55°   c) 280°   d) 90°

(D) If a=5° and radius of circle is 10cm then BC is

a) 5cm   b) 8cm   c) 10cm   d) 15cm

(E) If diameter of circle is 20cm and BC=12cm then the perpendicular distance of BC from O is

a) 6cm   b) 8cm   c) 10cm   d) 12cm

25. Three girls Reshma, Salma and Mandeep are playing a game by standing on a circle of radius 5m at R, S and M respectively as shown in figure. Reshma throws a ball to Salma, Salma to Mandeep, Mandeep to Reshma. The distance betweem Reshma and Salma and between Salma and Mandeep is 6m. 0 is the centre of the circle.
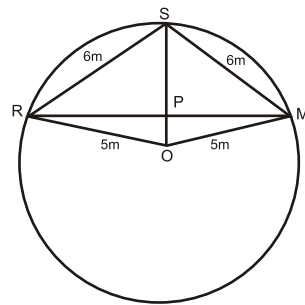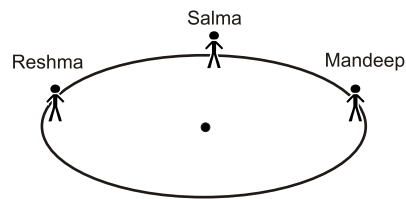
(A) The ratio of $\angle MOS : \angle MRS$ is :

a) 1 : 1   b) 1 : 2   c) 2 : 1   d) 3 : 2

(B) The quadrilateral RSMO is a :

a) Parallelogram

b) Rhombus

c) Square

d) Kite

(C) Length of perpendicular from O to the chord SM is :

a) 5m   b) 6m   c) 4m   d) 3.6m

(D) Length of OP is :

a) 1.4m   b) 4m   c) 36m   d) 3m

(E) Distance Reshma and Mandeep is :

a) 4.8m   b) 3.6m   c) 9.6m   d) 7.2m

Very Short Answer questions (2 marks)

26. Prove that cyclic parallelogram is a rectangle.

27. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

28. In the following figure. Find the value of ∠BCN.
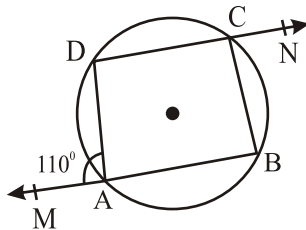
29. In the given figure. Find the value of reflex angle POR.
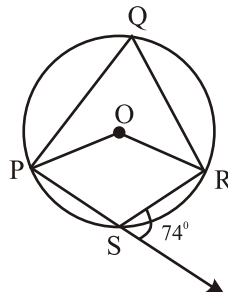
30. Find the value of x in figure if O is centre of circle and ∠OAB = 50°.
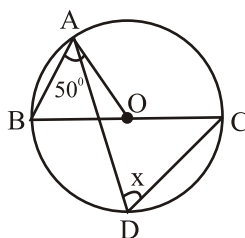
31. In the given figure, O is centre of the circle with radius 5 cm, OP ⊥ CD, OQ ⊥ AB, AB || CD, AB = 6 cm and CD = 8 cm. Determine PQ.
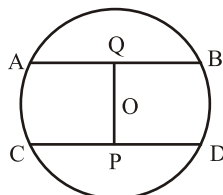
32. In the given figure, O is the centre of a circle, $\angle AOB = 90^0$, $\angle BOC = 120^0$, what is measure of $\angle ABC$?
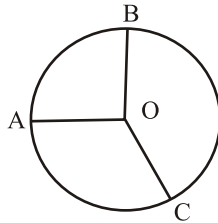
33. In the given figure AB and CD are parallel chords if the length of arc AC = 14 cm. What is length of BD?
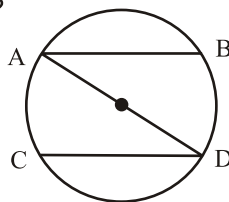
34. In given figure $\angle PQR = 100^0$ where P, Q & R are points on the circle with centre O. Find $\angle OPR$.
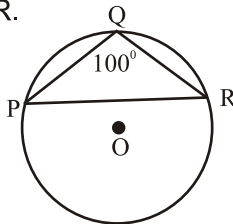
35. In the given figure O is centre of circle. If $\angle ABD = 35^0$ and $\angle BAD = 70^0$, find $\angle ACB$.
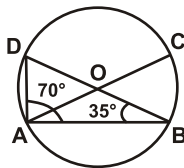
36. In fig, OD is perpendicular to the chord AB of a circle whose centre is O. If BC is a diameter. Show that CA=2OD.
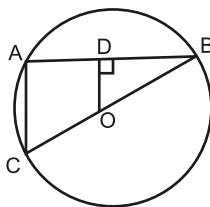
37. Two concentric circles with Centre O where AB is a chord of outer circle which intersects the inner circle at C and D are shown in figure. If AB = 12cm and CD=8cm find AD.
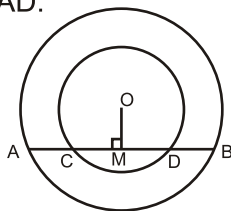
38. In figure, AB=BC and O is the centre is the circle. Prove that BO bisects ∠ABC.
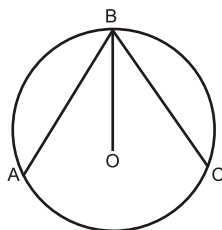
39. Prove that the diameter is the greatest chord of a circle.

40. In figure, PQRS is a cyclic quadrilateral. Find the value of x and y.
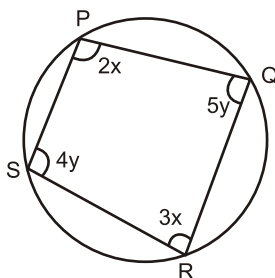
Short Answer Type Questions (3 marks)

41. In the given figure, O is the centre of a circle prove that $\angle x + \angle y = \angle z$.
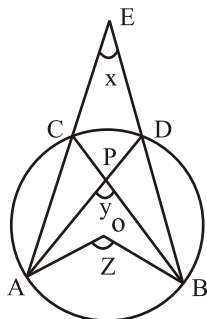
42. If two non parallel sides of a trapezium are equal prove that it is cyclic quadrilateral.

43. In the given figure, determine a, b & c if $\angle BCD = 43^0$, $\angle BAF = 62^0$.
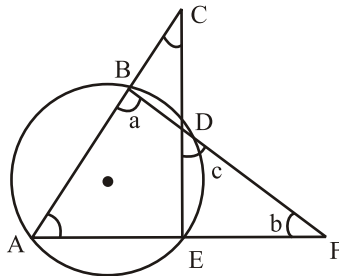
44. In the figure, P is the centre prove that $\angle XPZ = 2\,(\angle XZP + \angle YXZ)$
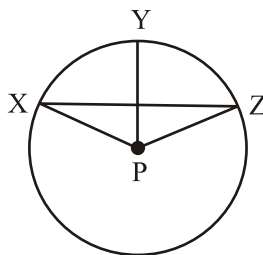
45. In the given figure AD is diameter of the circle whose centre is O and AB || CD prove that AB = CD.
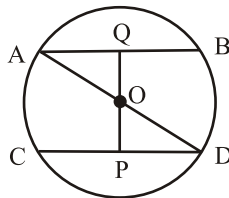
46. In an equilateral triangle, prove that the centroid and the circum centre coincide.

47. In the given figure A, B, C and D, E, F are two sets of collinear points. Prove that AD || CF.
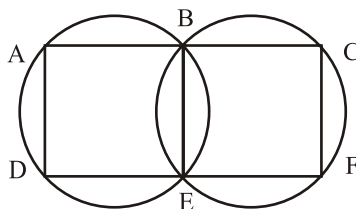
48. In given figure, O is centre of circle and $\angle DAB = 50^0$, calculate the value of x and y.
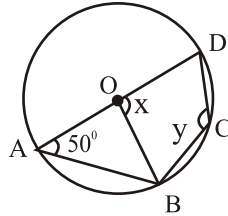
49. If two equal chords of a circle intersect within the circle prove that the segment of one chord is equal to corresponding segment of other chord.

50. Prove that if a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

51. Bisector of angle A, B and C of a $\triangle ABC$ intersect its circum circle at D, E and F respectively, prove that the angles of a triangle DEF are

$$90^0 - \frac{1}{2} A, \quad 90^0 - \frac{1}{2} B, \quad 90^0 - \frac{1}{2} C$$

52. Find the sum of the angles in the four segments exterior to a cyclic quadrilateral.

53. Let the vertex B of a triangle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

$$\angle ABC = \frac{1}{2} [\angle DOE - \angle AOC]$$
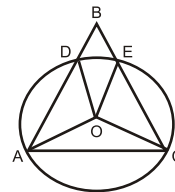
54. In the adjoining figure AC is diameter of a circle with centre O and chord BD $\perp$ AC, intersecting each other at E. Find out the values of p, q, r in terms of x if $\angle AOD = x^0$, $\angle BAC = p^0$, $\angle ACD = q^0$.
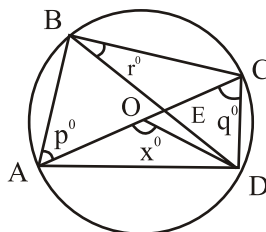
55. Prove that there is one and only one circle can pass through three non-collinear points.

56. In the given figure OPQR is a square. A circle drawn with centre O cuts the square in X and Y. Prove that QX = QY.
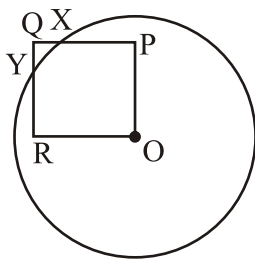
57. Prove that the opposite angles of a cyclic quadrilateral are supplementary.

58. In the given figure, AB is a diameter of a circle (O, r) and chord CD = radius OC. If AC and BD when produced meet at P. Prove that $\angle APB$ is constant.
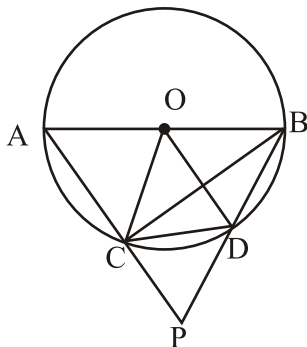
59. In the given figure, AB is a chord of a circle with centre O and AB is produced to C such that BC = OB. Also, CO is joined and produced to meet the circle in D. If $\angle ACD = y^0$ and $\angle AOD = x^0$. Prove that x = 3y.
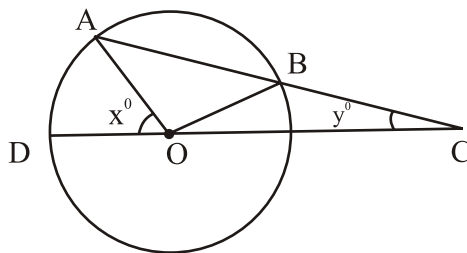
60. Two circles whose centres are O and O' intersect at P. Through P, a line l parallel to OO', intersecting the circle at C and D is drawn. Prove that CD = 2OO'.
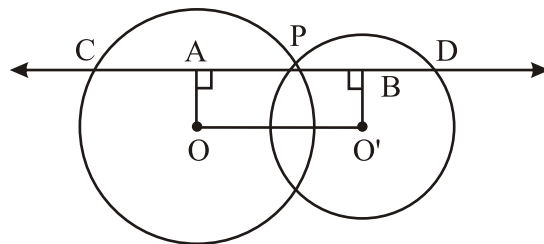
61. AB and CD are two parallel chords of a circle which are on opposite sides of the centre O such that AB = 10cm, CD = 24cm and the distance between AB and CD is 17 cm. Find the radius of the circle.
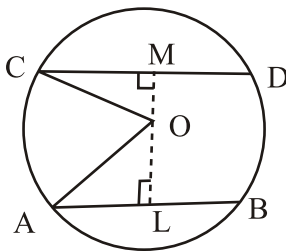
Long Answer type Questions (5 Marks)

62. AB and AC are two chords of a circle of radius r such that AB = 2 AC. If p and q are the distance of AB and AC from the centre, Prove that $4q^2 = p^2 + 3r^2$

63. In figure, O is the centre of the circle, $\angle$ BCO = 30°, AE $\perp$ BC and DO $\perp$ AE. Find x and y.
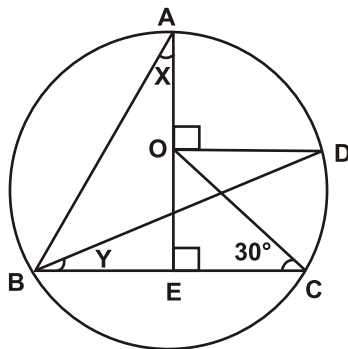
64. In figure, O is the centre of the circle, BD = OD and CD ⊥ AB, Find ∠CAB.
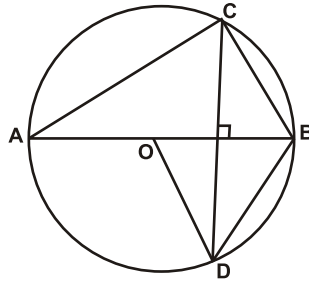
65. Prove that the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

66. Show that if two chords of a circle bisect one another they must be diameters.

67. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.

CIRCLES
ANSWERS

1. Cyclic
2. longest
3. Concentric Circles
4. Right angles or 90°
5. Congruent
6. 45°
7. 105°
8. 2cm
9. 12cm
10. 40°
11. 10cm
12. x=35°, y=35°
13. Sector
14. 8cm
15. 10cm
16. y=40°
17. 35°
18. x+40°=180°
    x = 140°
19. CB = 4cm
    ∴ AB = 8cm
20. ∠ABC = 115°
21. ∠APB = 90°
22. x = 60°
23. (A) (a) 20m
    (B) (b) 30m
    (C) (c) 120°
    (D) (b) 30°
    (E) (c) 20√3
24. (A) (d) a+25
    (B) (b) 55°
    (C) (c) 280°
    (D) (c) 10cm
    (E) (b) 8cm
25. (A) (c) 2 : 1
    (B) (d) Kite
    (C) (c) 4m

(D) (a) 1.4m
(E) (c) 9.6m

27. 30°, 150°
28. ∠DAM + ∠DAB = 180°
    ∠DAB = 70°
    ∠DAB + ∠DCB = 180°
    ∠DCB = 110°
    ∠BCN +∠DCB = 180°
    ∠BCN = 70°
29. ∠PSR + ∠RST = 180°
    ∠PSR= 106°
    reflex ∠POR = 2 ∠PSR
    = 212°
30. ∠AOB = 80°
    ∠AOB + ∠AOC = 180°
    ∠AOC = 100°
    ∠ADC = $\frac{1}{2}$ ∠AOC

    ∠ADC = 50
31. AQ = $\frac{1}{2}$ AB

    AQ = 3cm
    $(OA)^2 = (AQ)^2 + (OQ)^2$
    OQ = 4cm
    slly OP = 3cm
    PQ = 7cm
32. ∠AOB + ∠BOC + ∠AOC = 360°
    ∠AOC = 150°
    ∠ABC = $\frac{1}{2}$ ∠AOC = 75°

33. BD = 14cm
34. ∠OPR = 10°
35. ∠ABD + 70° + 35° = 180°
    ∠ADB = 75°
    ACB = ADB = 75°
36. OD || AC
    OD = $\frac{1}{2}$ CA

    CA = 2 OD

37. AM = 6cm
MD = 4cm
AD = (6+4)cm
= 10cm
38. Δ's AOB ≅ ΔCOB by sss
⇒ ∠OBA = ∠OBC
⇒ OB bisects ∠ABC
40. 2x + 3x = 180°
x = 36°
4y + 5y= 180°
y = 20°
41. ∠ACB = ∠ADB
∠z = 2 ∠ACB
∠z = ∠ACB + ∠ADB
∠y = ∠ACB + ∠CAD
∠z = ∠y − ∠DAC + ∠ADB
∠ADB − ∠DAC = ∠X
∠x + ∠y = ∠z
43. a = 105°, b = 13°, c = 62°
44. ∠XPY = 2 ∠XZY
∠YPZ = ∠YXZ
∠XPZ = 2 (∠XZY + ∠YXZ)
45. ΔAOQ ≅ ΔPOD
⇒ OQ = OP by CPCT
⇒ AB = CD
46. ΔBEC≅ΔBFC

⇒ BE = CF
slly ΔCAF ≅ ΔCAD
⇒ CF = AD
AD = BE = CF
$\frac{2}{3}$ AD= $\frac{2}{3}$ BE = $\frac{2}{3}$ CF

GA = GB = GC
Hence centroid and circumcentre are coincident
47. ∠DAB + ∠BED = 180°
∠BED = ∠BCF
⇒ ∠DAB + ∠BCF = 180°
Hence AD || CF
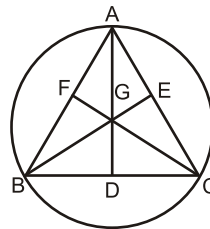
48. $\angle AOB = 80°$
    $x = 100°$
    $y = 130°$

52.
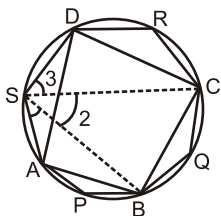
$\angle 1 + \angle P = 180°$
$\angle 2 + \angle Q = 180°$
$\angle 3 + \angle R = 180°$
$\angle 1 + \angle P + \angle 2 + \angle Q + \angle 3 + \angle R = 3 \times 180° \Rightarrow \angle P + \angle Q + \angle R + \angle S = 6 \times 90°$

54. $p = 90$
    $q = \frac{1}{2}x$
    $r = 90 - \frac{1}{2}x$

56.
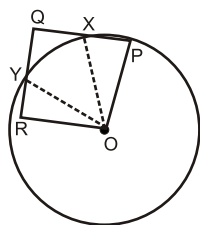
QR = QP
$\triangle ORY \cong \triangle OPX$
∴ RY = PX
QR – RY = QP – PX
QY = QX

59. $\angle BOC = y$
    $\angle ABO = 2y,\ \angle OAB = 2y$
    $(2y) + (2y) + (180 - x - y) = 180°$
    $x = 3y$

60. CA = AP
    $\Rightarrow$ CP = 2AP
    BP = BD
    $\Rightarrow$ PD = 2 PB
    CD = 2 AP + 2PB
    CD = 200

61. $OA^2 = OL^2 + AL^2$

   $r^2 = x^2 + 5^2$ —————————— 1

   $OC^2 = OM^2 + CM^2$

   $r^2 = (17-x)^2 + (12)^2$ —————————— 2

   from 1 & 2

   $34x = 408$

   $x = 12cm$

   $\therefore r = 13cm$

   [Where OL = x, OM = 17-x]

62.
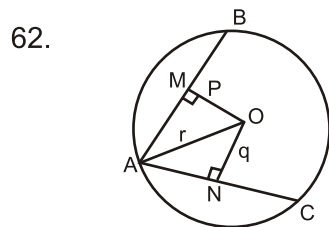
   $\left(\frac{AB}{2}\right)^2 = r^2 - P^2$

   $AB^2 = 4r^2 - 4p^2$

   slly $AC^2 = 4r^2 - 4q^2$

   As AB = 2AC

   $AB^2 = 4AC^2$

   $4r^2 - 4p^2 = 4[4r^2 - 4q^2]^2$

   $4q^2 = p^2 + 3r^2$

63. $\angle EOC = 180° - 30° - 90°$

   $= 60$

   $\angle COD = 180° - 60° - 90°$

   $= 30°$

   $2y = 30°$

   $y = 15°$

64. $\angle ABD = \frac{1}{2} \times 90°$

   $= 45°$

   $\angle ABC = 60$

   In $\triangle ABE$,

   $60° + x + 90° = 180°$

   $x = 30°$

   $x = 30°, y = 15°$

64. Since OB = OD = BD

   $\therefore \angle BOD = 60° \Rightarrow \angle AOD = 120°$

   Now $\angle ACD = 60° \Rightarrow \angle CBA = 60°$

   Hence CAB = 30 (Angle sum property)

PRACTICE TEST
Circles

Time : 1 hr   M.M.: 25

1.   In fig, $\angle DAB = 60°$ and $\angle ABD = 50°$. Find $\angle ACB$.   **1**
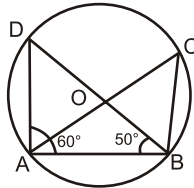
2.   The chord of a circle is equal to its radius. What is the angle subtended by this chord at the minor arc of the circle.   **1**

3.   Find the value of x in given figure if O is the centre of the circle.   **1**
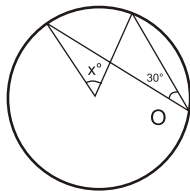
4.   Prove that equal chords of a circle subtend equal angles at the centre.   **2**

5.   A circle passes through A, B, C and D as shown in figure. If $\angle BAD = 93°$ Find x.   **2**
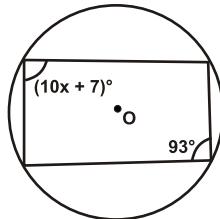
6.   In figure O is the centre of a circle. If $\angle AOC = 130°$ then find $\angle ABC$.   **2**
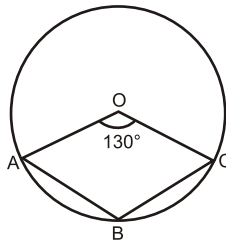
7.   Prove that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°.   **3**

8.   In fig, O is the centre of a circle prove that $\angle x + \angle y = \angle z$ **3**
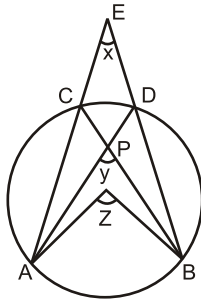
.9.   Prove that the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.   5

10.   In figure, AB is a chard of a circle with centre O and AB is produced to C. Such that BC=OB. Also, CO is joined and produced to meet the circle in D. If $\angle ACD = y$ and $\angle AOD = x°$. Prove that x = 3y   5
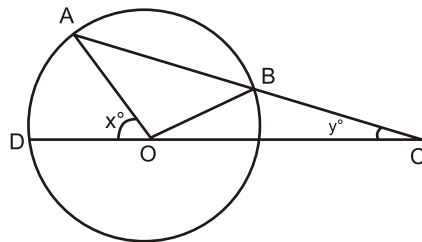
CONSTRUCTIONS
KEY POINTS

ANGLE BISECTOR

BISECTORS

PERPENDICULAR BISECTOR

CONSTRUCTION OF TRIANGLES

WHEN GIVEN

WHEN GIVEN

WHEN GIVEN
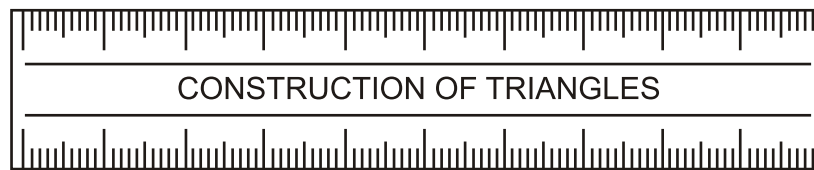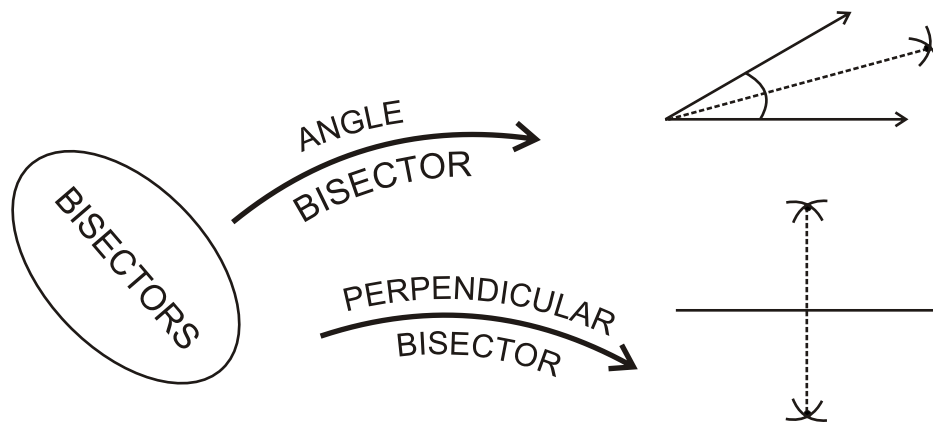
BASE, BASE ANGLE
& Sum of Two
OTHER SIDE

BASE, BASE ANGLE
& DIFFERENCE OF
OTHER TWO SIDE

TWO BASE
ANGLES &
PERIMETER

KEY-POINTS

1.   Angle-bisector : A line segment that divides the given angle into half i.e two equal parts.
2.   Perpendicular - bisector : A line segment that divides the given line segment into two equal parts.
3.   Angles of 60° and 120° can be constructed directly with compass without bisecting.
4.   When we bisect a given angle, we get half of it i.e. when we bisect 30°, we get 15° angle.
5.   Some angles can be obtained by bisecting two angles, like

Angles to be bisected	Angle obtained
30° and 60°	45°
120° and 180°	150°
60° and 90°	75°
90° and 120°	105°

VERY-VERY SHORT ANSWER QUESTIONS (1 MARK)

1.   What is the angle bisector of 50° ?
2.   The perpendicular - bisector divides a line segment of 8cm into two parts of _____ cm. each.
3.   We have to bisect _____ and _____ to get an angle of 135° in between them.
4.   A perpendicular - bisector divides a line – segment into _____.
5.   Which angle will be obtained in between by bisecting angles 60° and 90° ?

VERY SHORT ANSWER QUESTION (2 MARKS)

6.   Draw perpendicular bisector of AB = 6.4cm.
7.   Construct an angle of 15° using compass.
8.   Construct an angle of 90° using compass.

SHORT ANSWER QUESTION (3 MARKS)

9.   Draw a line segment of 7.2cm and bisect it. Also measure each part.
10.   Draw a line segment PQ = 8cm. Draw a perpendicular at P.
11.   Draw a line AB = 7.9cm and draw perpendicular at A and B. Are these two perpendiculars parallel to each other ?
12.   Draw $\angle ABC = 32°$ using protractor. Construct another angle equal to $\angle ABC$ using compass.
13.   Draw a line segment XY = 12.4cm. Find $\frac{1}{4}$ XY using rular and compass. Verify the same using scale.
14.   Construct an equilateral triangle the sum of its two sides is 10cm.

LONG ANSWER QUESTIONS (5 MARKS)

15. Construct $\triangle XYZ$ in which XY = 4.5 cm, YZ = 5.0 cm. and ZX = 6.0 cm. Also draw angle bisector of largest angle.

16. Construct an equilateral triangle of side 6 cm and label its vertices as P, Q and R. From point Q draw a median QT.

17. Construct a right triangle ABC, $\angle B = 90^0$, AB + AC = 10 cm., BC = 6 cm.

18. Construct a $\triangle PQR$ in which QR = 7 cm, $\angle Q = 75^0$ and PQ + PR = 13 cm.

19. Construct a $\triangle PQR$ in which QR = 6 cm, $\angle Q = 30^0$ and PQ − PR = 3 cm.

20. Construct a $\triangle XYZ$ in which YZ=4.1 cm, $\angle Y=45^0$, and XY + XZ = 6.7 cm.

21. Construct a $\triangle PQR$ in which QR = 5 cm, $\angle R=45^0$ and PR-PQ = 1.6 cm.

22. Construct a $\triangle XYZ$ in which $\angle Y = 30^0$, $\angle Z = 90^0$ and XY + YZ + ZX = 11cm.

23. Construct a triangle ABC in which $\angle B = 45^0$, $\angle C=60^0$ and the perpendicular from the vertex A to the base BC is 4.5 cm.

24. Construct a triangle with perimeter 12 cm and ratio of their angles are 3 : 4 : 5.

25. Government wish to make an old age home of right triangular shape. If one side is 13m and sum of hypotenuse and other side is 15 m then

    Construct the triangle taking measurement in cm.

26. Eco club of a school created a triangular park $\triangle ABC$ to maintain greenery of the school. If BC = 7m, $\angle B=75^0$, AB + AC = 13m then

    Construct $\triangle ABC$ taking measurement in cm.

27. Construct a triangle DEF in which DE = 5 cm, $\angle D = 120°$ and EF − DF = 3.6 cm.

28. Construct a right angled triangle with base 5.4 cm and difference of hypotenuse and perpendicular is 1.9 cm.

29. Construct a triangle PQR with PQ = 5 cm. $\angle P = 105°$ and PR + QR = 8 cm.

30. Construct a triangle whose perimeter is 11.9 cm and base angles are 80° and 60°.

31. Construct an isosceles triangle XYZ with YZ = ZX = 8 cm. and median YT = 4 cm.

CONSTRUCTIONS

Hint & Answers

1. 25°
2. 4°
3. 120° & 150° and other suitable answers
4. two equal parts
5. 75°

HERON'S FORMULA
MIND MAPPING

A

C $\quad$ b

B $\quad$ a $\quad$ c

Semi-Perimeter = $\dfrac{a+b+c}{2}$ = S

Area of Triangle = $\sqrt{S(S-a)(S-b)(S-c)}$

Heron's Formula

Perimeter and Area of Triangles

(a) Right angle Triangle

A

c $\quad$ b

B $\quad$ a $\quad$ C

$b^2 = a^2 + c^2$

(Pythagorus Thorem)

Perimeter = a+b+c

Area = $\frac{1}{2}$ x a x c

$\quad$ = $\frac{1}{2}$ x base x height

(b) Isosceles Right Triangle

A

a $\quad$ b

B $\quad$ a $\quad$ C

$b^2 = 2a^2$

(Pythagorus Thorem)

Perimeter = 2a+ b

Area = $\frac{1}{2}$ a^2

(c) Equilateral Triangle

A

a $\quad$ h $\quad$ a

B $\quad$ a/2 $\quad$ a/2 $\quad$ C

Perimeter = 3a

Height = $\dfrac{\sqrt{3}}{2}$ a

Area= $\frac{1}{2}$ × a X h

$\quad$ = $\frac{1}{2}$ ax $\frac{\sqrt{3}}{2}$ a

$\quad$ = $\frac{\sqrt{3}}{4} a^2$

CHAPTER-12
HERON'S FORMULA
KEY POINTS

- Rectangle : If length and breadth of a rectangle is 'l' and 'b' respectively then

    (i)   Perimeter of rectangle = 2 (l + b) units

    (ii)   Area of rectangle = l x b sq. units

    (iii) Diagonal of rectangle = $\sqrt{l^2 + b^2}$ units
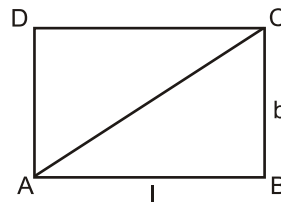
- Square : If 'a' is the length of side of a square

    (i)   Perimeter of square = 4a units

    (ii)   Area of square = (side)^2 = (a)^2 sq. units

    (iii) Area of square = $\frac{1}{2}$ x (diagonal)^2
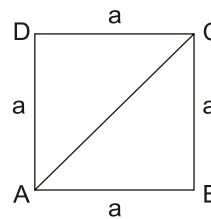
- Triangle :

    (A) Equilateral Triangle : In this triangle all three sides are equal. If the length of each side is 'a' then

        (i)   Perimeter = 3a units

        (ii)   Altitude = $\frac{\sqrt{3}}{2}$ a units

        (iii) Area = $\frac{\sqrt{3}}{4}$ a^2 or $\frac{\sqrt{3}}{4}$ (side)^2 sq. units
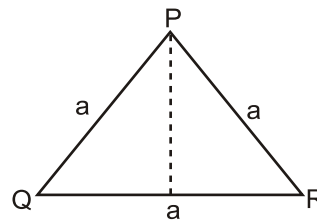
    (B) Right Angled Triangle : If one of the angles of a triangle is 90°.

        (i)   Hypotenuse k = $\sqrt{b^2 + h^2}$ units

        (ii)   Perimeter = b + h + k units

        (iii) Area = $\frac{1}{2}$ x b x h sq. units
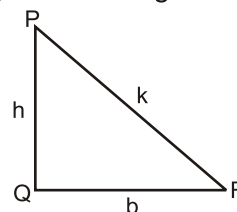
        Area of triangle (General Formula)

        = $\frac{1}{2}$ x base x Corresponding Altitude

        = $\frac{1}{2}$ x b x h sq. units
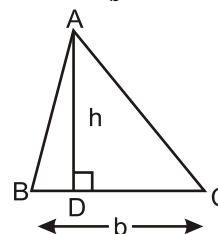
HERON'S FORMULA

- If the sides of triangle are a, b and c

    (i) Perimeter = a + b + c

    (ii) Semi Perimeter (S) = $\dfrac{a+b+c}{2}$

    (iii) Area of Triangle ($\triangle ABC$) = $\sqrt{s(s-a)(s-b)(s-c)}$
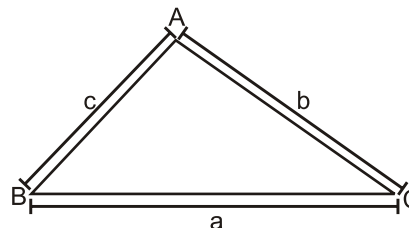
    Note : Heron's formula is applicable to all types of triangles.

- Area of Parallelogram : If a is the length and b is breadth of a parallelogram and h be the height or perpendicular distance between two parallel sides then

    Area of parallelogram (ABCD)

    = Base x Corresponding Height

    = AB x DE

    = a x h sq. units

    Area of $\triangle ABC$ = $\dfrac{1}{2}$ x Area of Parallelogram
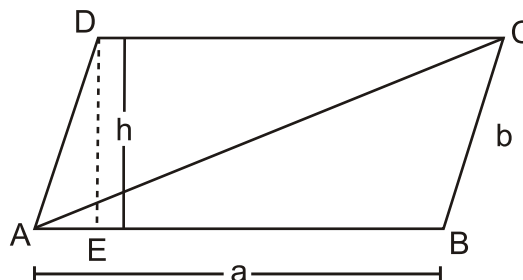
- Area of Trapezium : Trapezium with parallel sides a and b and the perpendicular distance between two parallel sides as h.

    Area of trapezium

    = $\dfrac{1}{2}$ x (a + b) x h

    = $\dfrac{1}{2}$ x (sum of parallel sides) x height
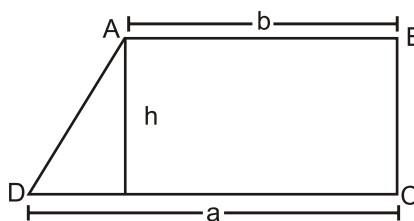
Very-Very Short Answer Type Questions (1 Mark)

1. Find the areas of a triangle whose base and attitude are 6cm and 3cm respectively.

2. The area of a triangle of base 35cm is 420 sq.cm. Find its altitude.

3. Find the area of a triangle whose base is 15cm long and the corresponding height is 9.8cm.

4. Find the area of an equilateral triangle with side $2\sqrt{3}$cm.

5. Find the area of an equilateral triangle of side 'a' units.

6. Find the area of an isosceles triangle each of whose equal sides is 13cm and base is 24cm.

7. The height of an equilateral triangle is 6cm. Find its side.

8. Find the semi-perimeter of an equilateral triangle of side 2a units.

9. For an isosceles triangle having base b and each of equal side a, find its perimeter.

10. Two sides of a triangle are 8cm and 11cm and perimeter of it is 32cm. Find the length of third side.

11. Find the length of the diagonal of a rectangle whose length is 'a' units and breadth is 'b' units.

12. Find the area of a regular hexagon, whose side is 'a' units.

13. Find the area of a parallelogram whose base is 12cm and altitude is 5.5cm.

14. The base and the corresponding altitude of a parallelogram are 10cm and 8.5cm respectively. Find the area of the parallelogram.

15. If the length of a median of an equilateral triangle is x cm, find its area.

16. If the sides of an equilateral triangle are tripled, then find its new area.

17. Each side of an equilateral triangle is $2x$ cm. If $x\sqrt{3} = 48$, then find its area.

18. If the sides of a triangle are 13cm, 14cm and 15cm. Then find the area of the triangle.

19. Find the area of a trapezium whose parallel sides are 25cm and 13cm long and distance between them is 8cm.

20. Find the area of the triangle having perimeter 32cm, one side 11cm and difference of other two sides is 5cm.

CASE STUDY BASED QUESTION (4 MARKS)

21. Sarla Devi has a triangular field with sides 240m, 200m and 360m, where

she grew wheat. In another triangular field with sides 240m, 320m and 400m adjacent to the previous field, she wanted to grow potatoes and onions. For this, she divided the field in two parts by joining the mid-point of the longest side to the opposite vertex and grew potatoes in one part and onions in the other part.
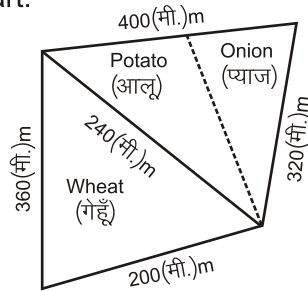
Based on the above information, answer the questions :-

(i) What is the area of the wheat field ?

a) 16000 m²   b) 16000 √2m²   c) 32000m²   d) 8000 m²

(ii) What is the area of the field used for growing potatoes ?

a) 3.8400m²   b) 38400m²   c) 38.400m²   d) 3.84m²

(iii) What is the area of the field used for growing onions ?

a) 3.84m²   b) 38.400m²   c) 38400m²   d) 3.84km²

(iv) The ratio of the areas of the fields used for growing potatoes and onions is

a) 1:1   b) 1 : 2   c) 2:1   d) 2 : 2

(v) What will be the total area of land she has in hectare?

a) 6 hectare   b) 6.1 hectare   c) 6.2 hectare   d) 6.3 hectare

22. During summer vacations, Rohit was getting bored due to lockdown in his city. Because of the COVID pandemic, he couldn't go out to play with his friends. His mother suggested him to start making some origami craft material. He learnt origami craft through internet and made a puppy as shown in figure.
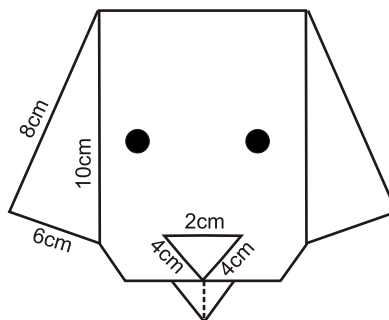
Based on the above information and measurements of different parts of the figure, answer the following questions :-

(i) What is the area of one of ears of the puppy ? (both ears are similar)

a) 20cm^2 b) 22cm^2 c) 24cm^2 d) 26cm^2

(ii) What is the area of the paper used to make nose of the puppy ?

a) 24$\sqrt{5}$cm^2 b) 12$\sqrt{5}$cm^2 c) $\sqrt{5}$cm^2 d) $\sqrt{15}$cm^2

(iii) If the tongue of the puppy is in the shape of an equilateral triangle, with side 2cm each, then what is the area of the paper used to make tongue?

a) $\sqrt{3}$cm^2 b) 3cm^2 c) 3$\sqrt{3}$cm^2 d) 9cm^2

(iv) What will be the length of the middle line of the tongue as shown in figure?

a) $\sqrt{3}$cm   b)3cm   c) 3$\sqrt{3}$cm   d) 9cm

(v) If the total area of paper used to make the puppy is 96cm^2, then find the area of paper used to make the face (except ears, nose and tongue) of the puppy.

a) (48-$\sqrt{15}$-$\sqrt{3}$)cm^2

b) (48- $\sqrt{15}$)cm^2

c) (48-$\sqrt{3}$)cm^2

d) 48cm^2

23. Vansh wanted to fly kite on the eve of Independence day. But the shops were all closed due to lockdown during COVID pandemic. His sister offered to make a kite using waste papers. Vansh agreed and they both made a beautiful Kite. The kite was in the shape of a square and an isosceles triangle. The diagonal of the square part was 40cm and the isosceles triangle was of base 6cm and sides 4cm. The diagonal was dividing the square part of the kite into two triangles. One triangle was of blue colour and the other one was of green colour. The isosceles triangle part was of yellow colour. Base on the above information, answer the following questions :-
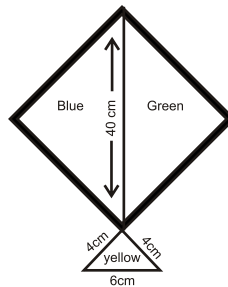
(i) What is the length of one side of the square ?

    (a) 20cm   (b) $20\sqrt{2}$cm   (c) $\sqrt{2}$cm   (d) 40cm

(ii) Vansh wanted to decorate the square part of the kite with paper strip. What length of the paper strip he had to use ?

    (a) $40\sqrt{2}$cm   (b) $80\sqrt{2}$cm   (c) 800cm   (d) 40cm

(iii) What is the area of the square part ?

    (a) $400\sqrt{2}$cm^2 (b) $800\sqrt{2}$cm^2 (c) 800cm^2 (d) 400cm^2

(iv) What is the area of the triangular part (yellow colour) ?

    (a) $14\sqrt{3}$cm^2 (b) 63cm^2 (c) $3\sqrt{7}$cm^2 (d) $7\sqrt{3}$cm^2

(v) What is the area of the blue part of the kite ?

    (a) $200\sqrt{2}$cm^2 (b) $400\sqrt{2}$cm^2 (d) 400cm^2 (d) 200cm^2

Very Short Answer Type Questions (2 marks)

24 Find the area of an equilateral triangle whose sides are 4cm each.

25. If sum of two sides of a triangle is 17cm and its perimeter is 30cm, then what is the length of third side?

26. If perimeter of a triangle is 24cm and sides are in the ratio 2:1:3, then find the longest side .

27. If each side of a triangle is double then how many times the perimeter of triangle increased ?

28. If area of a triangle is 50cm² and one of its sides is 10cm then find the length of corresponding altitude.

29. The area of an equilateral triangle is $16\sqrt{3}$ cm² then what will be the length of each side of that triangle?

30. Find the ratio between the area $\triangle ABC$ and area $\triangle ACD$ of the given rectangle.
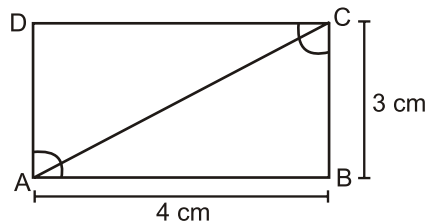
31. A square has each side of 5cm. Find the length of one of its diagonals.

32. If the length and corresponding height of a parallelogram are 10cm and 8cm then find the area of a triangle made by its diagonal.

33. The sides of a triangle are 11cm, 60cm and 61cm. Find the altitude to the smallest side.

34. The ratio between the sides of a triangle are 3 : 5 :7 and its perimeter is 300cm. Find the sides of triangle.

35. Find the area of isosceles triangle whose non equal side is of 12cm and its corresponding altitude is 7.5cm.

36. The sides of a triangular field are 51m, 37m and 20m. Find the number of flower beds that can be prepared if each bed is to occupy 9m^2 of area.

37. A rectangular plot is given for constructing a house having measurement of 40m long and 15m broad. According to the laws, a minimum of 3m wide space should be left in the front and back each and 2m wide space on each of other sides. Find the largest area, where house can be constructed.

38. Show that the Area of an equilateral triangle is $\dfrac{\sqrt{3}}{4}x^2$, where side is x.

39. Perimeter of an isosceles triangle is 32 cm. The ratio of equal side to its base is 3 : 2. Find area of this triangle.

40. The sides of a triangle are x, x+1, 2x-1 and its area is x $\sqrt{10}$ sq. units. Find the value of x.

Short Answer Type Questions (3 marks)

41. The area of a quadrilateral is 360m^2 and the perpendiculars drawn to one of the diagonal from the opposite vertices are 10m and 8m. Find the length of the diagonal.

42. The adjacent sides of a parallelogram ABCD are AB=34, BC=20cm and diagonal AC=42cm. Find the area of the parallelogram.

43. The perimeter of a triangle is 50cm. One side of a triangle is 4cm longer than the smaller side and the third side is 6cm less than twice the smaller side. Find the area of the triangle.

44. Find the area of shaded region in the figure.

    How may triangular flower beds of 6m^2

    can be made from this area?

    use $\sqrt{105}$ = 10.25
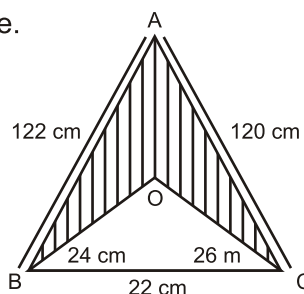
45. Find the area of rhombus whose perimeter is 100m and one of whose diagonal is 30m.

46. The sides of a triangular sheet are 5 cm, 12 cm and 13 cm. Find the cost of painting on the sheet at the rate of ₹ 30 per cm^2.

47. One side of a right angled triangle is 20 cm and the difference in lengths of its hypotenuses & other side is 8 cm. Find the other side and area of the triangle.

48. Find the ratio between the area of triangle $\triangle ABC$ and $\triangle DEF$.
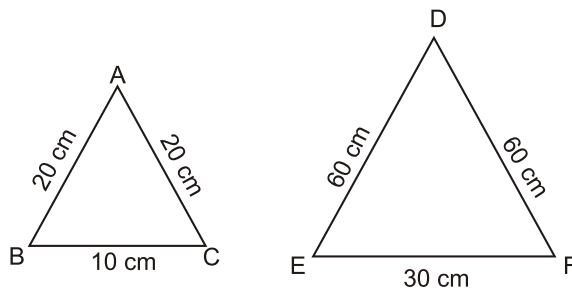
49. If perimeter of a triangle is x cm and its sides are p, q and r cm. What will be the area of triangle ? Use the Heron's formula.

50. If each side of equilateral triangle is doubled, then find the ratio of area of new triangle formed and the given triangle.

51. If every side of a equilateral triangle is doubled, then find the percent age increase in the area of the triangle.

52. The dimensions of a rectangle ABCD are 51cm x 25cm. A trapezium PBCQ with its parallel sides QC and PB in the ratio 9:8 is cut off from the rectangle as shown in figure. If the area of the trapezium PBCQ is $\frac{5}{6}$ part of the area of the rectangle, then find the lengths of QC and PB.
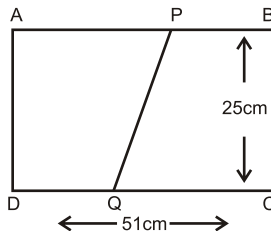
53. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 13cm, 14cm and 15cm and the parallelogram stands on the base 14cm, then find the height of the parallelogram.

54. In the given figure, $\triangle ABC$ has sides AB=7.5cm, AC=6.5cm and BC=7cm. On base BC, a parallelogram DBCE of same area as that of $\triangle ABC$ is constructed. Find the height DF of the parallelogram.
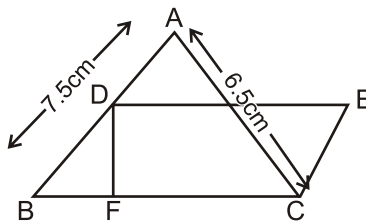
55. Find the area of the trapezium PQRS, with height PQ, which is given in the figure below.
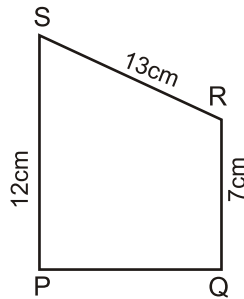
56. Anita has a piece of land which is in the shape of a rhombus. She has two children. one daughter Preetika and one son Naresh to work on the land and produce different types of crops. She divided the land in two equal parts. If the perimeter of the land is 200m and one of the diagonal is 70m then

(i) How much area will each of them get for their crops ?

(ii) Anita fenced the land by wires. How much wire is required to fence the land ?

57. How much paper of each shade is needed to make a kite given in the figure in which ABCD is a square with diagonal 44cm ?
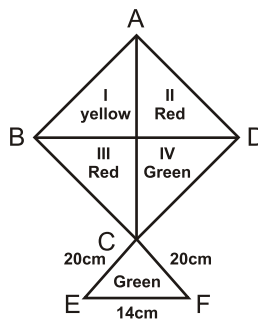
58. While selling clothes for making flags, a shopkeeper claims to sell each piece of cloth in the shape of an equilateral triangle of each side 10cm while actually he was selling the same in the shape of an isosceles triangle with sides 10cm, 10cm and 8cm. How much cloth was he saving in selling each flag ?

59. A piece of land is in the shape as given in the figure, has been cut along diagonal AC. The two pieces of land has been distributed between Ram and Sohan. Who will get larger piece of land in terms of area ? [Use $\sqrt{10}=3.15$]
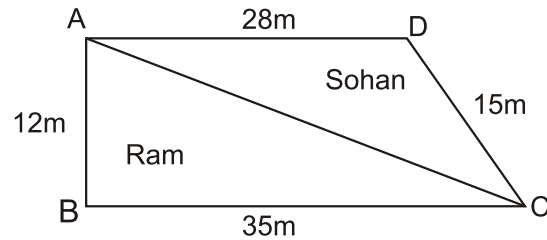
60. A triangular hoarding of dimensions 11m, 6m and 15m is used for commercial activities. The hoarding yield an earning of ₹ 5000 per m² per month.

Calculate the total earning by the hoarding in a month. [Use $\sqrt{2} = 1.41$]

CHAPTER-12
HERON'S FORMULA
ANSWERS

1. 9 sq cm
2. 24cm
3. 73.5 sq. cm
4. $3\sqrt{3}$ sq. cm
5. $\dfrac{\sqrt{3}}{4} a^2$ sq. units

6. 60 sq. units
7. $4\sqrt{3}$ cm
8. 3a units
9. 2a + b
10. 13cm
11. diagonal $= \sqrt{a^2 + b^2}$ units
12. $\dfrac{3}{2}\sqrt{3}a^2$
13. 66 sq. cm.
14. 35 sq. cm.
15. Hint :- for an equilateral triangle, altitude = median

    Ans : $\dfrac{x^2}{\sqrt{3}}$ cm^2

16. New area = 9 x area
17. $768\sqrt{3}$ sq. cm.
18. 84 sq. cm.
19. 152 sq. cm.
20. $8\sqrt{30}$ sq. cm.
21. (i) (b) $16000\sqrt{2}$ m^2

    (ii) (b) 38400 m^2

    (iii) (c) 38400 m^2

    (iv) (a) 1 : 1

    (v) (b) 6.1 hectare
22. (i) (c) 24 cm^2

    (ii) (d) $\sqrt{15}$ cm^2

(iii) (a) $\sqrt{3}\,\text{cm}^2$

(iv) (a) $\sqrt{3}$ cm

(v) (a) $(48 - \sqrt{15} - \sqrt{3})\,\text{cm}^2$

23.   (i) (b) $20\sqrt{2}\,\text{cm}$

(ii) (b) $80\sqrt{2}\,\text{cm}$

(iii) (c) $800\,\text{cm}^2$

(iv) (c) $3\sqrt{7}\,\text{cm}^2$

(v) (c) $400\,\text{cm}^2$

24. $4\sqrt{3}\,\text{cm}^2$

25.   13cm

26.   12cm

27.   2 times

28.   10cm

29.   8cm

30.   1 : 1

31. $5\sqrt{2}\,\text{cm}$

32. $40\,\text{cm}^2$

33.   60cm

34.   60cm, 100cm, 140cm

35. $45\,\text{cm}^2$

36.   Hint: No of flower beds $= \dfrac{\text{Area of field}}{\text{Area of 1 Flower Bed}}$

Ans : 34

37. $374\,\text{m}^2$

Hint :

Find area of EFGH
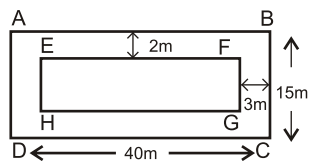
39. $32\sqrt{2}\,\text{cm}^2$

40.   6

41.   40m

42. $672\,\text{cm}^2$

43.   Hint : Let the length of smallest side = x m

        ∴ Other two sides will be x + 4 and 2x - 6

        ∴ Perimeter of triangle = x + x + 4 + 2x - 6

             50 = 4x - 2

             x = 13

        ∴ sides are 13, 17, 20cm

        Ans = 109.6 cm^2

44.   1047m^2, 179

45.   600m^2

46.   ₹900

47.   Hint :

        Let given side 'a' = 20

        hypotenuse = b

        other side = c

        $a^2 = b^2 - c^2$

        $a^2 = (b-c)(b+c)$

        $20^2 = 8 \times (b+c)$

        $\frac{400}{8} = b + c$

        b+c = 50

        <u>b-c = 8 (given)</u>

        2b = 58

        b = 29

        ∴ c = 21

        a=20, b=29, c=21

        Area = 210cm^2

48.   1 : 9

49. $\sqrt{\frac{x}{2}\left(\frac{x}{2}-p\right)\left(\frac{x}{2}-q\right)\left(\frac{x}{2}-r\right)}$

50.   4:1

51.   300% [Hint : Increase in area

        of triangle = Area of new triangle – Area of given triangle

        Percentage Increase = $\frac{\text{Increased Area}}{\text{Initial Area}}$ x 100

52.   Hint :

Area of trapezium = $\dfrac{5}{6}$ Area of rectangle .... (i)

Also Area of trapezium = $\dfrac{1}{2}$ x $\begin{pmatrix}\text{Sum of} \\ \text{parallel} \\ \text{sides}\end{pmatrix}$ X $\begin{pmatrix}\text{distance} \\ \text{between} \\ \text{parallel} \\ \text{sides}\end{pmatrix}$ .... (ii)

Take parallel sides of trapezium as 9x & 8x and equate equations (i) & (ii)

Ans.   QC = 45cm

PB = 40cm

53.   6cm

54.   3cm

55.
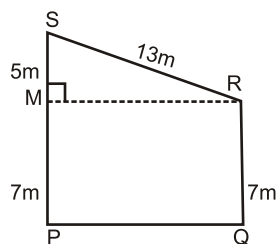
Hint : Find MR using pythagoras from $\triangle$SMR

$\Rightarrow$ MR = PQ

Then find area of trapezium

Ans. 114m^2

56.   (i) $175\sqrt{51}$cm^2

(iii) 200m

57.   Area of red paper = 484cm^2

Area of yellow paper = 242cm^2

Area of green paper = 373.14cm^2

58.   6.61cm^2

59.   Ram, 210m^2

60.   ₹141000

PRACTICE TEST

HERON'S FORMULA
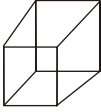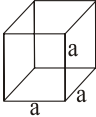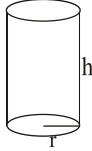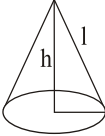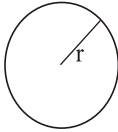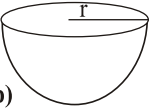
Time : 1 hour   M.M. 25

1. Find the length to sides of an equilateral triangle having area $a\sqrt{3}\ cm^2$.   1

2. If (s – a) = 5 cm, (s – b) = 10 cm, (S – C) = 1 cm. Find S.   1

3. The sides of triangle are 35cm, 54cm and 61cm. Find the length   1
   of its longest altitude.

4. Find the area of isosceles triangle whose equal sides are of length 15 cm each & the third side is 12 cm.   2

5. If each side of triangle is doubled, then find the ratio of area of new triangle thus formed & the given triangle.   2

6. A triangular park ABC has sides 120m, 80m and 50m. A gardner has planted some trees inside the park leaving 5m width along each side of park. Find the area in which he planted the trees.   2

7. The sides of a triangle are in the ratio 25 : 17 : 12 and its perimeter is 540 cm. Find the area of the triangle.   3

8. The area of trapezium is 475 cm^2 & height is 19 cm. Find length of its parallel sides if one side is 4 cm greater than the other.   3

9. The length of sides of a triangle are 7 cm, 12 cm & 13 cm. Find the length of perpendicular from opposite vertex to the side whose length is 12 cm.   5

10. The cost of fencing a field @ ₹ 5 per metre is ₹ 1920. If semi perimeter is 48 cm, find its area & all sides.   5

SURFACE AREAS AND VOLUMES

KEY POINTS

S. No.	Name	Figure	Lateral/ Curved Surface Area	Total surface Area	Volume	Symbols used for	
1.	Cuboid		$2(l+b) \times h$	$2(lb+bh+hl)$	lbh	l =Length b=breadth h=height	[Diagonal of cuboid $=\sqrt{l^2+b^2+h^2}$ Diagonal $=\sqrt{3} \times$ side
2.	Cube		$4a^2$	$6a^2$	a^3	a=side	
3.	Right Circular Cylinder		$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$	h=height r=radius of base	
4.	Right Circular Cone		πrl	$\pi r(l+r)$	$\dfrac{1}{3}\pi r^2 h$	h=height r=radius of base	
5.	Sphere		$4\pi r^2$	$4\pi r^2$	$\dfrac{4}{3}\pi r^3$	r = radius	
6.	Hemisphere Solid		$2\pi r^2$	$3\pi r^2$	$\dfrac{2}{3}\pi r^3$	r = radius	
7.	Hemisphere hollow **(Without top)**		$2\pi r^2$	$2\pi r^2$	$\dfrac{2}{3}\pi r^3$	r = radius	

- $1m^3 = 1000$ L
- $1l = 1000$ cm^3

Very Very Short Answer questions (1 mark)

1. If the volume of a sphere is numerically equal to its surface area then find the radius of the sphere.
2. Find the surface area of a solid hemisphere having radius r.
3. In a cylinder, if radius is halved and height is doubled then what will be the change in volume ?
4. Find the height of a cone of diameter 10cm and slant height 13cm.
5. The radius of a hemispherical balloon increase from 6cm to 12cm as air is being pumped into it. Find the ratios of the surface areas of the balloon in the two cases.
6. How many bricks will be required to construct a wall 13.5cm long; 6cm high and 22.5cm thick if each brick measures (27cm x 12.5cm x 9cm)?
7. If the radius of a sphere is 2r, then find its volume.
8. The radius of a sphere is 21cm. What is the surface area of the sphere?
9. What will be the length of the longest pole that can be put in a room of dimensions 10m x 10m x 5m?
10. A copper sphere of diameter 6cm is melted and drawn into a 36cm long wire of uniform circular cross-section. Find the radius of the wire.
11. Find The number of planks of dimension (4m x 50cm x 20cm) that can be stored in a pit which is 16m long, 12m wide and 4m deep .
12. The radius if a sphere is increased by 10%. Find the percentage increase in its volume.
13. In a cylinder, If radius is doubled and height is halved, then what will be the change in its volume?
14. Two cubes have their volumes in the ratio 1:27. Find the ratio of their surface areas.
15. A cone is 8.4cm high and the radius of its base is 2.1cm. It is melted and recast into a sphere. Find the radius of the sphere.
16. If the length of diagonal of a cube is 873cm then find its surface area.
17. The total surface area of a cube of 96cm. Find the volume of this cube.
18. If each side of a cube is doubled then find the change in its volume.
19. If a sphere is inscribed in a cube then find the ratio of the volume of the cube to the volume of the sphere.
20. If each edge of a cube is increased by 50% then find the percentage increase in its surface area.
21. The lateral surface area of a cube is 256 cm^2. Find its volume.
22. A matchbox measures 4cm x 2.5 cm x 1.5 cm. What will be the volume of a packet containing 12 such boxes ?

23. The ratio of height of two cylinders is 5 :3 as well as the ratio of their radii is 2 : 3. Find the ratio of the volumes of the cylinders.

24. Find the area of canvas required for a conical tent of height 24m and base radius 7m.

25. Find the ratio of total surface area of a sphere and a hemisphere of same radius.

26. The surface area of the cuboid is 1372 sq. cm. If its dimensions are in the ratio of 4:2:1 then find its length.

27. If the radius and slant height of a cone are r/2 and 2l then find its total surface area.

28. A cone and a hemisphere have equal base and equal volumes. Find the ratio of their heights.

29. The radius of a spherical balloon increases from 6cm to 12 cm as air is being pumped into it. Find the ratio of the surface areas of the balloon in two cases.

30. The largest possible right circular cone is cut out of a cube of edge r cm. What is the volume of cone ?

Case Study based Questions :-

31. Sana wanted to gift 3 cube puzzles to her friend on her birthday. Three cubes each of side 5cm are joined end to end. She arranged the cubes joining them end to end (as shown in figure).
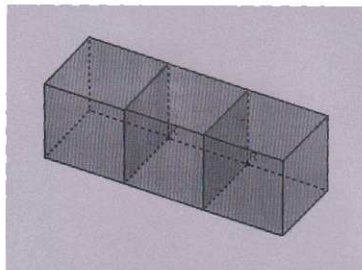
(i) The areas of the gift paper required to wrap the resulting shape if there is no overlapping of paper is :

   (a) 350cm^2 (b) 750cm^2 (c) 530cm^2 (d) 1000cm^2

(ii) The space occupied by this resulting shape is :

   (a) 573cm^2 (b) 370cm^2 (c) 375cm^2 (d) 570cm^2

(iii) The ratio of the total surface area of the new cuboid to that of the sum of the surface areas of the three cubes is :

   (a) 9 : 7   (b) 1:1   (c) 1:9   (d) 7:9

(iv) The ratio of the volume of new cuboid to that of the sum of the volumes of the three cubes is :

(a) 1:1   (b) 1:2   (c) 2:1   (d) 2:3

(v) Find total surface area of four such cubes joined end to end.

(a) 450cm^2 (b) 350cm^2 (c) 475cm^2 (d) 550cm^2

32. Rashmi wants to construct the hut given in the picture.
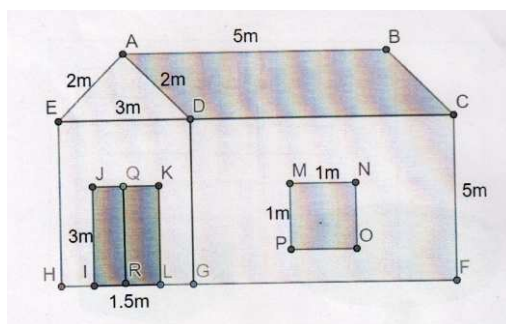
(i) She uses a tape on all joints of roof (such as AB, BC, CD, AD, and back side also) as well as the walls (such as DG, CF, EH and the black side). Find the length of the tape required.

(a) 42m   (b) 43m   (c) 34m   (d) 40m

(ii) The door is covered with tiles of measures 10cm by 15cm. How many such tiles are required to cover the door from both its sides ?

(a) 600 tiles   (b) 100 tiles   (c) 500 tiles   (d) 60 tiles

(iii) Find the area of the four walls to be white washed.

(a) 70.5m^2 (b) 75.4cm^2 (c) 74.5m^2 (d) 47.5m^2

(iv) Find the cost of white washing the four walls at the rate of Rs. 12 per m^2.

(a) Rs 894   (b) Rs 800   (c) Rs 900   (d) Rs. 984

(v) If the roof (ABCD and the back sides) is covered with tiles of size 20cm by 20m the find the number of tiles required.

(a) 100 tiles   (b) 500 tiles   (c) 400 tiles   (d) 540 tiles

33.

In the given picture, there are two windows EFGH and IJKL on the wall. MN, OP, QR, ST, UV, WZ are rods of the windows. A box and a chair with round seat are also shown.

(i) If the length of each rod is 6 units then the total length of the rods in two windows will be-

(a) 60 units   (b) 36 units   (c) 30 units   (d) 18 units

(ii) If the distance between two rods is 0.8 units then the height of each window will be-

(a) 3.2 units   (b) 4 units   (c) 2.4 units   (d) 5 units

(iii) Volume of the cuboidal box will be -

(a) 42 cubic units

(b) 48 cubic units

(c) 24 cubic units

(d) 52 cubic units

(iv) Total Surface area of the top seat of round chair if its height is 1 unit, will be-

(a) 1320 sq units

(b) 1230 sq units

(c) 2310 sq units

(d) 1200 sq units

(v) Find the area of wall leaving the two windows, if the length of the wall is 25 units and its breadth is 20 units.

(a) 500 Sq. Units   (b) 961.6 Sq. Units

(c) 960 Sq. Units   (d) 38.4 Sq. Units

Very Short answer type questions (2 marks)

34. A rectangular sheet of dimension 33 cm x 18 cm is rolled along its breadth to form a cylinder. Find the radius of the cylinder.

35. A roller 1.5 m long has a diameter of 70 cm. How many revolutions will it make to level a play ground measuring 50 m x 33 m ?

36. The dimensions of a cuboid are in the ratio of 1 : 2 : 3 and its total surface area is 88m^2. Find its dimensions.

37. A solid cylinder has a total surface area of 231 cm^2. The curved surface area is 2/3 of the total surface area. Find the volume of cylinder.

38. The total surface area of a cube is 150sq. cm. Find the perimeter of any one of its faces.

39. Three metal cubes whose edge measures 3cm, 4cm and 5cm respectively are melted to form a single cube. Find the edge of the cube.

40. The length, breadth and height of room are 5m, 4m and 3m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of ₹ 7.50 per m^2.

41. Three spheres of radii 3cm, 4cm and 5cm are melted together to form a single sphere. Find the radius of new sphere.

42. The curved surface area of a cylinder is 176 cm^2 and its base area is 38.5 cm^2. Find the volume of the cylinder.

43. A cylinder and a cone have the same height and the same radius. The volume of the cylinder is 24cm^3. What will be the volume of the cone ?

44. What is the volume of the largest cone that can be inscribed completely in a hollow hemisphere of radius 7 cm?

45. Find the maximum length of the rod that can be placed in a cuboid of dimesions 22.5 cm x 7.5 cm x 10 cm .

46. Which is false in case of a hollow cylinder? Write the correct answer.
    (a) curved surface area of a hollow cylinder = $2\pi h$ (R + r)
    (b) Total surface area of a hollow cylinder = 2π (R +r) (h+R-r)
    (c) Inner curved surface area of a hollow cylinder = $2\pi h$ (R -r)
    (d) Area of each end of a hollow cylinder = π (R^2 -r^2)

47. Which is false ? Write the correct answer.
    A metal pipe is 63cm long. Its inner diameter is 4 cm and the outer diameter is 4.4 cm. Then
    (a) Its inner curved surface area = 792 cm^2
    (b) Its outer curved surface area = 871.2 cm^2
    (c) Surface area of each end = 2.64 cm^2
    (d) Its total surface area = 1665.84 cm^2

48. Which is false ? Write the correct answer.
    (a) Volume of the hollow sphere = $\frac{4}{3}\pi$(R^3-r^3)
    (b) Volume of a hemisphere = $\frac{2}{3}\pi r^3$
    (c) Total surface area of a hemisphere = $3\pi r^2$
    (d) Curved surface area of a hemisphere = πr^2

49. Which is false ? write the correct answer.
    For a right circular cylinder of base radius = 7cm and height = 14 cm.
    (a) curved surface area = 616cm^2
    (b) Total surface area = 924 cm^2
    (c) Volume = 2156cm^3
    (d) Total area of the end face = 154 cm^2

50. Write true or false.

The largest possible right crcular cone is cut out of a cube of edge r cm. The volume of the cone is $\frac{1}{12}\pi r^3$. (T/F)

Short Answer Type Questions (3 Marks)

51. A cuboidal vessel is 10m long and 8m wide. How high must it be made to hold 380m^3 of a liquid ?

52. A wall of length 10m was to be built across an open ground. The height of the wall is 4m and thickness of the wall is 24cm. If this wall is to be built up with bricks whose dimensions are 24cm x10cm x 8cm, how many bricks would be required ?

53. 1.1 cm^3 of gold is drawn into a wire of 0.1 mm in diameter. Find the length of the wire in metre.

54. A hemispherical bowl of internal diameter 36cm contain a liquid. This liquid is to be filled in cylindrical bottles of radius 3cm and height 6cm. How many bottles are required to empty the bowl ?

55. Find the lateral curved surface area of a cylindrical petrol storage tank that is 4.2m in diameter and 4.5m high. How much steel was actually used if 1/12 of steel actually used was wasted in making the closed tank?

56. Water in a canal, 30 dm wide and 12 dm deep is flowing with a speed of 20 km per hour. How much area will it irrigate in 30 min if 9 cm of standing water is desired ? (10dm=1m)

57. The radius of a sphere is 10 cm. If the radius is increased by 1cm then prove that volume of the sphere is increased by 33.1%.

58. The diameter of a hemisphere is decreased by 30%. What will be the percentage change in its total surface area ?

59. A sphere and a cube have the same surface area. Find the ratio of their volumes.

60. The volume of a sphere is 4851 cm^3. How much should its radius be reduced so that its volume becomes $\frac{4312}{3}$ cm^3 ?

61. A semicircular sheet of paper of diameter 14 cm is bent to form an open conical cup. Find the capacity of the cup.

62. If c, t and v are curved surface area, total surface area and volume of a cylinder then show that
   $th^2 = ch^2 + 4v^2 + 8v^2rh$ where r is radius and h is height.

Long Answer Type Questions (5 Marks)

63. A cuboidal closed tank can store 5040 litres of water. The external dimensions

of the tank are 2.2m x 1.7m x1.7m. If the wall of the tank are 5 cm thick, then what is the thickness of the bottom of the tank ?

64. A metallic sheet is of the rectangular shape with dimensions 48cm x 36cm. From each one of its corners, a square of 8cm is cut off and an open box is made of the remaining sheet. Find the volume of the box.

65. A right triangle having side 6cm, 8cm and 10cm is revolved about the side of length 8cm. Find the volume of the solid so formed.

66. A right circular cone is 5.4 cm high and radius of its base is 2cm. It is melted and recast into another right circular one with radius of base as 1.5 cm. Find the height of new cone formed.

67. A cylindrical tub of radius 12cm contains water to the depth of 20cm. A spherical ball is dropped into the tub raising the level of water by 6.75cm. What is radius of the ball ?

68. A cylinder is within the cube touching all the vertical faces. A cone is inside the cylinder. If their height are the same with the same base find the ratio of their volumes.

69. A plot of land is in the form of rectangle with dimension 240m x 180m. A drainlet 10m wide is dug around it (on the outside), and the earth dug out is evenly spread out over the plot increasing its surface level by 25cm. Find the depth of the drainlet.

70. A residential colony has a population of 5400 and 60 litres of water is required per person per day. For the effective utilization of rain water, a group of people decided to do the WATER HARVESTING. They constructed a water reservoir measuring 48m x 27m x 25m to collect the rain water.

    For how many days the water of this tank is sufficient if during rain the height of water level is 5m?

71. 50 students of class IX planned a visit to an old age home and to spend the whole day with its inmates. Each one prepared a cylindrical flower vase using card board to gift the inmates. The radius of cylinder is 4.2cm and the height is 11.2 cm.

    What is the amount spent for purchasing the card board at the rate of 20 per $100 \, m^2$?

72 . Rahul wanted to make a temporary shelter for street dogs, by making a box like structure with tarpaulin that covers all the four sides and the top of the house. How much tarpaulin would be required to make the shelter of height 2.5 m with base dimensions 4m x 3m. Assuming stiching margin is negligible.

73. Twenty Seven solid iron spheres each of radius r and surface area S are melted to form sphere with surface area s'. Find the

(i) radius R of the new sphere.

(ii) Ratio of S and S'.

74. The diameter of a metallic ball is 4.2cm. What is the mass of the ball, if the density of the metal is 8.9g per cm³.

75. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior.

The diameter of the pencil is 7mm and the diameter of the graphite is 1mm. If the length of the pencil is 14cm, find the volume of the wood and that of the graphite.

76. A soft drink is available in two packs. (i) a tin can with a rectangular base of length 5cm and width 4cm having a height of 15cm and (ii) a plastic cylinder with circular base of diameter 7cm and height 10cm. Which container has greater capacity and by how much ?

77. A bus stop is barricaded from the remaining part of the road by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40cm and height 1m. If the outer side of each of the cone is to be painted and the cost of painting is ₹ 12per m², What will be the cost of painting all these cones ? (Use π=3.14 and √1.04 = 1.02)

78. A sphere of diameter 6cm is dropped in a right circular cylinder vessel partly filled with water. The diameter of the cylindrical vessel is 12cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel ?

79. Marbles of diameter 1.4cm are dropped into a cylindrical beaker of diameter 7cm containing some water. Find the number of marbles that should be dropped into the beaker so that the water level rises by 5.6cm.

80. Right circular cylinder having diameter 12cm and height 15 cm is full of ice-cream. The Ice-Cream is to be filled in cones of height 12cm and diameter 6cm having a hemispherical shaped on the top. Find the number of such cones which can be filled with Ice-Cream.

81. A toy is in the form of a cone mounted on a hemisphere of diameter 7cm. The total height of the toy is 14.5 cm. Find the volume and the total surface area of the toy. (Take $\pi = \frac{22}{7}$)

82. If h, c and v respectively, are the height, the curved surface and volume of the cone, prove that

$3\pi vh^3 - c^2h^2 + 9v^2 = 0$

83. A wooden box with dimensions 36 cm x 24 cm x 12 cm is 2cm thick. Find the weight of the wood if density of the wood is 100 gm/m³.

84. A rectangular reservoir is 210m long and 75m wide. Water is flowing into it through a square pipe of side 25 cm such that water rises to 3.5 m in 15 hours. Find the speed of the water.

85. A hemispherical bowl is to be painted from inside at the rate of Rs. 20 per 100m². The total cost of painting is Rs. 30.80. Find
    (i) Inner surface area of the bowl.
    (ii) Volume of air inside the bowl.

86. The volumes of the two spheres are in the ratio 64 : 27. Find the ratio of their surface areas.

87. A cube of side 4cm contains a sphere touching its sides. Find the volume of the gap in between.

88. A sphere and a right circular cylinder of the same radius have equal volumes. By what percentage does the diameter of the cylinder exceeds its heights ?

CHAPTER-13
SURFACE AREAS AND VOLUMES
ANSWERS

1. 3 Units

2. $3\pi r^2$

3. halved

4. 12cm

5. 1:4

6. 6000

7. $\dfrac{32}{3}\pi r^2$

8. $5544cm^2$

9. 15m

10. 1 cm

11. 1920

12. 33.1%

13. sam

14. 1:9

15. 2.1cm

16. $384cm^2$

17. $64cm^3$

18. becomes 8 times

19. $6:\pi$

20. 125%

21. $512\,cm^2$

22. $180cm^2$

23. 20:27

24. $550m^2$

25. 4:3

26. 28cm

27. $\pi r\,(l+r/4)$

28. 2:1

29. 1:4

30. Radius of cone = R = $\dfrac{r}{2}$

    Height of cone = h = r

    Ans. $\dfrac{1}{12}\pi r^3$
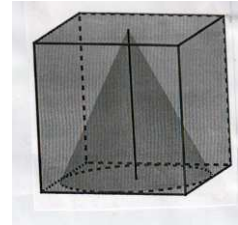
31. (i)   (a) $350cm^2$

    (ii) (c) $375cm^2$

    (iii) (d) 7 : 9

    (iv) (a) 1:1

    (v)   (a) $450cm^2$

32. (i)   (b) 43cm

    (ii) (a) 600 tiles

    (iii) (c) $74.5m^2$

    (iv) (a) Rs 894

    (v)   (b) 500 tiles

33. (i)   (b) 36 units

    (ii) (a) 3.2 units

    (iii) (c) 24 cubic units

    (iv) (a) 1320 sq. units

    (v)   961.6 sq. units

35. 500

37. $269.5\,cm^2$

39. Vol. of new single cube = Sum of volumes of 3 cubes.

    $v = s^3 = (3)^3 + (4)^3 + (5)^3$

    Ans - 6cm

41. 6cm

43. $8cm^3$

45. In rt $\triangle$BCD

    $BC^2 = (10)^2 + (7.5)^2$

    In rt $\triangle$ABC
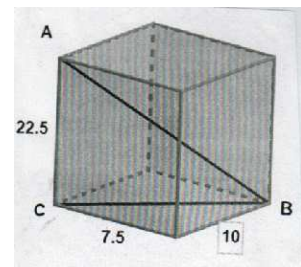
$AB^2 = AC^2 + BC^2$

$AB^2 = (22.5)^2 + (10)^2 + (7.5)^2$

$AB = 25.7 cm$

47. (d) $1665.84 cm^2$, $1668.48 cm^2$

49. (d) $154 cm^2$, $308 cm^2$

51. $4.75 cm$ $\dfrac{1}{2}$ $\dfrac{30}{100}$

53. $140 m$ $\dfrac{7}{10}$

55. $59.4 m^2$, $95.04 m^2$

58. Decreased Radius = $\dfrac{3\pi r^2 - 3\pi \left(\frac{7}{10}r\right)^2}{[2r\,3\pi r^2 \ x\ 2r]} \ x\ 100$

             =   r

% change in TSA   =

Ans. 51%

60. $3.5 cm$

61. Circumference of the base of the conical cup =

Length of the arc of the semi-circle

Ans. $79.2\ cm^3$

63. Let x be thickness of bottom & top

vd - $5040\ x\ 1000 cm^3$ = (220-5-5) (170-5-5) (170-x-x)

Ans. $10 cm$

64. $5120 cm^3$

65. $96\pi cm^3$

66. $9.6 cm$

67. $9 cm$

68. $V_1 : V_2 : V_3 = 42 : 33 : 11$

69. $1.227\ m$

70. 20 days

71. ₹3511.20

72. $47 m^2$

73. (i) $R = 3r$ (ii) s:s' = 1:9

74. $345.39 g$

75. $5.28 cm^3$, $0.11 cm^3$

Solution : Here the pencil is in the form of a cylinder which is made up of an outer cylinder of wood (radius "R") and an inner cylinder of graphite (radius "r") Height of the pencil is 14cm.

Therefore h = 14cm,

R = $\frac{7}{2}$ = 3.5 mm = 0.35cm and r = $\frac{1}{2}$ mm = 0.5mm = 0.05cm

Volume of graphite = V_G = $\pi r^2 h$ = $\frac{22}{7}$ x 0.05 x 0.05 x 14 = 0.11cm^3
Volume of wood = Total volume of the pencil - volume of graphite
V_w = $\pi R^2 h - \pi r^2 h$ = h (R^2 - r^2) = $\frac{22}{7}$ x 14 x [(0.35)^2) – (0.05)^2]
                    = 22 x 2 x 0.4 x 0.3 = 5.28cm^3
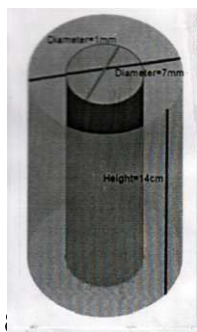
76. Plastic Cylinder
77. 384.34
78. 1cm
79. Hint
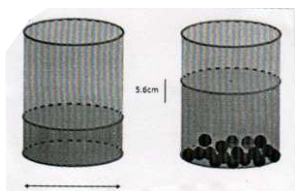
No. of marbles

= $\frac{\text{Vol. of water raised}}{\text{Vol. of 1 marble}}$

Ans :- 150

80. 10
81. 231cm^3, 204.05cm^2
83. Volume of wood = External Vol.- Internal Vol.
    Ans. 0.5248g
84. Vol of water in reservoir collected in 15 hours
    = 210 x 75 x 3.5m^3
    ATQ   210 x 75 x 3.5 =
    Where x km/h be the speed of water or 1000x m/h
    Ans. 58.8 km/hour $\frac{1}{4}$ x $\frac{1}{4}$ x 15000x
85. (i) 154m^2
    (ii) 251.5 m^3
86. 16:9
87. 30.48cm^3
88. 50%   Hint : $\frac{4}{3}\pi r^3 = \pi r^2 h$

PRACTICE TEST

Time : 1hr.   **Surface Areas and Volumes**   M.M. 25

1. If l, b and h are the length, breadth and height of a room then what will be the total area of the four walls? (1)

2. The volume of a sphere is 310.4 cm^3. Find its radius. (1)

3. Thee sphere of radii 3cm, 4cm and 5cm are metted together to form a single sphere. Find the radius of new sphere. (1)

4. The circumference of the base of a cylinder is 30.8 cm. Its curved surface area is 289.52 cm^2. Find the height of the cylinder. (2)

5. The side of a cube is double the length of the cuboid. The breadth and height of the cuboid are half of its length. Find the ratio of the curved surface area of cube to cuboid. (2)

6. A semcircular sheet of paper of diameter 14cm is bent to form a conical cup. Find the capacity of the cup. (2)

7. The seed of a corn has dimensions 1.8 cm × 0.8 cm × 0.2 cm. The height of the corn-tube is 13.7 cm and its radius is 4.2 cm. Assuming that the corn-seeds have negligible distance between them and all seeds are of same size, find the number of seeds on the corn-tube. (3)

8. The length, breadth and height of a cuboid are increased by 30%. Find the percent increase in the total surface area. (3)

9. Ajay prepared a dish and kept it in a hemispherical bowl of 30 cm diameter. He distributed the dish in cylinder cups of diameter 15 cm and height 4 cm among his friends and himself. How many friends were with Ajay? (5)

10. A river 15 m deep 50 m wide is flowing at the rate of 2 cm per second. How many litres of water will fall from the river into the sea in 9 hours? (5)

CHAPTER-14
STATISTICS
MIND MAP
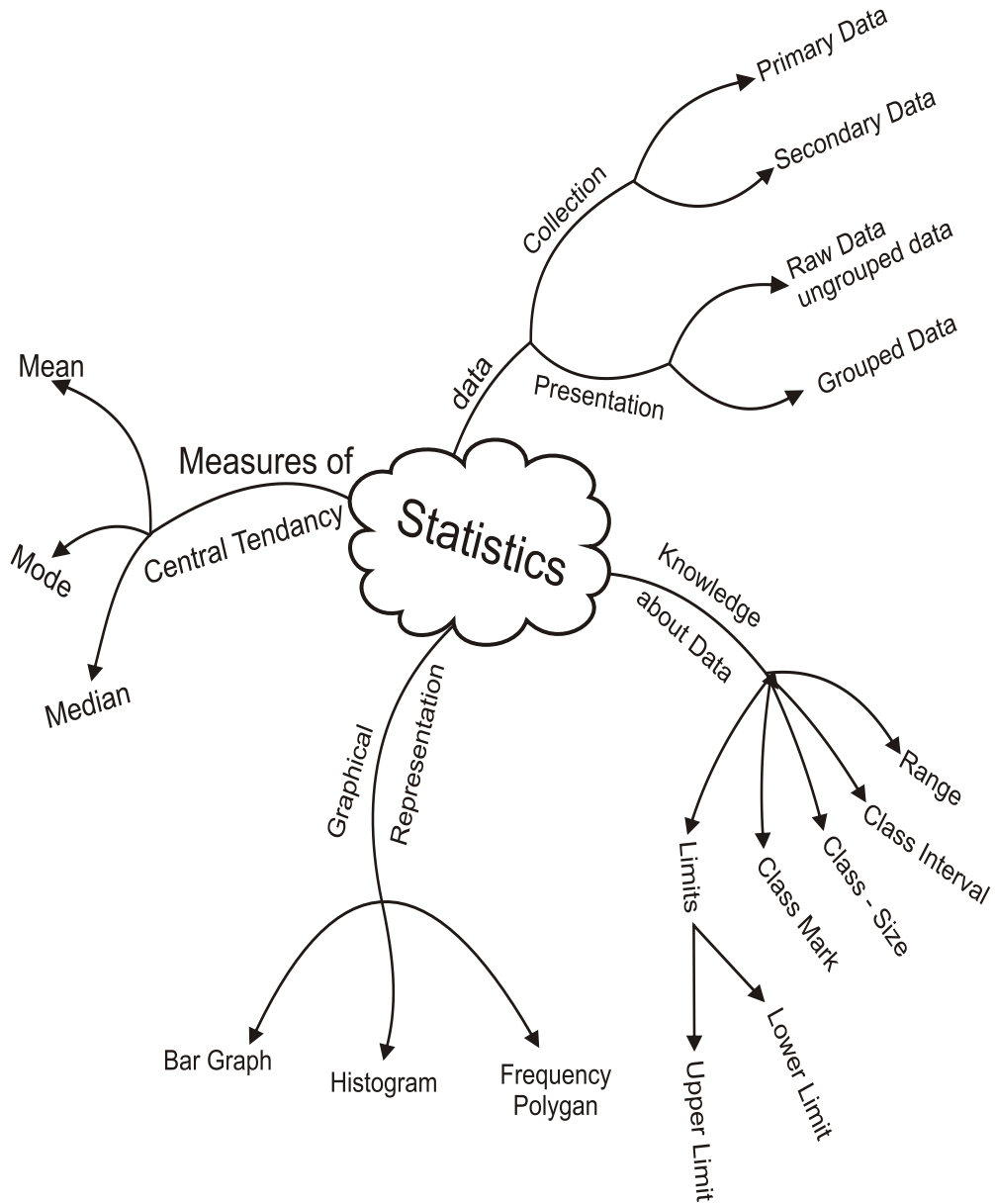
CHAPTER-14
STATISTICS
KEY POINTS

- In Statistics we study collection, presentation, analysis and interpretation of data.

- Facts or figures collected with a definite purpose are called data.

- The number of times an observation occurs in the given data is called frequency of the observation.

- Class intervals are the groups in which all observations are divided.

- For class-interval 20-30, 30 is called upper class limit and 20 is called lower class limit.

- Class mark = $\dfrac{\text{Lower class limit} + \text{upper class limit}}{2}$

- Average or mean = $\dfrac{\text{Sum of all observations}}{\text{number of observations}}$

- For raw data, mean $(\bar{x}) = \dfrac{\sum\limits_{i=1}^{n} xi}{n}$, Mean $(\bar{x}) = \dfrac{X_1 + X_2 + \dots + Xn}{n}$

- When frequency fi is given, Mean $\bar{x} = \dfrac{\sum\limits_{i=1}^{n} fi\, x_i}{\sum\limits_{i=1}^{n} fi}$

- Mode is the value of observation which occurs most frequently.

- For Median arrange the data in ascending order or descending order.

  If number of observation 'n' is odd

  Then Median $= \dfrac{(n+1)^{th}}{2}$

  If number of observation 'n' is even

  Then Median $= \dfrac{\left(\dfrac{n}{2}\right)^{th} \text{term} + \left(\dfrac{n}{2}+1\right)^{th} \text{term}}{2}$

Very-Very Short Answer Type (1 marks)

1. Facts or figures, collected with a definite purpose are called _____

2. The number of times an observation occurs in the given data is called _____ of that observation.

3. The difference between the highest and lowest values of the data is called _____ of the data

4. For class 5.5 - 15.5, 15.5-25.5, 25.5 - 35.5, Class size is _____

5. Class mark of class 7–12 is _____

6. The mean of 1, 3, 5, 7, 19, 28 is _____

7. The median of first 10 natural numbers is _____

8. The mode of 14, 25, 14, 28, 17, 18, 18, 14, 23, 25, 14, 18 is _____

9. Find the range of the data :
25, 20, 20, 19, 23, 26, 28, 22, 32, 45

10. Find the class size of the class intervals :
0-4, 5-9, 10-14, ...

11. The class marks of a continuous distribution are 3.05, 3.15, 3.25, 3.35, 3.45 and 3.55. Find the class interval corresponding to the class mark 3.35.

12. Find the mean of the prime factors of 24.

13. If the mode of the data 13, 27, 24, 13, 17, 16, 17, 21, 22, x, 13, 17 is 17 then find the value of x.

14. Find the mean of first 5 prime numbers.

15. The mean of x_1, x_2, x_3, x_4, x_5 is 10, then what is the mean of $3x_1, 3x_2, 3x_3, 3x_4, 3x_5$?

16. Write the class interval, whose class mark is 8.5 and class size is 5.

17. Write the upper limit of a class interval, whose lower limit is 13 and class mark is 15.

18. If 11 observations are arranged in descending order, which observation will be the median ?

19. In a data, 16 observations are arranged in ascending order. If the 10th observation is increased by 10, what will be the corresponding effect on the median ?

20. Find the median of first 9 whole numbers.

21. When a number a is added to each of the numbers 2, 7, 8, 12, 6, 10 and 11, the mean is 10. Calculate the value of a.

22. In a frequency distribution table, the frequencies of 3, 4 and 5 are 8, 19 and 13 respectively. Find their mean

23. Out of total of 20 observations arranged in ascending order 9th, 10th and 11th observations are 36, 40 and 44 respectively. Find the median.

24　The mean of first two observations is 6 and the mean of first three observations is 7. Find the third observation.

25.　The mean of first three observations is 15 and the mean of next two observations is 20. Find the mean of all five observations.

Case study based - problems (26 to 28)

26.　To identify the covid-19 symptomatic person, a health worker performed door to door survey. Record of one family is given below.

S.No.	Name	Gender	Age (in years)	Temperature ($^{\circ}$F)	SpO$_2$ level
1.	Roshan Lal	M	72	98.2	94
2.	Geeta Devi	F	70	98.3	95
3.	Rakesh	M	47	98.6	97
4.	Monika	F	43	98.0	97
5.	Ajay	M	16	98.4	99
6.	Ananya	F	13	98.5	98

(A)　The average temperature of the family is :
　　(a) 98.4° F (b) 98.2°F　　(c) 98.6° F　　(d) 98.3° F

(B)　The range of the ages is :
　　(a) 47 years　　(b) 59 years　　(c) 43 years　　(d) 50 years

(C) The mode of SpO2 level is :
　　(a) 97　　　　(b) 99　　　　(c) 98　　　　(d) 95

(D)　The median of the temperature is :
　　(a) 93.25° F　　(b) 98.35° F　　(c) 98.45°F　　(d) 98.55° F

(E)　The difference of average age of males and that of females is :
　　(a) 3 years　　(b) 4 years　　(c) 5 years　　(d) 6 years

<div align="center">

Case Study Based Questions

</div>

27.　During a Van Mahotsava a week, 50 trees each were planted by 25 schools. After one month, it was found that following number of trees were survived in the schools.

30	27	26	32	40
32	26	25	30	20
38	30	29	15	21
25	27	21	20	29
42	40	37	16	22

(A) The range of the data is :

(a) 27   (b) 30   (c) 25   (d) 32

(B) The mean of the data is :

(a) 27   (b) 30   (c) 25   (d) 28

(C) The median of the data is :

(a) 27   (b) 30   (c) 25   (d) 28

(D) The mode of the data is :

(a) 27   (b) 30   (c) 25   (d) 28

(E) Number of school where number of survived plant is more than their mean is :

(a) 10   (b) 12   (c) 14   (d) 16

28. A teacher keep records of various activities of his students. The record of watching television on a holiday by 40 students is shown by the following graph:
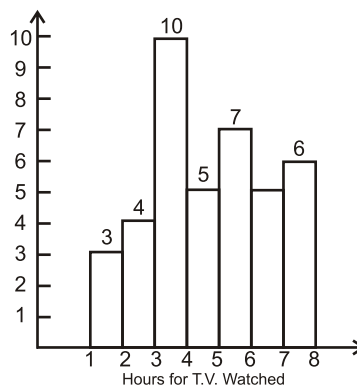
(A) Maximum number of students watched T.V. for :

(a) 3-4 hours   (b) 4-5 hours   (c) 5-6 hours   (d) 6-7hours

(B) Number of students watched T.V for less than 5 hours:

(a) 17   (b) 22   (c) 5   (d) 15

(C) The class-mark of the class 6-7 is :

(a) 6   (b) 7   (c) 6.5   (d) 1

(D) The class interval whose mark is 5.5 is:

(a) 2-3   (b) 3-4   (c) 4-5   (d) 5-6

(E) Number of students watched T.V. for more than equal to 6 hours is.

(a) 5   (b) 6   (c) 7   (d) 11

Very Short Answer Type (2 marks)

29. The mean of five numbers is 27. If one of the number is excluded, the mean gets reduced by 2. What is the value of the excluded number ?

30. Find the mode of the data 15, 14, 19, 20, 14, 15, 16, 14, 15, 18, 14, 19, 15, 17, 15, If last observation is changed to 14 then find the new mode.

31. The mean monthly salary of 40 workers of a factory is ₹ x in a particular year. Each one was given ₹ 3000 as Diwali Bonus. What will be the mean monthly salary in that month.

32. The median of following observations arranged in ascending order 14, 18 $x + 2$, $x + 4$, 30, 34 is 24. Find the value of x and hence find mean of the data.

33. For what value of x the mode of the following data : 13, 24, 13, 27, 17, 16, 17, x, 22, 21, 13, 17 is 17 ?

34. The average age of Shikha and her husband Amit is 48 years. The average age of Shikha, Amit and their daughter Advika is 39 years. Find the age of Advika.

35. The mean of 6, 10, 11, x, 12, y is 10. Also Y is 7 more than X. Find the value of x and y.

36. The weight (in kg) of 25 students are given below 35, 38, 36, 37, 38, 35, 37, 36, 35, 38, 36, 36, 35, 35, 38, 37, 35, 36, 38, 38, 35, 35, 36, 38, 37

Complete the following frequency table :

Weights	35	36	37	38
Frequences	____	____	____	____

37. The class marks of a distribution are 104, 114, 124, 134. Determine the class size and the class limits.

38. If the mean of the observations x, $2x + 1$, $2x + 5$, $2x + 9$ is 30. What is the mean of last two observations :

39. Find the mean of the ages (in year) of 25 people.

Age (In years)	25	26	27	28	29
No. of people	4	7	6	3	5

40. Following data gives the number of children in 30 families:

2, 1, 0, 3, 4, 2, 4, 3, 0, 1, 2, 4, 5, 3, 2, 2, 2, 1, 1, 1, 0, 2, 0, 3, 2, 1, 0, 4, 5, 1

Represent it in the form of a frequency distribution.

41. The following are the runs made by 18 players in one day cricket match:
3, 7, 16, 27, 46, 122, 73, 24, 7, 3, 0, 8, 46, 3, 99, 45, 28, 79
Form a frequency table for above data with equal class intervals one of these being 0-25 (excluding 25)

42. Find the mean of all the prime numbers lying between 11 and 41.

43. The mean of 15 observations is 18. If one number is included, the mean becomes 19. Find the included number.

44. In three unit tests of Mathematics Priyal got 75, 82 and 90 marks. How many marks must she obtain in Unit Test IV to have an average of 85 in all the four unit tests ?

45. Time taken in seconds by 25 students in an examination to solve certain question is given below.
20, 16, 20, 27, 27, 28, 30, 33, 37, 50, 40, 42, 46, 28, 43, 46, 46, 48, 49, 52, 58, 59, 60, 64, 52.
By, taking class interval of size 10, make a frequency distribution table.

46. Find the mean from the following table

xi	5	15	25	35	45
fi	6	4	9	6	5

47. Draw the histogram from the following data

Class	0-10	10-20	20-30	30-40	40-50
Frequency	8	15	20	12	16

48. Given below is a cumulative frequency distribution table showing the marks scored by 50 students of a class.

Marks	Number of students
Below 20	17
Below 40	22
Below 60	29
Below 80	37
Below 100	50

Form a frequency table from the above data.

49. Given below are the seats won by different political parties in a state assembly election.

Political Party	A	B	C	D	E	F	G
Seat Won	75	55	37	29	10	37	50

Draw a bar graph for above data.

50. Find the value of 'p' from the following distribution if the mean is 6.

xi	2	4	6	10	p+5
fi	3	3	3	1	2

51. Given below is the data of students who participated in different activities.

Activity	Sports	Meditation	Yoga	Walking
No. of Girls	42	35	100	120
No. of Boys	90	64	130	86

Draw double bar graph.

52. Draw histogram to represent the data given below.

Age (in years)	No of children
1 - 2	5
2 - 3	4
3 - 5	10
5 - 7	12
7 - 10	9
10 - 15	10
15 - 17	8

53. The mean marks of boys & girls in periodical test are 36 and 39 respectively. If the mean marks of all the students of class IX in that test are 37. Find the ratio of the number of boys to the number of girls.

Section-IV

54. In a mathematics best given to 15 students, the following marks (out of 80) are recorded : 49, 24, 51, 57, 43, 49, 71, 77, 49, 46, 43, 51, 20, 51, 57.
Find the mean, median and mode of the data.

55. The median of the following observations, arranged in ascending order is 25, find x, Also find the mean
11, 13, 15, 19, $x+2$, $x+4$, 30, 35, 39, 46

56. Construct a histogram from the following distribution of total marks obtained by 40 students of IX class in a test:

Class Marks (Mid Points)	5	15	25	35	45	55
No. of Students	3	7	6	14	8	2

57. For the following data, draw a frequency polygon:

Marks obtained	10-20	20-30	30-40	40-50	50-60
No. of Students	6	8	3	9	4

58. The mean of the following data is 15, find F.

xi	5	10	15	20	25
fi	6	F	6	10	5

Long Answer Type (5 Marks)

59. The mean of the following data is 50.

xi	10	30	50	70	90
fi	17	5a+3	32	7a-11	19

60. Draw a frequency polygon for the following data

Marks	Frequency
0 - 10	03
10 - 20	09
20 - 30	18
30 - 40	16
40 - 50	12
50 - 60	02

61. If the 26 English alphabets are taken such that A=1, B=2, C=3, ... Z=26 then find

    (I) The mean and median of the numbers corresponding to the vowels.

    (ii) Which vowel corresponds to the median ?

62. In a school a student who scored 80% or above in his/her previous class is eligible for "Merit scholarship". Marks obtained by two students Nishi and Vinayak of class IX in their previous class (VIII) in all subjects are given below.

Name	Hindi	English	Maths	Science	SSt.	Skt.
Nishi	78	74	86	85	73	83
Vinayak	79	76	88	83	71	85

Find average percentage score of Nishi and Vinayak. Which of the two are eligible for merit scholarship ?

63. The blood group of 30 students of class IX are recorded as follows.

    A, B, B, B, O, B, B, A, AB, A, O, B, O, AB, O

    AB, AB, B, AB, B, A, O, AB, B, A, O, AB, A, A, AB

    a) Make a frequency distribution table for the above data.

    b) Mr. 'X' meets an accident and needs blood, His blood group is AB. How many of these students are universal donors and how many are universal receipient ?

64. 15 students of Govt. school spend the following numbers of hours in a month for cleanliness of their street

25, 15, 20, 20, 9, 20, 25, 15, 7, 13, 20, 12, 10, 15, 8

Find mean, median and mode of the above data.

65. A doctor suggests two ways for treatment of a particular disease one by taking medicine only and other by doing meditation and yoga.

Age group	No. of patients taking medicines	No. of patients doing meditation & yoga
20-30	20	05
30-40	30	12
40-50	42	20
50-60	40	30
60-70	30	20

i) Draw Frequency polygons for the above data on the graph.

Represent the data of both the ways of treatment on the same graph by two frequency polygons.

66. The following table shows number of voluntary blood donors per day in voluntary blood donation camp organized in Delhi.

Days	No. of Donars
Sunday	100
Monday	80
Tuesday	110
Wednesday	80
Thursday	60
Friday	70
Saturday	120

i) Draw a bar graph showing above information.

ii) On which day donation was maximum and on which day it was minimum ?

STATISTICS
Answers

1. data
2. frequency
3. Class Size
4. 10
5. 9.5
6. 10.5
7. 5.5
8. 14
9. 26
10. 4
11. 3-30-3.40
12. 2.5
13. $x = 17$
14. 5.6
15. 30
16. 6-11
17. 17
18. 6th
19. No change
20. 4
21. 2
22. $\bar{x} = \dfrac{3 \times 8 + 4 \times 19 + 5 \times 13}{8 + 19 + 13}$
    $= \dfrac{165}{40}$
    $= 4.125$
23. Median $= \dfrac{40+44}{2}$
    $= 42$
24. $x_1 + x_2 = 12$
    $x_1 + x_2 + x_3 = 21$
    $x_3 = 9$
25. Mean $= 17$
26. (A) (d) 98.3°F
    (B) (b) 59 years

(C) (a) 97
(D) (b) 98.35°F
(E) (a) 3 years
27. (A) (a) 27
    (B) (d) 28
    (C) (a) 27
    (D) (b) 30
    (E) (b) 12
28. (A) (a) 3-4 hours
    (B) (b) 22
    (C) (c) 6.5
    (D) (d) 5-6
    (E) (d) 11
29. 35
30. Mode $= 15$
    New Mode $= 14$
31. ₹ (x + 3000)
32. $\dfrac{x+2+x+4}{2} = 24$
    $x = 21$
    Mean $= \dfrac{144}{6}$
    $= 24$
33. $x = 17$
34. Shikha + her husband = 96 years
    Shika + her husband + daughter = 117 years
    Daughter = 21 years
35. $\dfrac{39+x+x+7}{6} = 10$
    $2x = 14 \Rightarrow x = 7$
    $y = 14$
36.

Weights	35	36	37	38
Frequencies	8	6	4	7

37. Class Size $= 10$

Class limits :

99 – 109, 109 – 119, 119 – 129, 129 – 139

38. $\bar{x} = \dfrac{7x+15}{4}$

$x = 15$

Mean $= \dfrac{35+39}{2}$

$= 37$

39.

Age in yrs xi :	25	26	27	28	29
No. of people fi :	4	7	6	3	5
$x1fi$	100	182	162	84	145

$x = \dfrac{\varepsilon fixi}{\varepsilon fi}$

$= \dfrac{673}{25} = 26.92$

40.

No. of Children	Tally Marks	No. of Families
0	ℕ	5
1	ℕ ||	7
2	ℕ |||	8
3						4
4						4
5				2

41.

Class-Internal	Tally Marks	Frequency
0-25	ℕ ||||	9
25-50	ℕ	5
50-75			1
75-100	||	2
100-125			1

42. Mean $= \dfrac{169}{7}$

$= 2 \quad 4 . 14$

43. $\dfrac{270+x}{16} = 19$

$x = 34$

Included number $= 34$

44. $247 + x = 340$

$x = 93$

45.

Class-Internal	Tally Marks	Frequency
15-25	|||	3
25-35	|||	6
35-45	||||	4
45-55	||| |||	8
55-65	||||	4

46. $x = 25$

47.

48.
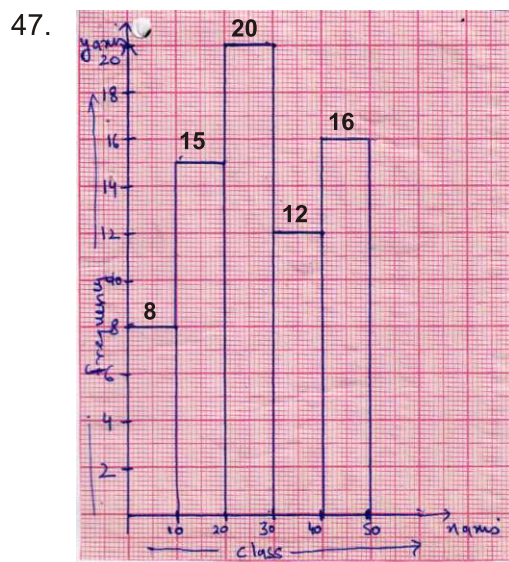
Class-Internal	Frequency
0-20	17
20-40	5
40-60	7
60-80	8
80-100	13

49.
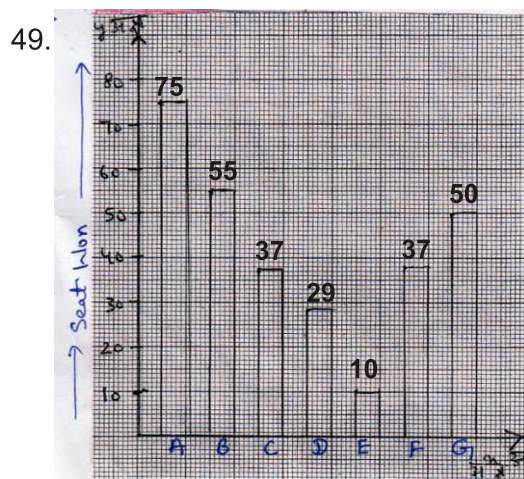
50.

x	f	fx
2	3	6
4	3	12
6	3	18
10	1	10
p+5	2	10+2P
	12	56+2P

$$6 = \frac{56+2P}{12}$$

$$2p = 16$$

$$p = 8$$

53. 2:1

54. Mean $= \frac{738}{15}$

$$= 49.2$$

Median = 49

Mode = 49, 51

55. Median $= \frac{x+2+x+4}{2}$

$$2x = 44$$

$$x = 22$$

Mean $= \frac{258}{10}$

$$= 25.8$$

58.

xi	fi	xifi
5	6	30
10	F	10F
15	6	90
20	10	200
25	5	125
	27+f	445+10f

$$15 = \frac{445+10f}{27+f}$$

$$F = 8$$

59.

xi	fi	fixi
10	17	170
30	5a+3	90+150a

50	32	1600
70	7a-11	-770+490a
90	19	1710
	12a+60	2800 + 640a

$$x = \frac{2800+640a}{12a+60}$$

$$600a + 3000 = 2800 + 640a$$

$$200 = 40a$$

$$a = 5$$

61.

A	E	1	0	U
1	5	9	15	21

Mean $= \frac{51}{5}$

$$= 10.2$$

Median = 9

(ii) I

62. Nishi's Avg.% score $= \frac{479}{6}$

$$= 79.83$$

Vinayak's Avg.% score $= \frac{482}{6}$

$$= 80.33$$

Vinayak is eligible

64. Mean $= \frac{234}{15}$

$$= 15.6$$

Median = 15

Mode = 20

PRACTICE TEST

Time : 1 hr.

Statistics

M.M. 25

1.   Write class limits of the following class marks :

47, 52, 57, 62, 67, 72, 77   1

2.   Find the value of 'x' if mode of the following data is 5. Find x.   1

2, 4, 3, 5, 4, 5, 6, 4, x, 7, 5

3.   Class-mark of class interval 8-15 is _____.   1

4.   The median of the following observations arranged in ascending order is 25. Find x.   2

11, 13, 15, 19, x+2, x+4, 30, 35, 39, 46

5.   Find the median of the first 10 natural numbers. Is it equal to their mean ?   2

6.   In the mean of x, x+3, x+6, x+9 and x+12 is 10. Find x.   2

7.   The mean of 40 observations was 160. It was detected on rechecking that the value of 165 was wrongly copied as 125 for computation of mean. Find the correct mean.   3

8.   A die is thrown 1000 times with the following frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given below. Draw the bar graph of the following data.   3

Outcomes	1	2	3	4	5	6
Frequency	175	125	250	150	100	200

9.   If the mean of the following data of 20.2. Find the value of a.   5

x	10	15	20	25	30
f	6	8	a	10	6

10.   Draw Histogram and Frequency polygon :-   5

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of Students	5	10	4	6	7	3	2

PROBABILITY

MIND MAP

PROBABILITY

EXPERIMENTS

Selection of an object

Organising Data

Tossing of Coins

Throwing of dice

All Possible Outcomes

Favourable Outcomes

$$P(E) = \frac{\text{No. of Favourable outcomes}}{\text{Total No. of Outcomes}}$$

$$O \leq P(E) \leq 1$$

P(Impossible Events) = O

P(Sure Events)=1
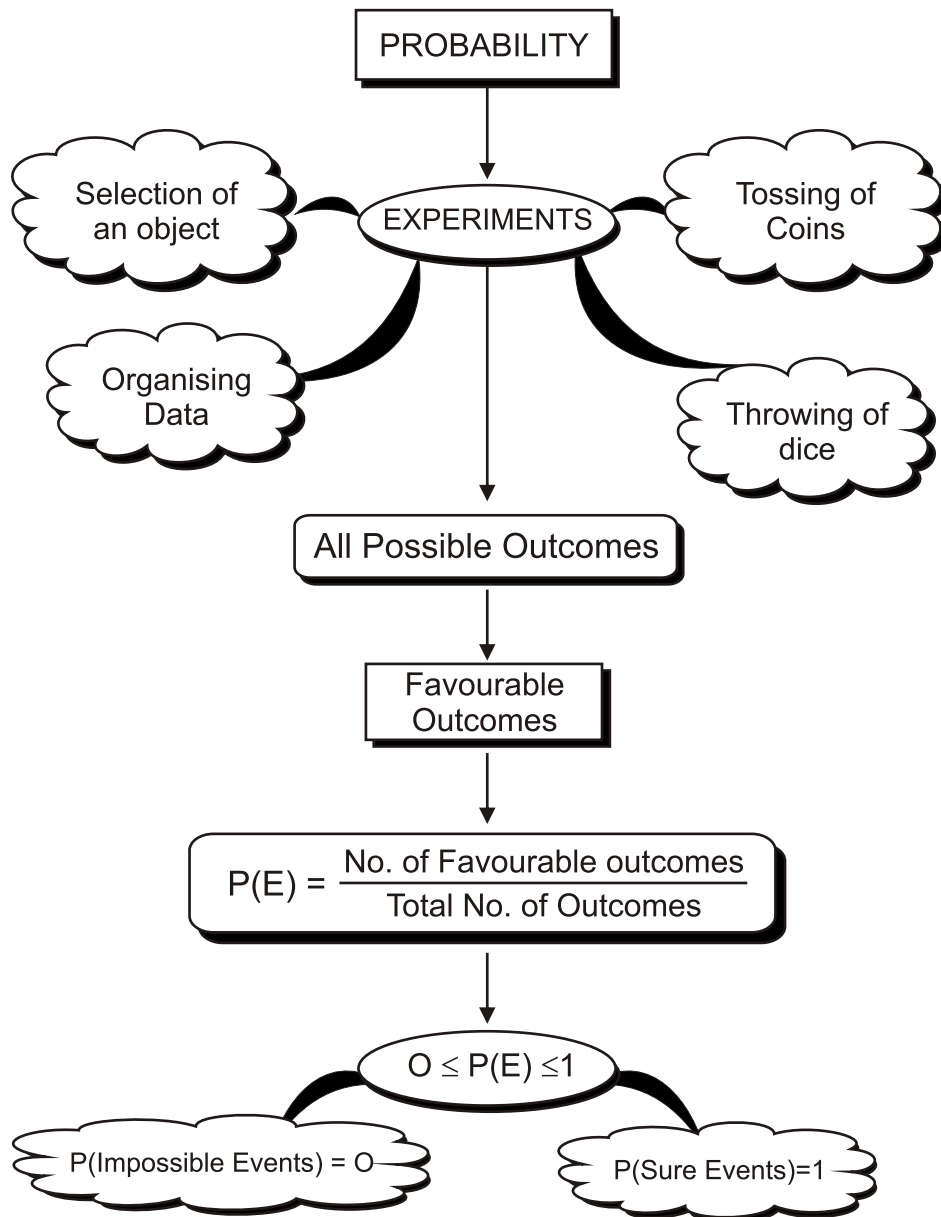
KEY – POINTS

Trials - Trial is an action which results in one or several outcomes.

Example :

(i)   Tossing of a coin every time is a trial.

(ii)   Throwing a dice every time is a trail.

• Probability of an event E is given by

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of trails}}$$

•The probability of an event always from 0 to 1.

$$0 \le P(E) \le 1$$

•The probability of any sure event is 1.

$$P(a) = 1$$

•The probability of an impossible event B is 0.

$$P(B) = 0$$

•The sum of all the probabilities of all the trials of an event is 1.

$$P(E_1) + P(E_2) + P(E_3) + \ldots\ldots\ldots = 1$$

• The sum of probabilities of happening and non-happening of an event is 1.

$$P(E) + P(\overline{E}) = 1$$

•

Deck of playing cards (52)

Red (26)		Black (26)	
<u>Heart 13</u>	<u>Diamond 13</u>	<u>Club 13</u>	<u>Spade 13</u>
King (K)	King (K)	King (K)	King (K)
Queen (Q)	Queen (Q)	Queen (Q)	Queen (Q)
Jack (J)	Jack (J)	Jack (J)	Jack (J)
Ace (A)	Ace (A)	Ace (A)	Ace (A)
2 to 10	2 to 10	2 to 10	2 to 10
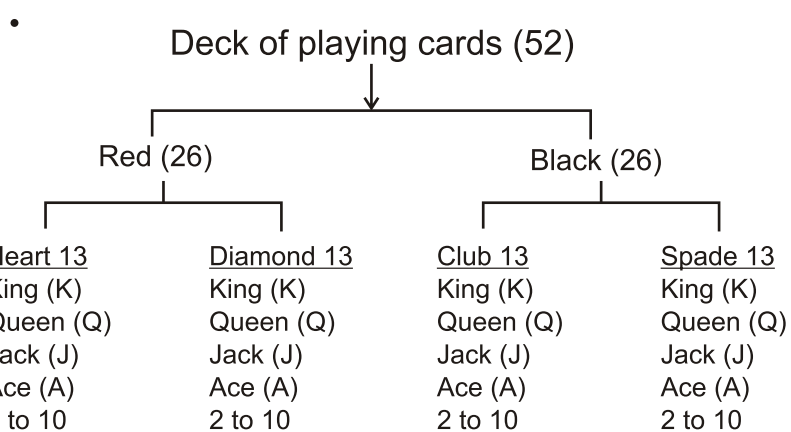
Very-Very Short Answer type (1 Mark)

1. A fruit basket contains 8 apples, 5 mangoes and 10 oranges. What is the probability of choosing a mango from the basket.

2. A letter is chosen at random from English alphabets. What is the probability that letter to be one of the letters of the word MATHEMATICS ?

3. The probability of not happening of an event is 0.73. What is the probability of happening of that event ?

4. A die is thrown. What is the probability of getting a multiple of 3 ?

5. What is the probability that diameter of a circle is the longest chord of the circle ?

6. A coin is tossed 200 times. If head appears 120 times what is the probability of getting tail ?

7. What is the sum of the probabilities of happening of an event and not happening of the event ?

8. What is the sum of the probabilities of all the possible events of a random experiment ?

9. What is the probability of getting a prime number on throwing a die ?

10. A coin is tossed once. What is the probability of getting a tail ?

11. A die is tossed once. What is the probability of getting an even number ?

12. A bag contains 2red, 3 green and 1 white ball. What is the probability that the ball picked up is not black ?

13. In the word MATHEMATICS, what is the probability of choosing a vowel?

14. A bag contains slips with numbers between 3 and 32. What is the probability that a slip choosen contains multiple of 4?

15. During an interview for estate manager 15 candidate appeared. Out of which 8 were retired army men, 4 were retired principals and 3 others were from different departments. What is the probability of selecting a retired army man for this post ?

6. A card is drawn at random from a well shuffed pack of 52 cards. Find the probability that the card drawn is a red card.

17. In a bag, there are 100 bulbs out of which 30 are defective ones. A bulb is taken out of the bag at random. Find the probability if the selected bulb to be a good one.

18. A bag contains x white, y blue and z red balls. A ball is drawn at random. Find the probability of drawing a red ball from the bag.

19. What is the range in which probability of an event lie ?

20. In a hockey match, a goalkeeper of a team can stop a goal 7 times out of 10 shots by other team. Find the probability that the team can make a goal.

Fill in the blanks (Ques 21 to 25)

21. Getting a number 11 on throwing a die is _____ event.

22. On tossing a coin, the possible outcomes are _____ and _____.

23. The probability of an impossible event is _____

24. The probability of happening of an event can never be more than _____

25. The probability of happening of an event is $\frac{3}{4}$ so the probability of not happening of this event is _____

CASE STUDY BASED PROBLEM

26. An insurance company selected 1000 drivers at random in a particular city to find a relationship between age and accidents in a year. The data obtained are given below.

Age group of driver (in years)	Number of Accidents				
	0	1	2	3	more than 3
18-25	212	74	62	34	18
25-40	195	62	45	25	12
40-60	110	45	50	12	7
60 and above	12	8	6	4	7

The probability of the event for a driver selected at random :

(A)   being 18-25 years of age and having exactly 2 accident is

(a) 0.034   (b) 0.045   (c) 0.062   (d) 0.110

(B)   being 25-40 years of age and having atleast 3 accidents

(a) 0.037   (b) 0.025   (c) 0.012   (d) 0.045

(C)   being more than 40 years of age and having no accident

(a) 0.053   (b) 0.012   (c) 0.110   (d) 0.122

(D)   having no accident

(a) 0.044   (b) 0.163   (c) 0.189   (d) 0.529

(E)   As per the above table, greatest probability of accident falls under the age group of

(a) 18-25 year   (c) 40-60 years

(b) 25-40 years   (d) 60 year and above

27. During festival season, a shopkeeper prepares gift packs of dry fruits. The net weights of dry fruits in 10 boxes of gross weight of 5kg each are : 4.798kg, 4.795kg, 4.801kg, 4.805kg, 4.810kg, 4.825kg, 4.798kg, 4.800kg, 4.806kg, 4.817kg. The remaining weight is the weight of the box without dry-fruits.

A box is chosen at random. The probability that its net weight of dry fruits is:

(A)   more than 4 kg 800g

   (a) 0.7   (b) 0.6   (c) 0.5   (d) 0.4

(B)   exactly 4.798 kg

   (a) 0.2   (b) 0.3   (c) 0.4   (d) 0.5

(C)   more than median weight

   (a) 0.48   (b) 0.52   (c) 0.51   (d) 0.5

(D)   equal to modal weight

   (a) 0.2   (b) 0.3   (c) 0.4   (d) 0.5

(E)   A box is selected at random. The probability of selecting a box having weight less than 200 grams without dry fruit is :

   (a) 0   (b) 0.6   (c) 0.5   (d) 0.7

28. A sports teacher maintains record of weights of 45 students in a frequency distribution table

Weight (in kg)	No. of students
30-35	8
35-40	6
40-45	16
45-50	9
50-55	3
55-60	1
60-65	0
65-70	2

The probability that the weight of a student selected at random:

(A)   lies in the interval 45-50 kg is

(a) $\frac{2}{15}$ (b) $\frac{1}{5}$ (c) $\frac{1}{15}$ (d) $\frac{2}{45}$

(B) is less than 45kg

(a) $\frac{2}{3}$ (b) $\frac{1}{5}$ (c) $\frac{1}{3}$ (d) $\frac{2}{15}$

(C) is almost 49kg

(a) $\frac{13}{15}$ (b) $\frac{12}{15}$ (c) $\frac{2}{15}$ (d) $\frac{1}{15}$

(D) In a particular sport, a player having weight atleast 60kg can participate. The probability that a randomly selected student can participate in the sport is.

(a) $\frac{43}{45}$ (b) 0 (c) $\frac{2}{45}$ (d) 1

(E) In a particular sport, only a student of weight 62 kg can participate. The probability that a randomly selected students can participate in that sport is

(a) $\frac{2}{45}$ (b) 1 (c) 0 (d) $\frac{43}{45}$

Very Short Answer Type (2 marks)

29. Below is the table showing marks secured in mathematics by students of class IX : What is

Marks secured	0-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	0	1	9	10	10	8	5	3	2

i) Probability of getting marks less than 50%

ii) Probability of getting marks 90% & above 90%

30. Cards numbered from 7 to 49 are put in a box & mixed thoroughly. A card is drawn from the box. What is the probability that the number written on it is:

i) A prime number

ii) A multiple of 7.

31. The number of hours spent by Ashu, a school student on various activities on a working day are given below:

Activity	Sleep	school	H.W.	Tuition out of home	Outdoor games	Other Acti. at home
No. of Hours	7	7	2	3	3	2

His friend Sonu came to his house to study together. What is the probability that

i)   Ashu is available at home.

ii) Ashu's friend will play outdoor games with Ashu.

32. The record of a weather station shows that out of the past 250 consecutive days, its weather forecast were correct 175 times. What is the probability that on a given day
(i) It was correct   (ii) it was not correct

33. The following data was collected from an old age home about their choices of drink.

Drink	Campa/Soft Drink	Shikanji	Milk	Canned Juice
No. of people	6	10	16	8

A person is selected at random from their old-age home. What is the probability that a person chosen likes.

i)   Natural drink

ii) Canned Juice

34. There are 35 students in class IX A, 34 in IX-B & 33 in IX C. The even roll numbers are allotted project on chapter 2, Polynomials & odd roll number are allotted for chapter-1, Number system. What is the probability that the student chosen

i)   Prepares project on chapter 1

ii) Prepares project on chapter 2

35. The difference between the probabilities of happening & non happening of an event E is $\frac{3}{7}$. Find the probability of happening of the event E.

36. Following table shows the birth month of 40 students of a class.

Jan	3	July	2
February	4	August	6
March	2	September	3
April	2	October	4
May	5	November	4
June	1	December	4

A students is chosen . What is the probability that

i)   its birth month is November

ii) The month contains 31 days.

SECTION-III

37. The probability of guessing the correct answer to a certain question is $\frac{x}{2}$. If the probability of not guessing the correct answer to the question is $\frac{2}{3}$, then find x.

38. A purse contains a number of ₹1, ₹2 and ₹5 coin as given below :

₹1	₹2	₹5
14	10	14

    If from the purse a coin is taken out of random, then find the probability that the coin.

    (a) is not a ₹5 coin

    (b) is a ₹1 coin

39. A box contains 130 bolts and 70 nuts. On checking the box, it was found that half of the bolts and half of the nuts are rusted. If one of them is chosen at random, find the probability that it is not rusted.

40. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability of drawing

    (i) an Ace card   (ii) a club card.

41. In a school library, a student find that in a book shelf, there are 30 English books, 20 literature books, 10 hindi books and 25 mathematics books. Find the probability of each book chosen at random. Find the sum of all probabilities.

42. A die is thrown 100 times. If the probability of getting an odd number is $\frac{3}{5}$, how many times an even number is obtained ?

Short Answer Type (3 Marks)

43. After a medical check up for HB level of 35 students of class IX , the following data was recorded.

HB Level	Below 8	Below 10	Below 12	Below 14	Below 16
No. of Students	3	7	13	23	35

    What is the probability that a student choosen has

    i) HB level less than 10.

    ii) HB level greater than or equal to 12 but less than 16.

44. To know the opinion of 35 students about sixth subject as automobile engineering or financial management a survey was done. The data is recorded in the following table in favour of choosing automobile engineering

    No of student like   20

    Dislike   15

Find the probability that a student will opt.

i) automobile engineering

ii) Financial management

45. A die is thrown 100 times by a player during a game. The data is recorded in the table given below.

Outcome	1	2	3	4	5	6
Frequency	20	12	18	19	16	15

A Player will get one more chance if he gets 1 or 6 & looses his/her next chance if 3 or 5 comes.

i) What is the probability of loosing the next chance?

ii) What is the probability of getting one more chance?

46. Following is the table showing marks obtained by 200 students out of 100 in an examination.

Marks	No. of Students
0-10	20
10-20	40
20-30	15
30-40	24
40-50	25
50-60	12
60-70	9
70-80	7
80-90	12
90-100	36

Find the probability that a student chosen.

i) Obtained less than 40 marks.

ii) Obtained greater than or equal to 60 but less than 80 marks.

iii) Obtained 80 & above marks.

47. Mathematics book of class IX contains 15 chapters. A maths teacher asked one of the students to write the name of each chapter on slips, One name on one slip. She mixed the slips thoroughly in a box.

She called a student to pick up one of the slips. What is the probability that the chapter written on it is from

(i) Geometry (ii) Algebra

48. A book of mathematics contains 400 pages numbered from 1-400 A page is selected at random. What is the probability that the page number contains.

    i)   9 at one's place.

    ii) multiple of 4

    iii) perfect square

49. The following table shows per day salary of 1000 workers.

Salary Per Day (I)	500-700	701-900	901-1100	1101-1300
No. of Workers	280	175	420	125

    If a worker is chosen at random, find the probability that he is getting.

    i)   at least ₹ 701 daily

    ii) at most ₹ 900 daily

    iii) at most ₹ 1300 daily

50. $BMI = \dfrac{Mass\ in\ Kg.}{(height\ in\ metres)^2}$

    The following table shows the BMI of different categories.

S.No.	Category	BMI (kg/m2)
1.	Under weight	16.0-18.5
2.	Normal weight	18.5-25.0
3.	Over weight	25.0-30.0
4.	Obesity	Above 30.0

    Three persons x, y,z have the same height 170 cm and their masses are 70 kg., 85 kg. & 65 kg. respectively.

    Find the probability that a person chosen is overweight.

51. Read the lines carefully

    Horse is horse, of course, of course.

    And no one can talk to horse of course.

    That is, of course, unless the horse is the famous mister ID.

    (i) Find the probability of the word 'course' from the above stanza.

    (ii) Name the word which has the same probability as the word 'course' has.

---

52. The bar graph below shows the number of students in different classes of a school.
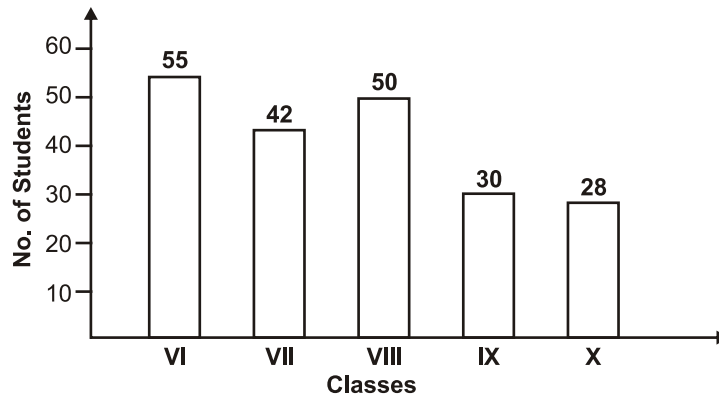
In the annual function of primary classes, class IX & X was deputed for discipline duty, students of class VII & VIII for sitting, class VI students were to welcome the chief guests.

Find the probability that a student chosen is

i)   Deputed for sitting

ii) a student of class X.

iii) member of welcome committee.

53. In a park, there is a right angled triangular flower bed. It's two small sides are 5m & 12m respectively. Along its all sides at a distance of 1/2m each, plants of different types are to be planted. Rose plants are to be planted along the shortest side, Marigold plants are to be planted along the longest side & sunflower plant along the third side. At each of the vertex a different type of flower plant is to be planted. A plant is chosen at random. Find the probability that the chosen plant is

(i) On the longest side.

(ii) Sun flower plants.

54. Out of 1000 small coloured bulbs $81^{3/2}$ are of white colour. 5^3 are red coloured, 2^6 are green coloured & rest are blue coloured. What is the probability that bulb chosen is.

(i) blue coloured

(ii) red coloured.

(iii) white coloured.

55. In a school there are 682 students. The mode of transport used by them is as follows:

Mode of Transport	Car with Parents	Bicycle	Pedestrian	DTC Bus	Bike with Parents	Van	Auto rickshaw
No. of Students	64	52	128	100	86	172	80

A student is chosen at random. What is the probability that he comes by:

(i) Four wheeler

(ii) Two wheeler

56. Cards with number 1, 2, 3 ----- 100 are placed in a box and mixed throughly. One card is drawn. What is the probability that the card drawn is

(i) a multiple of 5 or 7?

(ii) a multiple of 5 and 7?

(iii) a prime number less than 20?

57. The blood groups of 20 students of a class are recorded as below.

B, A, B, O, AB, O, A, AB, O, B, B, AB, AB, O, A, O, A, B, AB and O.

A student is selected at random. Find the probability that his blood group is

(i) A   (ii) AB   (iii) O

PROBABILITY

Hints and Solutions/ Answer

1. $\frac{5}{23}$

2. $\frac{4}{13}$

3. 0.27

4. $\frac{1}{3}$

5. 1

6. $\frac{2}{5}$

7. 1

8. 1

9. $\frac{1}{2}$

10. $\frac{1}{2}$

11. $\frac{1}{2}$

12. 1

13. $\frac{4}{11}$

14. $\frac{7}{28}$

15. $\frac{8}{15}$

16. $\frac{1}{2}$

17. $\frac{7}{10}$

18. $\frac{z}{x+y+z}$

19. 0 to 1

20. $\frac{3}{10}$

21. impossible

22. head, tail

23. 0

24. 1

25. $\frac{1}{4}$

26. (A) (c) 0.062

    (B) (a) 0.037

(C) (d) 0.122

(D) (d) 0.529

(E) (a) 18-25 years

27. (A) (b) 0.6

    (B) (a) 0.2

    (C) (d) 0.5

    (D) (a) 0.2

    (E) (b) 0.6

28. (A) (b) $\frac{1}{5}$

    (B) (a) $\frac{2}{3}$

    (C) (a) $\frac{13}{15}$

    (D) (c) $\frac{2}{45}$

    (E) (c) 0

29. (i) $\frac{5}{12}$

    (ii) $\frac{1}{24}$

30. (i) $\frac{12}{43}$

    (ii) $\frac{7}{43}$

31. (i) $\frac{11}{24}$

    (ii) $\frac{1}{8}$

32. (i) $\frac{7}{10}$

    (ii) $\frac{3}{10}$

33. (i) $\frac{13}{20}$

    (ii) $\frac{1}{5}$

34. (i) $\frac{26}{51}$

    (ii) $\frac{25}{51}$

35. $\frac{5}{7}$

36. (i) $\frac{1}{10}$

    (ii) $\frac{7}{12}$

37. $\frac{x}{2} + \frac{2}{3} = 1$

    $x = \frac{2}{3}$

38. (i) $\frac{12}{19}$

    (ii) $\frac{7}{19}$

39. $\frac{1}{2}$

40. (i) $\frac{1}{13}$

    (ii) $\frac{1}{4}$

41. $\frac{6}{17}, \frac{4}{17}, \frac{2}{17}, \frac{5}{17}$

    sum of probabilities

    $= \frac{6}{17} + \frac{4}{17} + \frac{2}{17} + \frac{5}{17}$

    $= \frac{17}{17}$

    $= 1$

42. $\frac{2}{5}$ x100=40

43. (i) $\frac{1}{5}$

    (ii) $\frac{22}{35}$

44. (i) $\frac{4}{7}$

    (ii) $\frac{3}{7}$

45. (i) $\frac{17}{50}$

    (ii) $\frac{7}{20}$

46. (i) $\frac{99}{200}$

    (ii) $\frac{2}{25}$

    (iii) $\frac{6}{25}$

47. (i) $\frac{7}{15}$

    (ii) $\frac{2}{15}$

48. (i) $\frac{1}{10}$

    (ii) $\frac{1}{4}$

    (iii) $\frac{1}{20}$

49. (i) $\frac{18}{25}$

    (ii) $\frac{91}{200}$

    (iii) 1

50. $\frac{1}{3}$

51. (i) $\frac{1}{7}$

    (ii) Horse

52. (i) $\frac{92}{205}$

    (ii) $\frac{28}{205}$

    (iii) $\frac{11}{41}$

53. (i) $\frac{5}{12}$

    (ii) $\frac{23}{60}$

54. (i) $\frac{41}{500}$

    (ii) $\frac{1}{8}$

    (iii) $\frac{729}{1000}$

55. (i) $\frac{118}{341}$

    (ii) $\frac{69}{341}$

56. (i) $\frac{8}{25}$

    (ii) $\frac{1}{50}$

    (iii) $\frac{2}{25}$

57. (i) $\frac{1}{5}$

    (ii) $\frac{1}{4}$

    (iii) $\frac{3}{10}$

PRACTICE TEST

Time : 1 hr.   **Probability**   M.M. 25

1. What is the probability that a number selected from the number 1, 2, 3, ... 15 is a multiple of 4?   1

2. What is the probability of a sure event.   1

3. In a throw of a die, find the probability of getting a prime number.   1

4. A letter of english alphabet is chosen at random calculate the probability that letter chosen in a vowel.   2

5. A bag contains cards numbered from 1 to 100. A card is drawn at random from the bag. Find the probability that the   2

    (i)   Card bears a number which is multiple of 5.

    (ii) Card bears a number which is greater than or equal to 80.

6. The probability of guessing the correct answer to a certain question is $\frac{x+1}{5}$.

    If the probability of not guessing the correct answer is $\frac{x}{5}$ then find value of x. 2

7. A card is drawn at random from a well - shuffled pack of 52 cards. Find the probability of drawing.   3

    (a) a club card

    (b) not a spade card

    (c) king card.

8. A die is thrown 400 times with the frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given in table.   3

Outcomes	1	2	3	4	5	6
Frequency	72	65	70	71	63	59

    Find the probability of

    (i)   getting a number less than 3

    (ii) getting an outcome 6

    (iii) getting a number more than 4.

9. In a one day international cricket match, a batsman play 50 balls. The run scored as follows.   5

Run Scored	0	1	2	3	4	5	6
No. of Balls	13	11	09	07	06	00	04

    Find the probability that batsman will score

    (a) 6 runs   (b) 4 or 6 runs   (c) Runs less than 2   (d) 3 runs

10. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes.   5

Outcomes	3 head	2 head	1 head	No head
Frequency	23	72	77	28

Find the probability of getting :

(a) Two heads

(b) Three heads

(c) Atleast two heads

(d) Atmost two heads

PRACTICE QUESTION PAPER - I
CLASS-IX
MATHEMATICS

Time: 3 Hrs.   **M.M. 80**

General Instruction:

1. This question paper contains 36 questions divided into two parts A and B. All the questions are compulsory.

2. Part A consists of two section I & II. Section I has 16 questions of 1 mark each and section II has 4 case study based questions. Each case-study based question has 5 sub-parts of 1 mark each.

3. Part B consists of 16 questions – 6 questions of 2 marks, 7 questions of 3 marks and 3 question of 5 marks each.

4. There is no overall choice. However internal choices are provided in 5 questions of 1 mark, 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks. You have attempt only one of the alternatives in all such question.

5. In case-study based questions, you have to attempt only four out of five sub-parts.

6. Use of calculator is not permitted.

7. Please write down the serial number of question before attempting it.

Part-A

Q.No. 1-16 are of 1 mark each.

1. Simplify : $(3-\sqrt{3})(3+\sqrt{3})$

2. Which is greater ? $\sqrt[3]{4}$ or $\sqrt[4]{5}$

3. Find the value of x if $\left(\frac{6}{5}\right)^{x}\left(\frac{5}{6}\right)^{2x}=\frac{125}{216}$

4. Find the remainder when $x^{2}-5x+7$ is divided by x-1.

5. Write the co-ordinate where the linear equation 2x-y=5 intersects at x-axis.

<div align="center">OR</div>

If the point (-4,2) lies on the graph of the linear equation 3y=ax+2, then find the value of a.

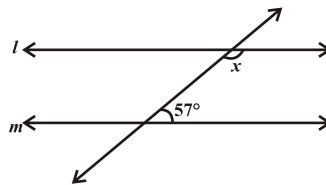
6. How many lines do pass through two distinct points.

OR

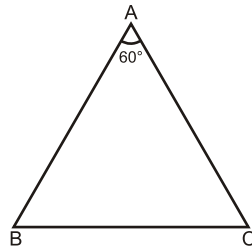Can two distinct intersecting lines be parallel to the same line ?

7. For an event E, If $P(E) = 0.37$ the find $P(\text{not } E)$.

8. An angle is 15° more than twice its complement. Find the angle.

OR

In figure, find the value of x for which the lines l and m are parallel.
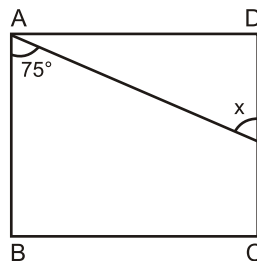

9. Two supplementary angles are in the ratio 3:2. Find the angles.

10. In the given figure, AB=AC and $\angle BAC = 60$, find $\angle ABC$.
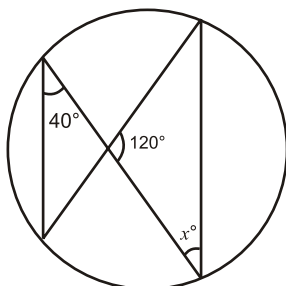

OR

In $\triangle ABC$, $\angle A = 62°$ and $\angle B = 55°$. Which side is largest ?

11. Is it possible to construct a triangle with length of its sides 6cm, 7cm and 13cm ? Give reason for your answer.

12. In the given figure, ABCD is a square. Find x.

13. One of the diagonals of a rhombus is equal to a side of rhombus. Find the angles of the rhombus.

14. In the given figure, find x.

OR

An arc is a _____ when its ends are the ends of a diameter

15. Find the length of a chord which is at a distance of 4cm from the centre of a circle of radius 6cm.

16. If each side of a triangle is doubled, then find the percentage increase in its area.

SECTION-II

Q.No. 17-20 are case study based questions. Attempt any 4 sub-parts from each question. Each sub-part carries one mark.

17. The marks obtained by 40 students of class-IX in an examination are given below :

18, 8, 12, 6, 8, 16, 12, 5, 23, 2, 16, 23, 2, 10, 20, 12, 9, 7, 6, 5, 3, 5, 13, 21, 13, 15, 20, 24, 1,7, 21, 16, 13, 18, 23, 7, 3, 18, 17, 16


(a) The range of above data is

(i) 21   (ii) 22   (iii) 23   (iv) 24

(b) Mean mark is

(i) 12.25   (ii) 12.35   (iii) 12.45   (iv) 12.55

(c) Median marks is

(i) 12.5   (ii) 13.5   (iii) 14.5   (iv) 15.5

(d)   Modal Marks is

       (i) 13   (ii) 14   (iii) 15   (iv) 16

(e)   Number of students who got marks more than mean marks is :

       (i) 21   (ii) 20   (iii) 23   (iv) 16

18.   A circus tent is cylindrical upto height of 3 meter and conical above it. The total height of the tent is 12 meter and diameter is 24 meter.

(a)   What is the slant height of the conical part ?

       (i) 12m   (ii) 15m   (iii) 18m   (iv) 24m

(b)   What is the curved surface area of conical part ?

       (i) $470.7m^2$ (ii) $512.8m^2$ (iii) $562.7m^2$ (iv) $572.6m^2$

(c)   What is the curved surface area of cylindrical part ?

       (i) $226.3m^2$ (ii) $279.2m^2$ (iii) $371.7m^2$ (iv) $340m^2$

(d)   What is the total surface area of the tent ?

       (i) $972m^2$ (ii) $729m^2$ (iii) $792m^2$ (iv) $912m^2$

(e)   What is the cost of cloth required to make this tent if rate of the cloth is ₹10 per square meter ?

       (i) ₹9120   (ii) ₹7290   (iii) ₹9720   (iv) ₹7920

19.   A student made a design with geometrical instruments, taking circle of radius 10cm and each point A, B, C, D, E and F is equidistant from its neighbouring points.
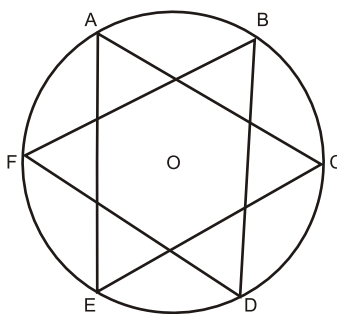
(a)   The measure of ∠AOC is

(i) 60°   (ii) 90°   (iii) 120°   (iv) 150°

(b)   The measure of ∠A+∠B+∠C is

(i) 120°   (ii)180°   (iii) 240°   (iv) 360°

(c)   The length of altitude from C on AE is

(i) 10cm   (ii) 12cm   (iii) 15cm   (iv) 18cm

(d)   The length of perpendicular drawn from O on AC is

(i) 3cm   (ii) 5cm   (iii) 7cm   (iv) 8cm

(e)   The length of chord AC is

(i) $5\sqrt{3}$cm   (ii) $8\sqrt{3}$cm   (iii) $10\sqrt{3}$cm   (iv) $12\sqrt{3}$cm

20.   Front face and side face of a house is plotted on graph sheet.
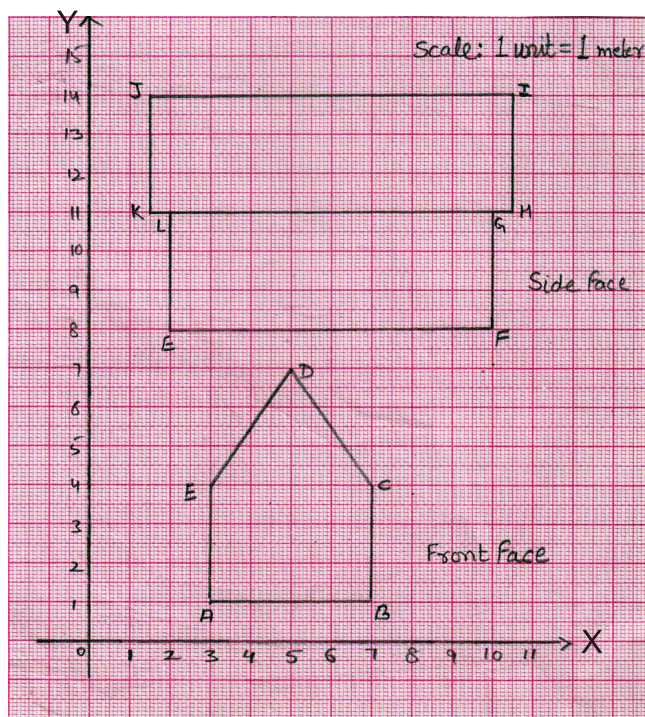
(a) The coordinates of D are

(i) (7,5)   (ii) (5,7)   (iii) (5,5)   (iv) (7,7)

(b) The length of AB is

(i) 3 meter   (ii) 7 meter   (iii) 4 meter   (iv) 4cm

(c) The length of IJ is

(i) 8m   (ii) 9m   (iii) 14m   (iv) 10.5cm

(d) The area of front face (in m^2) is

(i) 6   (ii) 12   (iii) 18   (iv) 24

(e) The Perimeter of side face is

(i) 46m   (ii) 28m   (iii) 29m   (iv) 30m

PART-B

Q.No. 21-26 are very short answer questions of 2 marks each.

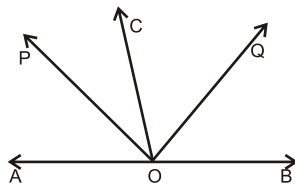21. Express $0.4\overline{7}$ in the form of $\frac{p}{q}$.

OR

Find the value of $(256)^{0.16} \times (256)^{0.09}$

22. If $x + \frac{1}{x} = 7$, find the value of $x^2 + \frac{1}{x^2}$

OR

Expand : $\left(\frac{1}{3}x + 3y\right)^3$

23. Find any four solution of the linear equation $2x-y=4$

24. In the given figure, OP bisects $\angle BOC$ and OQ bisects $\angle AOC$. Find $\angle POQ$.

25. If the angles of a triangle are in the ratio 2:3:4, find the measure of its angles.

26. Three coins were tossed 30 times. Each time the number of heads occuring was noted down as follows:

0	1	2	2	1	2	3	1	3	0
1	3	1	1	2	2	0	1	2	1
3	0	0	1	1	2	3	2	2	0

Prepare a frequency distribution table for the data given above.

Q.No. 27-33 are short answer questions of 3 marks each.

27. $\dfrac{3}{\sqrt{3}+1} + \dfrac{5}{\sqrt{3}-1} = a+b\sqrt{3}$, find a and b.

28. Factorise : $x^3 + 6x^2 + 11x + 6$

29. If a+b+c = 12 and $a^2 + b^2 + c^2 = 90$, find $a^3 + b^3 + c^3 - 3abc$

30. Prove that the sum of the four angles of a quadrilateral is 360°.

OR

Prove that a quadrilateral is a parallelogram, if the diagonals bisects each other.

31. Construct a triangle with the base of length 5cm. The sum of the other two sides is 7cm and one base angle is of 60°.

32. Find the area of a triangle whose sides are 16cm, 30cm and 34cm.

33. A die is tossed 100 times and outcomes are recorded as follows:

Outcomes	1	even no. <6	odd no.>	6
Frequency	15	30	39	16

Find the probability of getting

(i) an even number

(ii) getting 1 or 6

(iii) odd no.>1

OR

54 students of Class-IX appeared for mathematics olympiad. The marks obtained by them are given below :

48, 37, 26, 8, 64, 57, 46, 66, 45, 61, 54, 27, 49, 50, 76, 38, 47, 34, 46, 40, 59, 42, 46, 31, 74, 68, 42, 54, 14, 61, 93, 72, 53, 59, 38, 16, 88, 56, 27, 44, 63, 58, 43, 81, 64, 36, 77, 62, 53, 40, 71, 60, 45, 42.

One student is selected at random. Find the probability that selected student is :

(i) having marks more than 49.

(ii) having marks between 39 and 100.

Q.No. 34-36 are long answer questions of 5 marks each.

34. Draw the graphs of $2x+5y=10$ and $2x+3y=6$ on the same graph sheet. Also, write the coordinates of the point of intersection of these lines.

35. Prove that the angle subtended by an arc of the circle is double the angle subtended by it at only point on the remaining part of the circle.

Using the above, Find the value of x in the given figure.
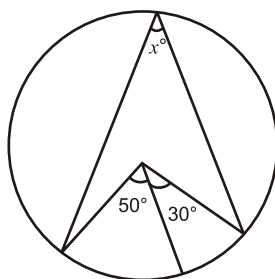
36. The diameter of roller 1.5m long is 84cm. If it takes 100 revolutions to level a playground, find the cost of levelling this ground at the rate of ₹2.50 per square metre.

OR

The radius and height of a cone are in the ratio 3:4. If its volume is 301.44cm^3, find its curved surface area. [use π = 3·14]

SOLUTIONS

1. $(3)^2 - (\sqrt{3})^2$

   $\Rightarrow 6$

2. $\sqrt[12]{4^4}$ or $\sqrt[12]{5^3}$

   $\sqrt[12]{256}$ or $\sqrt[12]{125}$

   $\therefore \sqrt[3]{4} > \sqrt[4]{5}$

3. $\left(\dfrac{5}{6}\right)^{-x} \left(\dfrac{5}{6}\right)^{2x} = \left(\dfrac{5}{6}\right)^3$

   $x=3$

4. $(1)^2 - 5(1) + 7$

   Remainder = 3

5. $\left(\dfrac{5}{2}, 0\right)$

   OR

   $a = -1$

6. One or No

7. P [not E] = 1-p [E]

   $= 0.63$

8. $3x+15 = 90°$

   $x = 25°$

   or

   $x+57=180°$

   $x = 123°$

9. $3x+2x = 180°$

   $x = \left(\dfrac{180}{5}\right)$

   $= 108°, 72°$

10. $\angle B = \angle C$

    $\therefore \, ABC = 60°$

    OR

    AB is largest side

11. No, because sum of two sides is equal to third side

12. $15°+90°+x = 180°$

    $x=75°$

13. $60°, 120°, 60°, 120°$

14. $x=20°$ or Semicircle

15. Chord = $4\sqrt{5}$ cm

---

16. 300%
17. (a) (iii) 23
    (b) (ii) 12.35
    (c) (i) 12.5
    (d) (iv) 16
    (e) (ii) 20 students
18. (a) (ii) 15m
    (b) (iii) 562.7m^2
    (c) (i) 226.3m^2
    (d) (iii) 792m^2
    (e) (iv) ₹7920
19. (a) (iii) 120°
    (b) (ii) 180°
    (c) (iii) 15cm
    (d) (ii) 5cm
    (e) (i) 5√3cm
20. (a) (ii) (5,7)
    (b) (iii) 4m
    (c) (ii) 9m
    (d) (iii) 18
    (e) (iv) 30m

Part B

21. Let $x = 0.4\overline{7}$
    $10x = 4.\overline{7}$
    $100x = 47.\overline{7}$
    $90x = 43$
    $x = \left(\dfrac{43}{90}\right)$

    or
    $(256)^{0.25}$
    $\Rightarrow (4^4)^{\frac{25}{100}}$
    $\Rightarrow (4^4)^{¼}$
    $\Rightarrow 4$ Ans

22. $x + \dfrac{1}{x} = 7$
    squaring both sides
    $x^2 + \dfrac{1}{x^2} + 2 = 49$

$x^2 + \frac{1}{x^2} = 47$

OR

$\left(\frac{1}{3}x + 3y\right)^3$

$= \left(\frac{1}{3}x\right)^3 + (3y)^3 + 3\left(\frac{1}{3}x\right)^2(3y) + 3\left(\frac{1}{3}x\right)(3y)^2$

$= \frac{1}{27}x^3 + 27y^3 + x^2y + 9xy^2$

23. $2x-y=4$

Put $x = 0 \Rightarrow 2(0) - y = 4$

$y = -4$

Put $y = 0 \Rightarrow 2x-0 = 4$

$x = 2$

Put $x = 1 \Rightarrow 2(1) - y = 4$

$y = -2$

Put $y = 2 \Rightarrow 2x - 2 = 4$

$x = 3$

x	0	2	1	3
y	-4	0	-2	2

24. AOC + BOC = 180 (Linear Pair)

$2\angle COP + 2\angle QOC = 180°$

$\left(\begin{array}{l}\therefore OP \text{ bisects } \angle BOC \\ \quad OQ \text{ bisects } \angle AOC\end{array}\right)$

$\angle COP + \angle QOC = 90°$

$\Rightarrow \angle POQ = 90°$

25. $2x+3x+4x = 180°$ (Angle sum property)

$x = 20°$

$\therefore$ Angles : 40°, 60°, 80°

26.

Result	Tally Marks	Frequency
0	⤿ I	6
1	⤿ ⤿	10
2	⤿ IIII	9
3	⤿	5
		30

27.

L.H.S.

$$= \frac{3}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} + \frac{5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{3\sqrt{3}-3}{2} + \frac{5\sqrt{3}+5}{2}$$

$$= \frac{3\sqrt{3}-3 + 5\sqrt{3}+5}{2}$$

$$= \frac{8\sqrt{3}+2}{2} \qquad = \qquad 4\sqrt{3}+1$$

Now $4\sqrt{3}+1 = a+b\sqrt{3}$

$\Rightarrow$ a=1; b=4

28. $(x+1)$, is one factor of $x^3+6x^2+11x+6$

$$x+1 \overline{)\, x^3+6x^2+11x+6\,} \big(\, x^2+5x+6$$
$$\underline{x^3 \pm x^2}$$
$$5x^2+11x+6$$
$$\underline{5x^2 \pm 5x}$$
$$6x+6$$
$$\underline{6x\pm6}$$
$$0$$

$x^3+6x^2+11x+6 \quad = (x+1)(x^2+5x+6)$
$\qquad\qquad\qquad\qquad = (x+1)\,(x+2)\,(x+3)$

29. $(a+b+c)^2 = (12)^2$

$a^2+b^2+c^2+2(ab+bc+ca) = 144$

$90+2\,(ab+bc+ca) = 144$

$ab+bc+ca = 27$

Now $a^3+b^3+c^3-3abc = (a+b+c)\,(a^2+b^2+c^2-ab-bc-ca)$
$\qquad\qquad = 12\,[90-27]$
$\qquad\qquad = 756$

30. In $\triangle ABC$

$\angle 1+\angle 2+\angle 3=180°$ —————— ①

(Angle sum property)

In ADC

$\angle 4+\angle 5+\angle 6 = 180°$ —————— ②

Adding 1 & 2

$\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6 = 180°+180°$

$\angle A + \angle B + \angle C + \angle D = 360°$

OR

In $\triangle$'s AOD and $\triangle$COB
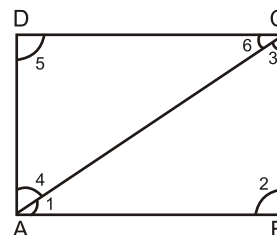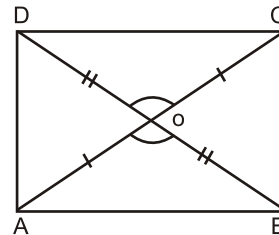
AO = OC (Given)

OD = OB (Given)

$\angle AOD = \angle COB$ (Vertically Opposite Angles)

$\Rightarrow \triangle AOD \cong \triangle COB$ by SAS

$\Rightarrow \angle OAD = \angle OCB$ by C.P.C.T

But this makes alternate angles

$\therefore$ AD $\parallel$ BC

Hence ABCD is a parallelogram

31.

32. $S = \dfrac{16+30+32}{2}$

$= 40$

ar of $\triangle = \sqrt{s(s-a)\,(s-b)\,(s-c)}$

$= \sqrt{40\,(40-16)\,(40-30)\,(40-34)}$

$= \sqrt{40 \times 24 \times 10 \times 6}$

$= \sqrt{2 \times 2 \times 2 \times 5 \times 2 \times 2 \times 2 \times 3 \times 5 \times 2 \times 2 \times 3}$

$= 240$ sq cm

33. (i) $\dfrac{46}{100} = 0.46$ (ii) $\dfrac{31}{100} = 0.31$ (iii) $\dfrac{39}{100} = 0.39$

OR

(i) $\dfrac{27}{54} = \dfrac{1}{2}$ (ii) $\dfrac{42}{54} = \dfrac{21}{27}$

34. $2x+5y=10 \Rightarrow y = \dfrac{10-2x}{5}$

$2x+3y=6 \Rightarrow y = \dfrac{6-2x}{3}$

x	0	5
y	2	0

x	0	3
y	2	0
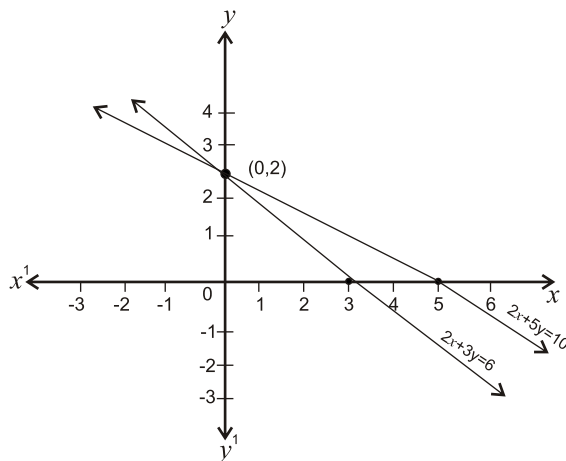
Point of intersection is (0,2)

35. Given : Arc ÃB of a circle with centre O subtends ∠AOB at the centre and ∠ACB on the remaining part of the circle

To prove : ∠AOB = 2 ∠ACB

Construction : Join CO and produce it to Q.

Proof :   In ∠OAC, OA=OC

∴ ∠3 = ∠5

Also, ∠1 = ∠3+∠5

or ∠1 = ∠3+∠3

∠1 = 2∠3 -------------(i)

Similarly, ∠2 = 2∠4 -------------(ii)

Adding (i) and (ii)

∠1 + ∠2= 2 (∠3+∠4)

⇒ ∠AOB = 2∠ACB Hence proved

$x = \dfrac{1}{2}$ (50°+30°)

$x = 40°$

36. Here, $r = \dfrac{84}{2}$ = 42cm = 0.42m

h = 1.5m

C.S.A of road roller $= 2 \times \dfrac{22}{7} \times 0.42 \times 1.5$

$= 3.96m^2$

Area of ground = 3.96 x 100 = 396m²

Cost of frequency = 396 x 2.50 = ₹990

OR
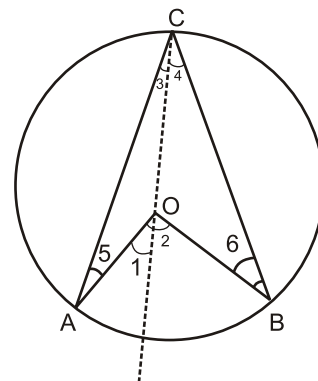
let radius of cone = $3x$

and height of cone = $4x$

$v = \frac{1}{3}\pi r^2 h$

$301.44 = \frac{1}{3} \times 3.14 \times 3x \times 3x \times 4x$

$301.44 = 3.14 \times 12x^3$

$x^3 = \frac{301.44}{3.14 \times 12} = 8$

$\Rightarrow x = 2$

radius = 3 x 2 = 6cm

height = 4 x 2 = 8cm

$l = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$cm

CSA $= \pi r l = 3.14 \times 6 \times 10 = 188.4$cm^2

PRACTICE QUESTION PAPER - 2
CLASS-IX
MATHEMATICS

Time: 3 Hrs.   **M.M. 80**

General Instruction:

1.   This question paper contains 36 questions divided into two parts A and B. All the questions are compulsory.

2.   Part A consists of two section I & II. Section I has 16 questions of 1 mark each and section II has 4 case study based questions. Each case-study based question has 5 sub-parts of 1 mark each.

3.   Part B consists of 16 questions – 6 questions of 2 marks, 7 questions of 3 marks and 3 question of 5 marks each.

4.   There is no overall choice. However internal choices are provided in 5 questions of 1 mark, 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks. You have attempt only one of the alternatives in all such question.

5.   In case-study based questions, you have to attempt only four out of five sub-parts.

6.   Use of calculator is not permitted.

7.   Please write down the serial number of question before attempting it.
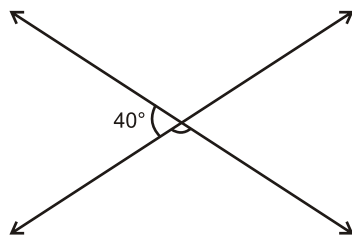
Part-A

Q.No. 1-16 are of 1 mark each.

1.   Find the value of a if (x-1) is a factor of the polynomial $2x^2$–2a.

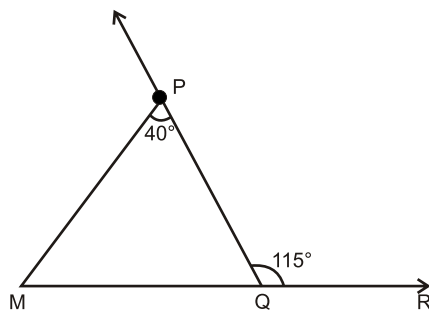2.   Name the congruency criteria for the following triangles.
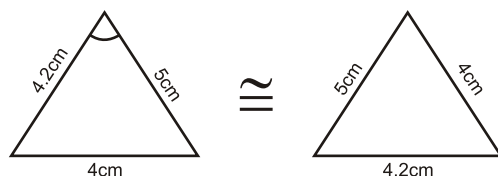
3.   Chords equidistant from the _____ of a circle are equal in length.

4.   Find the value of x from the following figure.

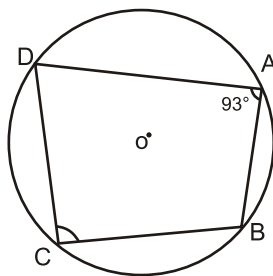
OR

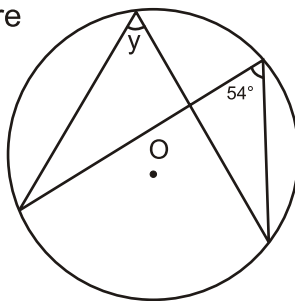Find the measure of ∠PMQ in the following figure.

5.   Examine if $3\sqrt{250}$ is rational or irrational.

6.   The total outcomes when two coins are tossed simultaneously are _____.

7.   ABCD is a cycle quadrilateral. Find ∠BCD.

OR
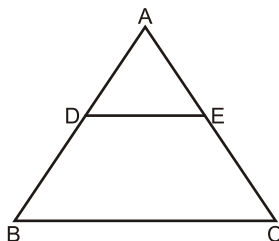
Find b in the following figure

8. What is the coefficient of x^2 in $2x^3-x^2+3$?

9. The perpendicular bisector MN intersects line segment PQ at E. If PQ = 10cm, find PE.

10. Find the decimal expansion of $\frac{31}{16}$.

11. The diagonal a parallelogram divides it into two _____ triangles.

OR

In $\triangle ABC$, D is mid-point of AB and DE||BC. Find AE if AC = 6cm.

12. Find the ratio of the curved surface area and total surface area of a hemisphere.

13. For a triangle a=8, b=7 and s=13. Find the value of c.

OR

Find the value of s for an equilateral triangle.

14. The base and height of a parallelogram are 10cm and 6cm respectively. Find the area of the parallelogram.

15. Find the value of $p(x) = x^5 - x^4 + 3$ at $x = -1$.

16. Find the class-mark of the interval 100-120.

OR

The mean of 6, 4, 7, 13 and p is 8. Find the value of p.

SECTION-II

Question Number 17-20 are case-study based questions.

Attempt any 4 sub-parts from each question. Each sub-part carries one mark.

17. Sonika went to market to purchase some neccessities. She purchased 2kg of wheat at the rate of ₹$40\sqrt{5}$ per kg. She bought a packet of juice for $51\sqrt{3}$ ₹ and four chocolates for ₹$72\sqrt{3}$. She paid total $[(1000)^{\frac{2}{3}} \times (125)^{\frac{1}{3}}]$ ₹ to the shopkeeper. The shopkeeper returned the balance amount to Sonika.

(i) How much is Sonika's total bill?

(a) $40\sqrt{5} + 123\sqrt{3}$ (b) $40\sqrt{5} \times 51\sqrt{3} \times 72\sqrt{3}$

(c) $80\sqrt{5} + 123\sqrt{3}$ (d) $80\sqrt{5} + 339\sqrt{3}$

(ii) What is the price of 1g wheat ?

(a) $\frac{1}{5\sqrt{5}}$ ₹   (b) $\frac{2}{5\sqrt{5}}$ ₹   (c) $\frac{2}{\sqrt{5}}$ ₹   (d) $\frac{1}{\sqrt{5}}$ ₹

(iii) The rationalisation of 1 gm of wheat price is :

(a) $\frac{2\sqrt{5}}{5}$ (b) $\frac{\sqrt{5}}{25}$ (c) $\frac{2\sqrt{5}}{25}$ (d) $\frac{\sqrt{5}}{5}$

(iv) How much money Sonika gave to shopkeeper ?

(a) 1250 ₹   (b) 250 ₹   (c) 200 ₹   (d) 500 ₹

(v) The difference in the price of juice and 3 chocolates is

(a) $21\sqrt{3}$ ₹   (b) $3\sqrt{3}$ ₹   (c) $33\sqrt{3}$ ₹   (d) $15\sqrt{3}$

18. A shopkeeper prepared a box of sweets for sale on Diwali. The dimensions of the sweet box are 24x20x3cm. The box contains laddoos, barfis and gulab-jamun. The diameter of laddoo is 2.1cm. The barfi is cuboidal in shape with dimensions 3.4x2x0.8cm. The gulab-jamun is cylindrical with base-radius 1.4cm and height 5.6cm. The shopkeeper divided the box into three equal parts and kept the three sweets.

(i) The shape of the sweet-box is :

(a) Cube   (b) Cuboid   (c) Square   (d) rectangle

(ii) The volume of the laddoo is :

(a) $2.3\,cm^3$ (b) $4.8\,cm^3$ (c) $2.4\,cm^3$ (d) $1.15\,cm^3$

(iii) How much space of the sweet-box is occupied by barfi and gulab-jamun together ?

(a) $480\,cm^2$ (b) $720\,cm^2$ (c) $360\,cm^2$ (d) $960\,cm^2$

(iv) The approximate number of gulab-jamuns in the box is :

(a) 34   (b) 18   (c) 13   (d) 15

(v) The total surface area (in cm^2) of the open-box is :

(a) 744   (b) 612   (c) 1224   (d) 1092

19. Manju and Suman live in 'Chitra Vihar' society in Delhi. The location of different buildings in the society in a cartesian plane is as shown in the given figure.
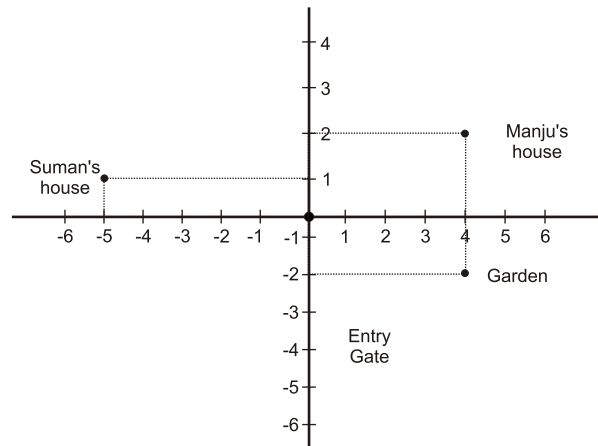
(i) The coordinates for garden are

(a) (-2,4) (b) (4,-2) (c) (-4, 0) (d) (0,-4)

(ii) Which of the following is located in II quadrant ?

(a) Suman's house

(b) Entry Gate

(c) Manju's house

(d) Garden

(iii) The Entry-Gate lies in

(a) x-axis (b) origin (c) y-axis (d) III quadrant

(iv) Which of the following is not true for Manju's house ?

(a) abscissa is positive (b) lies in I quadrant

(c) lies in IV quadrant (d) ordinate is positive

(v) The point having abscissa of Suman's house and ordinate of entry-gate will lie in

(a) II quadrant (b) origin (c) IV quadrant (d) III quadrant

20. Jyoti's office is at a 6km distance from his house. In the morning, she goes to his office by taxi. For the first kilometer, the taxi fare is ₹10 and for the subsequent distance the fare is ₹8 per km. Jyoti returns home by bus from her office. DTC is running the AC and non-AC buses. In a particular week, Jyoti took AC bus for two days and non-AC bus for 4 days. The total amount she spent on travelling by bus in that particular week is ₹70.

(i) Taking the fare for first kilometre as x and y for subsequent fare, the linear equation for taxi-fare is

(a) x+y=18 (b) x+5y = 50 (c) $5x$+y=50 (d) x+5y=18

(ii) One day, Jyoti walks 1 km distance and then took a taxi. The taxi-fare for that day is :

(a) 50₹   (b) 40₹   (c) 32₹   (d) 42₹

(iii) The linear equation for bus-fare taking x as fare for AC bus and y as fare for non AC bus is

(a) $2x+4y=70$ (b) $x+4y=70$ (c) $4x+2y=70$ (d) $2x+y=70$

(iv) If fare for AC bus is ₹15 then the fare for non-AC bus is

(a) 5₹   (b) 40₹   (c) 10₹   (d) 12₹

(v) Which of the following cannot be the taxi-fare?

(a) (10,8)   (b) (5,9)   (c) (15,7)   (d) (6,20)

PART-B

Q.No. 21-26 are very short answer questions of 2 marks each

21. The volume of a right circular cone is 9856cm^3. If the radius of the base is 14cm then find the height of the cone . (Use $\pi=\frac{22}{7}$)

22. Simplify $(\sqrt{5}+\sqrt{7})^2$

OR

Solve $(625)^{0.06} \times (625)^{0.19}$

23. Express $3x+15=0$ geometrically in two variables.

24. The side of a cube is 8cm. Find the lateral surface area of the cube.

25. If each side of a triangle is doubled then find the ratio of area of new triangle thus formed and the given triangle.

OR

In the given figure, MORE is a parallelogram, RN⊥ME and MP⊥ER. Find the value of ME if MO=16cm, MP=8cm and RN=10cm.
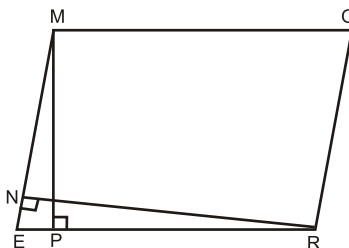
26. If p(x) = x-9 then find the value of p(x)-p($-x$).

Q.No. 27-33 are short answer questions of 3 marks each.

27. The sides of a triangle are in the ratio 11:19:24 and its perimeter is 540cm. Find the area of the triangle.

28. A dice is thrown 80 times. If the probability of having an even number is $\frac{7}{10}$ then how many times an odd number appears on dice ?
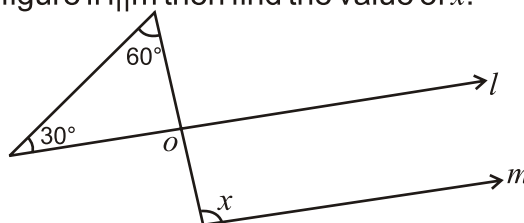
<p align="center">OR</p>

The blood-group of 450 students of a school is recorded as shown in the table below:

Blood Group	A	AB	O	B
No. of students	100	80	150	120

A student is selected at random. Find the probability that the blood-group of the student chosen is

(a) O (b) either A or B (c) not AB

29. In the given figure if l||m then find the value of x.

30. Divide the polynomial $9x^3 - 3x^3 + 15x - 3$ by $(3x-1)$ and find its quotient and remainder.

31. Prove that the angle opposite to the equal sides of an equilateral triangle are equal.

32. The cost of four chairs and five tables is ₹3200. Write a linear equation in two variable for this statement and find out its two solutions.

<p align="center">OR</p>

Solve for x : $5(x+1)(x+3) - 8 = (5x+1)(x+2)$

33. In the given figure, X and Y are the mid-points of sides AB and CD of a parallelogram ABCD. Prove that AXCY is a parallelogram.
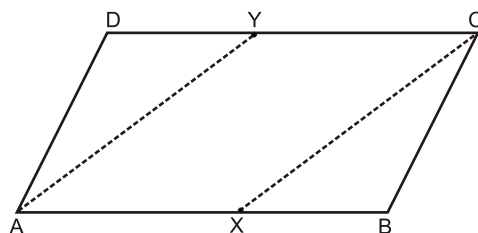
Q.No. 34-36 are long answer questions of 5 marks each.

34.   Draw the frequency-polygon for the following distribution.

Marks	No. of Students
0-10	7
10-20	10
20-30	6
30-40	8
40-50	12
50-60	3
60-70	2
70-80	2
Total	50

35.   Construct a triangle with base angles 60° and 45° and having perimeter 6.4cm.
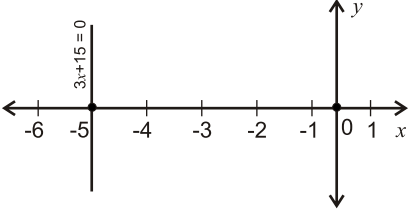
<div align="center">OR</div>

Construct a triangle PAR in which PA=5.4cm, ∠A=60° and PR-PA = 2.3cm.

36.   Prove that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle. Hence find the angle ∠ADC in the given figure.
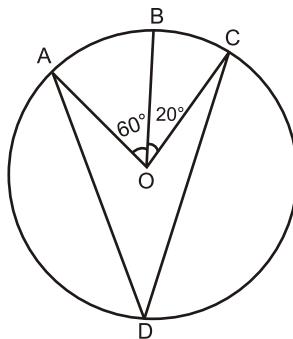
ANSWERS
PRACTICE PAPER 2

1. a=1
2. SSS congruency
3. Centre
4. $x=140°$ or $\angle PMQ=75°$
5. Irrational
6. 4
7. 87° or y=54°
8. -1
9. PE=5cm
10. 1.9375
11. Congruent OR AE=3cm
12. 2:3
13. C=11 or S = $\frac{3}{2}$ x side

14. 60cm²
15. p(-1) = 1
16. 110 or p=10
17. (i) (c) $80\sqrt{5} + 123\sqrt{3}$
    (ii) (a) $\dfrac{1}{5\sqrt{5}}$ ₹
    (iii) (b) $\dfrac{\sqrt{5}}{25}$
    (iv) (d) 500 ₹
    (v) (b) $3\sqrt{3}$ ₹
18. (i) (b) Cuboid
    (ii) (b) 4.8cm³
    (iii) (d) 960cm²
    (iv) (c) 13
    (v) (a) 744
19. (i) (b) (4,-2)
    (ii) (a) Suman's house
    (iii) (c) y-axis
    (iv) (c) lies in IV quadrant
    (v) (d) III quadrant
20. (i) (b) x+5y = 50

(ii) (d) 42₹
(iii) (a) 2x+4y = 70
(iv) (c) 10₹
(v) (d) (6,20)
21. h = 48cm
22. $12+2\sqrt{35}$ OR 5
23.

24. 256cm³
25. 4:1 OR ME = 12.8cm
26. 2x
27. $7200\sqrt{2}$cm²
28. 24 times OR
    (a) $\dfrac{1}{3}$ (b) $\dfrac{22}{45}$ (c) $\dfrac{37}{45}$
29. x=90°
30. quotient = 3x²+5
    remainder = 2
31. Correct proof
32. Let no. of chair =x
    no. of table = y
    4x+5y=3200
    Any two solutions
    or
    x = $\dfrac{-5}{9}$

34. Correct frequency polygon
35. $\angle AOC = 60°+20° = 80°$
    $\angle ADC = \dfrac{1}{2} \angle AOC = 40°$

PRACTICE QUESTION PAPER - 3
CLASS-IX
MATHEMATICS

Time: 3 Hrs.   **M.M. 80**

General Instruction:

1.   This question paper contains 36 questions divided into two parts A and B. All the questions are compulsory.

2.   Part A consists of two section I & II. Section I has 16 questions of 1 mark each and section II has 4 case study based questions. Each case-study based question has 5 sub-parts of 1 mark each.

3.   Part B consists of 16 questions – 6 questions of 2 marks, 7 questions of 3 marks and 3 question of 5 marks each.

4.   There is no overall choice. However internal choices are provided in 5 questions of 1 mark, 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks. You have attempt only one of the alternatives in all such question.

5.   In case-study based questions, you have to attempt only four out of five sub-parts.

6.   Use of calculator is not permitted.

7.   Please write down the serial number of question before attempting it.

Part-A

Q.No. 1-16 are of 1 mark each.

1.   At what point the graph of $2x-y=6$ cuts x-axis ?

2.   Write two numbers whose decimal expansions are non-terminating non-repeating (non-recurring)

3.   If $2^{5x} \div 2^x = \sqrt[5]{32}$. Then find the value of x.

4.   The ordinate of any point on the x-axis is _____

<div align="center">OR</div>

Which of the points A(-5,0), B(0,-3), C(3,0), D(0,4) are closer to the origin ?

5.   If the base of a parallelogram is 8cm and its altitude is 5cm then find its area.

<div align="center">OR</div>

The area of a parallelogram PQRS is 36cm². M is any point on the side RS. Find area of △PMQ.

6. Find an angle which is equal to its complement.

7. Find the zero(s) of the polynomial z^2-2z.

8. Find the perpendicular distance of point P (-7,2) from y-axis.

<div align="center">OR</div>

In which quadrant does point (a,b) lie ?

9. If two consecutive sides of a rhombus are represented by $3x$-6 and x+14 then find the perimeter of the rhombus.

<div align="center">OR</div>

Two adjacent angles of a parallelogram are in the ratio 2:4. Find the values of these angles.

10. If AB=12cm, BC=16cm and AB is perpendicular to BC, then find the radius of the circle passing through the point A, B and C.
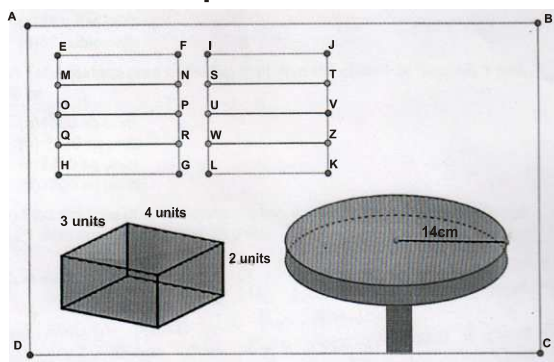
11. State Euclid's first postulate.

<div align="center">OR</div>

If a+b = 15 and a+b+c=15+c which axiom of Euclid does the statement illustrate ?

12. The area of a triangle is 150cm^2 and its sides are in the ratio 3:4:5. Find its perimeter.

13. If mode of 4, 9, 5, 4, 9, 4, 9 and x –10 is 9, find x.

14. 15 observations are arranged is ascending order. If 9th observation is increased by 2 then find the change in new median.

15. The class mark of a class-interval is 8.5. The class limits of the corresponding class interval will be ------------ if class size is 5.

16. A fruit basket contains 8 apples, 5 mangoes and 10 oranges. Find the probability of choosing a mango from the basket.

<div align="center">

SECTION-II

</div>

Q.No 17-20 are case study based questions, attempt any four sub parts from each question. Each sub part carries 1 mark.

In the given picture there are two windows EFGH and IJKL on the wall. MN, OP, QR, ST, UV, WZ are rods of the windows. A box and a chair with round seat are also shown.

(i) If the length of each rod is 6 units then the total length of the rods in two windows will be-

(a) 60 units   (b) 36 units   (c) 30 units   (d) 18 units

(ii) If the distance between two rods is 0.8 units then the height of each window will be-

(a) 3.2 units   (b) 4 units   (c) 2.4 units   (d) 5 units

(iii) Volume of the cuboidal box will be-

(a) 42 cubic units

(b) 48 cubic units

(c) 24 cubic units

(d) 52 cubic units

(iv) Total surface area of the top seat of round chair if its height is 1 unit, will be-

(a) 1320 sq units

(b) 1230 sq units

(c) 2310 sq units

(d) 1200 sq units

(v) Find the area of wall leaving the two windows, if the length of the wall is 25 units and its breadth is 20 units.

(a) 500 sq. unit   (b) 961.6 sq. unit

(c) 960 sq. unit   (d) 38.4 sq. unit

18. A farmer has a circular, garden as shown in the picture. He has different types of trees, plants and flowers plants in his garden

In the garden there are two mango trees A and B at a distance of AB=10m. Similarly he has two ashoka trees at the same distance of 10m as shown at C and D. AB the same distance of 10m as shown at C and D. AB subtends ∠AOB=120° at the center O, the perpendicular distance of AC from center is 5m. The radius of the circle is 13m.
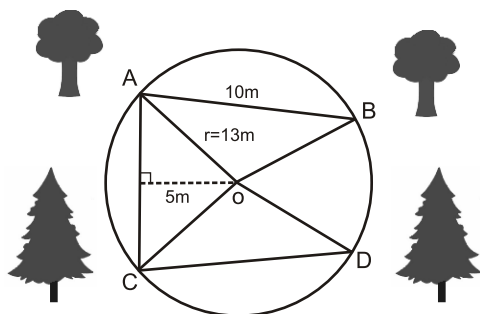
(i) What is the value of ∠COD

    (a) 60°   (b) 120°   (c) 100°   (d) 80°

(ii) What is the distance between mango tree A and Ashoka tree C ?

    (a) 12m   (b) 24m   (c) 13m   (d) 15m

(iii) What is the value of ∠OAB ?

    (a) 60°   (b) 120°   (c) 30°   (d) 90°

(iv) What is the value of ∠OCD?

    (a) 30°   (b) 120°   (c) 60°   (d) 90°

(v) What is the value of ∠ODC?

    (a) 90°   (b) 120°   (c) 60°   (d) 30°

19. A school wants to plant trees in a row as distances shown in the figure. The Height of each tree is considered equal and is taken as 1 unit and the distance OA is also 1 unit. First Tree is planted at Point B & second at C. Consider point 'O' as the zero of the line and OB=OP, OC=OQ, then
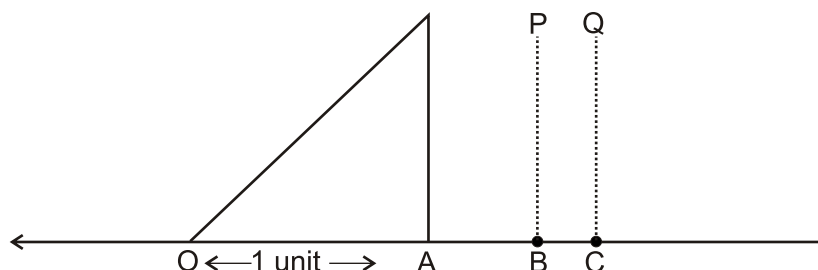

[Neglect width of the tree]

(i) Distance OB will be

    (a) 1 unit   (b) 2 units   (c) $\sqrt{1}$ unit   (d) $\sqrt{2}$ units

(ii) Distance OC will be

    (a) $\sqrt{3}$ units   (b) $\sqrt{2}$ units   (c) 2 units   (d) 3 units

(iii) Rationalising $\frac{1}{OB}$, we get

    (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\frac{2}{\sqrt{2}}$

(iv) Making denominator of $\frac{OB}{OC}$ rational, we get

    (a) $\frac{\sqrt{6}}{3}$ (b) 2   (c) $\frac{\sqrt{2}}{3}$ (d) $\frac{\sqrt{3}}{3}$

(v) Simplifying the reciprocal of difference (OC-OB)

(a) $\dfrac{1}{\sqrt{3}-\sqrt{2}}$ (b) 1 (c) $\sqrt{3}+\sqrt{2}$ (d) $-(\sqrt{2}+\sqrt{3})$

20. There is Diwali celebration is the school. The children were asked to prepare Rangoli in a triangular shape. They made Rangoli in the shape of triangle ABC. The sides of $\triangle$ABC are 26cm, 28cm and 25cm.
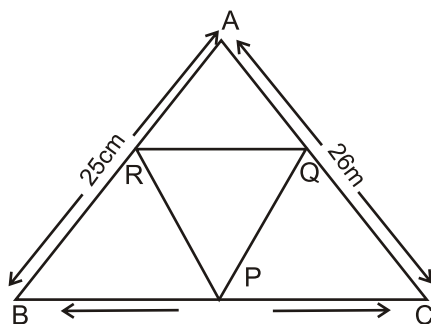
(i) In fig R is mid point of AB and RQ||BC then AQ is equal to

(a) BC (b) RB (c) QC (d) AD

(ii) In fig R is mid point of AB and Q is mid point of AC then the length of RQ is

(a) 14cm (b) 13cm (c) 12cm (d) 10cm

(iii) R, P, Q are mid points of AB, BC, CA. Then $\triangle$QPR$\cong\triangle$B____

(a) PR (b) RP (c) AC (d) None of these

(iv) $\triangle$ARQ $\cong\triangle$ _____

(a) PQR (b) QPR (c) RQP (d) RPQ

(v) Which will be greater out of $\angle$A and $\angle$C of $\triangle$ABC ?

(a) $\angle$C (b) $\angle$A (c) both are equal

(d) Information is not sufficient

Q.No. 21-26 are very short answer questions of 2 marks each.

21. Find any two solutions of equations 3x-4y=24.

22. If E, F, G and H are respectively the midpoints of the sides of a parallelogram ABCD then show that area (EFGH) = ½ area (ABCD)

OR

Show that a median of a triangle divides it into two triangles of equal areas.

23. Find the volume of the hemisphere of radius 3.5cm.

24. Simplify $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{80}+\sqrt{48}-\sqrt{45}-\sqrt{27}}$

25. Factorize $x^2 + 3\sqrt{3}x + 6$

     OR

     Show that (x-5) is a factor of $x^3-3x^2-4x-30$

26. The volume of a cube is 1000cm³. Find its total surface area.

Q.No. 27-33 are short answer question of 3 marks each.

27. After a medical check up for HB level of 35 students of class-IX following data was recorded.

| HB Level	Below 8	Below 10	Below 12	Below 14	Below 16
No. of students	3	7	13	23	35

What is the probability that a student choosen has

i) HB level less than 10.

ii) HB level greater than or equal to 12 but less than 16.

28. The taxi fare in a city as follows : for the first kilometre, the fare is ₹25 and for the subsequent distance it is ₹14 per km. Taking the distance covered as x km and total fare as ₹y. write the linear equation for this information and draw its graph.

29. In the given figure if AB||DC and ∠BDC=30° ∠BDC=80° find ∠x, ∠y, ∠z.
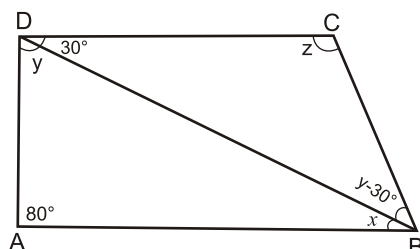
30. A field is in the shape of trapezium whose parallel sides are 25cm, and 10cm and the non-parallel sides are 14cm and 13cm. find the area of the field.

31. Factorize $2y^3 + y^2 - 2y - 1$

32. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see Fig.) Show that.

     (i)   ΔABM ≅ ΔPQN

     (ii) ΔABC ≅ ΔPQR
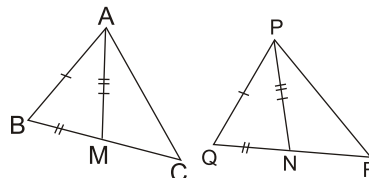
                    OR

     Prove that the sum of all the angles of a triangle is 180°.

33.	Show that the bisectors of angles of parallelogram form a rectangle.

OR

Show that the line segment joining the mid-points of two sides of a triangle is parallel to the third side.

Q.No. 34-36 are long answer questions of 5 marks each.

34.	The run scored by two teams A and B on the first 60 balls in a cricket match are given below :

Number of balls	Team A	Team B
1-6	2	5
7-12	1	6
13-18	8	2
19-24	9	10
25-30	4	5
31-36	5	6
37-42	6	3
43-48	10	4
49-54	6	8
55-60	2	10

Represent the data of both the terms on the same graph by frequency polygons.

OR

The mean of 100 items was found to be 30. If at the time of calculation two items were wrongly taken as 32 and 12 instead of 23 and 11 then the correct mean.

35.	Construct a $\triangle ABC$ in which BC = 5.6 CM, AC-AB = 1.6cm and $\angle B = 45°$.

36.	Find the values of a and b, if (x+1) and (x-1) both are factors of $ax^3 + x^2 - 2x + b$.

Answer (Part-A)

1. (3,0)
3. $x = \frac{1}{4}$
4. O or B and C
5. $40cm^2$ or $18cm^2$
6. 45°
7. 0,2
8. 7 units or Ist quadrant
9. 96 or 60°, 120°
10. 10cm
11. 2nd
12. 60cm
13. 19
14. No change
15. 6-11
16. 5/23
17. (i) (b) 36 units
    (ii) (a) 3.2 units
    (iii) (c) 24 cubic units
    (iv) (a) 1320sq. units
    (v) 480.80 sq. units
18. (i) (b) 120°
    (ii) (b) 24m
    (iii) (c) 30°
    (iv) (a) 30°
    (v) (d) 30°
19. (i) (d) $\sqrt{2}$ units
    (ii) (a) $\sqrt{3}$ units
    (iii) (c) $\frac{\sqrt{2}}{2}$
    (iv) (a) $\frac{\sqrt{6}}{3}$
    (v) (c) $\sqrt{3} + \sqrt{2}$

20. (i) (c) QC
    (ii) (a) 14cm
    (iii) (b) RP
    (iv) (a) PQR
    (v) (b) $\angle A$

Part B

23. $89.83cm^3$
24. 1
25. $(x + 2\sqrt{3})(x + \sqrt{3})$
26. $600cm^2$
27. (i) 2/7   (ii) 22/35
28.
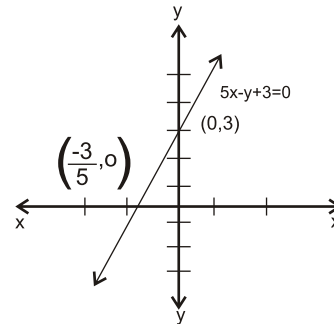
29. x=30°, y=70°, z=110°
30. $196m^2$
31. (y+1)(y-1)(2y+1)
34. 29.9
36. a=2, b=-1